Simple quantitative assessment of the outdoor versus indoor airborne transmission of viruses and covid-19

Supplementary materials

B.R. Rowe[a], A. Canosa[b], J.M. Drouffe[c] and J.B.A. Mitchell[d]*

[a]*Rowe Consulting, 22 chemin des moines, 22750 Saint Jacut de la Mer (France).*

[b]*CNRS, IPR (Institut de Physique de Rennes)-UMR 6251, Université de Rennes, 35000 Rennes, (France).*

[c]*31B Chemin du Couvent, 91190 Gif-sur-Yvette (France)*

[d]*MERL-Consulting SAS, 21 Rue Sergent Guihard, 35000 Rennes (France).*

* corresponding author: bertrand.rowe@gmail.com

S1. Micro droplet evaporation.

An aerosol is released while coughing, sneezing, speaking or simply breathing. The corresponding droplets, mainly in the range from a fraction of micrometre to one hundred, enclose the viral content. It might be believed that most of the mass of these droplets fall to the ground and therefore that there is no airborne contamination. As already recognized by Wells in 1934 (Wells, 1934) this is false because these water droplets can evaporate in the ambient atmosphere; they shrink until their falling speed becomes so small that they are carried by the small movement of the atmosphere, would it be very quiet.

Quantitatively, due to the Stokes law, the droplets of radius *r* reach a limit speed in the (viscous) air of

(S1-1) $$v_{lm} = \frac{2}{9}\frac{r^2 \rho_{water} g}{\mu_{air}}$$

where ρ_{water}, g and μ_{air} are respectively the volume mass of water, gravity acceleration and air viscosity. This means that a non-evaporating droplet of diameter 100 µm falls from a height of 1.5m in 5 s; with a diameter of 30 µm, 55 s are necessary; and with a diameter of 10 µm, the falling speed lowers to 3 mm/s so that this droplet may be dragged by the movements of the atmosphere.

Now consider the evaporation. The models are quite complex, and we refer for instance to reference (Lefevre and McDonel, 2017) for the details. The main result is that the squared diameter of a drop shrinks linearly in time:

(S1-2) $$\frac{dD^2}{dt} = -\lambda$$

where λ depends in particular on the relative humidity *RH* and is falling to 0 in a saturated air. Hence plugging in the limiting fall speed above, one finds:

(S1-3) $$\frac{dz}{dt} = \frac{1}{18}\frac{\rho_{water} g}{\mu_{air}}(D_0^2 - \lambda t)$$

Integrating this differential equation up to the lifetime of the droplet $\tau = D_0^2/\lambda$, one finds the falling height *h* between its production and its complete drying

(S1-4) $$h = \frac{\rho_{water} g}{36 \mu_{air} \lambda} D_0^4$$

If this height is greater than the production height (typically 1.5 m), the particle deposits on the ground; otherwise, it becomes so light due to the evaporation that it participates to airborne contamination.

We do not enter in the evaluation of the parameter λ which expresses an equilibrium between heat transfer towards the droplet and water vapour mass transfer from the droplet. It depends on the temperature which governs the saturated water vapour pressure near the droplet surface and on *RH* which commands the mass transfer of water vapour from the droplet (see (Lefevre and McDonel, 2017) for the detailed formulae). Hence we give only a final table (T1-1) below for the results at the temperature of 20 °C for various *RH*. The second column displays the limiting diameter under which the droplet evaporates completely before reaching the ground supposed to be 1.5 m under the droplet production. Below this diameter, all droplets would participate to airborne transmission; the airborne dispersed droplet proportion that would be obtained with these calculation hypotheses is given in the last column, assuming a flat distribution of droplets between 30 µm and 100 µm. The interest of table T1-1 is to show the importance of relative humidity on droplet evaporation.

RH	D_0 (µm)	τ (s)	Airborne dispersion
0%	103	9	100%
25%	94	11	78%
50%	84	14	49%
90%	55	33	8%
100%	31	105	0.08%

Table T1-1 Droplet airborne evaporation for pure water.

Note that indoor conditions lead generally to dry air (*RH* = 15-20%), enhancing airborne dispersion while wet outdoor conditions favour deposition. There are a very large number of papers in the scientific literature on this topic, see for example (Morawska, 2006).

