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SCATTERING ACOUSTIC FIELD ABOVE A RIGID RECTANGULAR PARALLELEPIPEDIC SHAPE

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ABSTRACT

Noise at work is a real health problem that affects many employees at their workstations. Irregular reliefs of walls facings that delimit industrial premises disturb the acoustic field and make the sound levels control difficult to preserve the employees. This requires development of theoretical and experimental methods allowing acoustic characterization of these facings. Such walls, which possess periodic or aperiodic relief (rectangular cavities or volumes as furniture), scatter acoustic waves. This work consists in developing a theoretical model to predict the acoustic pressure field reflected and scattered over a rigid rectangular parallelepipedic shape on an infinite rigid screen. Originally, the model was based on a study of strips in electromagnetism. It was adapted to study the acoustic behavior of a rigid rectangular parallelepipedic shape by considering the specular reflection and the diffracted reflection and transmission of the incident field. The model was compared with the results of a numerical simulation (Finite Element Method) and the experimental ones obtained in the semi-anechoic room for an elementary rigid rectangular parallelepiped shape insonified by an incident spherical acoustic field. The observed concordance between the numerical and experimental results supports the validity of our model over a wide spectral range.

1. INTRODUCTION

The problem of sound wave scattering by rough surface has been studied by many authors. A. Bolghasi et al. [1] used an extended Hall-Novarini model and optimized Helmholtz-Kirchhoff-Fresnel method to study the scattered sound from the rough bubbly sea surface. H. Lee et al. [2] investigate the characteristics of random- and normal-incidence scattering coefficients of one-dimensional periodic surfaces, focusing on the effects of shape, height and width of surface profile. Y. Liu et al. [3] presented a high-order accurate boundary-based solver to study through the 3D frequency-domain the acoustic scattering generated by a doubly-periodic grating of smooth axisymmetric sound-hard or transmission obstacles. W. Lee et al. [4] used the collocation multipole method to model the scattering of an acoustic plane wave by multiple elliptical cylinders. EL. Richards et al. [5] used the Kirchhoff approximation and the Rayleigh-Fourier method to investigate the scattering acoustic field above a sinusoidal surface waves at low grazing angles. A comparison of these two techniques was made. J. Ducourneau et al. [6] used the wave guide method to investigate the scattered acoustic field above a double-periodic wall facing featuring rectangular cavities. The Kobayashi Potential method was used by many authors in electromagnetism and acoustic to investigate scattering problems [7, 8, 9]. In this paper, we propose a hybrid model based on the KP method and the image sources technique to model the scattering acoustic field above a rectangular parallelepipedic shape on a rigid screen.

2. STATEMENT OF THE PROBLEM

Consider an infinitely rigid large screen containing a (2a, 2b, 2d) rigid rectangular parallelepipedic shape. The reference frame is centered on the upper facet of the shape. The expressions of $\Phi^\text{inc}$ and $\Phi_r$ are given by the relationships (1) and (2).

$$\Phi^\text{inc} = A e^{ikz} e^{ik_2 y} e^{ik_2 x}$$
$$\Phi_r = A e^{ik_3 x} e^{ik_2 y} e^{ik_2 z}$$

In the case of normal-incidence only the upper facet of the shape is insonified. In the case of oblique-incidence where
3. MODEL DEVELOPMENT

3.1 The diffracted field generated by the upper facet of the shape

According to the KP method [9] the diffracted acoustic field generated by a rigid rectangular plate is given by the relationship (3).

\[
\Phi_d^{\pm,0} = \sum_{\mu, \nu=0}^{\infty} A_{\mu, \nu, n=0}^{\mu, \nu} \int A_{m, n, z=0}^{\mu, \nu} \frac{f_{m+1+\mu}(a)}{\beta} \cos(\alpha x - \mu y) \cos(\beta y - \nu z) e^{\frac{i}{\beta} \sqrt{(\alpha^2 + (\beta^2/k^2) - k^2) \cos(\theta) d\theta d\beta}, \pm z > 0}
\]

where \( \xi = \frac{x}{a}, \eta = \frac{y}{b} \) normalized coordinates according to the dimensions of the shape. \( \alpha \) and \( \beta \) are integration variables. \( \mu, \nu, m, n \) are summation indices. The plus sign (respectively the minus sign) in \( \Phi_d^{\pm,0} \) refers to the diffracted acoustic field that propagates in the direction of positive \( z \) (respectively negative \( z \)). \( A_{m, n, z=0}^{\mu, \nu} \) are modal amplitudes to determine by enforcing boundary condition given by equation (4) traducing a null velocity at the domain of the upper facet.

