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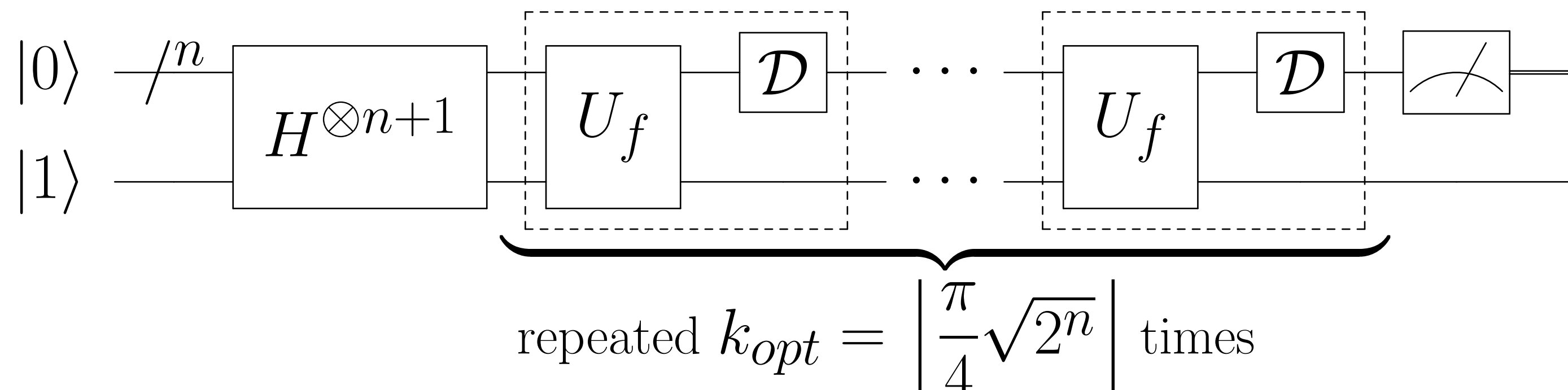
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NON-LOCALITY AND ENTANGLEMENT DETECTION WITH MERMIN POLYNOMIALS FOR GROVER'S ALGORITHM AND QUANTUM FOURIER TRANSFORM

Henri de Boutray, Hamza Jaffali, Frédéric Holweck, Alain Giorgetti & Pierre-Alain Masson

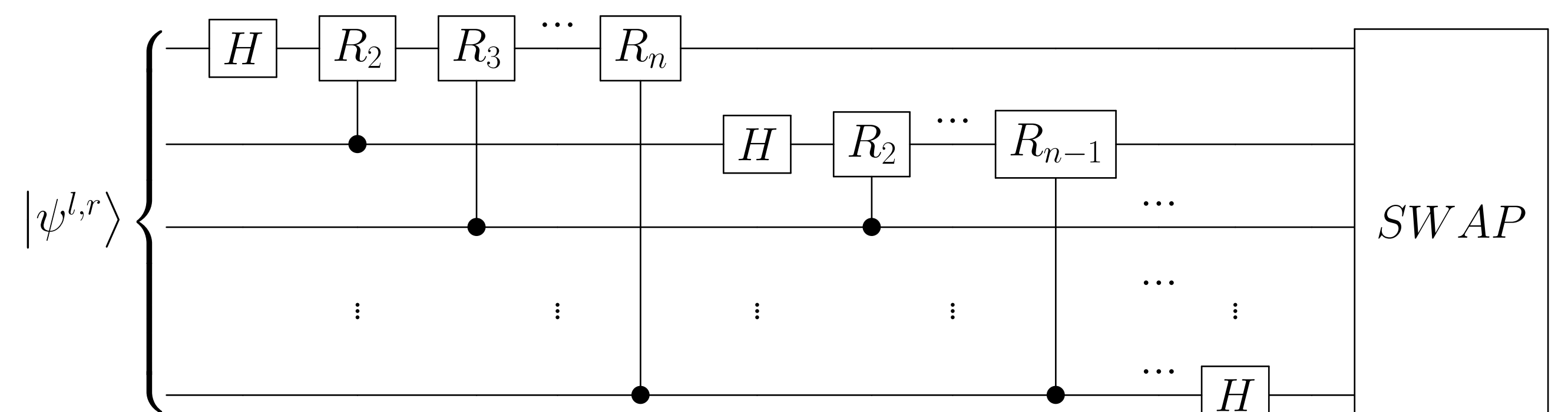
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Grover's quantum algorithm



