



**HAL**  
open science

# Parameter determination of the Compressible Packing Model (CPM) for concrete application

Mokrane Bala, Rachid Zentar, Pascal Boustingorry

## ► To cite this version:

Mokrane Bala, Rachid Zentar, Pascal Boustingorry. Parameter determination of the Compressible Packing Model (CPM) for concrete application. *Powder Technology*, 2020, 367, pp.56-66. 10.1016/j.powtec.2019.11.085 . hal-03224977

**HAL Id: hal-03224977**

**<https://hal.science/hal-03224977>**

Submitted on 24 Nov 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# 1 **Parameter determination of the Compressible Packing Model (CPM)**

## 2 **for concrete application**

3 **Mokrane BALA <sup>a,\*</sup>, Rachid ZENTAR <sup>a</sup>, Pascal BOUSTINGORRY <sup>b</sup>**

4 <sup>a</sup> Institut Mines Telecom Lille-Douai, LGCgE-GCE, 764 bd Lahure, BP 10838, 59508 Douai, France

5

6 <sup>b</sup> CHRYSO France, 7 rue de l'Europe - Z.I. - 45300 Sermaises du Loiret, France

7

8

### 9 **Abstract**

10 The packing density of granular materials is in the centre of interest of many industries. For concrete  
11 industry, the packing density of aggregates is the most representative physical parameter of the  
12 granular mixtures: it combines the grain size and the morphology of grains. Compressible Packing  
13 Model (CPM) accurately predicts packing density by involving three parameters: wall effect coefficient,  
14 loosening effect coefficient and compaction index. The identification of these parameters has been  
15 made on elementary granular classes (narrow particle size distributions) which can be time consuming  
16 and difficult to achieve. In this research work, we have elaborate a strategy to determine CPM  
17 parameters for wide particle size distribution of granular mixtures. This allows reducing the number of  
18 tests to optimize packing density of granular mixtures. The results of the modelling show a good  
19 agreement with the experiment. The work will be undertaken on two typologies of grains: crushed and  
20 rolled aggregates.

21

22

23

24

---

\* Corresponding author.  
Tel: +33 327 712 214  
E-mail address: mokrane.bala@imt-lille-douai.fr

**1 Keywords**

2 Packing density

3 Interaction coefficients

4 Compaction index

5 Crushed aggregates

6 Rolled aggregates

7

8

9

10

11

**12 1. Introduction**

13 In the field of civil engineering, concrete is an indispensable raw material which often offering the best  
14 technical and economic choice for the construction of buildings and infrastructures. Aggregates in  
15 concrete reach up to 80% of the total volume and come in different sizes and shapes. The remaining  
16 volume is occupied by a cement paste. Aggregates can arise from natural, artificial or recycled  
17 (industrial by-product and demolition source materials). The most important property in the granular  
18 mixture of concrete is its packing density. Packing density optimization for the granular mixtures allows  
19 not only minimizing the quantity of cement incorporated in the concrete but also enhances its  
20 performance and durability [1], [2].

21

22 In conventional concrete mix design methods, granular mixtures are determined empirically, often from  
23 particle size distribution curves (ideal distribution curves), as for the works of Fuller & Thompson  
24 (1907), Andreasen & Andersen (1929), Faury (1958) and Dreux (1970) [3], [4]. These methods make it  
25 possible to determine the ideal proportions of each grain size to approach the maximum packing  
26 density of the mix, but do not allow to predict accurately the packing density [3] and may require  
27 several series of experiments for the optimization of the granular mix [2], [5].

28

1 With the emergence of modern concretes and special concretes in the 20th century (such as High  
 2 Performance Concretes (HPC), Self-Compacting Concrete (SCC)...), concrete mix design by "ideal"  
 3 particle size distribution curves proved to be difficult or unsuccessful [2], [5]. For these concretes,  
 4 several objectives are aimed at the same time: obtaining high workability, without risk of segregation  
 5 and good mechanical properties. Therefore, a wide variety of distributions are possible and there is no  
 6 ideal curve that suits all requirements [3].

7

8 In order to appear to these difficulties, several models have been developed to predict the packing  
 9 density of a granular mixture. Modelling of packing density was developed first by highlighting the  
 10 interaction effects between grains of different sizes, in particular by the work of Caquot (1937), which  
 11 shows the major influence of the wall effect on granular mixtures [3]. It has prompted researchers to  
 12 study granular classes in pairs. Inspired by Mooney viscosity model (1950), Stovall studied binary  
 13 mixtures, with and without interaction. He has developed the linear packing model [6] for granular  
 14 mixtures with multiple classes, taking into account both the wall effects and the loosening effect  
 15 between the granular classes of different sizes. The loosening effect appears when a small grain is  
 16 inserted in a dominant large grains population and the wall effect appears when some quantities of  
 17 large grains isolated are immersed in fine grains agglomerate [7].

18

19 The linear packing model was refined to build the virtual packing model that predicted virtual packing  
 20 density (i.e. an orderly packing of grains with the least voids), including mixtures of grains of the same  
 21 size but of different shapes. The virtual packing density for a mixture of "n" granular fractions where  
 22 the class (i) is dominant, is given by Equation 1 [5]. It involves the volume proportions of each of the  
 23 granular classes ( $y_i$ ), their packing density when they are arranged separately ( $\beta_i$ ), the loosening effect  
 24 coefficient ( $a_{ij}$ ) given by Equation 2 and the wall effect coefficient ( $b_{ji}$ ) given by Equation 3 [5]. "e" is  
 25 the void ratio of the granular mixture defined as the ratio of void and solid fraction (given by Equation  
 26 4,  $\emptyset$  is the solid fraction).

27

28

$$\gamma = \gamma_i = \frac{\beta_i}{1 - \sum_{j=1}^{i-1} y_j \left( 1 - \beta_i + \beta_i b_{ji} \left( 1 - \frac{1}{\beta_j} \right) \right) - \sum_{j=i+1}^n y_j \left( 1 - a_{ij} \frac{\beta_i}{\beta_j} \right)} \quad \text{Equation 1}$$

$$1 \quad a_{ij} = \beta_j \left( \left| \frac{\partial e}{\partial y_j} \right|_{y_j=0} + \frac{1}{\beta_i} \right) \quad \text{Equation 2}$$

$$2 \quad b_{ji} = \frac{\frac{1}{\beta_j} - 1 - \left| \frac{\partial e}{\partial y_j} \right|_{y_j=1}}{\frac{1}{\beta_i} - 1} \quad \text{Equation 3}$$

$$3 \quad e = \frac{1-\emptyset}{\emptyset} \quad \text{Equation 4}$$

4

5 A first attempt to predict the packing density by the Solid Suspension Model [8], [9] was limited by two  
6 defects related to the notion of reference viscosity (considered as a description of the degree of  
7 compaction of the system) and the interaction functions that were not satisfactory in their mathematical  
8 form [10]. This compaction energy was introduced later in CPM model by the concept of the  
9 compaction index "K". This model makes it possible to predict the real packing density of a mixture of  
10 several granular classes from the knowledge of the compactness of each one-dimensional class and  
11 the energy of the setting up. The real packing density depends on the compaction energy. The real  
12 packing density "C" of a mixture of aggregates is connected to "K" by the expression given in Equation  
13 5. Moreover, simplified formulas (Equation 6 and Equation 7) of the granular interaction coefficients (a  
14 and b) was proposed by de Larrard [5] after calibration of the CPM on different series of experimental  
15 data.

16

$$17 \quad K = \sum_{i=1}^n K_i = \sum_{i=1}^n \frac{y_i / \beta_i}{\frac{1}{c} \frac{1}{\gamma_i}} \quad \text{Equation 5}$$

$$18 \quad a_{i,j} = \sqrt{1 - \left(1 - \frac{d_j}{d_i}\right)^{1,02}} \quad \text{Equation 6}$$

$$19 \quad b_{j,i} = 1 - \left(1 - \frac{d_i}{d_j}\right)^{1,5} \quad \text{Equation 7}$$

20

21 In the framework of the CPM, the prediction of the packing density is therefore possible after  
22 determination of the coefficients a, b (by the calculation of the void index at the limits of coarse grains  
23 dominant and fine grains dominant) and K. These parameters are calibrated on binary mixtures of  
24 aggregates of different natures (crushed and rolled) [5], [11]. The error between measured and

1 calculated packing densities after calibration does not exceed 0.77% for rolled aggregates and 1.71%  
2 for crushed aggregates [5].