\[
\frac{\partial (\Phi_d^{\pm,0} - \Phi_{inc}^{\pm,0})}{\partial z} = 0, z = 0, |\xi|, |\eta| < 1
\]

The parity of the function on \( \xi \) and \( \eta \) gives four sub-equations. To which we apply the orthonormality relations of the Jacobi polynomials: \( G^{(1,1/2)} \) for pair functions and \( G^{(2,3/2)} \) for uneven functions. A linear system given by the relationship (5) is obtained allowing determination of the modal amplitudes \( A_{m, n, z=0}^{\mu, \nu} \).

\[
[A_{m, n, z=0}^{\mu, \nu} ] = [G_{m, n, z=0}^{\mu, \nu} ]
\]

where:

\[
G_{m, n, z=0}^{\mu, \nu} = \int \frac{f_{m+1+\mu}(a)}{\beta} \cos(\alpha x - \mu y) \cos(\beta y - \nu z) e^{\frac{i}{\beta} \sqrt{(\alpha^2 + (\beta^2/k^2) - k^2) d\alpha d\beta}}, \pm z = 0
\]

\[
A_{m, n, z=0}^{\mu, \nu} = j^{\mu+\nu+1} k_4 A \frac{f_{m+1+\mu}(a)}{\beta} \cos(\beta y - \nu z) e^{\frac{i}{\beta} \sqrt{(\alpha^2 + (\beta^2/k^2) - k^2) \cos(\theta) d\theta d\beta}}, \pm z = 0
\]

3.2 The diffracted field generated by the lateral facet of the shape

In this section we'll discuss how to determine the scattered acoustic field by the lateral facet. So let's deal first with the facet at \( y = b \). The complexity here comes from the existence of the infinite rigid screen at \( z = -2d \). Indeed, we'll combine the KP method and the image sources method to investigate the diffracted field generated by this type of geometry. Two sources will insonify the lateral facet: the incident field and the specular reflected field. The incident plane field \( \Phi_{inc}^{\pm,0} \) will insonify the lateral facet and generate a diffracted field \( \Phi_d^{\pm,0} \). This field will be reflected then by the infinite rigid surface at \( z = -2d \). Let \( \Phi_{dr}^{\pm,0} \) be this reflected diffracted field. Similarly, the specular reflected field will insonify the image of the lateral facet according to the plane \( z = -2d \) and generate a diffracted field \( \Phi_d^{\pm,0} \).

\[
\Phi_d^{\pm,0} = \sum_{\mu, \nu=0}^{\infty} A_{\mu, \nu, n=0}^{\mu, \nu} \int B_{m, n, z=0}^{\mu, \nu} \frac{f_{m+1+\mu}(a)}{\beta} \cos(\alpha x - \mu y) \cos(\beta y - \nu z) e^{\frac{i}{\beta} \sqrt{(\alpha^2 + (\beta^2/k^2) - k^2) \cos(\theta) d\theta d\beta}, \pm z > 0}
\]

\[
\Phi_{dr}^{\pm,0} = \sum_{\mu, \nu=0}^{\infty} A_{\mu, \nu, n=0}^{\mu, \nu} \int B_{m, n, z=0}^{\mu, \nu} \frac{f_{m+1+\mu}(a)}{\beta} \cos(\alpha x - \mu y) \cos(\beta y - \nu z) e^{\frac{i}{\beta} \sqrt{(\alpha^2 + (\beta^2/k^2) - k^2) \cos(\theta) d\theta d\beta}, \pm z > 0}
\]

