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Characterization of open woodwind toneholes by the tube reversed method

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1 Woodwind tonehole's linear behavior is characterized by two complex quantities: the
2 series and shunt acoustic impedances. A method to determine experimentally these
3 two quantities is presented. It is based on two input impedance measurements. The
4 method can be applied to clarinet-like instruments. The robustness of the method
5 proposed is explored numerically through the simulation of the experiment when con-
6 sidering geometrical and measurement uncertainties. Experimental results confirm
7 the relevance of the method proposed to estimate the shunt impedance. Even the
8 effect of small changes in the hole's geometry, such as those induced by undercutting,
9 are characterized experimentally. The main effect of undercutting is shown to be a
10 decrease of the tonehole's acoustic mass, in agreement with theoretical considerations
11 based on the shape of the tonehole. Experimental results also reveal that losses in
12 toneholes are significantly higher than those predicted by the theory. Therefore the
13 method is suitable for the experimental determination of the shunt impedance, but
14 it is not convenient for the characterization of the series impedance.

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15 **I. INTRODUCTION**

16 For woodwind instruments, the effect of toneholes on the intonation and the ease of
17 playing is essential. The present paper focuses on linear behaviour of toneholes, which
18 is especially important for the playing frequencies. The characterization of holes can be
19 independent of the geometry of the resonator (either cylindrical or conical¹). The first
20 theory was given by Keefe², and completed later^{3,4}. It is based on matching plane waves
21 within the resonator and the tonehole. The tonehole is characterized by a transfer matrix or
22 an impedance matrix of order 2. Because of reciprocity, only three elements of the matrix
23 are necessary. In the present paper, the tonehole is assumed to be symmetrical, and two
24 elements (i.e., two complex impedances) are sufficient³ for asymmetrical toneholes). The
25 theory, based upon modal expansion, assumes the tonehole to be cylindrical, and this leads
26 to a difficulty of the geometric matching between two cylinders. However, the number and
27 nature of the matrix elements does not depend on the shape of the toneholes, and they
28 can be determined either by experiment or numerical discretization⁵⁻⁷. The Finite Element
29 Method can be used, but the modeling of boundary layers⁶ and nonlinear behaviour is not
30 straightforward. Acoustic experiment can be also used for the computation of the input
31 impedance of an instrument by using the transfer matrix method: the measurement of the
32 two acoustic impedances make unnecessary the knowledge of the precise geometry. For the
33 computation of the input impedance of an instrument, the acoustic characterization of the
34 toneholes is sufficient.

35 Considering the impedance matrix of the tonehole, the elements are essentially acoustic
36 masses. One is in series, modifying the acoustic pressure, and the other is in parallel, mod-
37 ifying the acoustic flow rate. They can be regarded as length corrections to the main tube
38 and to the tonehole, respectively. Nevertheless, for high (i.e., long) toneholes, compress-
39 ibility (and propagation) effects can appear. Moreover, for both the impedances in series
40 and in parallel, losses (i.e., resistances) exist. Losses added to the series mass are generally
41 ignored, and no theoretical determination exists, while experimental evidence was found by
42 Dalmont⁷ in a nonlinear regime. At low frequencies, the two masses are almost independent
43 of frequency, but they increase when approaching the first cutoff of the main tube³ for the
44 2D, rectangular case). Other shunt acoustic masses intervene, in particular that of the plane
45 mode in the hole, and a resonance of the total shunt mass can occur at high frequency: this
46 is detailed in Section II.

47 Previous articles⁷⁻⁹ took advantage of the tonehole symmetry to limit the experiment
48 to simultaneous measurement of two quantities, the input impedance of a tube with one
49 tonehole at its middle, and a transfer impedance. This allows avoiding dismantling the
50 apparatus during the measurement. The present paper aims at exploring another method.
51 It limits the measurement to two input impedances, by turning the cylindrical tube, the
52 extremity being open. Thus the termination impedance is unchanged when turning the
53 tube. The drawback is the need of dismantling the set up.

54 In Sect. II, the direct calculation is performed by using the theoretical, known model of
55 a cylindrical tonehole on a cylindrical tube. In Sect. III, the inverse problem is computed,
56 and a simulation of experiment is done, assuming wrong values of some parameters, such as

57 the main tube length, the location of the tonehole and the accuracy of the input impedance
 58 measurement. This allows choosing appropriate geometric parameters of the main tube, in
 59 order to achieve results with high accuracy. Sect. IV describes the experimental method
 60 and results for cylindrical toneholes, with similar dimensions to those of oacclarinet. Sect. V
 61 presents the results for examples of undercut toneholes. Sect. VI discusses the validity and
 62 interest of the method.