- Search of an item $|\mathbf{x}_0\rangle$ in a list, based on an oracle
- Complexity: $\mathcal{O}(\sqrt{2^n})$ (against $\mathcal{O}(2^n)$ for classical search algorithms)

Quantum Fourier Transform



Generally performed on periodic states defined by:
 $|\varphi^{l,r}\rangle = \frac{1}{\sqrt{A}} \sum_{i=0}^{A-1} |l + ir\rangle$

- Quantum analogue of the discrete Fourier transform
- Complexity: $\mathcal{O}(n^2)$ (against $\mathcal{O}(n^{2n})$)

Mermin's polynomials

$$\begin{cases} M_1 = a_1 \\ \forall n \geq 2, M_n = \frac{1}{2} M_{n-1} \otimes (a_n + a'_n) + \frac{1}{2} M'_{n-1} \otimes (a_n - a'_n) \end{cases}$$

(a_i) and (a'_i) are one-qubit observables with eigenvalues in $\{-1, 1\}$

$\langle \varphi | M_n | \varphi \rangle > 1$ implies that $|\varphi\rangle$ is non-local.

Example:

With $(a_1, a_2, a'_1, a'_2) = (Z, X, (Z + X)/\sqrt{2}, (Z - X)/\sqrt{2})$, M_2 corresponds to the operator used for Bell inequalities.

Grover's algorithm evaluation

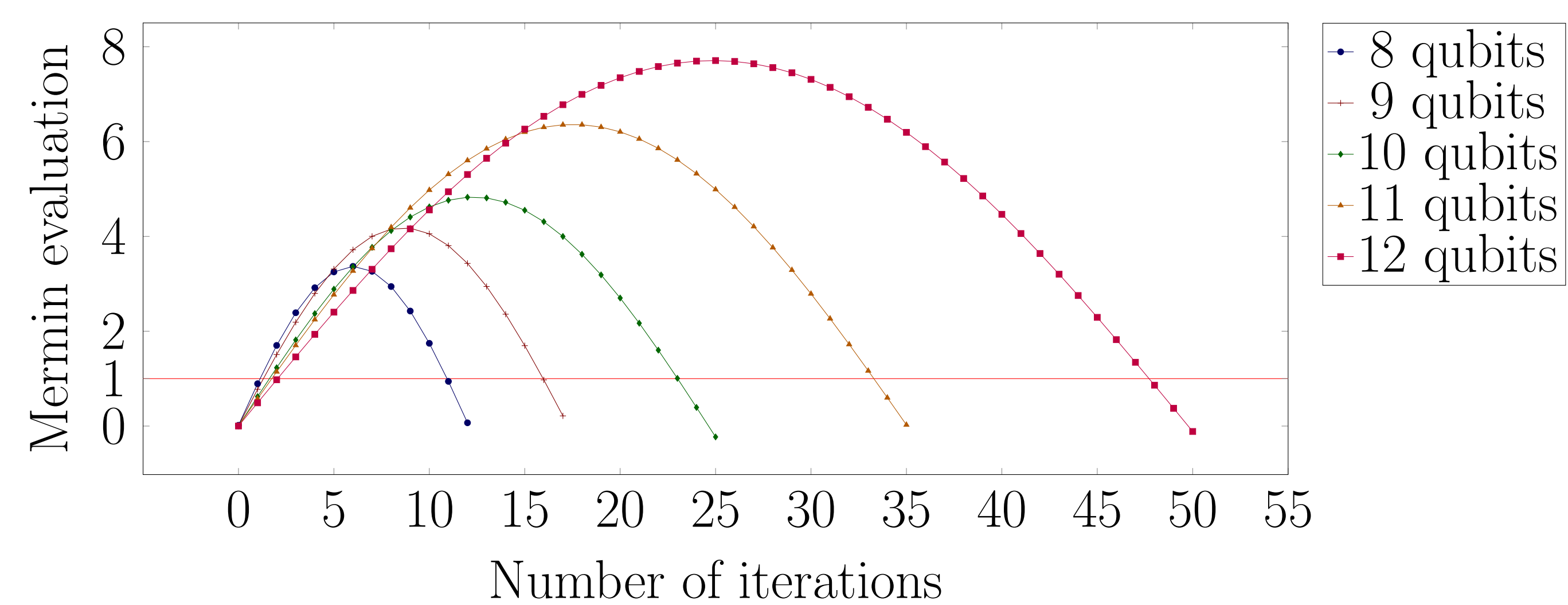
Proposition ([JH19]):