3

4 In order to improve the prediction of the packing density, several models and calculation approaches  
5 have been proposed [12]–[18]. Some authors introduced new interaction parameters [15], [17].  
6 Roquier [17] introduced the interference effect that occurs when coarse grains, in a growing number,  
7 become too close to each other by trapping some fine grains in small spaces between them [3], [17].  
8 However, despite these various attempts to improve the packing density prediction, the CPM remains  
9 among the most accurate models and the most simplest to apply [19], [20]. Moutassem [19] had  
10 compared 9 packing density models and he found that the CPM predict correctly the packing density  
11 of granular mixtures used in concrete.

12

13 Other research works has focused on the CPM parameters in order to propose optimized values for  
14  $a_{ij}$ ,  $b_{ji}$  and  $K$  [11], [20]. The work of Lecomte [11] has slightly improved the calculation of the interaction  
15 coefficients through new simplified formulas of calculation. Indeed, the simplified formulas of the  
16 interaction coefficients are probably valid only for one type of aggregates which justifies the  
17 differences between the formulas proposed by de Larrard [2] and by Lecomte [11]. In fact, for the  
18 CPM, the functions of  $a_{ij}$  and  $b_{ji}$  have been calibrated on elementary granular classes ( $d$  and  $D$  are the  
19 minimum and maximum grain sizes respectively) while respecting on the one hand a ratio  $d/D_i > 0.1$  (so  
20 that the elementary granular classes are the more unimodal possible) [2], [5] and on the other hand a  
21 ratio  $d_j/d_i < 4$  [2], [5]. In the case of granular mixtures optimization for concrete, this work is long and  
22 expensive to achieve because it requires a long process of material preparation and sieving  
23 elementary granular classes.

24

25 In the present paper we will calibrate the CPM parameters in the case of large granular classes with  
26  $d_i/D_i < 0.1$  (especially for sand) and for some ratios  $d_j/d_i > 4$ . This study does not require any sieving work  
27 to obtain elementary granular classes that must have narrower particle size distributions. Our goal is to  
28 save time and materials in packing density studies of granular mixtures for concrete without affecting  
29 the accuracy of the model. First, we will apply the CPM as developed by de Larrard in order to qualify

1 the prediction of the model and determine the most influential parameters. After that, we will optimize  
2 the parameters of the CPM for wide particle size distributions of granular mixtures of crushed and  
3 rolled aggregates. The coefficients  $a$ ,  $b$  and  $K$  will be calibrated on binary mixtures. Finally, we will  
4 evaluate the accuracy of the model applied to large granular classes of binary and ternary mixtures  
5 compared to previous studies.

6

## 7 **2. Materials and methods**

8 In Table 1 are listed the aggregates of the present study and few characteristics. The size, shape and  
9 roughness of the grains are the three main parameters that affect the packing density [3], [5], [21] that  
10 has influenced our choice. Rolled aggregates come from “Chevrières” quarry (“Hauts-de-France”  
11 region) and “Decize” quarry (“Bourgogne-Franche-Comté” region). The crushed aggregates come  
12 from the quarries of “Boulonnais” (“Nord Pas-de-Calais” region). The granular fractions adopted for  
13 this study range from sand (0/4 mm) to gravel (4/10 and 10/20 mm). The absolute density of the  
14 aggregates was measured according to the European standard NF EN 1097-6 [22] and the particle  
15 size analysis was carried out according to the European standard NF EN 933-1 [23].

16 In order to measure the packing density of the mono-granular classes, the LPC procedure No. 61 was  
17 followed [24]. After weighing 7 kg of one mono-granular material, the sample is placed in a cylindrical  
18 mould in three equivalent layers. Each layer is subjected to 20 shocks in the shaking table before  
19 putting the next layer. The sample is then subjected to 40 shocks under a pressure of 10 kPa  
20 (equivalent weight of 20 kg can be placed above the sample). The energy transmitted by the shocks  
21 allows the granular material to be into a more dense configuration, which improves its packing density.  
22 The compaction index ( $K$ ) of this process was calibrated after several tests. Using Equation 5,  $K = 9$   
23 was found [24].

24

25 The packing densities were measured on binary granular mixtures of crushed and rolled aggregates to  
26 determine the interaction coefficients for each type of aggregates [2], [11]. Different combinations  
27 between sand and gravels were analysed. As shown in Table 2, considering the maximum nominal  
28 diameter of each class, the effect of the size ratio  $D_1/D_2$  and the grain shape on the packing density  
29 was experienced. Otherwise, the variation of the proportions of the smaller granular class in the binary

1 mixture equal to 5% in the extreme parts of the curve of packing density and to 10% in the centre of  
2 the curve. This is in order to measure the impact of the granular interaction effects that have defined  
3 previously (wall effect and loosening effect coefficients). Finally, packing density of ternary granular  
4 mixtures was measured for crushed and rolled aggregates. This is in order to evaluate the efficiency of  
5 the interaction parameters determined on the binary mixtures. The measurements were realized by  
6 increment of 10%. The different combinations of the experimental program on ternary mixtures are  
7 shown in Fig. 1.

8

### 9 **3. Results and discussions**

10 The main properties of the aggregates used in this study are listed in Table 3. Otherwise, the particle  
11 size distribution of the different materials are shown in Fig. 2. Particle size analysis showed a high  
12 fines content in crushed sand (5% of grains are smaller than 63 microns and 16% smaller than 125  
13 microns).

14

15 Packing densities measurement tests were realised using the compaction table according to LPC  
16 procedure No. 61 [24] on binary and ternary mixtures of crushed and rolled aggregates. The different  
17 materials are used in their raw state without any prior sieving to remove the fine fraction. For each test,  
18 two different measurements of compactness were made. In each case, the average packing density  
19 and the variances are calculated. It has been observed through the experimental program that the  
20 repeatability of the tests is ensured [25] (maximum standard deviation recorded is 0.017 for crushed  
21 aggregates and 0.008 for rolled aggregates). The use of the aggregates in their raw state without any  
22 prior sieving (fine particles content up to 7%) did not influence the packing density measurements by  
23 compaction table [25].

24

25 The evolution curves of the packing density measured by the compaction table as a function of the  
26 proportion of the addition of fine material are shown in Fig. 3. From the curves in Fig. 3, it is observed  
27 a rapid increase of the packing density when the sizes ratio of the mixed materials is large ( $D_1/D_2 = 5$ ).  
28 The packing density curves exhibits an optimum before decreasing when wall effects occur in the area  
29 where the small grains material becomes dominant. Even for smaller ratio ( $D_1/D_2=2.5$ ), the evolution of



1 the packing density still important (Fig. 3-b). For  $D_1/D_2=2$ , the packing density curve evolution is almost  
2 nil (Fig. 3-c). These results show that the packing density optimum of the binary mixtures increases  
3 when  $D_1/D_2$  ratio increases from 2 up to 5. This is in agreement with the results found by de Larrard  
4 [2], [5] and McGeary [26]. Finally, the size ratio  $D_1/D_2=2$  seems to define the limit for materials which  
5 are in total interaction [2], [5].

6

7 Despite their low fines content, the rolled aggregates (Fig. 3. right) reaches the highest packing  
8 density in comparison to crushed aggregates. The packing of rolled aggregates is composed of grains  
9 of various sizes (wide particle size distribution) that their shape is close to the sphere. This packing  
10 approaches the Apollonian packing (packing of circles) which gives the highest packing density [27],  
11 [28]. The high fines content (particle content <63 microns) in the crushed aggregates (Fig. 3. left) was  
12 not able to counterbalance the shape effects in terms of packing density (Fig. 3a and Fig. 3b).

13

14 From these results, we deduce that the maximum packing density that can be achieved on binary  
15 mixtures is influenced by the combination of the following parameters (regardless of the compaction  
16 index): the packing density of the elementary granular classes, the shape and roughness of the grains  
17 as well as the fines content and the size ratio. The packing density of binary mixtures can be  
18 increased by improving the packing density of the elementary classes as found by de Larrard [2], [5].  
19 Similarly, some authors have shown that the packing density increases when the size ratio of  
20 elementary classes increases [2], [17], [29]. The effect of the presence of ultra-fine particles (powders)  
21 can also be related to the large size ratio between the powders and coarse grains.

22

23 For the ternary mixtures (of crushed aggregates) studied in this research work, the results are shown  
24 in Fig. 4. The packing density measured decreases in the area of low sand content (dark area) and  
25 increases with the introduction of sand (white area). The presence of intermediate gravel (G 4/10) with  
26 the coarser gravel (G 10/20) disturbs the granular mixture because of the observed interaction  
27 between these materials as seen in Fig. 3-c.