63 II. MODEL OF A TUBE WITH AN OPEN TONEHOLE

64 The radii of the main tube and the hole are denoted a and b , respectively. The wavenum-
 65 ber in free space is denoted $k = \omega/c$; ω is the angular frequency, and c is the sound speed in
 66 free space. The wavenumber involving viscous-thermal losses in the main tube is given by a
 67 standard expression¹⁰:

$$k_a = k \left[1 + 1.044\sqrt{-2j/r_v} - 1.08j/r_v^2 \right] \quad (1)$$

68 where $r_v = a\sqrt{\omega\rho/\mu}$ for the main tube. ρ is the air density, and μ the air viscosity. The same
 69 formula holds for the tonehole, with the notations k_b and b . The characteristic impedances
 70 are $Z_c = \rho c/\pi a^2$ and $Z_{ch} = \rho c/\pi b^2$. The quantities at the left (resp. right) of the tonehole
 71 are denoted with subscript 1 (resp. 2). The lengths of the main tube on the two sides of
 72 the tonehole are L_1 and L_2 . The height of the tonehole is t . The schematic of the tonehole
 73 geometry and the acoustic variables are shown in Fig. 1. In both the main tube and the
 74 tonehole, only the plane mode propagates, i.e., higher order modes are evanescent, i.e., the
 75 frequency is low enough. The plane mode can be matched on the two sides of the tonehole

76 symmetry axis by a second order transfer matrix². The effect of the tonehole is described
 77 by the following equation:

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = M_h \begin{pmatrix} P_2 \\ U_2 \end{pmatrix}, \quad (2)$$

78 where acoustic pressure and volume velocity are denoted P and U , respectively. M_h is a
 79 symmetrical matrix with unity determinant¹¹. It corresponds to the T-circuit^{3,10} shown in
 80 Fig. 2. It is written as follows:

$$M_h = \frac{1}{1 - Y_s Z_a / 4} \begin{pmatrix} 1 + Y_s Z_a / 4 & Z_a \\ Y_s & 1 + Y_s Z_a / 4 \end{pmatrix} \quad (3)$$

81 The series impedance Z_a and the shunt impedance $Z_s = 1/Y_s$ are the impedances corre-
 82 sponding to the anti-symmetric and symmetric parts of the velocity at the input of the
 83 tonehole^{2,10}, respectively. For an open tonehole, they are given by the following equations³:

$$Z_a = jkZ_c t_a \quad (4)$$

$$Z_s = jZ_{ch}(kt_i + \tan[k_b t + k(t_m + t_r)]). \quad (5)$$

84 In the equivalent circuit and the transfer matrix the quantity Z_h appears. It is defined by:

$$Z_h = Z_s - Z_a / 4. \quad (6)$$

85 The lengths included in the above expressions are given hereafter. tonehole height. It
 86 can be written as³:

$$Z_a = jkZ_c t_a \quad (7)$$

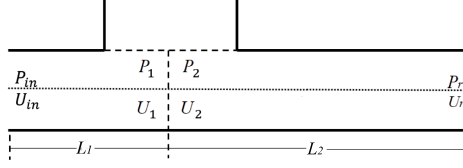


FIG. 1. Scheme of the tonehole geometry and acoustic variables.

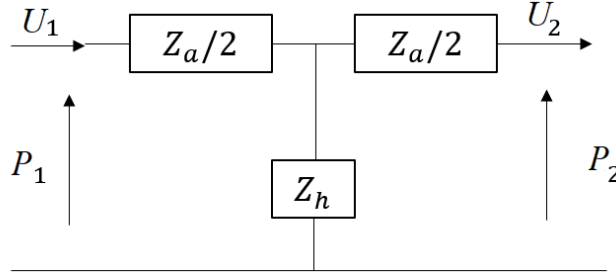


FIG. 2. Equivalent circuit for the tonehole

87

$$Z_s = jZ_{ch}(kt_i + \tan[k_b t + k(t_m + t_r)]). \quad (8)$$

88 In the equivalent circuit the quantity Z_h is defined by:

$$Z_h = Z_s - Z_a/4. \quad (9)$$

89 The lengths included in the above expressions are given hereafter. If $\delta = b/a$, the series

90 length correction t_a is given by³:

$$t_a = -b\delta^2 / [1.78 \tanh(1.84t/b) + 0.94 + 0.540\delta + 0.285\delta^2]. \quad (10)$$

91 This quantity is very small (a typical value is 0.5 mm). For this reason several authors
 92 neglect the corresponding term in Eq. 6. However, in the matrix M_h it is not consistent to
 93 ignore a quantity in one element while keeping it in the other elements. This remark can
 94 be related to the dual role of pressure and volume velocity in Eq. (3). At low frequencies,
 95 the length t_i , due to evanescent modes, is independent of frequency and can be regarded as
 96 an internal length correction for the tonehole height. It was written in Dubos et al³, and
 97 corrected by Dalmont⁷:

$$t_i = b(0.82 - 0.193\delta - 1.09\delta^2 + 1.27\delta^3 - 0.71\delta^4). \quad (11)$$

98 The length t_m is related to the matching volume between the tonehole and the main tube,
 99 and cannot be exactly computed with the modal matching method, except when the main
 100 tube is rectangular (in which case it vanishes). Its value is given by⁴:

$$t_m = b\delta(1 + 0.207\delta^3)/8. \quad (12)$$

101 The length t_r is the (complex) radiation length given by $t_h = Z_{rh}/(jkZ_{ch})$, where r is the
 102 subscript for the tube end, and Z_{rh} the radiation impedance of the tonehole. Different
 103 expressions exist in the literature. For the sake of simplicity, we assume that it is equal to
 104 the radiation of a tube without flange¹²). At low frequencies, the order of magnitude of the
 105 uncertainty concerning the length correction is $0.2b$, if losses near the walls are ignored, the
 106 total equivalent height of the tonehole is defined as:

$$t_s = \text{Im}(Z_s/(kZ_{ch})). \quad (13)$$

107 At low frequencies, it is equal to:

$$t_s = t_i + t + t_m + \text{Re}(t_r). \quad (14)$$

108 The geometric values chosen in this paper are the tonehole radius $b = 4$ mm (the main tube
 109 radius is $a = 7.3$ mm), and height $t = 8.5$ mm; the matching length correction is $t_m = 0.3$
 110 mm. The length correction for radiation is $t_r = 2.5$ mm (with a significant uncertainty of
 111 $0.2b = 0.8$ mm) and the internal length correction is $t_i = 2.1$ mm. The total equivalent height
 112 is therefore $t_s = 13.4$ mm. This quantity is of major interest for the computation of the
 113 input impedance of an instrument. Using the standard transmission line theory, a difference
 114 can be computed: it is of 1 mm, and implies a typical shift of the first impedance peak of a
 115 clarinet-like instrument by 0.5% to 1% (i.e., 9 to 17 cents). Therefore the cumulative shift
 116 for several toneholes can be rather high.