- The states in Grover's algorithm are $|\varphi_k\rangle = \alpha_k |\mathbf{x}_0\rangle + \beta_k |+\rangle^{\otimes n}$, with $(\alpha_0, \beta_0) = (0, 1)$ and $(\alpha_{k_{opt}}, \beta_{k_{opt}}) \approx (1, 0)$.
- For k close to $k_{opt}/2$, $|\varphi_k\rangle$ comes close to a state $|\varphi_{ent}\rangle = (|\mathbf{x}_0\rangle + |+\rangle^{\otimes n})/K$ maximizing $\langle \varphi | M_n | \varphi \rangle$.

Evaluation method: find M_n such that $\langle \varphi_{ent} | M_n | \varphi_{ent} \rangle$ is maximal.

Computing $\langle \varphi_k | M_n | \varphi_k \rangle$ for every k with this M_n **positively** answers the following question:

"Is Grover's algorithm using entanglement to achieve quantum speedup?"

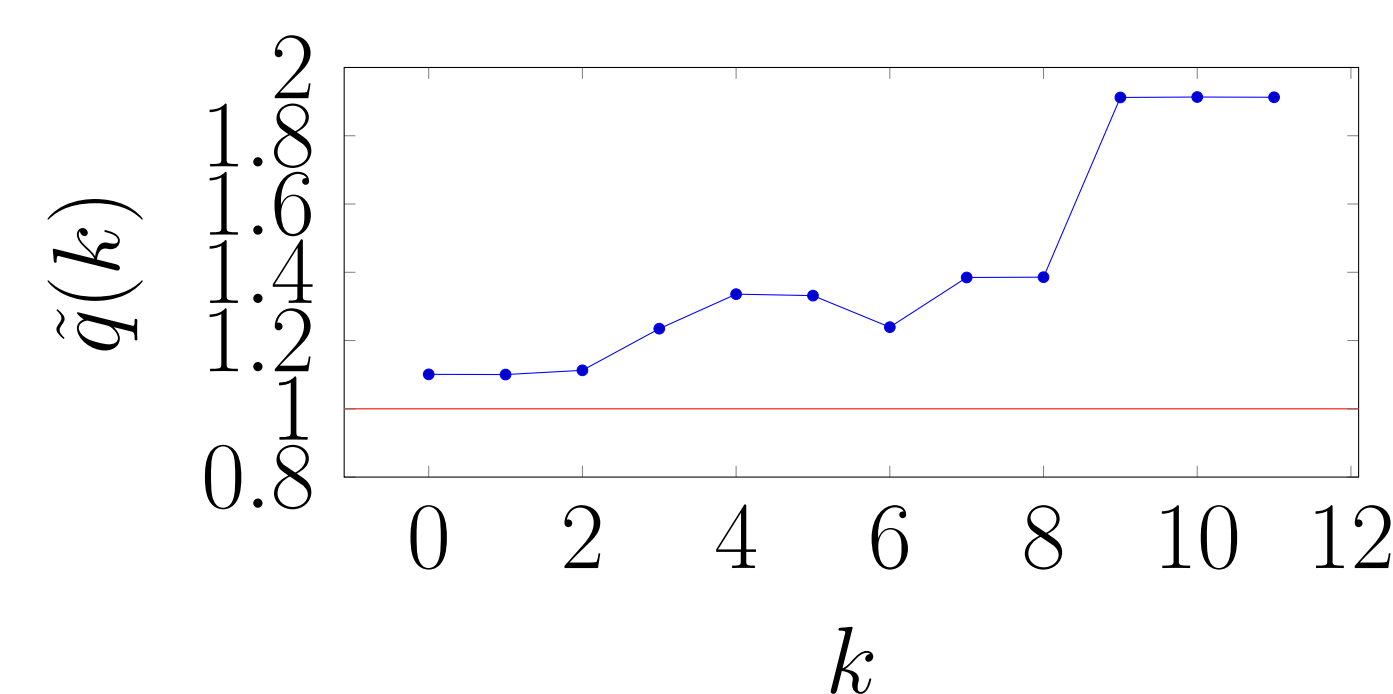


QFT evaluation

$q : |\varphi\rangle \mapsto \max_{M_n} \langle \varphi | M_n | \varphi \rangle$ is a measure of entanglement.