The above results assume that the non-volatile part of the droplet can be neglected which is not true. Considering this point, it can be shown that the final diameter cannot shrink below 26-44% of its initial value, see for example (Nicas *et al.*, 2005), which implies that for most of the diameters in table T1-1 the dry nuclei will finally fall to the ground. This final diameter corresponds to "dry nuclei" with a complicated composition (salt, protein, viruses). Note that there is no reason to assume that these dry nuclei are hard solid spheres. It is known that evaporation of solutions can lead to hollow or fractal structures which will have a much higher drag coefficient than a compact sphere, leading to an easier aerosolization.

S2- Transient effects and time of inactivation for the indoor case.

The following equation, developed in the main paper, determines the indoor temporal evolution of the concentration of inhaled particles n_i that have been exhaled by human emitters:

(S2-1) $$\frac{dN_i}{dt} = V \times \frac{dn_i}{dt} = N_p \times q_1 \times n_1 - q_2 \times n_i$$

where V is the building volume, N_p the number of people, q_1 the mean exhaled flow rate per person, n_1 the concentration of particle in the exhaled air.

Its solution takes clearly into account transient effects which occur when the time t is not $>> \tau_1$ with τ_1 defined by equation (S2-3). This is the case for example at the opening of a supermarket. Of course, as the flow rate of fresh air q_2 could be regulated as a function of the number of people following equation (S2-1) this could make transient analysis rather complicated. However, in many cases this flow rate is adjusted constant, taking into account a "normal" number of people. Equation (S2-1) yields for n_i:

(S2-2) $$n_i(t) = n_i^{\infty} \times \left[1 - exp\left(-\frac{t}{\tau_1}\right)\right]$$

and involves mainly:

(S2-3) $$\tau_1 = \frac{V}{q_2}$$

and:

(S2-4) $$n_i^{\infty} = \frac{N_p \times q_1 \times n_1}{V} \times \tau_1$$

Considering some typical values leads to typical times τ_1 around one hour or below. Clearly the risk is higher toward the end of the day. Note that monitoring CO_2 is clearly a good way to track transient effects. These transient effects have already been studied in the literature (Rudnick and Milton, 2003).

Another point which could introduce indoor time dependent effect is the time of inactivation (or lifetime) of a virus. If we assume that infective particles loss their infective power with an exponential lifetime τ_2 then it can be added a term in equation (S2-1).

(S2-5) $$\frac{dN_i}{dt} = V \times \frac{dn_i}{dt} = N_p \times q_1 \times n_1 - q_2 \times n_i - \frac{n_i}{\tau_2} \times V$$

Of course, equation (S2-5) considers all particles of human origin, infective or not. But if we consider that infective particle concentration is just proportional to the total particle concentration, and infector number proportional to the number of people N_p, with the same coefficient of proportionality, then it is justified to consider that equation (S2-5) mirror the equation for infective particles and that the derived n_i can be used to calculate a modified IFREP (Inhaled Flow Rate of Exhaled Particles) which takes into account the lifetime. Doing this results in a modified characteristic time τ_m and stationary concentration value n_i^∞ .

(S2-6) $$\tau_m = \frac{\tau_1 \times \tau_2}{(\tau_1 + \tau_2)}$$

and:

(S2-7) $$n_i^\infty = \frac{N_p \times q_1 \times n_1}{V} \times \tau_m$$

Note that τ_1 can also be written:

(S2-8) $$\tau_1 = \frac{h_b}{D_p \times q_{norm}}$$

where h_b is the ceiling height of the building, D_p being as in the main paper the people density.

In many situations this leads to $\tau_1 \ll \tau_2$ hence $\tau_m \sim \tau_1$ which means that there is little influence of the virus inactivation on the stationary concentration of particles to consider for the comparison of outdoor to indoor situation, the principal purpose of the main paper.

Note that outdoor for most situations the hydrodynamic time $\tau_h = l/V_\infty$ is such that viruses have no time at all to be inactivated ($\tau_h \ll \tau_2$).

S3- Evaluation of the relative level of exposure for any indoor ventilation rate.

In the main paper we have established a simple formula for the relative level of exposure between outdoor and indoor situations:

(S3-1) $$R = \frac{q_{norm}}{V_\infty} \times D_p \times \frac{l}{H}$$

Under this form the only quantity which refers to the indoor situation is q_{norm}. This is due to the fact that the ventilation rate of fresh air is normally proportional to the number of people indoors. D_p is the number density of people outdoors (person/square meter), $D_p(outdoor) = N_p(outdoor)/A$, the quantity H/l is the ratio of the height of dispersion to the length along the wind in the area *A*.