117 III. TUBE REVERSED METHOD

118 A. From the radiation impedance of the main tube to its input impedance

119 For the present method, the main tube is open and the tonehole is not located at the
 120 middle of the tube, in order to obtain two different input impedances when the tube is
 121 reversed. The two different situations are $L_1 < L_2$, and $L'_1 < L'_2$, when in the second case
 122 the tube is reversed such that $L'_1 = L_2$, and $L'_2 = L_1$. The apostrophe indicates the reverse
 123 situation. M_1 and M_2 are the transfer matrices of the cylindrical sections of the tube
 124 ($i = 1, 2$):

$$M_i = \begin{pmatrix} A_i & B_i \\ C_i & A_i \end{pmatrix} = \begin{pmatrix} \cos k_a L_i & jZ_c \sin k_a L_i \\ jZ_c^{-1} \sin k_a L_i & \cos k_a L_i \end{pmatrix}. \quad (15)$$

125 Finally the input impedance is derived from (*in* is the subscript of the tube input):

$$\begin{pmatrix} P_{in} \\ U_{in} \end{pmatrix} = M_1 M_h M_2 \begin{pmatrix} P_r \\ U_r \end{pmatrix}. \quad (16)$$

126 The radiation impedance Z_r is projected back to the right of the tonehole, as follows:

$$Z_2 = \frac{A_2 Z_r + B_2}{C_2 Z_r + A_2}. \quad (17)$$

127 Similarly, the impedance Z_1 at the left of the tonehole and the input impedance Z_{in} are
 128 calculated by using the projection formula.

129 B. Inverse problem

130 The input impedance Z_{in} , assumed to be known, is projected to the left of the tonehole,
 131 multiplying by the inverse matrix of M_1 as:

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = \begin{pmatrix} A_1 & -B_1 \\ -C_1 & A_1 \end{pmatrix} \begin{pmatrix} P_{in} \\ U_{in} \end{pmatrix} \quad (18)$$

132

$$\Rightarrow Z_1 = \frac{A_1 Z_{in} - B_1}{-C_1 Z_{in} + A_1}. \quad (19)$$

133 Following Fig. 2, the equations for the 3 elements of the electrical equivalent circuit can be
 134 written: Defining $P = Z_h(U_1 - U_2)$; $P_1 = Z_1 U_1 = P + Z_a/2 U_1$; and $P_2 = Z_2 U_2 = P - Z_a/2 U_2$,
 135 the following equation is obtained:

$$\frac{1}{Z_h} = \frac{1}{Z_1 - Z_a/2} - \frac{1}{Z_2 + Z_a/2}. \quad (20)$$

136 A similar equation holds for the second situation (reversed tube), replacing Z_1 and Z_2 by
 137 Z'_1 and Z'_2 , respectively.

$$\frac{1}{Z_h} = \frac{1}{Z'_1 - Z_a/2} - \frac{1}{Z'_2 + Z_a/2}. \quad (21)$$

138 The following quadratic equation is obtained by eliminating Z_h :

$$AZ_a^2/4 + BZ_a/2 + C = 0, \quad (22)$$

139

$$A = (Z'_1 - Z_1) - (Z'_2 - Z_2);$$

$$B = 2(Z'_1 Z'_2 - Z_1 Z_2); \quad (23)$$

$$C = Z'_2 Z_2 (Z'_1 - Z_1) - Z'_1 Z_1 (Z'_2 - Z_2)$$

140 Eq. (22) can be solved for Z_a , then Z_h is derived from Eq. (20) or Eq. (21). However a
 141 simpler solution is obtained by expressing Z_h with respect to Z_a . Using Eqs. (20 and 22)
 142 and eliminating Z_a^2 , it can be written as:

$$Z_h = -\frac{B}{2A} - \frac{Z_a}{2}. \quad (24)$$

143 Then, introducing this result in the quadratic equation (22), the following result is obtained:

144

$$Z_h^2 = \frac{B^2}{4A^2} - \frac{C}{A}. \quad (25)$$

145 Two solutions exist for this equation. The solution with a negative real part can be elimi-
 146 nated because the physical system is passive. Z_s can be deduced from Eq. (6):

$$Z_s = Z_h + Z_a/4. \quad (26)$$

147 Throughout this paper, the results are focussed on 3 quantities: the total equivalent height
 148 of the tonehole t_s , given by Eqs. (13, 25, 26); the real part of Z_h and the imaginary part of
 149 Z_a , from Eq. (24). The results of the inverse problem were checked by using computed input
 150 impedances, and the order of magnitude of the numerical error is smaller than 10^{-14} . Fig. 3
 151 shows the comparison between the direct and the inverse computations for the equivalent
 152 height of the tonehole $t_s = Re(Z_s/(jkS_{ch}))$. For Z_a , the numerical error is smaller than
 153 10^{-12} . For other choices of termination impedance, such as an infinite impedance or the
 154 characteristic impedance, the entire computation remains valid. When the frequency tends