\[
\Phi_{image}^{\pm,0} = \sum_{\mu, \nu=0}^{\infty} A_{\mu, \nu, n=0}^{\mu, \nu} \int B_{m, n, z=0}^{\mu, \nu} \frac{f_{m+1+\mu}(a)}{\beta} \cos(\alpha x - \mu y) \cos(\beta y - \nu z) e^{\frac{i}{\beta} \sqrt{(\alpha^2 + (\beta^2/k^2) - k^2) \cos(\theta) d\theta d\beta}, \pm z > 0}
\]
The determination process of this different diffracted fields consists in determining the modal amplitudes $B_{\mu,n}^{\nu,v}$ and $C_{\mu,n}^{\nu,v}$ of the fields $\Phi_d^{1,direct}$ and $\Phi_d^{1,image}$. This can be done by the application of the same process described in the subsection before. Finally, we determine $B_{\mu,n}^{\nu,v}$ and $C_{\mu,n}^{\nu,v}$ of the fields $\Phi_d^{2,direct}$. This can be done by the application of the same process described in the subsection before. Finally, we determine $B_{\mu,n}^{\nu,v}$ and $C_{\mu,n}^{\nu,v}$ are given by the following relationships.

\[
\begin{align*}
[B_{\mu,n}^{\nu,v}] [C_{\mu,n}^{\nu,v}] &= [A_{\mu,v,x,t}^{1,direct}] \\
[C_{\mu,n}^{\nu,v}] [C_{\mu,n}^{\nu,v}] &= [A_{\mu,v,x,t}^{1,image}] 
\end{align*}
\]

where:

\[
A_{\mu,v,x,t}^{1,direct} = j^{1+\mu+v} k_2 e^{jk_2b} e^{-jk_3d} \frac{f_{2x1+1}(k_3d)}{k_3d}
\]

\[
A_{\mu,v,x,t}^{1,image} = (-1)^j j^{1+\mu+v} k_2 e^{jk_2b} e^{-jk_3d} \frac{f_{2x1+1}(k_1a)}{k_1a}
\]

4. MODEL VALIDATION

4.1 FEM Modeling

COMSOL software (5.2) was used to model the problem of the diffraction of a spherical source by a 90° corner. The aim of this step is to validate the model presented before. Steel was used to model the infinite surface and the bulky rectangular plate on to form a 90°. An air volume was placed above the steel in order to model the propagation medium. All the structure was covered then with Perfectly Matched Layers in order to model free field conditions. These features are shown in figure 3. A spherical source was placed above the steel materials. Boundary conditions on the steel blocks were applied to eliminate any structural vibration.

4.2 FEM Validation

Figures 4-7 present the normalized acoustic pressure 0.2 m above a (46 × 21) cm² 90° corner for many frequencies: 230 Hz, 500 Hz, 780 Hz and 1100 Hz. The source was positioned at (0, 0.5, 0.98) m. The curves are not symmetric due to the eccentricity of the acoustic source. The curves show a good agreement between our model and the FEM results. Some disparities exist between the two numerical results especially in low frequencies. We think that is due to non-perfect PML boundary condition (anechoic termination conditions) in the case of the FEM method [10]. The use also of a geometrical method could also cause some disparities.

![Figure 3. Principle of finite element modeling.](image)

![Figure 4. Normalized acoustic pressure 20 cm above a (46 × 21) cm² 90° corner at 230 Hz.](image)

![Figure 5. Normalized acoustic pressure 20 cm above a (46 × 21) cm² 90° corner at 500 Hz.](image)
4.3 Experimental validation of the model

A first validation step of our model has been carried out by measuring the sound pressure profiles above a rigid rectangular parallelepipedic shape. The source coordinates were (0, 0, 0.86) m. The infinite surface at $z = -2d$ were in steel. The profile was formed by tiled polystyrene block to ensure a high acoustic reflection coefficient.

The sound source used was a horn with a 15 mm outlet diameter. A 10 cm diameter Pioneer TS E1077 loudspeaker was fixed between the horn and a cylindrical body. The source emitted spherical waves [6, 7, 8]. The acoustic source was connected to a B&K 1405 noise generator through a Power APK 2000 amplifier and a Yamaha GQ 1031 Graphic Equalizer to generate white noise. Fifteen B&K 4935 1/4” microphones were used for acquisition. They were connected to a B&K 2694 Deltatron conditioner. The acquisition system was a NetdB of the 01dB-Metravib company (Figure 8). Signal acquisition was performed at a 25600 Hz sampling frequency for 30 s.