28

1 The evolution curves of the experimental packing density for binary and ternary mixtures allow to  
2 proceed to the modelling of these packing densities by the CPM in which we will analyse the different  
3 parameters of the model which are: the coefficients of granular interactions ( $a_{ij}$  and  $b_{ji}$ ) and the  
4 compaction index ( $K$ ). This work will be undertaken for large granular classes.

5

6 In order to determine the packing densities by the CPM, the virtual packing densities are calculated  
7 using Equation 1 where the coefficient of loosening effect ( $a_{ij}$ ) and wall effect ( $b_{ji}$ ) have to be  
8 determined. In the first method, these parameters are calculated using the simplified formulas  
9 (Equation 6 and Equation 7) proposed by de Larard [5]. The real packing density of the mixtures is  
10 subsequently calculated through the compaction index which is taken equal to 9 [24]. In the second  
11 method and for comparison, the parameters  $a_{ij}$  and  $b_{ji}$  are determined on the experimental curves and  
12 the compaction index value is maintained equal to 9 as in the first method.

13

14 The results in terms of experimental data in comparison to the model prediction following the first  
15 method are shown in Fig. 5.

16

17 In this case the CPM allows to predict experimental packing densities with an average error of 3.2%  
18 for crushed aggregates and 1.7% for rolled aggregates. We note through the curves shown in Fig. 5  
19 that the largest differences are found in the area of optimal packing density. In this zone, the packing  
20 density is overestimated by the CPM for crushed aggregate mixtures and underestimated in the case  
21 of rolled aggregate mixtures. This leads to say that the efficiency of the compaction mode by the  
22 compaction table is influenced not only by the particle size as found by Sadok et al. [16], [30], but also  
23 by the shape of the grains. The parameters  $a$ ,  $b$  and  $K$  used in this first modelling are given in Table 4.

24

25 In the previous studies performed by de Larrard, an average error of 1.71% for crushed aggregates  
26 and 0.77% for rolled aggregates is found [2], [5].

27

28 In the second method, the parameters of the CPM are determined from experimental data. According  
29 to the basic formulas of interaction coefficients (Equation 2 and Equation 3), it could be seen that the

1 slope of the experimental data in terms of void ratio (given by Equation 4) versus the percentage of  
2 fine material at the origin of the curve and at the extremity of the curve are respectively proportional to  
3 the parameters  $a_{ij}$  and  $b_{ji}$ . These latter are calculated using Equation 6 and Equation 7 after calculating  
4  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  by Equation 5 ( $K$  is taken equal to 9) and evaluating graphically the slope of the curve  
5 evolution of the void ratio as a function of the percentage of fine grains. The parameter determination  
6 for binary mixtures is highlighted in Fig. 6.

7

8 As it could be seen in Fig. 6, due to the discrepancy of the data, for some curves it is difficult to define  
9 the slope at the origin or at the extremity of the curve. In this order, the slope is evaluated on 25% of  
10 the curve (at the origin and at the extremity) in area considered as linear part. The values of the  
11 coefficients  $a_{ij}$  and  $b_{ji}$  determined (given in Table 5) were compared to those obtained by de Larrard  
12 [2], [5] and Lecomte [11] as shown on Fig. 7. The results of the modelling are presented in Fig. 8.

13

14 As shown in Fig. 7, the coefficients  $a_{ij}$  and  $b_{ji}$  obtained on large granular classes follow the same  
15 evolution as those obtained by de Larrard [2], [5] and Lecomte [11]. They increase when the ratio  
16  $D_2/D_1$  increases. This shows that the granular interactions are stronger when the granular classes are  
17 of the same size (until total interaction as shown in Fig. 3-c). However, Fig. 7 shows some differences  
18 between the interaction coefficients determined experimentally and the functions of the simplified  
19 formulas proposed by de Larrard or Lecomte. This allows to say that there is no unique function  
20 which makes it possible to calculate the interaction coefficients accurately for different type of  
21 aggregates and without causing errors in the prediction of the packing density. Moreover, when we  
22 analyse in more details the obtained results from our experimental data we can point out that the  
23 values of  $a_{ij}$  measured on rolled aggregates are systematically lower than those measured on crushed  
24 aggregates. This result could be observed for most data published by de Larard and is in line with the  
25 published paper of Roquier [18]. In this later, the estimates of the loosening effect parameter  
26 calculated from two different models (4- parameter CPM as 3PPM) show clearly that the values  
27 calculated on rolled aggregates are lower than those calculated for crushed aggregates. It is also  
28 interesting to note that for small diameter ratios ( $D_2/D_1$ ), the loosening effect parameter evolves very  
29 quickly with increasing the diameter ratios.

1 For the wall effects parameter  $b_{ji}$ , in terms of aggregates shape effects, the same conclusions than  
2 those addressed for the loosening effect parameter could be drawn. The measured parameters values  
3 on rolled aggregates are systematically lower than those measured on crushed aggregates. This is  
4 also in line with most published data from de Larrad and other researchers.

5 The comparisons of absolute values of the wall effect parameter seems to be systematically lower than  
6 the values of the loosening effect parameter in the whole range of the diameter ratios.

7 Finally the comparison of the measured values for both parameters with the calculated values using  
8 the proposed relationships by de Larrard [2], [5] and Lecomte [11] shows that a big differences could  
9 be induced.

10

11 Comparisons between the predictions using CPM with identified  $a_{ij}$  and  $b_{ji}$  on experimental data and  
12 experimental results (Fig. 8) show better agreement than in the first method (modelling through  
13 simplified formulas of  $a_{ij}$  and  $b_{ji}$ ). The difference between the model prediction and the experimental  
14 data decreases to 2.0% for crushed aggregates and 1.1% for rolled aggregates. These errors are  
15 similar to those found in the work by de Larrard [2], [5]. From these results, we can deduce that  
16 identification on elementary granular classes obtained by sieving is unnecessary to achieve the best  
17 performances of the CPM. However, it seems necessary to define interaction coefficients on  
18 experimental data.

19

20 In order to assess the effect of the compaction index ( $K$ ) on the prediction of the CPM, in the last  
21 section of this study a back calculation of  $K$  on experimental data is undertaken. In this procedure, the  
22 difference between the model prediction and the experimental data is decreased by changing the  
23 value of  $K$ . The parameters  $a_{ij}$  and  $b_{ji}$  are kept as defined in the second method. The obtained results  
24 in terms of compaction index and errors between experimental data and prediction of the model is  
25 summarised in Table 6 and Fig. 9.

26

27 The compaction index obtained after calibration is lower than 9 (especially for crushed aggregates).  
28 Different values of the compaction index are obtained (given in Table 6) depending on the granular  
29 mixture. The compaction index depends to the experimental process, other values of  $K$  was recorded

1 with others experimental processes [31], [32]. But for compaction mode by shaking table,  $K$  must be  
2 constant because the energy used was constant [7]. We conclude that for crushed aggregates the  
3 compaction by the shaking table is not effective [25], [33]. This is due to their high fines content [5],  
4 [30]. With regard to the rolled aggregates, the compaction efficiency was better since we could reach a  
5 compaction index close to 9. However, the difference is not very significant in terms of packing density  
6 prediction, from which we retain the compaction index  $K = 9$  for modelling of ternary mixtures.

7  
8 In the case of ternary mixtures, the prediction of the experimental data is undertaken using CPM with  
9  $a_{ij}$  and  $b_{ji}$  as defined in method 2 on binary mixtures. The compaction index, following the observed  
10 results on the binary mixtures and the results of several authors [2], [3], [30] the result is value is fixed  
11 to 9. To ease the comparisons, the predicted results versus experiments are shown on Fig. 10.

12  
13 The errors observed between the model and the experiments for ternary mixtures is about 2.4% for  
14 crushed aggregates and 1.6% for rolled aggregates. From the results shown on Fig. 10, we note that  
15 in the majority of ternary mixtures of crushed and rolled aggregates, CPM overestimates the packing  
16 density.

#### 17 18 **4. Conclusion and perspectives**

19 In this paper, the prediction of packing density of binary and ternary mixtures of crushed and rolled  
20 aggregates was investigated. From this study, we can draw the following conclusions:

- 21
- 22 - For packing density measurements at the compaction table made with materials including the  
23 fine fraction (<63 microns), the repeatability of the tests is ensured (maximum standard  
24 deviation of 0.015 between two tests). The segregation effects of fine particles can be  
25 neglected for crushed and rolled aggregates with fine particles (less than 63 microns) content  
26 up to 7%.
  - 27 - The modelling of the packing density by the CPM have shown that the use of the granular  
28 interaction coefficients determined by the simplified formulas is insufficient to have a good  
29 prediction.