Note that if the indoor ventilation rate is not fixed accordingly to a number of persons then *R* can write:

(S3-2) $$R = \frac{q_2}{V_\infty} \times \frac{D_p(outdoor)}{N_p(indoor)} \times \frac{l}{H}$$

Since q_2, the building fresh air renewal is (as stated above) $q_2 = q_{norm} \times N_p$

Note that *R* can also be written:

(S3-3) $$R = \frac{q_2}{V_\infty} \times \frac{N_p(outdoor)}{N_p(indoor)} \times \frac{l}{H} \times \frac{1}{A}$$

Again in both equations we can see that the parameter difficult to evaluate for the relative level of exposure is the height of dispersion H, completely dependent of meteorological and climatic conditions.

S4- Correlations between "increase of contamination" and "fine particle pollution episodes"

A correlation between episodes of fine particle (PM hereafter) pollution and the increase in contamination cases has been observed on several occasions (Rohrer *et al.*, 2020; Zoran *et al.*, 2020). It could be deduced from this that atmospheric PM pollutants act as vectors for viruses because of surface aggregation and reaction phenomena. However, it can also be stated: "Atmospheric conditions that are prone to the accumulation of fine particles (such as soot particles) in the atmosphere at ground are favourable to the accumulation of any kind of aerosols, including viral particles in suspension."

Therefore, even in an un-polluted zone, these same atmospheric conditions favour the transmission of the virus.

Of course, atmospheric pollution is not healthy for the lungs of individuals and could possibly predispose them to suffer more to viral contamination.

However, the calculation presented below shows that it is indeed the second statement, *atmospheric conditions favourable to aerosols,* that should probably be retained. Thus, the message concerning airborne transmission of the virus should not be mixed in with that on pollution: in a contaminated area with specific meteorology and geography, airborne transmission could be effective outdoor with or without PM pollution.

The calculation:

This calculation has been performed considering a "gas of particles" where between two interactions, the particles follow a straight trajectory. In reality, particles undergo "Brownian" (or diffusion) motion where the trajectories are not straight, however, taking this complexity into account changes the results by less than an order of magnitude. The calculation is performed assuming the following data:

- For the sake of the calculation, fine particles (known also as PM for Particulate Matter) are considered to have a density ρ of 2000 kg/m^3 (this is typical soot density but could be a little bit different for another material, which will not alter the conclusions).
- Their diameter in the calculation will be taken equal to three successive values i.e. 10, 2 and 0.2 μm.
- The mass per unit volume of fine particles in the air is 40 μg/m^3
- The air temperature T = 290 K (17°C).

It should be noted that this mass per unit volume of fine particles is what is retained in the standards (not the number of particles/m^3) and what is usually measured. Obviously, the choice of a size of the particles will then considerably influence their calculated concentration per unit volume. Usually standard by mass/volume is given per size of particles, i.e. the allowable total

mass of particles below a certain size. Here we use the maximum mass concentration of PM10 as given by the European directive 2008/50/CE.

From the above data, supposing that the particles are mono-disperse and in the assumption of a "gas of particles" the number of particles per unit volume n (particles/m^3) can be calculated together with the following quantities:

- Their cross section $\sigma=\pi d^2/4$ (m^2)
- The mass of a particle ($m=4\rho\pi r^3/3$, $r=d/2$)
- A thermal agitation speed ($v = \sqrt{8 \times k_B \times T/\pi \times m}$ with k_B, Boltzmann's constant, m particle mass)
- The agglomeration reaction rate coefficient in m^3/s, *i.e.* $k_{agg} = \sigma \times v$
- The characteristic reaction time for agglomeration ($\tau = 2/k_{agg} \times n$) seconds)

From all these parameters, the most significant is the characteristic time τ

The presence of air will change the results somewhat but not by an order of magnitude. The input parameter that affects the results most is the size retained for the particles (through the derived number concentration from the adopted mass concentration).

The calculation of k_{agg} and hence τ is an order of magnitude in agreement with more sophisticated calculations that can be found in the literature, taking into account the influence of the air, Brownian motion, Van der Waals forces and the Knudsen number. Such a refined calculation is shown in figure F-S4-1.