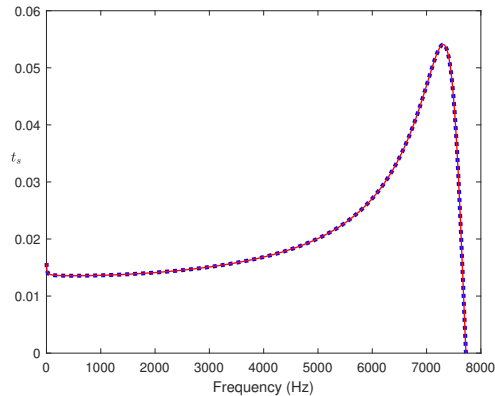


FIG. 3. (Color online) Equivalent height t_s of the tonehole (in m). Solid, red line: model; blue, dotted line: inverse problem (from Eq. (25)). Dimensions $a = 7.3$ mm, $b = 4$ mm, $t = 8.5$ mm, $L_1 = 44$ mm, $L_2 = 74$ mm.

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157 to zero, the small increase is due to the visco-thermal dispersion, which diminishes the
 158 sound speed, and increases the equivalent length. Furthermore, the strong variation at
 159 higher frequencies is due to the propagation of the planar mode in the tonehole (see the

160 function $\tan(x)$ in Eq. (6)). The resonance near 7540 Hz corresponds to the minimum of
 161 the input impedance of the tonehole.

162 **C. Numerical simulation of the experiment: effect of uncertainty on the main**
 163 **tube length**

164 In order to simulate the experiment, errors are introduced on the data of the inverse
 165 problem. The input impedance is first computed, and the values are treated as experimental
 166 data. We start with an error of 0.2 mm on the length L_1 . For the second case, an error
 167 of 0.2 mm on the length L_1 is considered together with an opposite error on the length
 168 L_2 (the later case corresponds to an error on the location of the tonehole, without change
 169 in the total length $L_1 + L_2$). For the equivalent height of the hole t_s , Fig. 4 shows the
 170 comparison between results for the two cases simulated and the theoretical result (without
 171 errors introduced). Between 1550 Hz and 1650 Hz, the error on the result is very large.
 172 Because this also happens at other higher frequencies, the figure is limited to 2000 Hz. The
 173 frequency ranges with large error are close to the input impedance minima of the main tube
 174 (1560 Hz for Z_{in} and 1610 Hz for Z'_{in}). A simple qualitative interpretation is the following:
 175 suppose that the radiation impedance of the tube is 0 (whatever the frequency), and that
 176 the input impedance vanishes at a given frequency, therefore the eigenfrequencies of the
 177 tube in the two positions are equal, and the problem becomes ill-posed (one equation for
 178 two unknowns): the solutions tend to infinity. This reasoning is not exact, because the
 179 radiation impedance is small, but not 0. The variations of t_s are very small up to 1400
 180 Hz, as well as the discrepancies with the theoretical values. Concerning the real part of the
 181

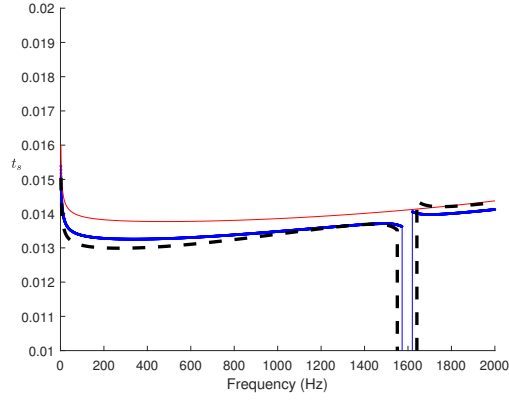


FIG. 4. (Color online) Equivalent height t_s (in m). Red, solid thin line: theory without length errors. Blue, thick, solid line: inverse problem (Eq. (25) with 0.2 mm error on L_1 . Black, dashed line: inverse problem with 0.2 mm error on L_1 and -0.2 mm error on L_2 .

182 shunt impedance Z_h , it can be seen in Fig. 5 that the accuracy of the simulated results is
 183 satisfactory up to 1400 Hz.

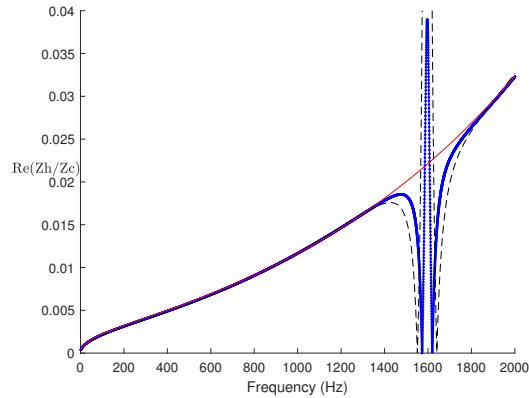


FIG. 5. (Color online) Real part of the shunt impedance Z_h . See line definitions in the caption of Fig. 4.