- 1        - The back calculation of the compaction index (K) may vary from one mixture to another  
2            (especially for crushed aggregates).
- 3        - It has been demonstrated in this study that the CPM can be applied to large granular classes  
4            without creating elementary subclasses. This save time and materials in packing density  
5            studies, especially for concrete mix design.
- 6        - The identification of the model coefficients following the procedure suggested improved the  
7            prediction of the CPM.

8

9        The experimental program and the modelization realized allow to improve the precision of the CPM in  
10        the determination of the packing density of granular mixtures for concrete. In the perspective of this  
11        study, the packing density will be exploited to go back to the yield stress and the compressive strength  
12        of concrete following the approaches of Chateau et al. [34] and de Larrard [2], [35] respectevly by  
13        combining the results of this study with the study of the properties of the cement paste.

14

#### 15        **Acknowledgements**

16        The authors thank the R & D department of CHRYSO France for their technical and financial support  
17        for this research work.

18

#### 19        **References**

- 20        [1] P. J. Andersen, "Particle packing and concrete properties," *Mater. Sci. Concr. II*, pp. 111–146,  
21            1995.
- 22        [2] F. de Larrard, *Concrete mixture proportioning: a scientific approach*. CRC Press, 1999.
- 23        [3] G. Roquier, "Etude de la compacité optimale des mélanges granulaires binaires: classe  
24            granulaire dominante, effet de paroi, effet de desserrement," PhD Thesis, Paris Est, 2016.
- 25        [4] S. A. Fennis and J. C. Walraven, "Using particle packing technology for sustainable concrete  
26            mixture design," *Heron 57 2012 2*, 2012.
- 27        [5] F. de Larrard, "Structures granulaires et formulation des bétons," *Etudes Rech. Lab. Ponts*  
28            *Chaussées*, vol. OA 34, p. 414 p., 2000.

- 1 [6] T. Stovall, F. de Larrard, and M. Buil, "Linear packing density model of grain mixtures," *Powder*  
2 *Technol.*, vol. 48, no. 1, pp. 1–12, Sep. 1986.
- 3 [7] M. Miraoui, R. Zentar, and N.-E. Abriak, "Road material basis in dredged sediment and basic  
4 oxygen furnace steel slag," *Constr. Build. Mater.*, vol. 30, pp. 309–319, 2012.
- 5 [8] T. Sedran, F. de Larrard, and D. Angot, "Prévision de la compacité des mélanges granulaires  
6 par le modèle de suspension solide -I- Fondements théoriques et étalonnage du modèle," *Bull.*  
7 *Liaison Lab. Ponts Chaussées*, no 194, 1994.
- 8 [9] T. Sedran, F. de Larrard, and D. Angot, "Prévision de la compacité des mélanges granulaires  
9 par le modèle de suspension solide -II- Validation, cas des mélanges confinés," *Bull. Liaison*  
10 *Lab. Ponts Chaussées*, no 194, 1994.
- 11 [10] F. de Larrard and T. Sedran, "Une nouvelle approche de la formulation des bétons," in *Annales*  
12 *du BTP*, 1999, vol. 6, pp. 39–54.
- 13 [11] A. Lecomte, A. Zennir, and F. de Larrard, "Modèle de suspension solide et formulation de bétons  
14 calcaires en Lorraine," *Bull.-Lab. Ponts Chaussées*, pp. 41–52, 1997.
- 15 [12] W. Toufar, M. Born, and E. Klose, "Contribution of optimisation of components of different  
16 density in polydispersed particles systems," *Freib. Bookl. A*, vol. 558, pp. 29–44, 1976.
- 17 [13] J. D. Dewar, "Ready-mixed concrete mix design," *Munic. Eng.*, vol. 3, pp. 35–43, 1986.
- 18 [14] M. Reisi and D. M. Nejad, "A numerical method to predict packing density of aggregates in  
19 concrete," in *Advanced Materials Research*, vol. 337, pp. 313–316, 2011.
- 20 [15] A. K. H. Kwan, K. W. Chan, and V. Wong, "A 3-parameter particle packing model incorporating  
21 the wedging effect," *Powder Technol.*, vol. 237, pp. 172–179, Mar. 2013.
- 22 [16] A. Sadok, R. Zentar, and N.-E. Abriak, "Genetic programming for granular compactness  
23 modelling," *Eur. J. Environ. Civ. Eng.*, pp. 1–13, 2016.
- 24 [17] G. Roquier, "The 4-parameter Compressible Packing Model (CPM) including a new theory about  
25 wall effect and loosening effect for spheres," *Powder Technol.*, vol. 302, pp. 247–253, Nov.  
26 2016.
- 27 [18] G. Roquier, "A Theoretical Packing Density Model (TPDM) for ordered and disordered packings,"  
28 *Powder Technol.*, 2018.

- 1 [19] F. Moutassem, « Assessment of Packing Density Models and Optimizing Concrete Mixtures »,  
2 Int. J. Civ. Mech. Energy Sci. (IJCMES), vol. 2, no 4, p. 29-36, 2016.
- 3 [20] M. R. Jones, L. Zheng, and M. D. Newlands, "Comparison of particle packing models for  
4 proportioning concrete constituents for minimum voids ratio," *Mater. Struct.*, vol. 35, no. 5, pp.  
5 301–309, 2002.
- 6 [21] A. Lecomte and J.-M. Mechling, "Compacité des mélanges et propriétés des grains," *Bull.-Lab.*  
7 *Ponts Chaussées*, pp. 21–34, 1999.
- 8 [22] Association Française de Normalisation (AFNOR), " Tests for mechanical and physical  
9 properties of aggregates - Part 6: determination of particle density and water absorption",  
10 Standard No. 1097-6, 2014.
- 11 [23] Association Française de Normalisation (AFNOR), " Tests for geometrical properties of aggrega-  
12 tes - Part 1: determination of particle size distribution - Sieving method", Standard No. 933-1,  
13 2012.
- 14 [24] V. Ledee, F. de Larrard, T. Sedran, and F. Brochu, « Essai de compacité des fractions  
15 granulaires à la table à secousses: Mode opératoire », Tech. Méthodes Lab. Ponts Chaussées,  
16 2004.
- 17 [25] M. Bala, R. Zentar, and P. Boustingorry, "Parameter analysis of the compressible packing model  
18 for Concrete application," 12th fib International PhD Symposium in Civil Engineering, Prague,  
19 Czech Republic, Unpublished results, pp. 1–8.
- 20 [26] R. K. McGeary, "Mechanical packing of spherical particles," *J. Am. Ceram. Soc.*, vol. 44, no. 10,  
21 pp. 513–522, 1961.
- 22 [27] G. W. Delaney, S. Hutzler, and T. Aste, "Relation between grain shape and fractal properties in  
23 random Apollonian packing with grain rotation," *Phys. Rev. Lett.*, vol. 101, no. 12, p. 120602,  
24 2008.
- 25 [28] D. Weaire and T. Aste, *The pursuit of perfect packing*. CRC Press, 2008.
- 26 [29] A. B. Yu and N. Standish, "Porosity calculations of multi-component mixtures of spherical  
27 particles," *Powder Technol.*, vol. 52, no. 3, pp. 233–241, 1987.
- 28 [30] S. Abdelfeteh, "Formulation de matériaux de construction à base de sous-produits industriels  
29 avec des méthodes issues de l'intelligence artificielle," PhD Thesis, Univ. Lille 1, 2016.



- 1 [31] A. Sadok, R. Zentar, and N. E. Abriak, "Modélisation de la compacité granulaire par une  
2 approche d'Apprentissage Automatique," in *Rencontres Universitaires de Génie Civil*, 2015.
- 3 [32] T. N. Thanh, "Valorisation de sédiments marins et fluviaux en technique routière," PhD Thesis,  
4 Univ. d'Artois - Ecole des Mines de Douai, 2009.
- 5 [33] M. Bala, R. Zentar, and P. Boustingorry, "Étude d'impact de la forme des granulats sur les  
6 paramètres du modèle d'empilement compressible," presented at the 36èmes Rencontres  
7 Universitaires de Génie Civil de l'AUGC, Saint-Etienne, France, 2018, pp. 1–4.
- 8 [34] X. Chateau, G. Ovarlez, and K. L. Trung, "Homogenization approach to the behavior of  
9 suspensions of noncolloidal particles in yield stress fluids," *J. Rheol.*, vol. 52, no. 2, pp. 489–506,  
10 2008.
- 11 [35] F. de Larrard and A. Belloc, "L'influence du granulat sur la résistance à la compression des  
12 bétons," *Bull. Lab. Ponts Chaussées*, no. 219, 1999.