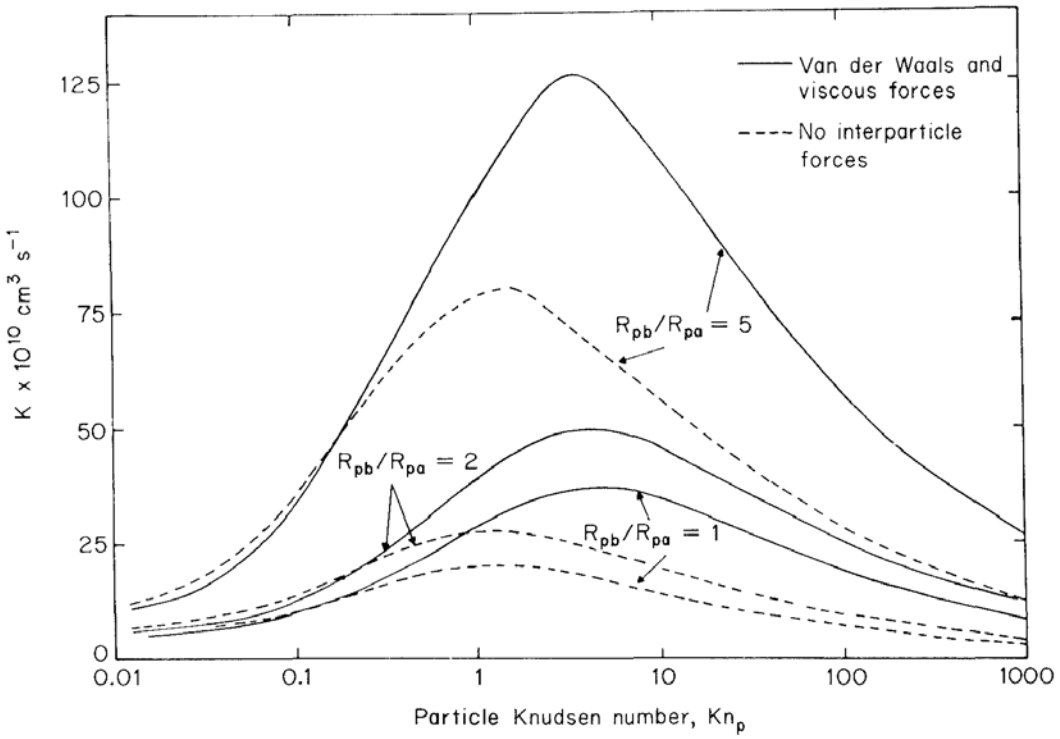

Figure F-S4-1: Variation of the agglomeration coefficient as a function of the Knudsen number for different assumptions (from reference (Fuchs, 1989).

Thus, with the input parameters given above, the characteristic time for coagulation is very long. If the size of the given particles (e.g., soot) is 10 microns, it is on the order of a century, for 2 microns, on the order of one year and for 0.2 microns (200 nm), about 2 days. This must be compared with the lifetime of a virus in an aerosol particle (dry or wet nuclei) which is much shorter for most situations.

These order of magnitude calculations are for a mono-disperse (single size) aerosol and a concentration equal to the maximum mass concentration of PM10 as given by the European

directive 2008/50/CE. It should be noted that exceeding reasonably this standard value does not change the conclusion, which is in agreement with the discussion made by other researchers (Doussin, 2020).

S5 - Atmospheric considerations.

The wind.

The meteorological wind is normally measured ten meters above ground surface. Usually, its horizontal component is much higher than the vertical one. Close to the ground its variation follows a logarithmic law which includes a variety of parameters such as the roughness length z_0, the friction velocity V_∞^* and displacement height D. z_0 depends on the nature of the surface and is usually much lower than ten meters for open lands with few barriers such as trees or buildings. In this case the wind around 2 m height is quite close (within a factor of 2) to the 10 m wind. D is grossly a displacement of the ground surface, for example in the case of a forest.

It is beyond the scope of the main paper and of this supplementary material to examine the variety of situations that can be found in real circumstances. Therefore, wind values refer always to the meteorological ones and calculations does not consider the complexity linked to tall buildings for example.

Atmospheric stability.

