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# SEQUENTIAL SENSOR PLACEMENT USING BAYESIAN COMPRESSIVE SENSING FOR DIRECTION OF ARRIVAL ESTIMATION

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## 1. INTRODUCTION

In this contribution, we propose a sequential sensor placement to design an array geometry for underwater acoustic source localization.

Inspired by the compressed sensing (CS) framework, we consider the source localization as a sparse problem and solve it in its undetermined form with a Bayesian method, taking a sparse prior into account. In order to improve the localization, we design the array geometry following the Bayesian experimental design [1]. The sensor placement becomes then an optimization problem. The proposed approach solves it with a greedy data-dependent method, that answer the following question: according to what is measured by the current array, which new sensor position could improve the source localization at most? The proposed procedure iterates a sparse direction of arrival (DOA) estimation followed by the sensor placement in the D-optimal sense [2]. Each iteration leads to position one sensor at a time.

In this extended abstract, we present the main theoretical components grounding the proposed method and illustrate its performance in a setup of practical interest.

## 2. MODEL

We consider a classic underwater source localization problem with a linear array of aperture  $D$  and  $M$  sensors with their positions  $\mathbf{p} \in [-\frac{D}{2}, \frac{D}{2}]^M$ . The  $S$  sources are uncorrelated narrowband and considered far-field. The sparse vector  $\mathbf{x} \in \mathbb{C}^N$ , with  $N$  the number of angles observed, contains the complex amplitude of the sources. The measured signal  $\mathbf{y}(\mathbf{p}) \in \mathbb{C}^M$  can be modeled, using a on grid approach, as follow:

$$\mathbf{y}(\mathbf{p}) = \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{w}, \quad (1)$$

where  $\mathbf{w}$  is an uncorrelated noise, supposed to be a zero-mean complex Gaussian with variance  $\alpha_0^{-1}$ . The matrix  $\mathbf{A}(\mathbf{p}) \in \mathbb{C}^{M \times N}$  contains  $N$  steering vectors associated to plane waves propagating from angles  $\theta_1, \dots, \theta_N \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ . For convenience we define  $\mathbf{A}(\mathbf{p})$  by its rows, such that  $\mathbf{A}(\mathbf{p}) = [\mathbf{r}(p_1)^\top, \dots, \mathbf{r}(p_M)^\top]^\top$  and

$$\mathbf{r}(p_m) = [e^{-j\frac{2\pi}{\lambda} p_m \sin(\theta_1)}, \dots, e^{-j\frac{2\pi}{\lambda} p_m \sin(\theta_N)}], \quad (2)$$

with  $\lambda$  the source wavelength.

## 3. PROPOSED STRATEGY

The proposed iterative sensor placement method follows the Bayesian experimental design. The optimization criterion is the volume of the ellipsoid error [2]. Meaning that the new sensor has to minimize the determinant of the error covariance matrix estimated a posteriori using the measure acquired from the previous sensor array.

The proposed global procedure to sequentially add new sensors, one at the time, is summarized in Algorithm 1. The main steps are the Bayesian source localization, to estimate the error covariance matrix, and the computation of the sensor placement. These steps are detailed in the next sections.

### 3.1 Source Localization

The first step consist in estimating the source locations from a given number of sensors. Consider the estimation of  $\mathbf{x}$  in a *maximum a posteriori* sense:

$$\tilde{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmax}} \mathcal{P}(\mathbf{x}|\mathbf{y}(\mathbf{p}), \boldsymbol{\alpha}, \alpha_0), \quad (3)$$

with  $\boldsymbol{\alpha}$  the vector containing the variances of the complex normal distribution priors for each value of  $\mathbf{x}$ . In order to exploit a sparse prior on the number of sources, the estimation of  $\mathbf{x}$  is performed via the so-called Sparse Bayesian Inference (SBI) [3]. The algorithm considers that the variances,  $\alpha_0$  and  $\boldsymbol{\alpha}$  are unknown, and uses a non-informative Gamma prior on both variables. Taking the complex normal distribution and Gamma distribution into account, a closed-form solution of  $\tilde{\mathbf{x}}$  can be found since all hyperpriors are conjugates of the Gaussian priors on  $\mathbf{x}$  and  $\mathbf{w}$ . In fact, the posterior distribution is also Gaussian and writes  $\mathcal{P}(\mathbf{x}|\mathbf{y}(\mathbf{p}), \boldsymbol{\alpha}, \alpha_0) = \mathcal{CN}(\tilde{\mathbf{x}}_{\mathbf{p}}, \tilde{\boldsymbol{\Sigma}}_{\mathbf{p}})$ . Its mean and covari-

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**Algorithm 1** Sequential sensor placement procedure.