13

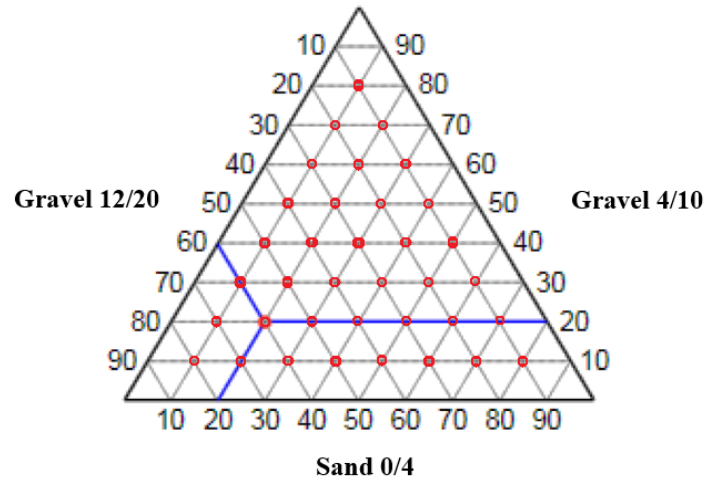
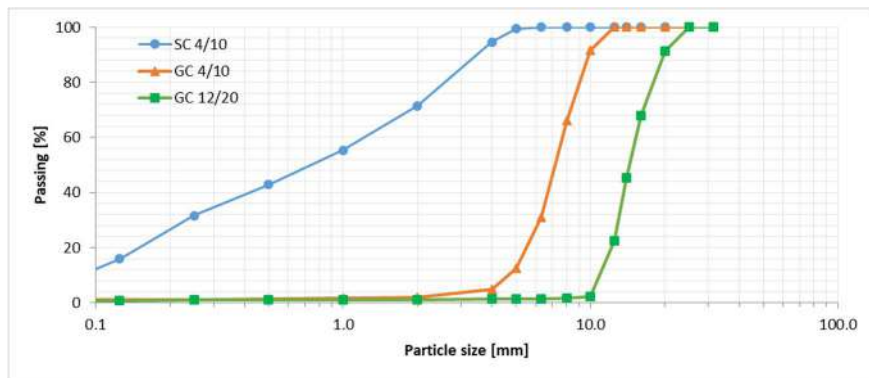
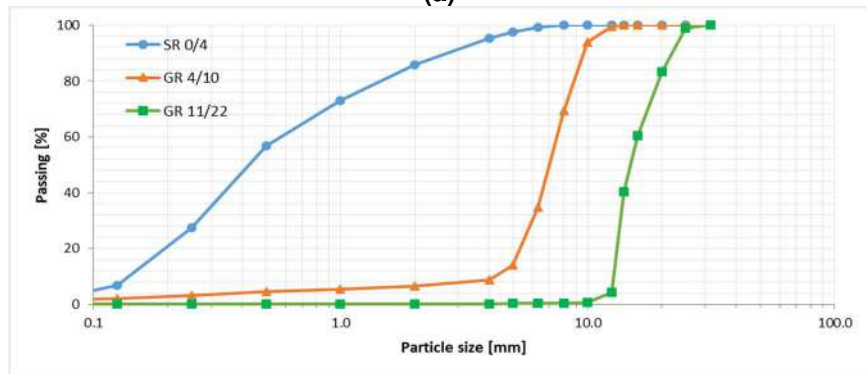


Fig. 1: Experimental program of ternary mixtures (Crushed aggregates)

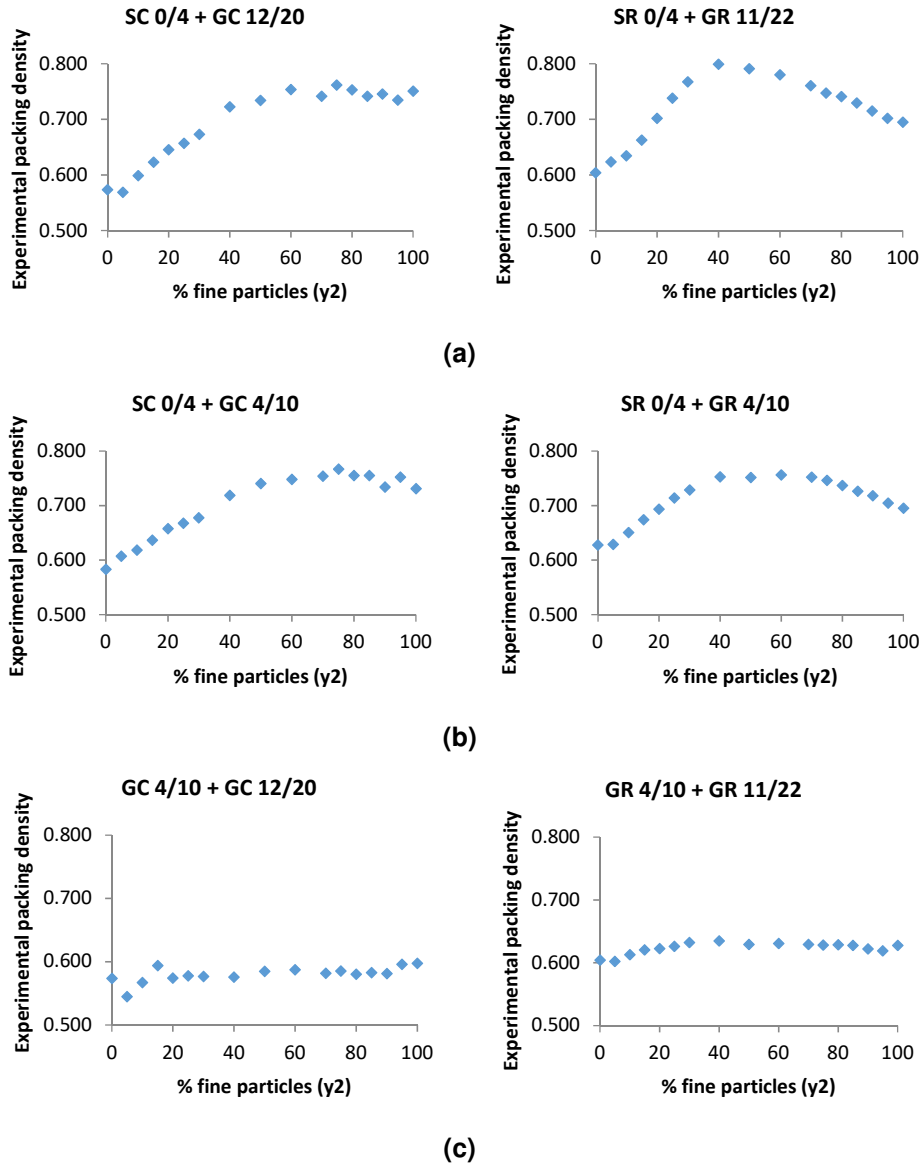


(a)

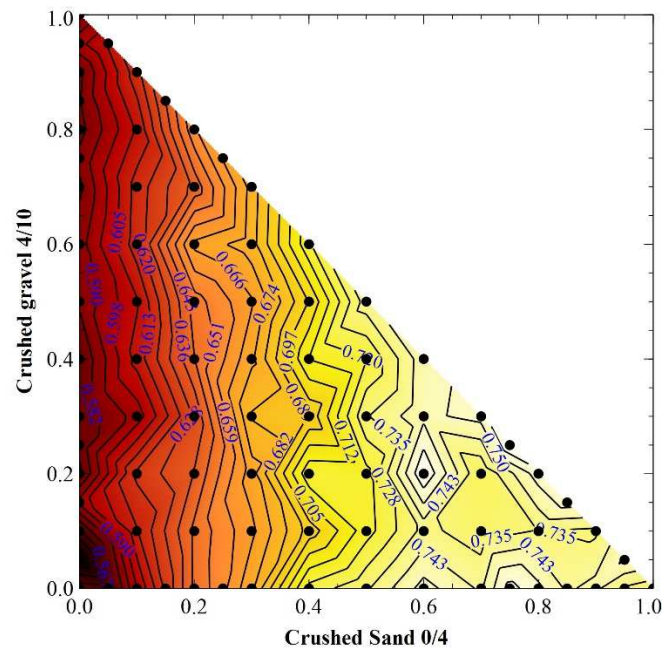


(b)