Vertical dispersion is strongly due to convective instabilities induced by the thermal profile of the atmosphere. Let us recall the argument. If a volume of air at altitude *z* is displaced adiabatically by an amount δz, a hydrostatic change of pressure is induced with altitude. This yields a change of temperature with altitude which can be evaluated by thermodynamics to be $\delta T = -\Gamma \times \delta z$ with:

(S5-1) $$\Gamma = \frac{g}{c_p}$$

Where g is the acceleration of gravity and c_p the specific heat of air at constant pressure. The quantity Γ is known as the dry adiabatic lapse rate. If this change of temperature δT is less than the real decrease of temperature in the atmosphere, the displaced volume is denser and will return by buoyancy to its original place: the atmosphere is stable. Otherwise, the displacement will be amplified: there is a lot of up and down movements tending to restore the mechanical stability. In this situation of instability, pollutants are also moved up and down, amplifying the length of dispersion σ_z.

Common atmospheric conditions have been classified (the so-called Pasquil– Gifford– Turner scheme, (Pasquill, 1961; Gifford, 1961; Turner, 1994) from A to F, running from very unstable conditions to very stable ones; D is the neutral case. They are shown in table T-5-1 and correspond to figure F-S5-1.

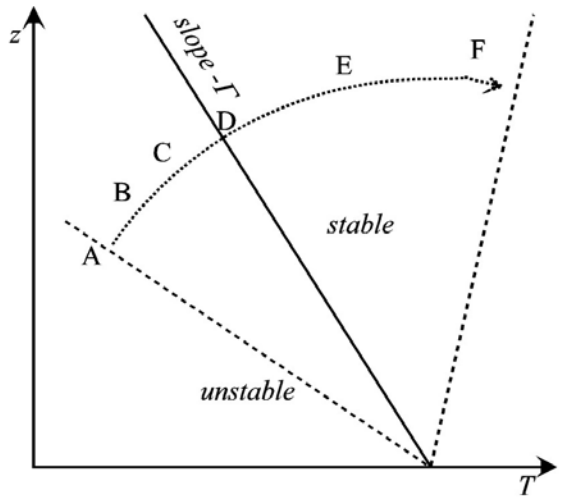

Figure F-S5-1: atmospheric stability following the temperature lapse rate. D is the neutral case.

V_∞ (m/s) At 10 m	Daytime , solar insulation				Nighttime, cloudiness		
	strong	moderate	slight	overcast	overcast	Thin overcast	clear
0-2	A	A-B	B	D	D	E	F
2-3	A-B	B	C	D	D	E	F
3-5	B	B-C	C	D	D	D	E
5-6	C	C-D	D	D	D	D	D
6-	C	D	D	D	D	D	D

Table T5-1: The Pasquil-Giffort-Turner class of atmospheric stability following meteorological conditions. Neutral (D) conditions prevail nights and days for overcast conditions.

Explanations of this table are rather simple. During the day, sun exposure will heat up the ground, leading to higher temperatures near it and therefore a strong decrease of temperature with altitude; hence $dT/dz \ll -\Gamma$ with $dT/dz < 0$ leading to high instability, *i.e. a* large characteristic length of dispersion. Conversely, in a clear night, heat is released by the ground without being trapped by cloud cover, leading to high stability. The influence of the wind speed V_∞ is explained by stirring the atmosphere which is rapidly led to recover the equilibrium thermal profile. Note that a situation of inversion can occur within the day, especially in wintertime for mid latitudes where the sun is low in the sky and supplies less warmth to the Earth's surface. A so-called subsidence inversion, *i.e.* above the ground, can act as a lid and trap cold air at the ground. These effects can be strongly amplified in mountain valleys.

Dispersion of pollutants downwind of a stack.

Downwind of a stack there is a stream of polluted air in the form either of puffs or of a plume. Plumes are classified in a variety of ways, depending on meteorological conditions, as shown in figure F-S5-2.

Assuming a quantitatively stochastic random walk leads to a Gaussian distribution downstream of a stack which has been used widely in the field and is appropriate for a number of situations. Description of spatial concentration of pollutants takes the origin at the bottom of the stack, x the distance downwind of the stack, y and z respectively lateral and vertical coordinates. Lateral and vertical dispersion lengths σ_y and σ_z are introduced. They are different since they are produced by different causes. With several sources, lateral dispersion helps to mix and homogenize the particle concentration and is not important for our purpose. We are interested in the vertical

dispersion which is strongly affected by the stability of the atmosphere which has been discussed previously.

