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Fast solvers for Hybrid High-Order discretizations*

Pierre Matalon, Daniele A. Di Pietro, Frank Hülsemann, Paul Mycek, Ulrich Råde, Daniel Ruiz

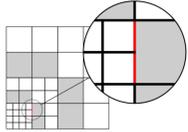
Problem/context: solving large scale CFD problems on complex geometries

Introduction to the Hybrid High-Order (HHO) discretization [1]

Characteristics

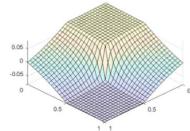
Applies to general polyhedral meshes

- Easily approximates complex geometries
- Non-conforming junctions perceived as coplanar faces

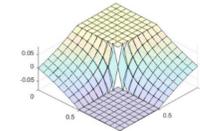


Discontinuous approximation:

suitable to approximate non-smooth solution



Exact solution



HHO linear approximate solution

Handles high orders

The solution can be approximated by a polynomial of arbitrary degree $p \geq 1$

Superconvergence

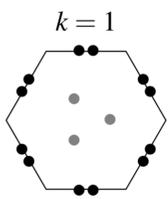
For an approximate of degree $p \geq 1$, if $u \in H^{p+1}(\Omega)$, convergence in $\mathcal{O}(h^{p+1})$ in L^2 -norm

Construction

Degrees of Freedom (DoFs)

Given a mesh $(\mathcal{T}_h, \mathcal{F}_h)$ and a polynomial order $k \in \mathbb{N}$:

- the DoFs are located in cells and on faces, defining polynomials of degree k

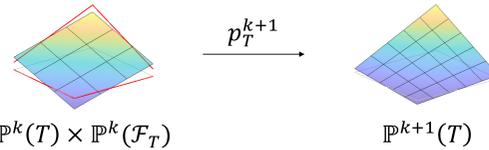


- Space of unknowns local to $T \in \mathcal{T}_h$:

$$\{(v_T, (v_F)_{F \in \mathcal{F}_T}) \mid \begin{array}{l} v_T \in \mathbb{P}^k(T) \\ v_F \in \mathbb{P}^k(F) \quad \forall F \in \mathcal{F}_T \end{array}\}$$

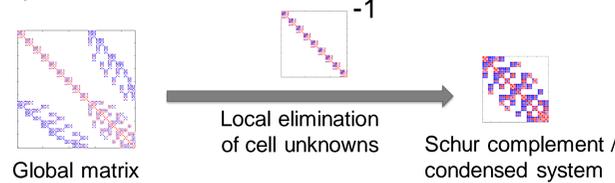
Higher-order reconstruction

The local reconstruction operator p_T^{k+1} allows to gain one order of approximation