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187 However, concerning the imaginary part of the series impedance Z_a , even a very small
 188 error on the lengths causes large errors on the result (see Fig. 6). Even the sign of the

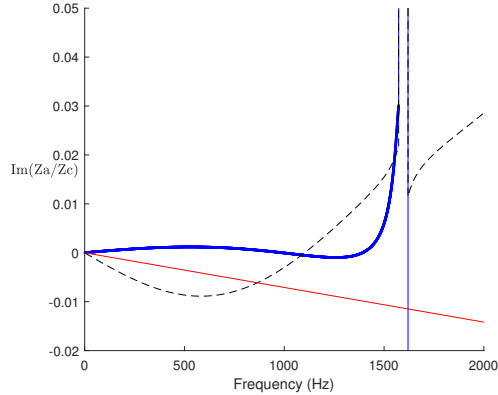


FIG. 6. (Color online) Imaginary part of the series impedance Z_a . See line definitions in the caption of Fig. 4.

189 quantity is not determined. This result suggests that it is extremely difficult to expect a
 190 precise measurement of the series impedance. From this perspective, the method is less
 191 robust than the method of the input and transfer impedance⁷, even if the later is not very
 192 precise (the uncertainty is almost 35%). The present method is probably not suitable for
 193 measuring this element through experimentation.

194 **D. Numerical simulation of the experiment: effect of the uncertainty on the mea-**
 195 **sured input impedance**

196 A second attempt to simulate the experiment is based on the introduction of a random
 197 error on the input impedance (for the two configurations of the main tube Z_{in} and Z'_{in}).
 198 The input impedance is modified as follows:

$$\widetilde{Z}_{in} = Z_{in}\{1 + 0.005[rand(N) - 0.5]\}. \quad (27)$$

199 The number N is the size of the input impedance vector. *rand* is a Matlab function that gen-
 200 erates uniform pseudo-random numbers in the interval $[0, 1]$. The value 0.005 is determined
 201 by the measurement of many input impedances. It means that the error modelled ranges
 202 from -0.25% to 0.25% of Z_{in} . The three figures 7 to 9 show a confirmation of the previous
 203 observations: the measurement can be accurate up to 1400 Hz for the shunt impedance, but
 204 the measurement of the series impedance is not possible (see Figs. 7 to 9).

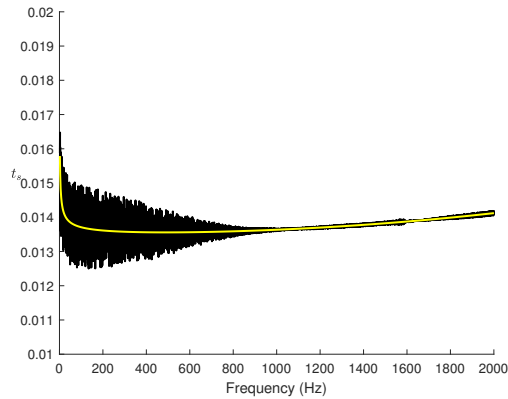


FIG. 7. (Color online) Tonehole equivalent height t_s (in m). Black lines: result of a simulation with a random error on the input impedance of the tube (Eq. (27)). Yellow line: no random error.

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209 E. Practical considerations for the dimensions of the main tube

210 A conclusion of the simulation study implies that the main tube has to be chosen to be as
 211 short as possible. In order to avoid the coupling of evanescent modes between the tonehole
 212 and the radiating termination, the distance L_1 between the tonehole and the termination
 213 can be chosen between 2 and 3 times the main tube diameter. Furthermore the value of the
 214 first minimum frequency implies a small total length $L_1 + L_2$. However it is essential that

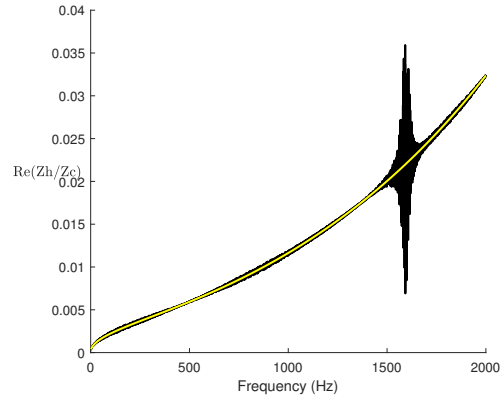


FIG. 8. (Color online) Real part of the shunt impedance Z_h . Black lines: result of a simulation with a random error on the input impedance of the tube (Eq. (27)). Yellow line: no random error.

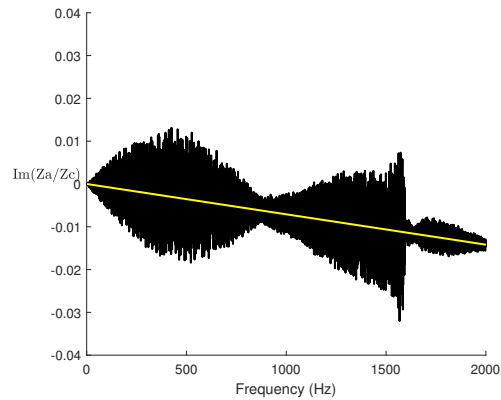


FIG. 9. (Color online) Imaginary part of the series impedance Z_a . Black lines: result of a simulation with a random error on the input impedance of the tube (Eq. (27)). Yellow line: no random error.

215 the two lengths are sufficiently different, in order to avoid the quadratic equation to become
 216 degenerate. The convenient choice for the L_2 is between 2 and 3 times the length L_1 .