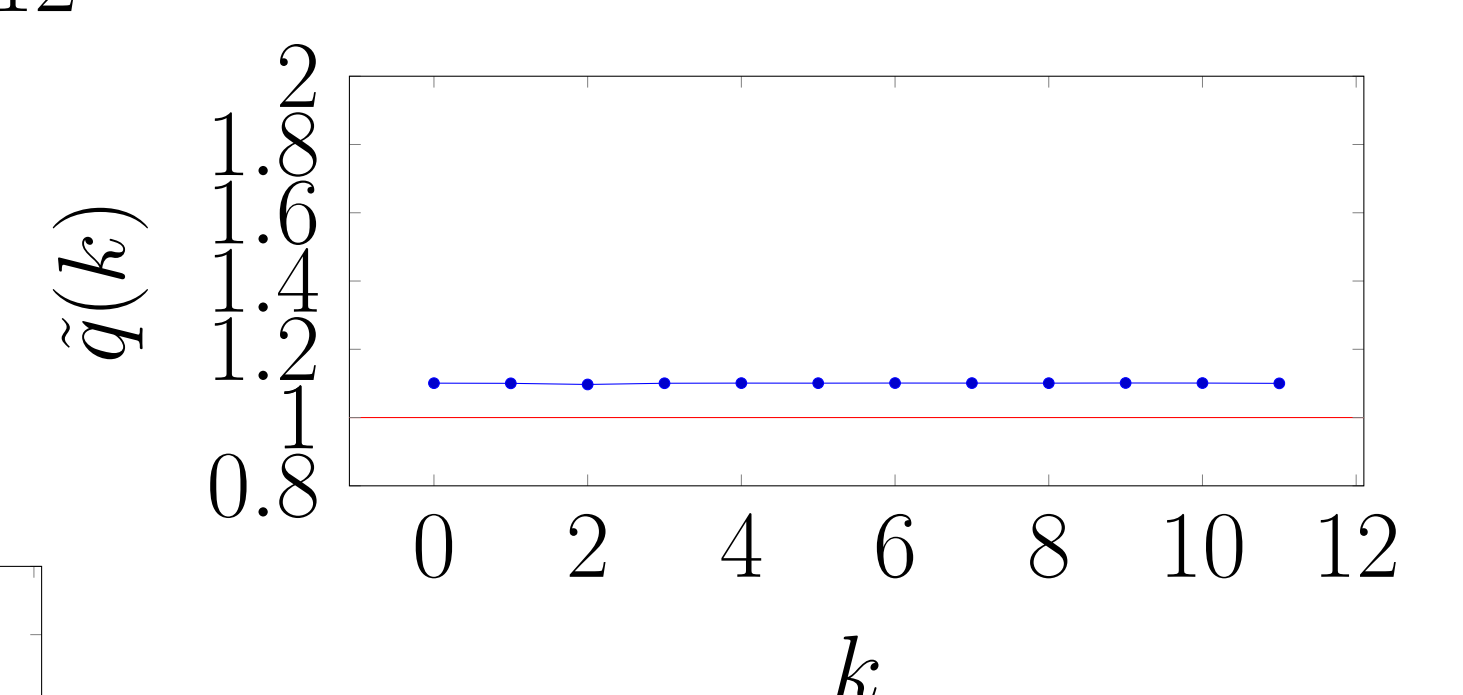
Evaluation method: for each state $|\varphi_k\rangle$, find \tilde{M}_n such that $\langle \varphi_k | \tilde{M}_n | \varphi_k \rangle$ is maximal.