Measurement was performed in three stages, for three juxtaposed positions of an antenna fitted with the 15 microphones spaced apart at 5.5 cm intervals. Their positions were chosen such that the central microphone of the 45-sensor virtual array was positioned directly below the loudspeaker at 0.2 m above the studied bulky rectangular parallelepipedic shape (Figure 9).

In order to be comparable to experiment, we decomposed the incident spherical source into a plane wave spectrum. The total diffracted field was obtained then by summing all fields generated by the respective plane waves [7, 8]. In simulation, the matrices $[G_{m,n,s,t}]$ were truncated by $S^2 \times S^2$. Where $S$ is the maximum value of $m, n, s$, and $t$, i.e., it is a sufficiently large integer such that for all $m,n,s,t \geq S$ the series defining the diffracted fields converge [7, 8, 9]. The decomposition into plane wave spectrum were integrated into the systems of the matrix equations (5)-(8), (12)-(15). The sizes of the matrices $[A_{m,n}]$, $[B_{m,n}]$, $[c_{m,n}]$, $[A_{1,\text{direct}}]$, $[A_{1,\text{imag}}]$ and $[A_{0,\text{real}}]$ are $n_x n_y \times S^2$. $n_x$ and $n_y$ are the numbers of FFT samples along the $x$ and $y$ axis respectively.

Figures 10-13 show a comparison of the scattered acoustic field obtained with our model and the experiment for the frequencies 230 Hz, 780 Hz, 1100 Hz and 1500 Hz along the $x$-axis 20 cm above the bulky rectangular
parallelepipedic shape. The spherical source was centered 86 cm above the shape. All the acoustic pressure profiles were normalized with respect to the signal of the central microphone. The different profiles show a good agreement supporting the validity of our model. The curves are symmetric due to the symmetry of the problem. The acoustic pressure curves become more irregular as the frequency increases. This phenomenon is justified by the scattering increasing directivity at these frequencies. Another reason is that the size of the edges becomes large compared to the wavelength, increasing the scattering. Other experiments are in progress to study scattering acoustic field above a rigid rectangular parallelepipedic shape in the case of an eccentric sound source to validate the diffracted field generated by the lateral facet.

Figure 10.Normalized acoustic pressure 40 cm above a (46 x 48 x 21) cm³ bulky rectangular parallelepipedic shape at 230 Hz

Figure 11. Normalized acoustic pressure 20 cm above a (46 x 48 x 21) cm³ bulky rectangular parallelepipedic shape at 780 Hz

Figure 12. Normalized acoustic pressure 20 cm above a (46 x 48 x 21) cm³ bulky rectangular parallelepipedic shape at 1100 Hz

In Figure 13 the KP model was compared only to the experiment at the frequency 1900 Hz. At this frequency, it was difficult to obtain a sound pressure profile with the finite element model because it requires a significant spatial resolution and therefore a too long computation time.

Figure 13. Normalized acoustic pressure 20 cm above a (46 x 48 x 21) cm³ bulky rectangular parallelepipedic shape at 1500 Hz

5. CONCLUSIONS

A new hybrid technique based on the Kobayashi Potential method and the image sources was developed in order to investigate the scattered acoustic field above a rigid bulky rectangular parallelepipedic shape on a rigid infinite plane. The problem was divided into two problems: diffraction by a rectangular plate and diffraction by a 90° corner. The superposition was then applied. The model was first compared to numerical results obtained using an FEM method to study diffracted field generated
by a 90° corner of the shape. This comparison showed a good agreement supporting the validity of our model. Above a rigid bulky rectangular parallelepipedic shape (for a sound source centered) and despite some low differences observed between the theoretical results and the experimental and numerical data, the model developed is promising. This study will be generalized in the future to the case of gratings of rectangular parallelepipedic shapes. Parallel and non-parallel configurations will be studied. An analysis of the model with and without coupling between the blocks leads to understand the influence of the frequency and the spacings between them.

6. REFERENCES