Assuming a height z_c for the center of the pollution cloud, the polluting concentration at a distance *x* from the source can be expressed as (Turner, 1994):

(S5-2) $$c(x,y,z) = \frac{Q}{2\pi \times V_\infty \times \sigma_y \times \sigma_z} \times \exp\left(-\frac{y^2}{2\times {\sigma_y}^2}\right) \times \left[\exp\left(-\frac{(z-z_c)^2}{2\times {\sigma_z}^2}\right) + \exp\left(-\frac{(z+z_c)^2}{2\times {\sigma_z}^2}\right)\right]$$

Now the problem is to evaluate σ_y, σ_z as a function of x, and especially σ_z which is the major parameter for an evaluation of H. z_c is the center of the plume and Q the source term in unit (particles, mass or others) per second consistent with the concentration unit.

The vertical dispersion will depend strongly on the conditions. In the neutral case D, pollutants are emitted in a cone ("coning" in the figure below), due to similarity considerations. When the stability increases (F), the cone angle shrinks, showing better stability (parcels of deviant air are more strongly pushed back towards their original position). This corresponds for example to the case of "fanning" in the figure. Conversely, in unstable conditions, fluctuations of the transporting air appear, first in large fluctuations (looping in the figure), then with more and more turbulence leading to a large dispersion and cone (not shown in the above figure).

In a first estimation, it can be expected that σ_y and σ_z are proportional to *x* which means that the polluting plume is conical. This is due to a dimensional analysis since *x* seems to be the only fundamental length in the problem. Also, the wind induces turbulence and, in a second approach, the relation between the dispersions and *x* is no longer linear. The following curves (Turner, 1994) show this dependency (figure F-S5-3), but are focused over a large *x* (from 100 m to 20 km) since the paper was written for the description of industrial pollution. However, they are based on outdoor experiments conducted for a distance comprised between 50 and 800 meters ((Barad, 1958; Gifford, 1961; Turner, 1994) and references therein).