The corresponding experimental approximation \tilde{q} of q allows us to distinguish between three types of QFT runs in our experiments with $n = 4$:

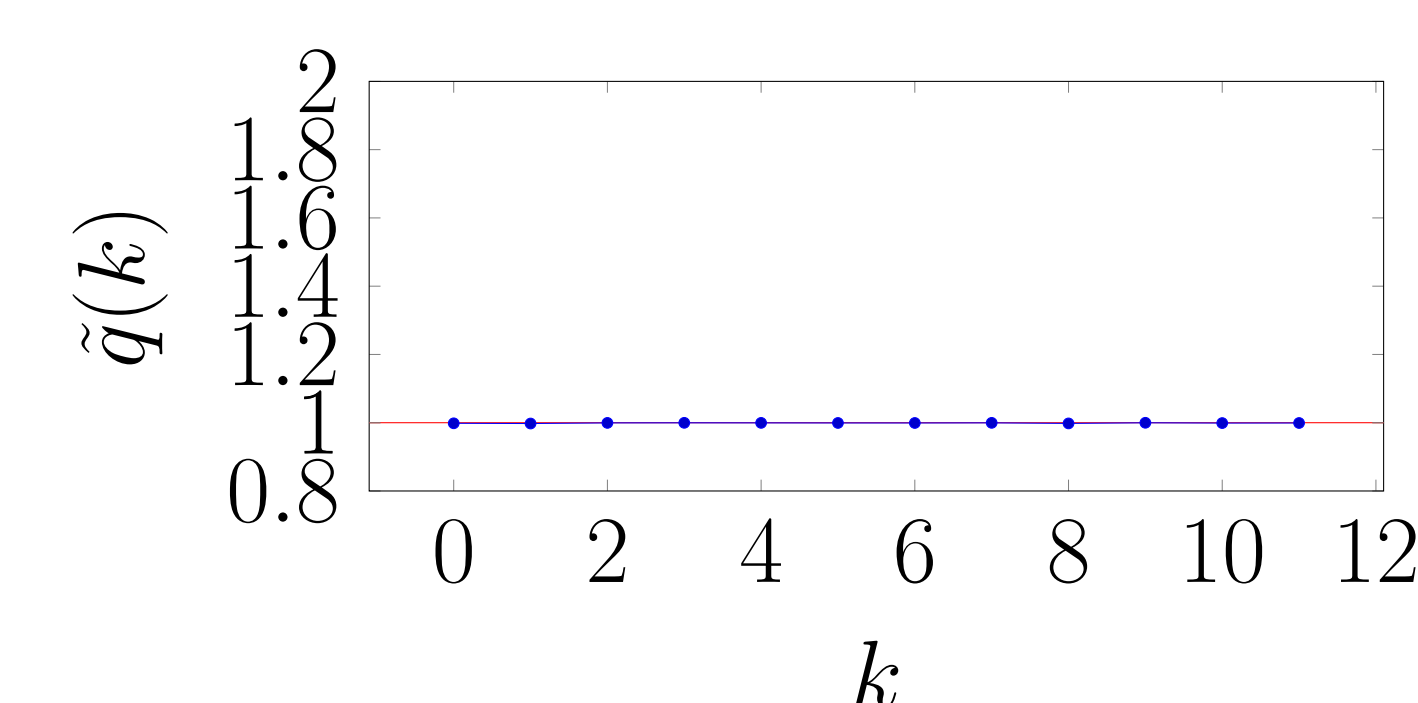


Entangled states and variable measure (here for $(l, r) = (9, 1)$).

Entangled states and constant measure (here for $(l, r) = (2, 2)$).



Separable states (here for $(l, r) = (8, 5)$).



One can also check some key points such as the fact that entanglement evaluation doesn't change during LOCC operations (H gates in this case).

References

[dJH⁺20] H. de Boutray, H. Jaffali, F. Holweck, A. Giorgetti, and P.-A. Masson. Non-locality and Entanglement Detection with Mermin polynomials for Grover's algorithm and Quantum Fourier Transform. *Submitted in September 2020*, previous version at arXiv:2001.05192.

[JH19] H. Jaffali and F. Holweck. Quantum Entanglement involved in Grover's and Shor's algorithms: The four-qubit case. *Quantum Information Processing*, 18(5):133, May 2019.

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