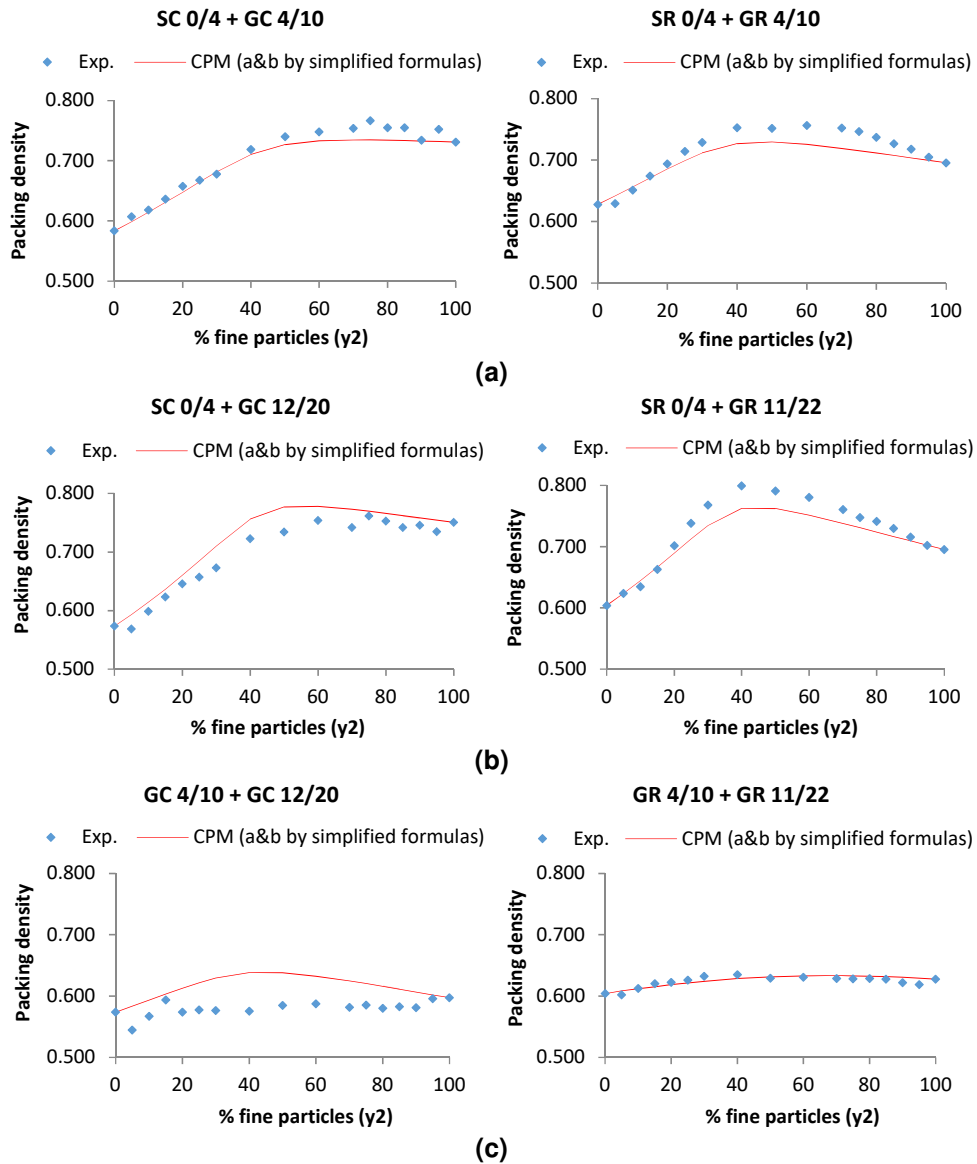
Fig. 2: Particle size distribution of crushed aggregates (a) and rolled aggregates (b)



**Fig. 3: Packing density measured by the compaction table of binary mixtures of crushed aggregates (left) and rolled aggregates (right) for: (a)  $D_1/D_2=5$ , (b)  $D_1/D_2=2.5$  and (c)  $D_1/D_2=2$**



**Fig. 4: Packing density measured by the compaction table of ternary mixtures of crushed aggregates**



**Fig. 5: Measured and modelled packing density of crushed (left) and rolled (right) binary Mixtures via simplified formulas of interaction coefficients for: (a)  $D_1/D_2=5$ , (b)  $D_1/D_2=2.5$  and (c)  $D_1/D_2=2$**

