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Localization of exceptional points and modal branch tracking for acoustic waveguides applications

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functions are defined

$$g(\nu) = \lambda_1 + \lambda_2, \quad \text{and} \quad h(\nu) = (\lambda_1 - \lambda_2)^2. \quad (3)$$

By construction, these functions are holomorphic in the vicinity of ν^* (as this was already mentioned in [7] and in [2, p. 66]). The method consists of 3 steps:

1. Computation of the derivatives of two selected eigenpairs $(\lambda_i, \mathbf{x}_i)$ ($i = 1, 2$) following the technique presented in [9]. This serves to construct the truncated Taylor series T_g and T_h of g and h with respect to the arbitrary value ν_0 . This step is the most computationally expensive.
2. Application of standard root-finding algorithms for polynomials in order to find the zeros of T_h and locate the branch point ν^* .
3. Find the connecting coefficients between the Taylor series T_g and T_h with those given by the Puiseux series (2).

To illustrate the method, we consider a bidimensional acoustic waveguide with one treated wall (at $y = 0$). The eigenvalue problem is obtained from the discretization of the weak formulation

$$(k^2 - \lambda)\langle \psi, \phi \rangle - \langle \psi', \phi' \rangle + \frac{1}{\nu} \psi(0)\phi(0) = 0. \quad (4)$$

As illustrated in Fig. 2, the spurious roots of T_h tend to be aligned on a circle (which is known to be connected with the radius of convergence of the series) and genuine zeros corresponding to the EP located inside the circle of convergence. The computed value ν^* is a good approximation of the true value with a relative error of 0.4% for a significantly distant initial value $\nu_0 \approx 2.3\nu^*$.

The proposed approach has been implemented in a dedicated open source python library available at github.com/nennigb/EasterEig. This lined waveguide example is included in `EasterEig` the test suite.

Work is ongoing to extend the radius of converge of the approximation and to tackle several parameters to locate higher order EPs. This algorithm can also be used to extend perturbation methods for real random eigenvalue problems, when *veering* is present as in [10], or more generally, to speed up parametric eigenvalue problem with explicit dependencies.

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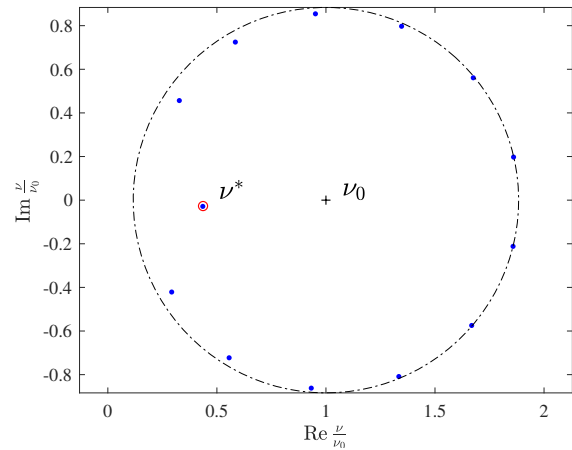


Figure 2. Roots of T_h . Example of EP localization in the case of locally reacting liner with 13 derivatives with $\nu_0 = 0.5825 - 0.6412i$. Roots of T_h (\cdot), initial value ν_0 ($+$) and Tester’s reference solution $\nu_{\text{ref}}^* = (1.6506 + 2.0599i)^{-1}$ (\circ) for the merging of the two least attenuated modes.