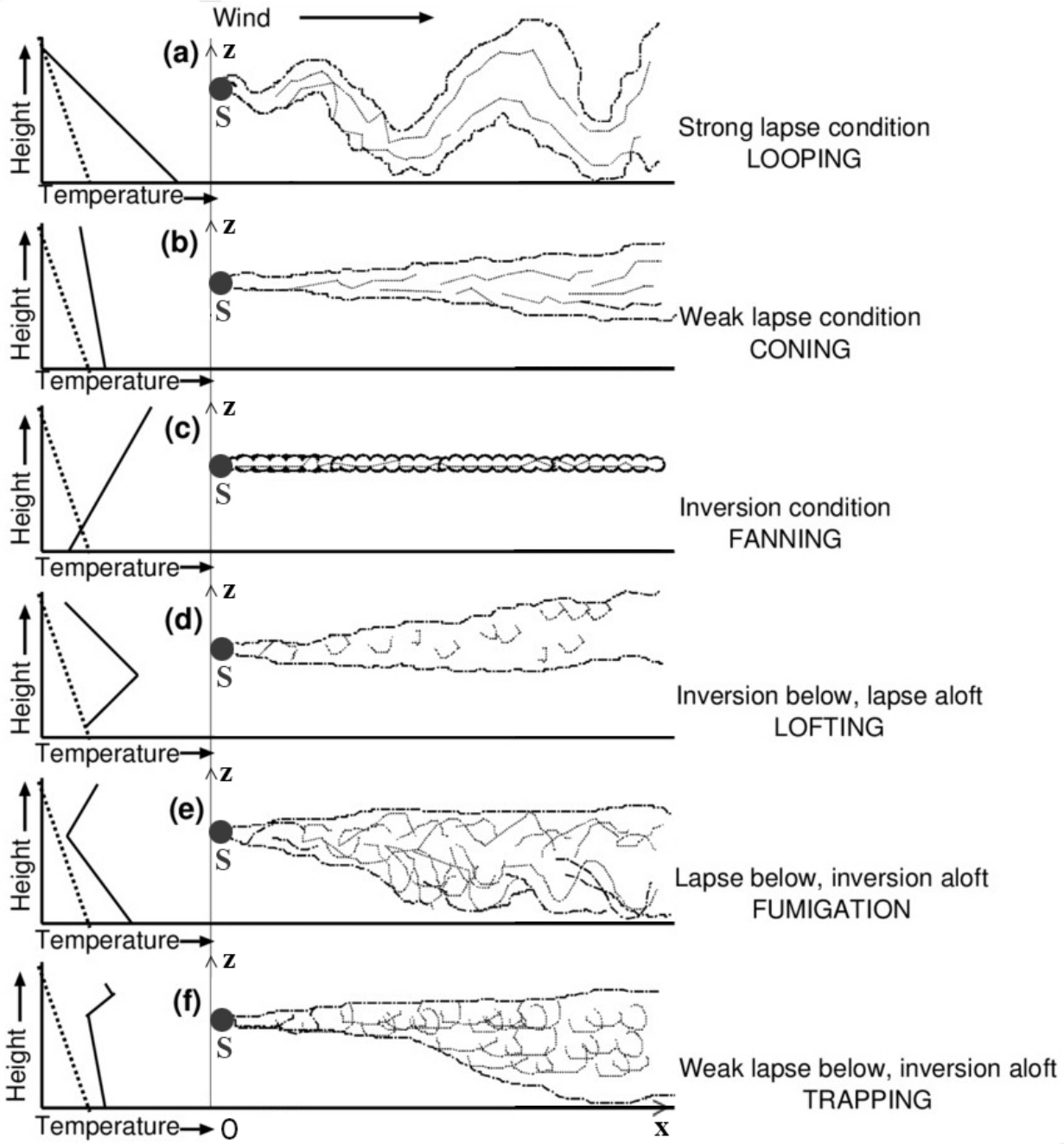

Figure F-S5-2: different kind of plumes following atmospheric conditions, especially lapse rate. The dotted line for the temperature is the dry adiabatic lapse rate, letters in minuscule do not refer to the stability although (a) is equivalent to A very unstable conditions. (b) refers to moderately stable or unstable conditions, cases from (c) to (d) refer to inversion, either from the ground (radiation inversion) or aloft.

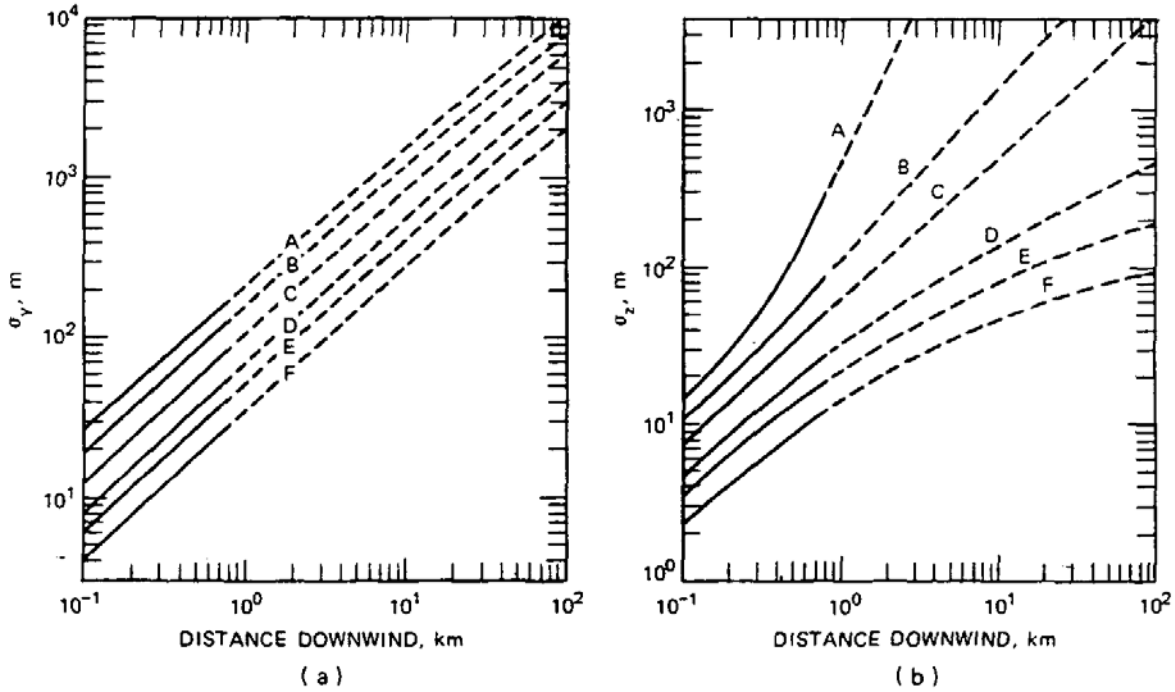

Figure F-S5-3: horizontal and vertical dispersion length for Gaussian plumes (Turner, 1994).