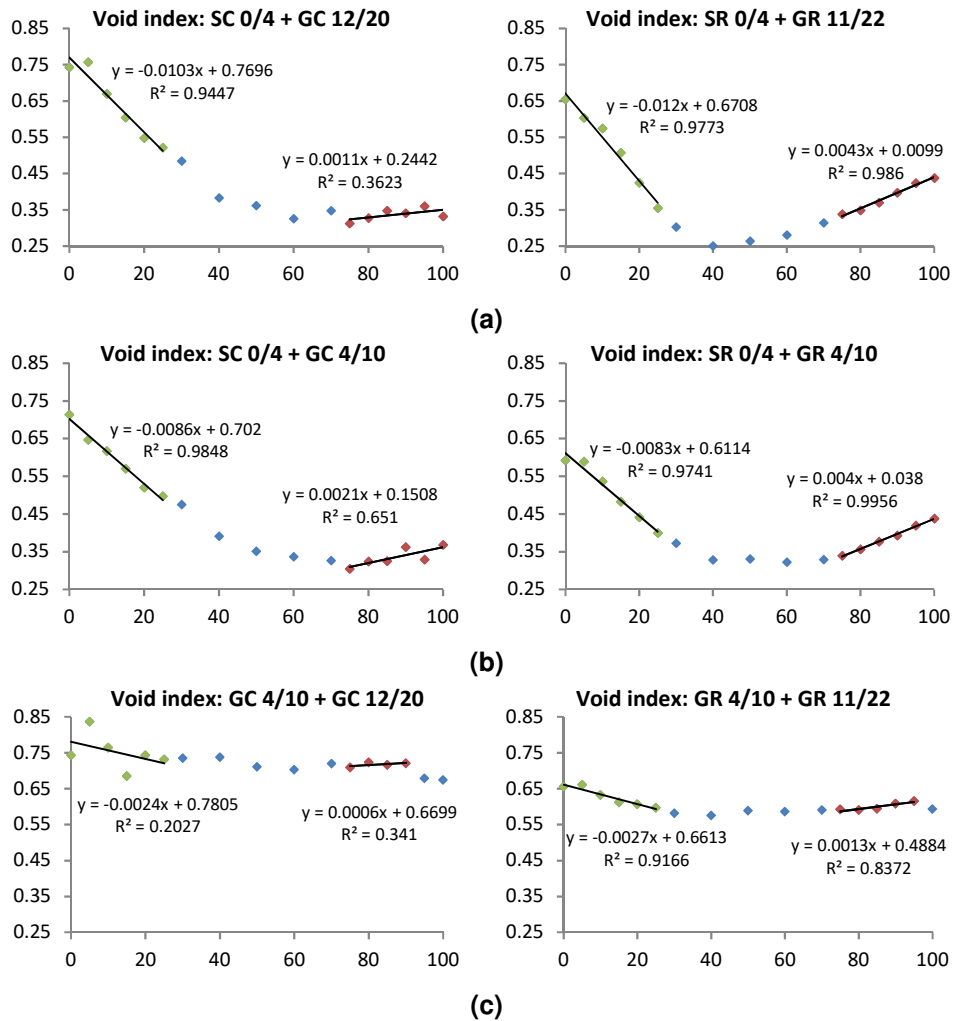
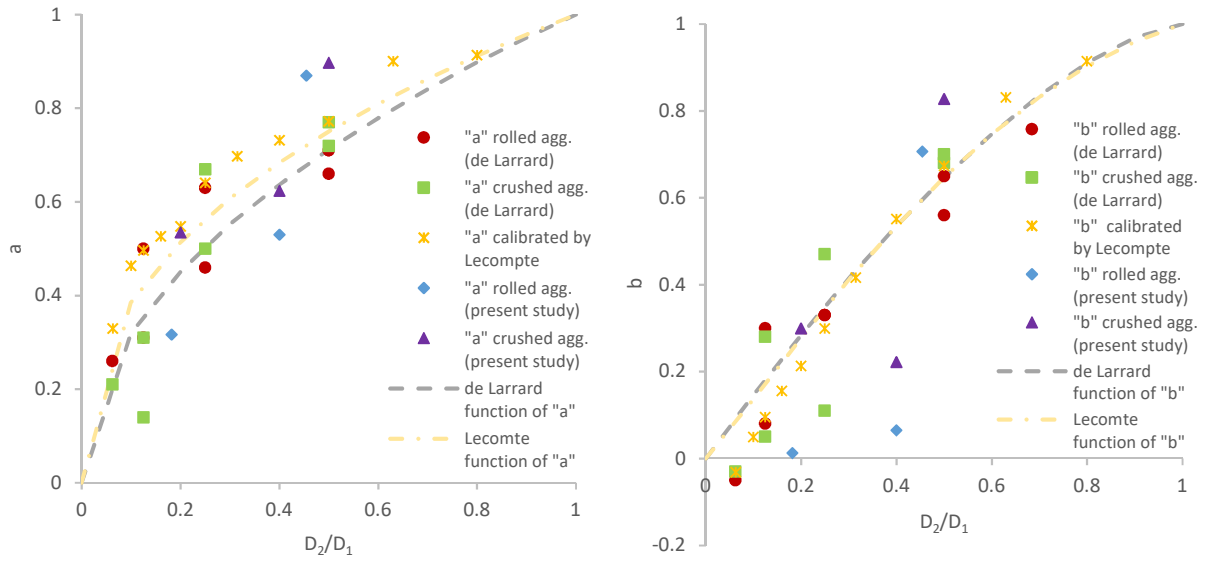
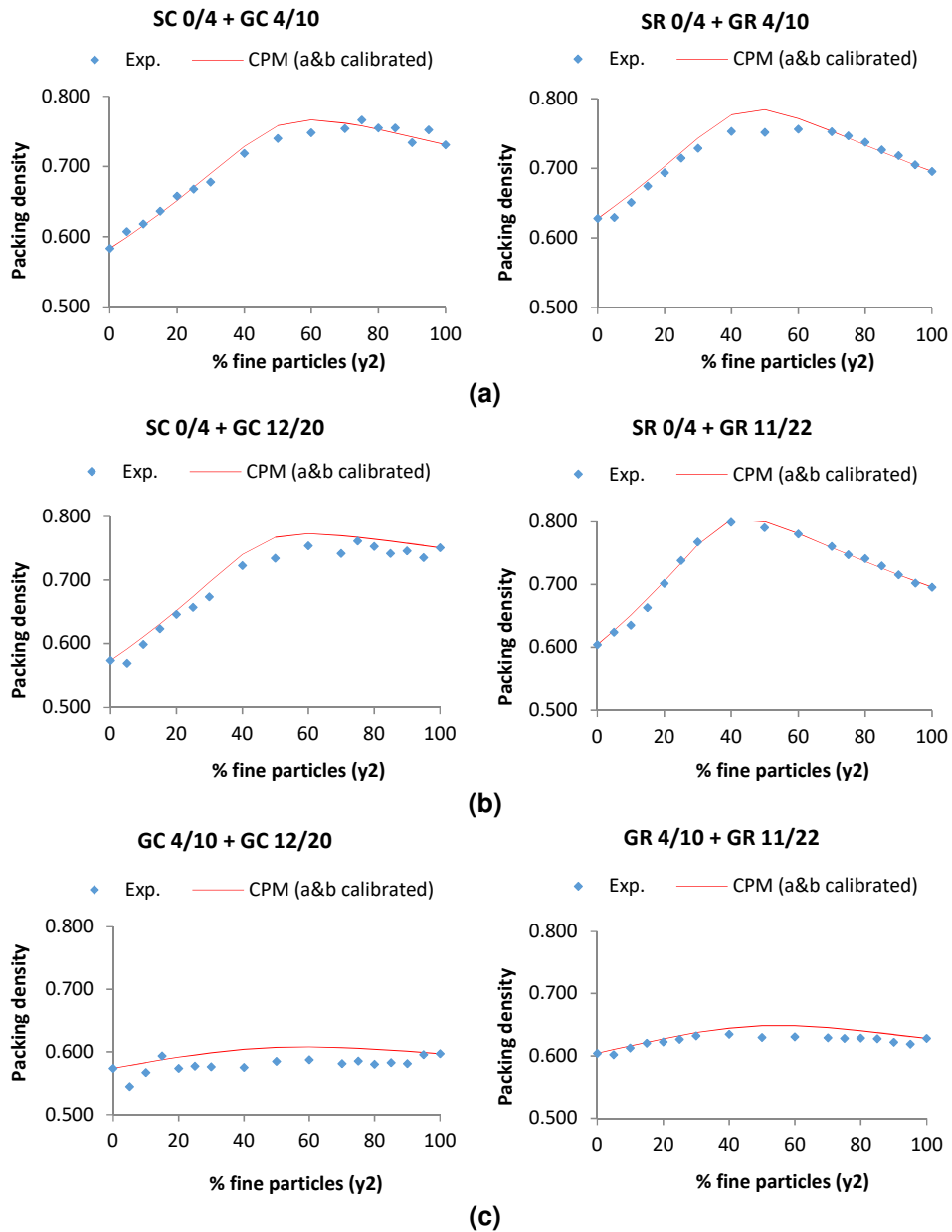


Fig. 6 : Determination of the granular interaction coefficients a and b for crushed (left) and rolled (right) aggregates for: (a)  $D_1/D_2=5$ , (b)  $D_1/D_2=2.5$  and (c)  $D_1/D_2=2$

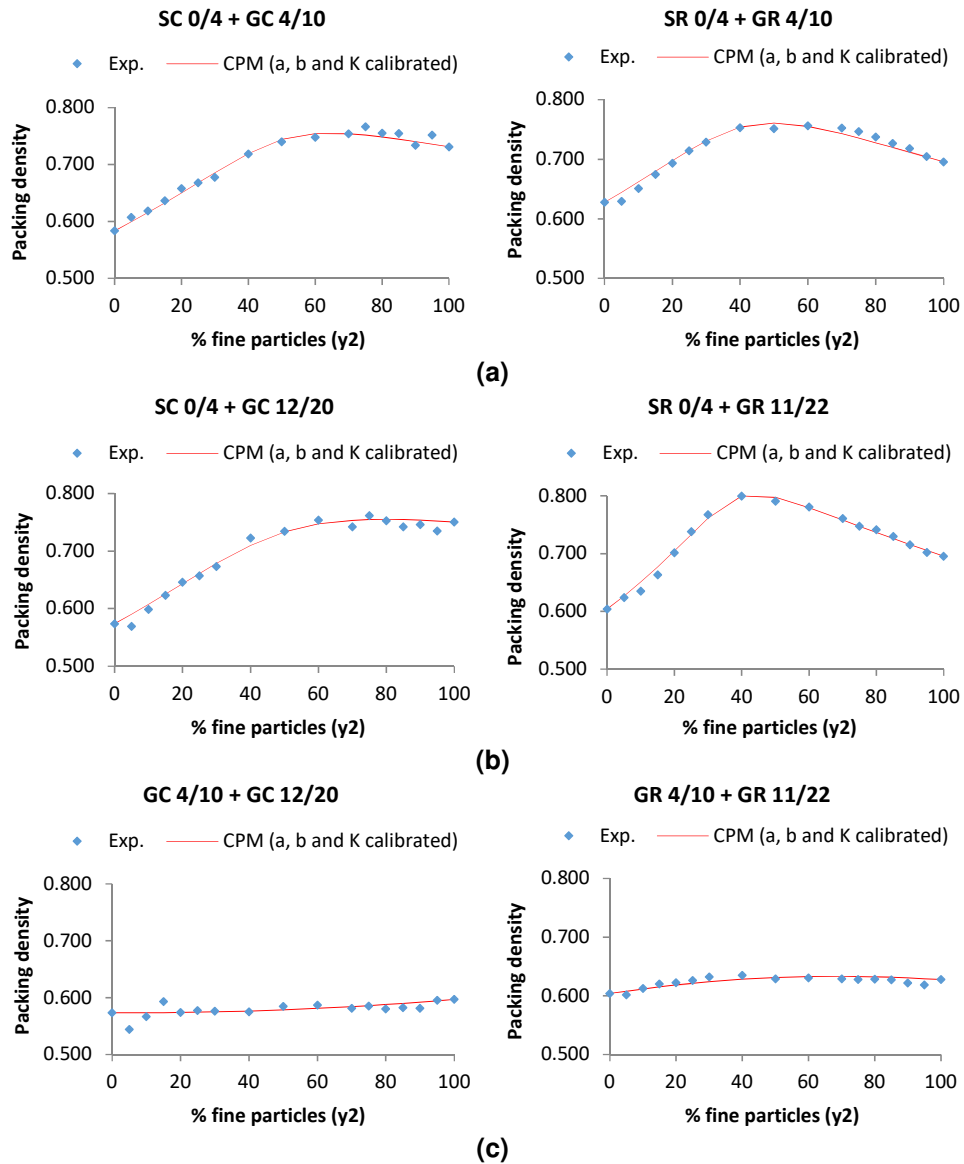


**Fig. 7: Interaction coefficients vs. size ratio for crushed and rolled aggregates (comparison between experimental data of the present study and data from the literature): loosening effect coefficient "a" (left) and wall effect coefficient "b" (right)**

