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# Stochastic modeling of mesoscale eddies

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## LU formalism

Stochastic material derivative

$$\frac{\mathbb{D}q}{\mathbb{D}t} = \frac{\partial q}{\partial t} + (\mathbf{u} + \dot{\boldsymbol{\eta}}) \cdot \nabla q - \frac{1}{2} \nabla \cdot \nabla \cdot (\mathbf{a}q)$$

Noise and variance

$$\dot{\boldsymbol{\eta}}(\mathbf{x}, t) = \bar{\boldsymbol{\eta}}(\mathbf{x}) + \sum_k \boldsymbol{\phi}_k(\mathbf{x}, t) \xi_k, \quad \mathbf{a} = \delta t \sum_k \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T$$

$\xi_k \sim \mathcal{N}(0, 1)$

Stochastic evolution of layered PV

$$\frac{\mathbb{D}\mathbf{q}}{\mathbb{D}t} = F(\mathbf{q}) + D(\nabla^2 \mathbf{p}) + \dot{S}(\nabla^2 \mathbf{p})$$

QG kinematic relations

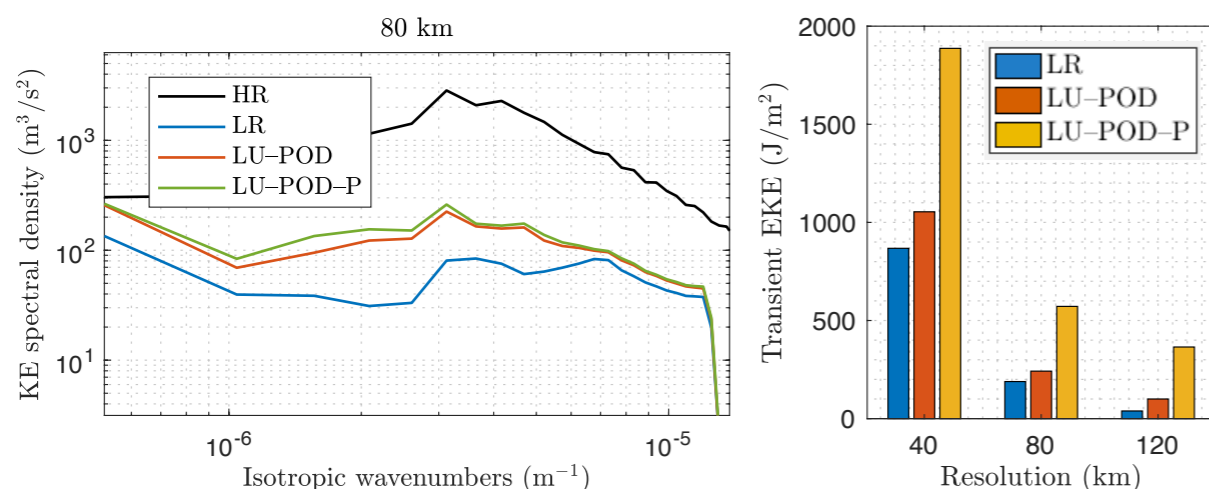
$$f_0(\mathbf{q} - \beta \mathbf{y}) = \nabla^2 \mathbf{p} - f_0^2 \mathbf{A} \mathbf{p}, \quad f_0 \mathbf{u} = \nabla^\perp \mathbf{p}$$

