



**HAL**  
open science

# Acoustic fields in thermo-viscous binary gas mixtures: finite element modelling on adapted meshes

Cécile Guianvarc'H, Nicolas Joly

► **To cite this version:**

Cécile Guianvarc'H, Nicolas Joly. Acoustic fields in thermo-viscous binary gas mixtures: finite element modelling on adapted meshes. Forum Acusticum, Dec 2020, Lyon, France. pp.3467-3470, 10.48465/fa.2020.0499 . hal-03220567

**HAL Id: hal-03220567**

**<https://hal.science/hal-03220567>**

Submitted on 28 May 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# ACOUSTIC FIELDS IN THERMO-VISCOUS BINARY GAS MIXTURES: FINITE ELEMENT MODELLING ON ADAPTED MESHES

**Cécile Guianvarc'h**  
LCM LNE-Cnam  
La Plaine Saint Denis, France  
cecile.guianvarch@lecnam.net

**Nicolas Joly**  
LAUM UMR CNRS 6613  
Le Mans, France  
nicolas.joly@univ-lemans.fr

## ABSTRACT

The acoustic behaviour of thermo-viscous gas mixtures, both in proximity of walls and far from them, differs significantly from those of gases that are assumed to have homogeneous combination of constituents. Indeed, an acoustic field propagating in a gas mixture composed of multiple molecular species having different physical properties, involves several diffusive fields, namely momentum by shear, entropic and concentration variation fields. Unified analytical solutions to the fundamental equations of acoustics in thermo-viscous binary gas mixtures, subjected to non-slip, isothermal and vanishing concentration flux boundary conditions, have already been proposed in an infinite, semi-infinite and finite medium, throughout and beyond the boundary layers. These solutions highlight the strong coupling between the acoustic propagation and the heat and concentration diffusive fields in the boundary layers, while their effects appear to be simple additive processes in the bulk of the medium. However, relevant experimental validations for these analytical solutions are still missing.

A finite element solution for the acoustic behaviour of the gas mixture is computed in using a coupled formulation based on particle velocity, temperature and concentration variations. Because of the strong variations of all of these variables inside the boundary layers, anisotropic meshing technique with an adaptive process is required to interpolate the solution.

This numerical modelling procedure is applied here to describe a 1D acoustic propagation and the acoustic field in a 2D axisymmetrical enclosure, both considering a helium-argon gas mixture. The results then obtained are compared to analytical solutions in order to check their validity.

## 1. INTRODUCTION

When dealing with acoustic propagation in a gas mixture, one has to take into account not only the specific physical properties of the mixture (that are not all reduced to the simple weight mean of the properties of each component), but also specific and complex physical phenomena strongly coupled to the acoustic movement, including viscous, thermal, and molecular diffusion effects.

Basically, in a binary gas mixture submitted to a dynamic movement and/or a heat flow, the molecular species

constituting the mixture react in a different way depending almost on their respective molar masses and interatomic potentials. As a consequence, there is a mutual diffusion of one specie into the other one, involving a molecular flow (called concentration flow in the following) and local variations of each component of the mixture. Then, as described by Landau and Lifshitz [1], three different kinds of diffusive effects occur, related respectively to a gradient of concentration, a gradient of temperature (thermodiffusion) and a gradient of pressure (barodiffusion). Since an acoustic motion involves local variations of pressure, temperature and concentration, they are thus coupled to these three molecular diffusion phenomena.

Actually, molecular diffusion effects on sound absorption in air are negligible and have usually not to be considered in airborne acoustic applications [2]. However these effects become significant almost in disparate gas mixtures, such as He-Ar or He-Xe mixtures that are commonly used in thermoacoustic engines for their interesting Prandtl number [3, 4], and they have already been experimentally evidenced in such mixtures used for other purposes [5].

A unified analytical modelling procedure has previously been suggested for acoustic fields in thermoviscous binary gas mixtures, taking into account viscous, thermal, and molecular diffusion effects throughout and beyond the boundary layers [6, 7]. A numerical method is presented here to describe the acoustic behaviour of thermoviscous binary gas mixtures at rest, relying on this coupled formulation involving particle velocity, temperature and concentration variations. The use of adaptive and anisotropic meshing techniques is required to efficiently interpolate the solution [8, 9]. The numerical solutions computed here for describing acoustic fields in 1-D and 2-D axisymmetrical geometries can be used to check the validity of the analytical solutions previously suggested [6, 7].