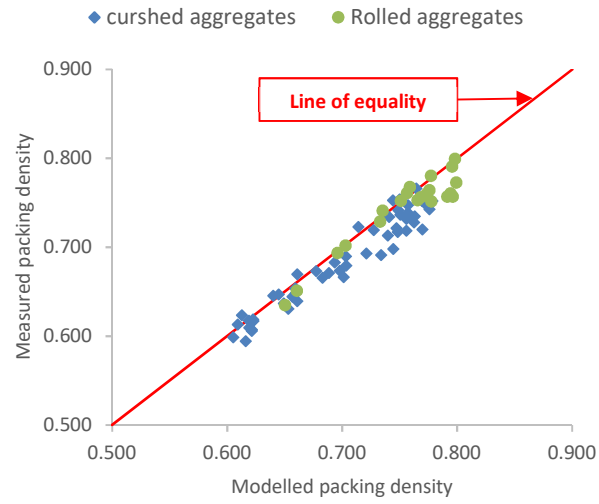


**Fig. 8: Measured and modelled packing density of crushed (left) and rolled (right) binary mixtures after calibration of interaction coefficients for: (a)  $D_1/D_2=5$ , (b)  $D_1/D_2=2.5$  and (c)  $D_1/D_2=2$**





**Fig. 9 : Measured and modelled packing density of crushed (left) and rolled (right) binary mixtures after calibration of a, b and K for: (a)  $D_1/D_2=5$ , (b)  $D_1/D_2=2.5$  and (c)  $D_1/D_2=2$**



**Fig. 10: Modelled vs. Measured packing density of ternary mixtures after calibration of  $a$  and  $b$  ( $K=9$ )**

**Table 1: Origin and nature of the studied aggregates**

Type of aggregates	Origin	Granular class d/D	Density [t/m <sup>3</sup> ]
Crushed	Boulonnais	Sand 0/4	2.69
		Gravel 4/10	2.67
		Gravel 12/20	2.67
Rolled	Chevrières	Sand 0/4	2.55
	Decize	Gravel 4/10	2.43
		Gravel 11/22	2.54

**Table 2: Experimental program of binary mixtures**

Type of aggregates	Granular class d/D	Binary mixtures	D <sub>1</sub> /D <sub>2</sub> ratio
Crushed	Sand 0/4	SC 0/4 + GC 12/20	5
	Gravel 4/10	SC 0/4 + GC 4/10	2.5
	Gravel 12/20	GC 4/10 + GC 12/20	2
Rolled	Sand 0/4	SR 0/4 + GR 11/22	5
	Gravel 4/10	SR 0/4 + GR 4/10	2.5
	Gravel 11/22	GR 4/10 + GR 11/22	2

**Table 3: Characteristics of the aggregates of the study**

d/D	Crushed aggregates			Rolled aggregates		
	SC 0/4 (Sand)	GC 4/10 (Gravel)	GC 12/20 (Gravel)	SR 0/4 (Sand)	GR 4/10 (Gravel)	GR 11/22 (Gravel)
Fine content < 63 μm [%]	6.2	1.0	0.8	1.3	1.7	0.1
d <sub>50</sub> [mm]	0.7	7.2	15.2	0.4	7.0	16.0
Fineness modulus (for sands)	3.2	-	-	3.4	-	-
Absolute density [g/cm <sup>3</sup> ]	2.726	2.735	2.711	2.626	2.648	2.626
Water absorption [%]	0.43	0.59	0.49	1.18	3.06	1.79
Shape and roughness	Angular aggregates, flat and rough surface			Rounded shape, flat or elongated aggregates, smooth surface		

**Table 4: Parameters used for modelling by simplified formulas of a and b (K = 9)**

Type of aggregates	D <sub>1</sub>	D <sub>2</sub>	Mixtures	D <sub>2</sub> /D <sub>1</sub>	a & b by simplified formulas			
					a	b	K	Average error
Crushed	10	4	SC 0/4 + GC 4/10	0.40	0.64	0.54	9	1.5%
	20	4	SC 0/4 + GC 10/20	0.20	0.45	0.28		2.8%
	20	10	GC 4/10 + GC 10/20	0.50	0.71	0.65		5.2%
	Average error for crushed aggregates							3.2%
Rolled	10	4	SR + GR 4/10	0.40	0.64	0.54	9	2.2%
	22	4	SR + GR 10/20	0.18	0.43	0.26		2.1%
	22	10	GR 4/10 + GR 10/20	0.45	0.68	0.60		0.7%
	Average error for rolled aggregates							1.7%

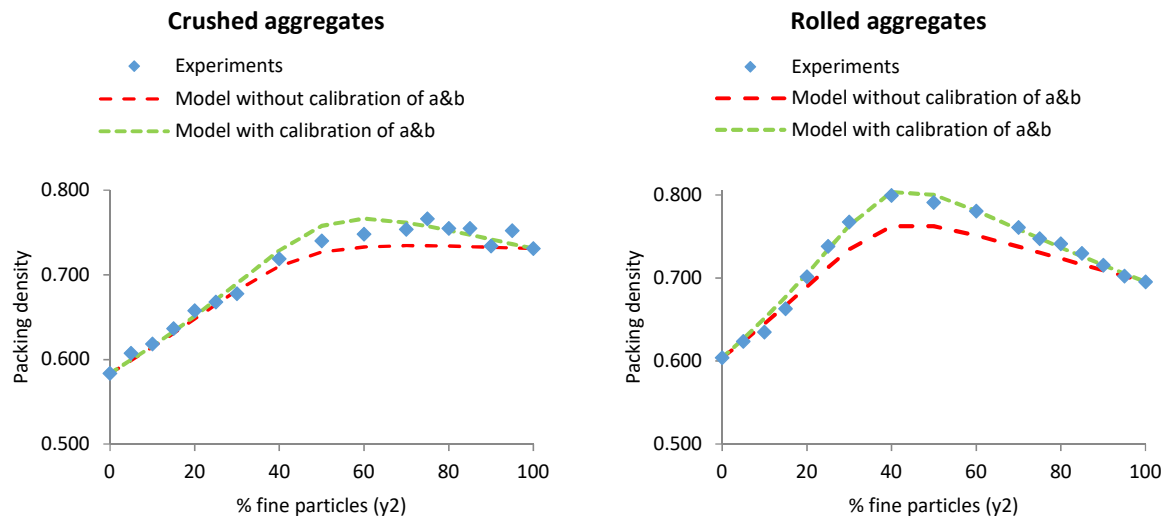
**Table 5: Parameters of the modelling by calibrated formulas of a and b (K = 9)**

Type of aggregates	D <sub>1</sub>	D <sub>2</sub>	Mixtures	D <sub>2</sub> /D <sub>1</sub>	a & b after calibration			
					a	b	K	Average error
Crushed	10	4	SC 0/4 + GC 4/10	0.40	0.62	0.22	9	1.1%
	20	4	SC 0/4 + GC 10/20	0.20	0.54	0.30		2.1%
	20	10	GC 4/10 + GC 10/20	0.50	0.90	0.83		2.9%
	Average error for crushed aggregates							2.0%
Rolled	10	4	SR + GR 4/10	0.40	0.53	0.06	9	1.2%
	22	4	SR + GR 10/20	0.18	0.32	0.01		0.6%
	22	10	GR 4/10 + GR 10/20	0.45	0.87	0.71		1.4%
	Average error for rolled aggregates							1.1%

**Table 6: Parameters of the modelling after calibration of a, b and K**

Type of aggregates	D <sub>1</sub>	D <sub>2</sub>	Mixtures	D <sub>2</sub> /D <sub>1</sub>	a, b and K after calibration			
					a	b	K	Average error
<b>Crushed</b>	10	4	SC 0/4 + GC 4/10	0.40	0.62	0.22	5.5	0.8%
	20	4	SC 0/4 + GC 10/20	0.20	0.54	0.30	3.2	1.1%
	20	10	GC 4/10 + GC 10/20	0.50	0.90	0.83	0.3	1.1%
	Average error for crushed aggregates							1.0%
<b>Rolled</b>	10	4	SR + GR 4/10	0.40	0.53	0.06	4.8	0.8%
	22	4	SR + GR 10/20	0.18	0.32	0.01	8.2	0.6%
	22	10	GR 4/10 + GR 10/20	0.45	0.87	0.71	2.0	0.7%
	Average error for rolled aggregates							0.7%

## Graphical abstract



Effect of calibration of a and b on modeled packing density by CPM