217 IV. EXPERIMENTAL RESULTS FOR CYLINDRICAL TONEHOLES

218 A. Input impedance measurement

219 The previous analysis encourages us to study an experiment based upon the method
220 presented in the present paper. The method is tested experimentally by using wood pieces,
221 and the CTTM sensor¹³ for the impedance measurement. A piezoelectric buzzer is used as
222 a source. The pressure in the back cavity of the buzzer is measured by a microphone, which
223 gives an estimation of the volume velocity. The measured pipe is connected to the front of
224 the buzzer via a small open cavity in which a second microphone measures the pressure. The
225 input impedance of the pipe is at first order proportional to the transfer function between
226 the two microphones. The comparison with theoretical results for cylindrical tubes (without
227 toneholes) is satisfactory: the discrepancy for a closed tube is 4 cents for the resonance
228 frequencies and 1 dB for the peak heights, except at very low frequencies. For this reason,
229 measurements are done above 200 Hz.

230 B. Preliminary results concerning the repeatability of the measurement

231 We first study the repeatability for a tube and a tonehole with dimensions equal to those
232 previously considered. For the frequency range 200 to 1400 Hz, the equivalent height t_s of
233 the tonehole is found to be between 14.4 mm and 15.5 mm, while the theoretical value (from
234 Eq. (13) is 13.4 mm. For 4 measurements after disassembly and assembly, the uncertainty is
235 found to be about 1 to 2% (see Fig. 10). Furthermore Fig. 11 shows the comparison between
236 the measurements of 4 tubes built with the same tools. The results are distributed on both

237 sides of the theoretical one. This is an effect of the manufacturing tolerance, which is of the
 238 same order of magnitude as the measurement uncertainty, or higher. For all experimental
 239 results, the Matlab function *smooth* has been used. We remark measurements are not
 240 necessarily taken on the same day and at the same temperature, but the computation took
 241 it into account.

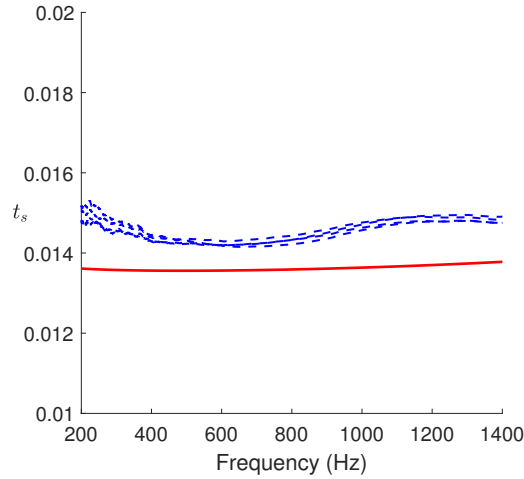


FIG. 10. (Color online) Tonehole equivalent height measured 4 times after disassembly. Blue, dashed lines: measurements. Red, solid line: theory. Dimensions $a = 7.3$ mm, $b = 4$ mm, $t = 8.5$ mm, $L_1 = 44$ mm, $L_2 = 74$ mm.

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243

245 C. Comparison between two tubes of different lengths

246 Two tubes of total length $L_1 + L_2 = 118$ mm and 162 mm are compared. The tonehole
 247 is located at the same distance of one of the ends of the two tubes (44 mm). This value is
 248 chosen to be 2.7 times the hole diameter. The dimensions of the hole are identical for the
 249 two tube lengths ($b = 4$ mm; $t = 8.5$ mm). As expected, Fig. 12 shows a small increase

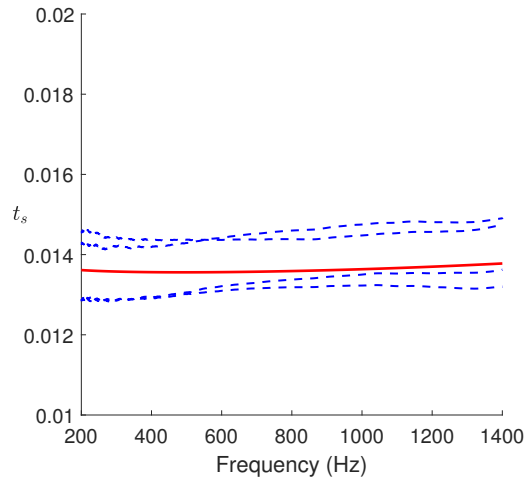


FIG. 11. (Color online) Tonehole equivalent height t_s for 4 tubes built with the same tool. Blue, dashed lines: measurements. Red, solid line: theory.

250 when the frequency approaches the eigenfrequency of the tubes. As explained above, the
 251 short tube yields better results on a wider frequency range and the results are closer to
 252 the theoretical value, in particular near the measured minimum. The discrepancy between
 253 the results of the two tubes is about 3%, except near the eigenfrequency. Concerning the
 255 real part of the shunt impedance, it appears that the two tubes yield very similar values,
 256 except in the vicinity of the eigenfrequency. Fig. 13 shows that they are higher than the
 257 theoretical values. Remember that for a linear functioning, radiation losses are proportional
 258 to ω^2 , while visco-thermal losses increase as $\sqrt{\omega}$. We refer to⁷ for a discussion about the
 259 theoretical aspects. Finally, the experiment confirms that the series impedance cannot be
 262 measured by the tube reversed method, as shown in Fig. 14.

