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LOCALIZATION OF EXCEPTIONAL POINTS AND MODAL BRANCH TRACKING FOR ACOUSTIC WAVEGUIDES APPLICATIONS

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ABSTRACT

For applications dealing with dissipative acoustic waveguides strong modal attenuation are often achieved close to exceptional points (EP). These EP correspond to a particular tuning of certain design parameters which render the associated eigenvalue problem degenerate where both eigenvalues, i.e. the wavenumber, and eigenvectors of a pair of guided modes coalesce. These non-Hermitian degeneracies have raised considerable attention in the scientific community as these can have a great impact in a variety of physical problems. Here, a new algorithm is proposed to quickly explore the parametric space and to locate EPs. The method requires the computation of successive derivatives of two selected eigenpairs with respect to the parameter so that, after recombination, regular functions can be constructed. This algebraic manipulation permits the EP localization, using standard root-finding algorithms and the computation of the associated Puiseux series up to an arbitrary order, useful to follow modal branches. Examples related to the acoustic propagation in straight ducts with absorbing walls and in periodic guiding structures are given to illustrate the versatility of the proposed method and its ability to handle large size matrices arising from finite element discretization techniques.

1. INTRODUCTION

In presence of losses, gain or with open systems, the finite element discretization of the wave equation typically yields sparse non-hermitian eigenvalue problem depending smoothly on a single complex parameters ν

$$\mathbf{L}(\lambda(\nu), \nu)\mathbf{x}(\nu) = \mathbf{0}. \quad (1)$$

Here $\lambda(\nu)$ is the eigenvalue, $\mathbf{x}(\nu) \neq \mathbf{0}$ is the right eigenvector. In the context of duct acoustics, the description of waves is usually accomplished using mode decomposition and \mathbf{L} takes the form of a quadratic eigenvalue problem [1] and λ represents the axial wave number.

Depending on the configuration, the parameter can be, for instance, a wall impedance associated with a locally reacting liner or the effective density of a porous material as illustrated in Fig. 1. For some specific values ν^* the matrix (1) is defective, and in the case of an isolated defective

eigenvalue λ^* of algebraical multiplicity 2, we can anticipate the local behavior of the two branches of solution by using Puiseux series expansion [2]. These can be written formally as

$$\lambda_1(\nu) = \lambda^* + \sum_{k=1}^{\infty} a_k \left((\nu - \nu^*)^{\frac{1}{2}} \right)^k, \quad (2a)$$

$$\lambda_2(\nu) = \lambda^* + \sum_{k=1}^{\infty} a_k \left(-(\nu - \nu^*)^{\frac{1}{2}} \right)^k. \quad (2b)$$

As stated in [3], as long as $a_1 \neq 0$ the two branches coalesce at a branch point singularity as ν tends to ν^* . The branch point ν^* in the ν -complex plane is called an *exceptional point* [2].

In the vicinity of the EP, eigenvalues coalesce in opposite directions, and this has remarkable effects on modal attenuation [1, 4, 5] and on stability issues like the flutter phenomenon [6]. EPs also arises in quantum mechanics [7] or in phase transition in PT-symmetric systems.

In this work, we are interested in finding ν^* as well in computing the coefficients of the Puiseux series up to a certain order.

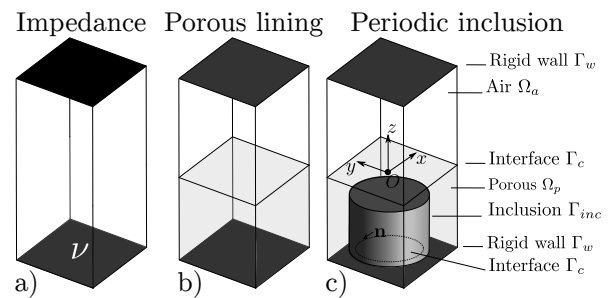


Figure 1. Examples of acoustical waveguides.

2. EP LOCALIZATION

The proposed algorithm, detailed in [8], exploits the knowledge of high order derivatives of two selected eigenvalues (λ_1, λ_2) calculated at an arbitrary value ν_0 . In order to circumvent the branch point singularity (2) two auxiliary

functions are defined

$$g(\nu) = \lambda_1 + \lambda_2, \quad \text{and} \quad h(\nu) = (\lambda_1 - \lambda_2)^2. \quad (3)$$

By construction, these functions are holomorphic in the vicinity of ν^* (as this was already mentioned in [7] and in [2, p. 66]). The method consists of 3 steps:

1. Computation of the derivatives of two selected eigenpairs $(\lambda_i, \mathbf{x}_i)$ ($i = 1, 2$) following the technique presented in [9]. This serves to construct the truncated Taylor series T_g and T_h of g and h with respect to the arbitrary value ν_0 . This step is the most computationally expensive.
2. Application of standard root-finding algorithms for polynomials in order to find the zeros of T_h and locate the branch point ν^* .
3. Find the connecting coefficients between the Taylor series T_g and T_h with those given by the Puiseux series (2).

To illustrate the method, we consider a bidimensional acoustic waveguide with one treated wall (at $y = 0$). The eigenvalue problem is obtained from the discretization of the weak formulation

$$(k^2 - \lambda)\langle \psi, \phi \rangle - \langle \psi', \phi' \rangle + \frac{1}{\nu} \psi(0)\phi(0) = 0. \quad (4)$$

As illustrated in Fig. 2, the spurious roots of T_h tend to be aligned on a circle (which is known to be connected with the radius of convergence of the series) and genuine zeros corresponding to the EP located inside the circle of convergence. The computed value ν^* is a good approximation of the true value with a relative error of 0.4% for a significantly distant initial value $\nu_0 \approx 2.3\nu^*$.

The proposed approach has been implemented in a dedicated open source python library available at github.com/nennigb/EasterEig. This lined waveguide example is included in `EasterEig` the test suite.

Work is ongoing to extend the radius of converge of the approximation and to tackle several parameters to locate higher order EPs. This algorithm can also be used to extend perturbation methods for real random eigenvalue problems, when *veering* is present as in [10], or more generally, to speed up parametric eigenvalue problem with explicit dependencies.

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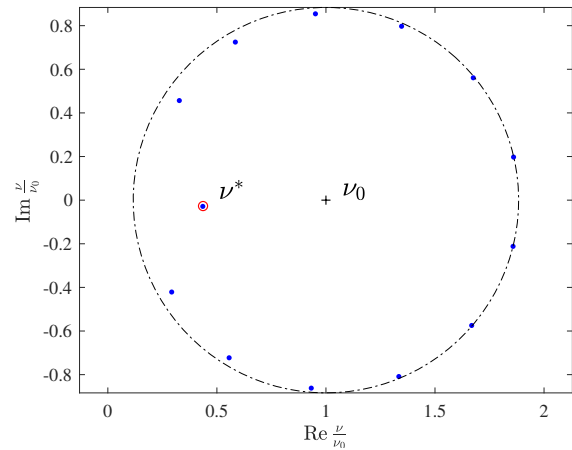


Figure 2. Roots of T_h . Example of EP localization in the case of locally reacting liner with 13 derivatives with $\nu_0 = 0.5825 - 0.6412i$. Roots of T_h (\cdot), initial value ν_0 ($+$) and Tester’s reference solution $\nu_{\text{ref}}^* = (1.6506 + 2.0599i)^{-1}$ (\circ) for the merging of the two least attenuated modes.