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**Require:**  $M_0, M, \mathbf{y} \in \mathbb{C}^{M_0}, \mathbf{p} = [p_1, \dots, p_{M_0}]^\top$

**for**  $m = M_0 + 1, M_0 + 2, \dots, M$  **do**

- Localize sources by SBI to compute  $\tilde{\boldsymbol{\Sigma}}_{\mathbf{p}}$

- Find the next optimal sensor position  $\tilde{p}_*$  – eq. (7)

- Acquire the new measurement  $y_*$

- Update variables  $\mathbf{y} \leftarrow [\mathbf{y}^\top, y_*]^\top$  and  $\mathbf{p} \leftarrow [\mathbf{p}^\top, \tilde{p}_*]^\top$

**end for**

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ance read [4]:

$$\tilde{\mathbf{x}}_{\mathbf{p}} = \alpha_0 \tilde{\Sigma}_{\mathbf{p}} \mathbf{A}(\mathbf{p})^H \mathbf{y}, \quad (4)$$

$$\tilde{\Sigma}_{\mathbf{p}} = (\alpha_0 \mathbf{A}(\mathbf{p})^H \mathbf{A}(\mathbf{p}) + \text{diag}(\boldsymbol{\alpha})^{-1})^{-1}. \quad (5)$$

### 3.2 Sensor placement

The goal of the second step is to choose the sensor position  $p_*$  that minimize the determinant of the covariance matrix or maximize the reduction of the ellipsoid error volume [2]. Following a greedy procedure and as in [5], this step can be written as:

$$\tilde{p}_* = \underset{p_* \in [-\frac{D}{2}, \frac{D}{2}]}{\text{argmin}} \log \left| \Sigma \left( \begin{bmatrix} \mathbf{P} \\ p_* \end{bmatrix} \right) \right|, \quad (6)$$

by keeping the previous positions  $\mathbf{p}$  constant so that the covariance is parametric to  $p_*$  only. As in [5], the greedy sensor placement in (6) can be rewritten as:

$$\tilde{p}_* = \underset{p_* \in [-\frac{D}{2}, \frac{D}{2}]}{\text{argmax}} \delta h(p_*), \quad (7)$$

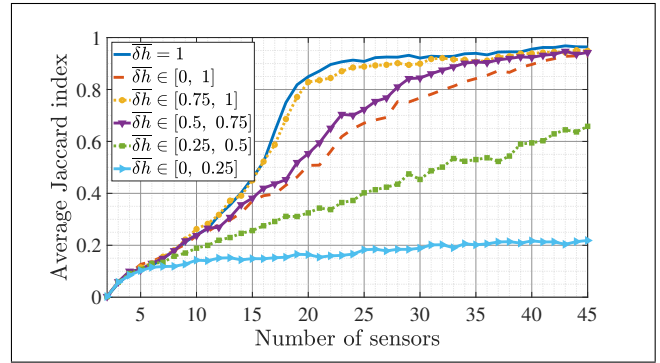
with  $\delta h(p_*) = \log(1 + \alpha_0 \mathbf{r}(p_*)^H \tilde{\Sigma}_{\mathbf{p}} \mathbf{r}(p_*))$  and  $\mathbf{r}(p_*)$  the candidate row vector computed at the position  $p_*$ . Finally, finding  $p_*$  is done by calculating  $\delta h(p_*)$  for a high number of candidate positions regularly sampled in the array.

## 4. EXPERIMENTS

The proposed iterative sensor placement procedure starts with  $M_0 = 2$  sensors located at  $\pm D/2$ , where  $D = 10$  m is the largest aperture of the array. The algorithm increase sequentially the number of sensors. In this experiment we fix to  $M = 45$  the maximal number of sensors. The sensors have to localize  $S = 10$  far-field uncorrelated sources emitting at the wavelength  $\lambda = 1.0$  m. We force the angular positions of the sources to be sufficiently separated to respect the necessary condition for the sparse source recovery [6]. The source positions are randomly selected from  $N = 181$  angles regularly sampled in the interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ . We adjust the sensor noise variance in order to maintain the SNR = 12 dB.

The chosen performance score is the Jaccard index  $J$ , a measure of similarity between 2 sets of samples. It quantifies the detection and the localization in a sparse problem [7]. Note that  $0 \leq J \leq 1$ :  $J = 1$  reveals an exact detection, whereas  $J = 0$  if none of the sources is correctly detected. To quantify the performance of the sensor placement, we compute the Jaccard index for each sensor added.

To analyze how much the criteria from the experimental design  $\delta h(p_*)$  sequentially improve the detection score, we choose to estimate a sensor placement as a function of  $\delta h(p_*)$ . To do so,  $\delta h(p_*)$  is linearly scaled between 0 and 1 to obtain  $\bar{\delta h}(p_*)$ . According to the D-design,  $\bar{\delta h}(p_*) = 1$  is the most informative position candidate. Then we select randomly a position that fit the threshold chosen, for example  $\bar{\delta h}(p_*) \in [0, 0.25]$ . To quantify the criteria we test 4 different thresholds from the least to the most informative.



**Figure 1.** Jaccard index (averaged on 100 realizations) according to the number of new sensor placement.

We also test the optimized placement (i.e.  $\bar{\delta h}(p_*) \in [1, 1]$ ) and the random placement (i.e.  $\bar{\delta h}(p_*) \in [0, 1]$ ).

Fig. 1 corresponds to the averaged Jaccard index on 100 realizations, where each realization contains different noise  $\mathbf{w}$ , but with the source positions fixed. The results show the more we maximize the criterion (i.e.  $\bar{\delta h}(p_*) = 1$ ), the quicker the performance score increases with the number of sensors. Note that if you have enough sensors, you can place them randomly to obtain the same result as our iterative approach. But if you want to reduce the number of sensors, you can see that our approach only needs 25 sensors to reach the best detection performance against 40 for the random placement. We also note that the best detection performance does not reach 1 in average, since in a small portion of the realizations, the SBI algorithm doesn't detect all the sources.

We have evaluate the robustness of our approach according to the SNR level. We have also done some comparison with state of the art approaches. These results will be discussed during the presentation.

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