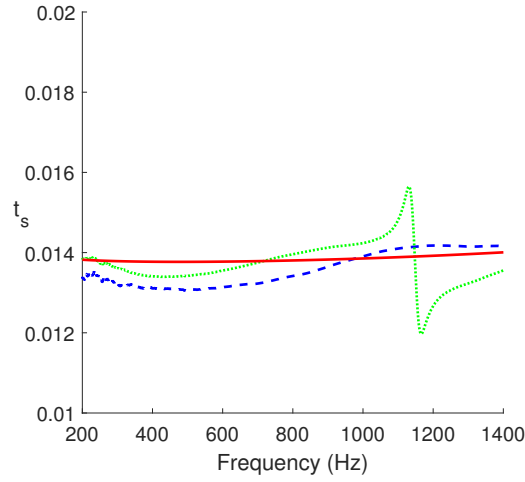


FIG. 12. (Color online) Measured value of the equivalent height t_s of the hole. Green, dashed lines: long tube. Blue, dotted line: short tube. Red, solid line: theory

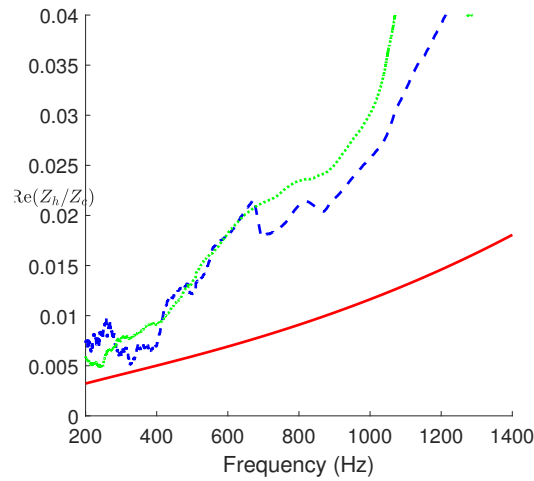


FIG. 13. (Color online) Measured value of the real part of the shunt impedance $\text{Re}(Z_h)$. See the line definitions in the caption of Fig. 12.

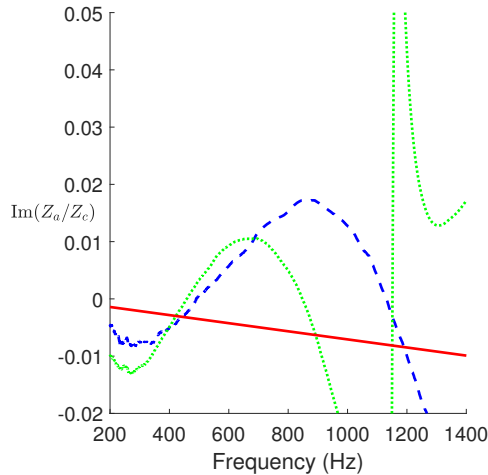


FIG. 14. (Color online) Measured value of the imaginary part of the series impedance $Im(Z_a)$. See the line definitions in the caption of Fig. 12.

263 V. EXPERIMENTAL RESULTS FOR UNDERCUT TONEHOLES

264 Undercutting toneholes was studied for high excitation level by Dalmont et al⁷ and
 265 McDonald¹⁴ (see also¹⁵) for rectangular geometry. Nine short tubes of length 118 mm are
 266 holes drilled at $L_1 = 44$ mm that have three different geometries: three are straight (but
 267 the hole is deburred), three are undercut by 2 mm and two are undercut by 3 mm. Figs. 15
 268 and 16 show the effect of undercutting the toneholes. The quantity shown by Fig. 15 is
 269 slightly different from that shown previously (see e.g. Fig. 13), because considering the
 270 length correction in Eq. (13) implies a division by the cross-section area S_h , but for the
 271 case of undercut toneholes, the area is not constant. For this reason, we choose the acoustic
 272 mass (per unit density) m_s :

$$m_s = Z_h / (j\omega\rho). \quad (28)$$

273 The figures represent the average quantities for each geometry. The effect of undercutting is
 274 a decrease of $10m^{-1}$ to $20m^{-1}$ for the acoustic mass when the undercutting becomes wider.
 275 The jump below 400 Hz in Fig. 13 remains unexplained. Two causes for this mass increase
 276 can be analyzed. The widening implies a decrease of the acoustic mass of the plane mode,
 277 and also of the internal length correction due to the discontinuity between the main tube and
 278 the tonehole. The first of these causes can be modelled. Considering the acoustic mass for
 279 the cylindrical tonehole case, the result seems to be close to the theoretical result between
 280 400 Hz and 600 Hz. Calculating the average value, we obtain $280 m^{-1}$. For the cases of
 281 undercutting, we obtain $270 m^{-1}$ and $264 m^{-1}$.

282 An elementary model can be made in order to interpret these results. The shape of
 283 the most undercut tonehole is close to a cylinder extended in a truncated cone joining the
 284 internal wall of the main tube. For the cases studied, the lengths ℓ of the cylinder and ℓ'
 285 of the cone are approximately equal to $\ell = \ell' = 5.5$ mm. The radius of the cylinder is
 286 $b = 4$ mm, the small radius of the cone is $R_1 = b$ and its large radius is $R_2 = 5.4$ mm.
 287 The calculation of the mass of a tube with variable cross section is done by integrating the
 288 inverse of the area along the axis. For a cone, the result is published in¹⁰, p. 325. It is that
 289 of a cylinder with a cross section equal to the geometric average of the radius: $S = \pi R_1 R_2$.
 290 The difference between the cylindrical tonehole and the undercutting one is:

$$\delta_m = \frac{\ell'}{\pi b^2} \left[\frac{b}{R_2} - 1 \right]. \quad (29)$$

291 The result of this formula is $26m^{-1}$. This result, based on approximate geometric and
 292 acoustic models, is consistent with the experimental data. This is encouraging for the
 293 use of an accurate measurement method for the computation of the input impedance of