## Parameterizations

LU-POD:  $\bar{\boldsymbol{\eta}}, \{\boldsymbol{\phi}_k\}$  learned from data

LU-POD-P:  $\boldsymbol{\phi}_k \perp \nabla(\mathbf{A} \mathbf{p})$

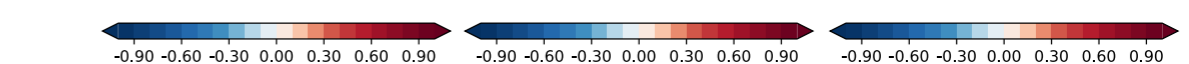
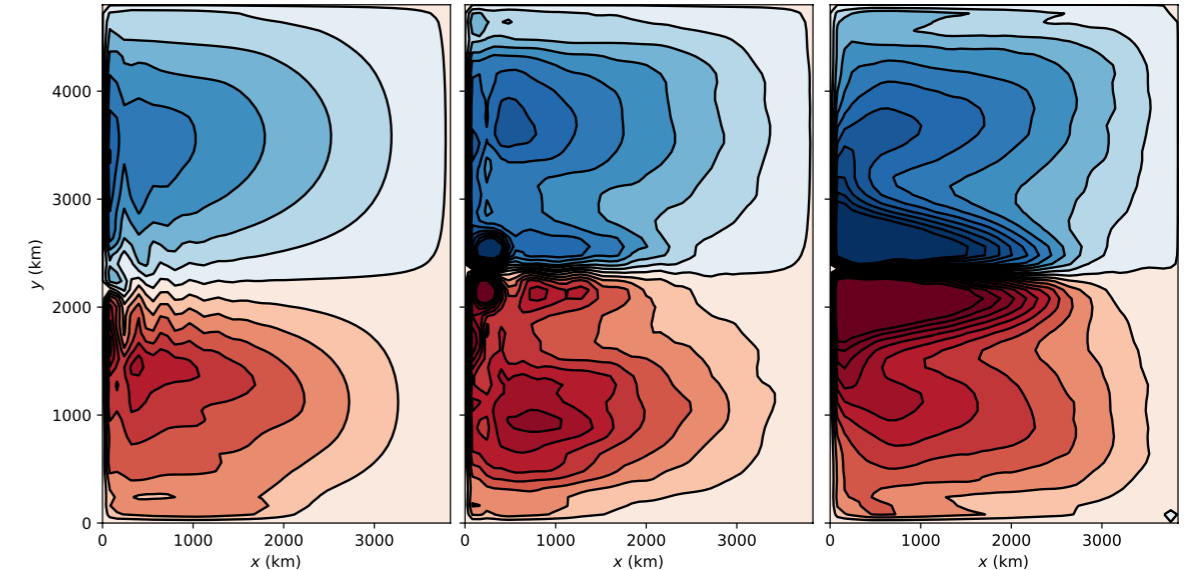
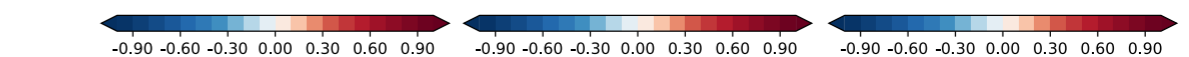
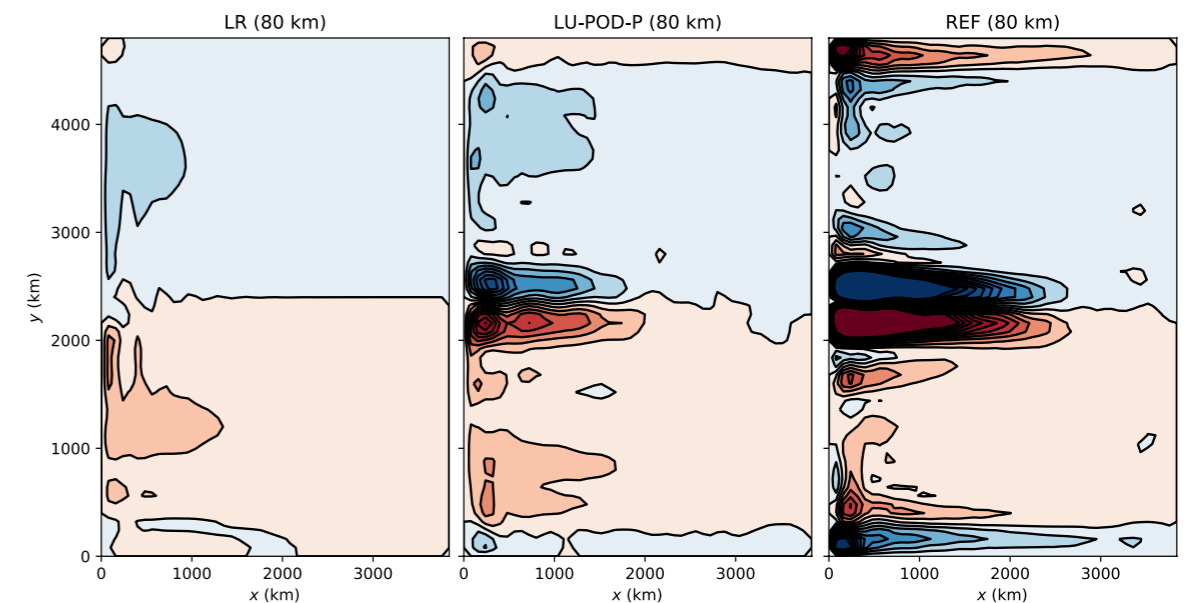
## Energy budget



Benefits: reproduction of jet and recirculation, energy backscattering and internal variability

## Long-term mean

Barotropic mode of  $p$



Reference: Li 2021, Stochastic modeling and numerical simulation of ocean dynamics, Chap. 8, <https://hal.archives-ouvertes.fr/tel-03207741>