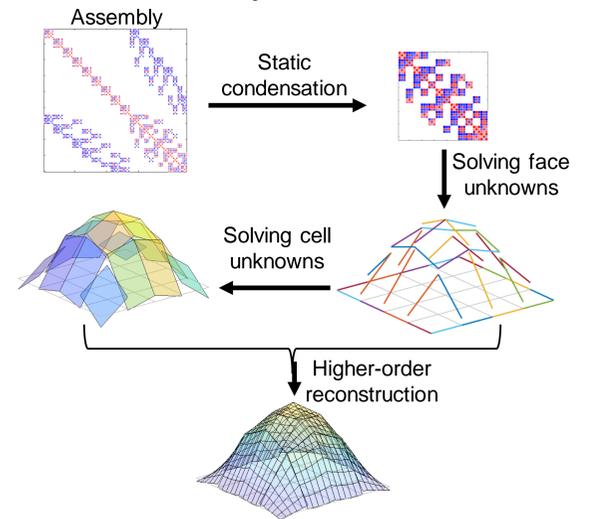


Static condensation

Allows to reduce the number of DoFs in the linear system



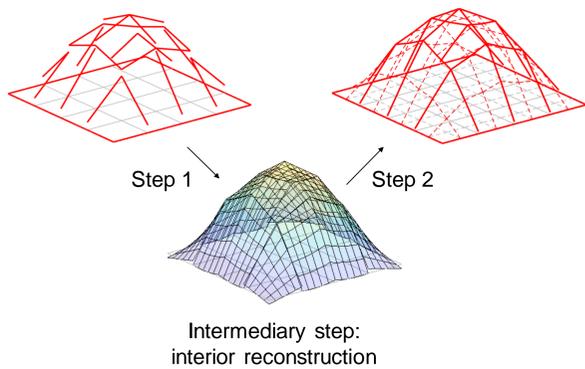
Process summary



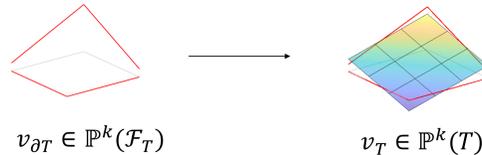
Research goal: fast linear solvers targeting the statically condensed system arising from the HHO discretization of elliptic equations

Multigrid solver for the statically condensed system [2]

Face-defined prolongation operator



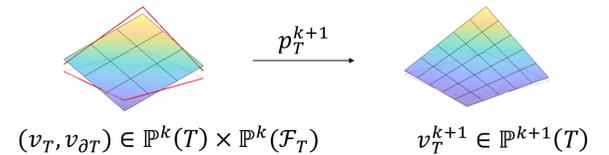
Step 1a: reconstruction of degree k



The static condensation is reversed by the solution of the local problem:

$$A_T := \begin{pmatrix} A_{TT} & A_{TF_T} \\ A_{F_T T} & A_{F_T F_T} \end{pmatrix} \quad v_T := -A_{TT}^{-1} A_{TF_T} v_{\partial T}$$

Step 1b: reconstruction of degree $k+1$



Step 2a: if the meshes are nested, canonical injection from the coarse cells to the fine ones; if not, L^2 -orthogonal projection

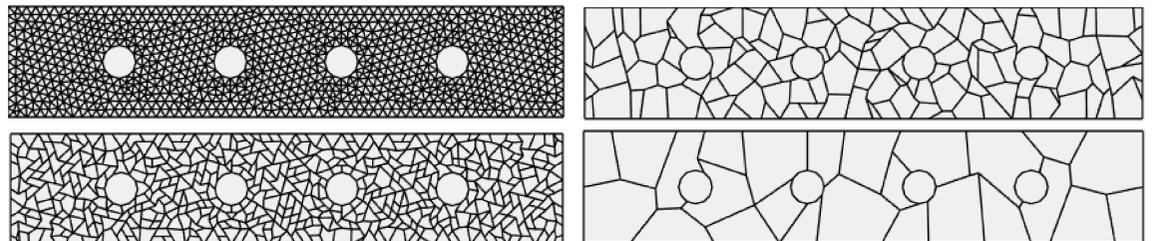
Step 2b: weighted average trace on the fine faces and L^2 -projection onto the lower order space

Coarsening strategy for unstructured meshes

Constraint: the faces must also be coarsened!

Algorithm:

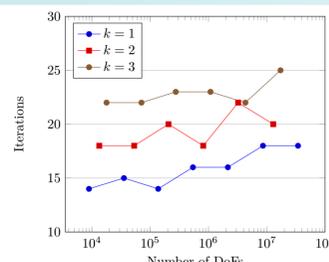
1. For each element, **agglomerate** it with all of its non-agglomerated neighbours
2. **Collapse** into one single face the interfaces between two coarse elements that are made of multiple faces



Numerical results

Test setting

- Diffusion problem
- V(0,3) cycle
- Block Gauss-Seidel smoother
- Stopping criterion: normalized residual $< 10^{-8}$



Results

- Fast and scalable
- Robust to discontinuities in the diffusion coefficient
- Limited overhead induced by the increase of the polynomial order

[1] D. A. Di Pietro and J. Droniou, *The Hybrid High-Order method for polytopal meshes*, no. 19 in Modeling, Simulation and Application, Springer International Publishing, 2020.

[2] P. Matalon, D. A. Di Pietro, F. Hülsemann, P. Mycek, D. Ruiz, U. Råde, *An h-multigrid method for Hybrid High-Order discretizations*, preprint available on HAL, 2020.