294 an instrument with undercut holes or other deviations from the cylindrical shape, such as
 295 holes with keypads. Furthermore, Fig. 16 shows that the effect of undercutting on the real
 296 part of the shunt impedance is small, but significant: it causes a decrease in resistance by
 297 approximately 10 % as the undercut is increased from 0 to 2 and 3 mm. . It is difficult to
 298 interpret the differences between the three geometries and their variation with frequency,
 299 and the influence of nonlinear effects cannot be ignored. However, a linear reasoning can
 300 be applied here: undercutting a tonehole broadens the effective radius, and visco-thermal
 301 effects diminish.

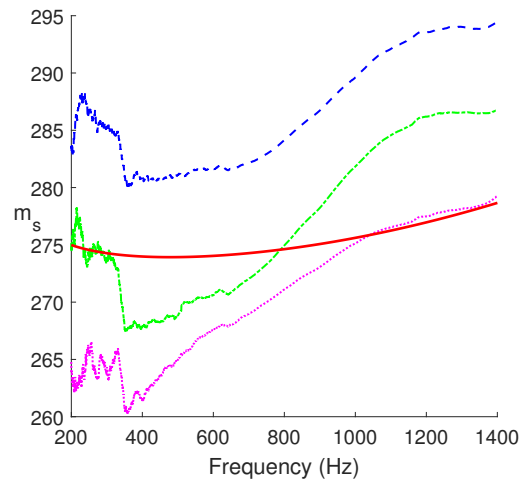


FIG. 15. (Color online) Measured value of the acoustic mass per unit density m_s of the hole. Red, solid line: theory of a cylindrical tonehole in m^{-1} . From top to bottom, 3 geometries of the tonehole: Blue, dashed line: no undercutting; Green, dash-dot line: undercutting by 2 mm; Magenta, dotted line: undercutting by 3 mm

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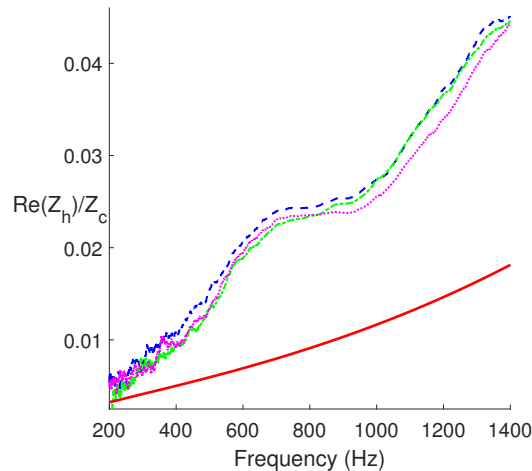


FIG. 16. (Color online) Measured value of the real part of the shunt impedance $\text{Re}(Z_h)$. Red line: theory of a cylindrical tonehole. See line definitions in the caption of Fig. 15.

305 VI. CONCLUSION

306 The method presented in this paper allows an evaluation of the effect of the complex
 307 shunt impedance of an open tonehole. We recall that the aim is to insert the experimental
 308 value in the computation of the input impedance of an instrument. The effect of a hole
 309 modification on the input impedance of an instrument is significant: a difference of 1 mm
 310 for the equivalent height may imply a shift of the first impedance peak. The cumulative
 311 shift for several toneholes can be rather high (see e.g. an article on the clarinet tuning¹¹).
 312 It is important to use a short tube for this method, due to antiresonances associated to the
 313 total tube length. We remark that a similar problem concerning the “forbidden” frequency
 314 ranges is encountered in other methods. Moreover the distance of the hole to the tube end
 315 needs to be short. Concerning the real part of the shunt impedance, the results appear
 316 to be robust, and suggest further studies on the theoretical aspects, even for cylindrical

317 toneholes in the linear regime. Concerning the imaginary part of the shunt impedance,
318 the primary quantity studied here, the results seem to be very sensitive to small geometric
319 differences. The relative variation of the equivalent height with frequency is small, and the
320 absolute variation remains small. For a cylindrical tonehole, at approximately 500 Hz, the
321 discrepancy between experiment and theory is very small for the equivalent height (0.1 mm),
322 and is of the same order of magnitude as the result obtained by other method⁷. The paper
323 is limited to the frequency range [200 Hz, 1400 Hz] for the measurements. It is concluded
324 that the variation with frequency is mainly due to the measurement method. Assuming
325 that the true value of the tonehole equivalent height is independent of frequency, the choice
326 of an average of the values between 400 and 600 Hz as appropriate can be extended to
327 any hole geometry. This result of the different cases examined in the present work can be
328 used for including the acoustic characteristic of undercut tonehole in a computation of input
329 impedances of an instrument.

330 The method is not convenient for measuring the series impedance. Actually this quantity
331 is very small, but for this quantity the methods proposed in previous publications seem to be
332 better. Concerning undercut toneholes, which are generally not symmetrical, in certain cases
333 it could be useful to search for a circuit with 3 unknowns³. The aim of the present paper
334 is not to improve a model, but it is useful in that it highlights some of the complications
335 inherent in existing open tonehole models. The main improvement to existing models could
336 be done on the radiation impedance of a tonehole.

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