## 2. EQUATIONS OF ACOUSTICS IN THERMO-VISCOUS BINARY GAS MIXTURES AT REST

### 2.1 Formulation for linear, harmonic acoustics

The mixture considered here is composed of two gases denoted 1 and 2, of molar masses  $M_1$  and  $M_2$  and molar concentrations  $x$  and  $(1 - x)$  respectively.

The set of equations describing the acoustic behaviour of the binary gas mixture are derived from the fundamen-

tal equations of hydrodynamics and thermodynamics available in references [1, 6, 7]:

- (a) the Navier-Stokes equation remains unchanged in a gas mixture respect to a pure gas;
- (b) the mass conservation equation is written for the mixture and for each component;
- (c) the molecular (or mutual) diffusion equation of component 1 in component 2 is related to the pressure variations of the mixture (barodiffusion) and to the heat flows (thermodiffusion) in the mixture during an acoustic cycle;
- (d) the heat diffusion equation involves the concentration flows of components 1 and 2 related to the heat flows in the mixture;
- (e) the equation of state is written assuming the gas mixture to be an ideal gas.

For the sake of modelling acoustics in thermo-viscous binary gas mixture, we only need to keep the variables associated to the particle velocity  $\mathbf{v}$ , the temperature variation  $\tau$  and the mass concentration variation of component 1  $c$ . Considering an harmonic movement of angular frequency  $\omega$ , the acoustic behaviour of the gas mixture at rest is then fully described by a set of linear equations as follows (at the first order of small perturbations):

$$\omega^2 \mathbf{v} + B \nabla (\nabla \cdot \mathbf{v}) + C \nabla \wedge \nabla \wedge \mathbf{v} = D \nabla \tau + E \nabla c, \quad (1)$$

$$Nc + M \nabla \cdot (\nabla c) = U \nabla \cdot (\nabla \tau) + V \nabla \cdot \mathbf{v}, \quad (2)$$

$$S\tau + Q \nabla \cdot (\nabla \tau) = Pc + R \nabla \cdot \mathbf{v}, \quad (3)$$

all the coefficients of these equations being expressed as functions of  $\omega$  and of the gas mixture's physical properties, namely static pressure, temperature, and density, molar masses of mixture and each component, molar fraction of each component, heat capacity ratio and transport properties of the mixture, mutual diffusion coefficient, barodiffusion and thermodiffusion ratios [1, 10].

If considering a propagation domain limited by a rigid wall, the boundary conditions for the particle velocity  $\mathbf{v}$  are usual Dirichlet conditions.

The previous equations are also subjected to homogeneous Dirichlet conditions for the temperature variation (isothermal condition) and for the concentration flow in the direction normal to the wall (denoted  $u$ ). Finally, the vanishing mass flow for each species allows us to write a linear relation linking the normal derivatives for variables  $\tau$  and  $c$  and the normal velocity at boundaries.

## 2.2 Finite element implementation

The operators of linear equations (1-3) are easily implemented in using the Finite Element Method, with 4 dofs per node in 2D or axisymmetrical models (for the two components of particle velocity  $\mathbf{v}$  and the two scalar fields  $\tau$  and  $c$ ). Homogeneous Dirichlet boundary conditions for the

normal component of  $\mathbf{v}$  and  $\tau$  are easily implemented, and the remaining linear condition linking the normal derivatives for variables  $\tau$  and  $c$  to the normal velocity at the boundary is taken into account in using the Lagrange multiplier technique. The numerical results presented are obtained with the FreeFEM code, including the anisotropic adaptive meshing procedure that allows to catch both propagative acoustic waves at large scale in the bulk of the fluid, and confined diffusive processes in the viscous, thermal and concentration inside the boundary layers, at much smaller scale [9].

## 3. APPLICATIONS

The problem presented in § 2.1 is solved here by the FEM suggested in § 2.2 for a couple of simple cases (1-D and 2-D axisymmetric acoustic fields). The results are compared to exact or simplified analytical solutions of the problem in order to highlight the validity of the approach.

The calculations are computed at 1 kHz considering a He-Ar mixture, with a helium molar concentration of 0.9, at a static pressure of 100 kPa and static temperature of 273 K. The physical properties of this mixture are calculated using the procedures and data from references [10–17]

### 3.1 Reflexion of a plane wave on a rigid wall

The case considered here is the 1-D propagation of an acoustic wave in the direction normal ( $u$  coordinate) to a rigid wall located at  $u = 0$ . In such a simple case, exact analytical solutions of the formulation described in § 2.1 can be written for variables  $\mathbf{v}$ ,  $\tau$  and  $c$ . The results then obtained are in perfect agreement with the numerical ones, allowing us to check the validity of the numerical implementation for solving equations (1-3). The numerical results obtained for an excitation velocity of 1 m/s at 20 cm of the rigid wall are plotted on Figure 1 close to the boundary layers for variables  $\mathbf{v}$ ,  $\tau$  and  $c$ .