In any case, at short distances, it appears that the experimental measurements reveal a quasi-linear dependence. For small *x* (20 m to 1 km), σ_y and σ_z can be approximated as linear in *x* and the extrapolation of the above curves yields the following table:

	A	B	C	D	E	F
σ_y/x	0.276	0.199	0.136	0.085	0.065	0.042
σ_z/x	0.2	0.097	0.074	0.051	0.034	0.023

Table T5-2: linear variation of dispersion lengths with distance downwind.

Note that σ_y, essentially due to the fluctuations of the wind direction, is greater than σ_z but as stated above we are mainly interested in σ_z. Taking a value of the height of dispersion H equal to σ_z clearly minimizes its real value and therefore, doing so leads to an over-estimation of the outdoor risk which goes clearly in the direction of the main conclusions of the main paper.

However, figure F-S5-3 given above and widely used in the literature does not cover all the situations that a plume can encounter. They are adapted to the “coning” case but not for vertical dispersion in special cases referenced as “fanning”, “fumigation” or “trapping” which can occur in cases of extremely stable inversion. Lateral dispersion may always be described by a Gaussian but in these cases there is hardly a vertical dispersion above the height of the inversion, if any, and its prediction is exceedingly difficult. In these cases, we will assume following Turner (Turner, 1994) that dispersion does not occur above this height. We choose a constant value for H equal to z_e, the mean height of an emitter, which again clearly maximizes the evaluated risk outdoor.

Buoyancy effects.

An infector expels polluted air at a temperature near 32 °C while the external temperature may be lower, say 15 °C. The puff (cough or sneeze) or the plume (speaking, breathing, or singing) are at the very beginning turbulent, emitted in a cone with an apex angle about 24°. Part of the droplets are ejected out of the plume by falling or turbulences. The smallest few ones, too small for falling on the ground, are carried away by the wind in a secondary horizontal plume; the largest droplets which have not enough time to evaporate, deposit on the ground. The remaining plume is raised

by buoyancy, but this is less and less efficient because it cools down due to external air mixing. Figure displays this behavior.

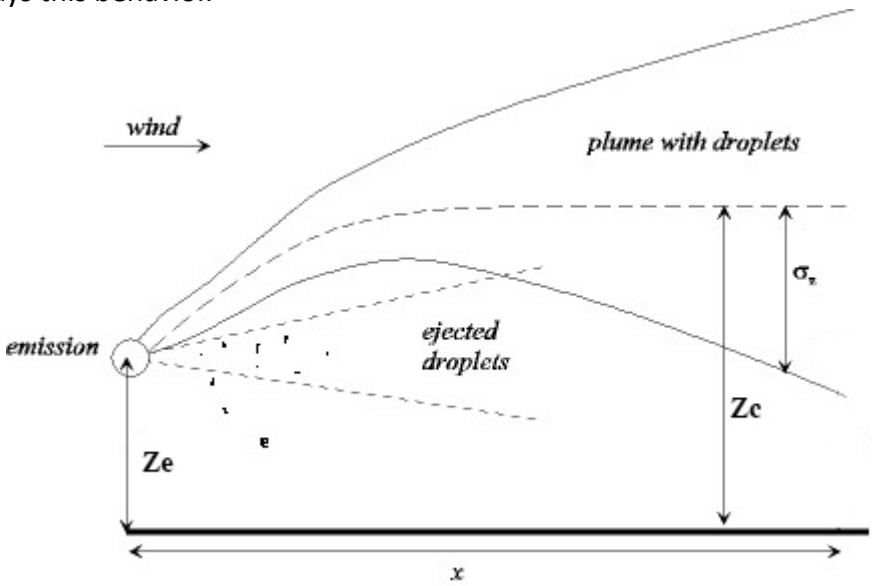

Figure F-S5-4: overview of a plume with buoyancy.

In a stable atmosphere the vertical dispersion remains small and outdoor the plume will eventually rise sufficiently high to pass over an exposed person. This is not the case for the secondary plume of small, ejected particles. Indoor buoyancy can bring the aerosol particles to the inlet of recirculated air.

Note therefore that the phenomena of buoyancy, which has not been taken into account in the main paper, can increase the probability of being infected indoor while most often reducing it outdoor.

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