These results illustrate for the first time the gradient of concentration variations close to the wall.

### 3.2 Acoustic field in a cylindrical duct

We consider here the acoustic field in a closed cylindrical duct of 1 cm radius and 5 cm length. The duct is closed with rigid walls, one end generating a vibration normal velocity (cosinus shaped) of maximum amplitude 1 m/s.

The problem is solved by the Finite Element Method described in § 2.2 in the 2-D axisymmetric domain and the results obtained close to the rigid end of the duct are illustrated on Figure 2 for the particle velocity, temperature variation and concentration variation fields, highlighting the viscous, thermal and molecular diffusive effects in the boundary layers.

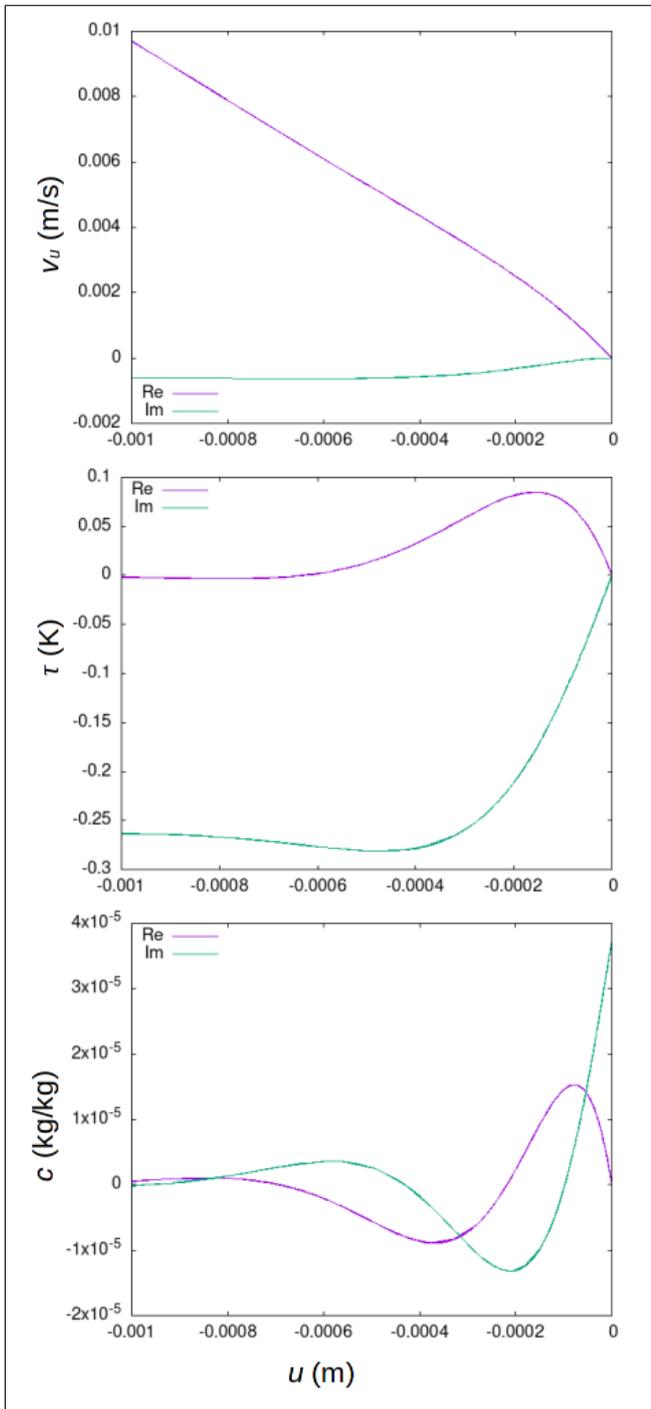
#### 4. CONCLUSION

The formulation suggested here offers analytical and numerical tools for solving practical problems involving acoustics in binary gas mixtures as described in References [6]. The analytical and numerical results obtained from this formulation in a couple of simple cases highlight the coupled viscous, thermal and molecular diffusive effects in the boundary layers. Such a tool will also allow us to check the validity of simplifying assumptions done until now in order to solve more complex problems.

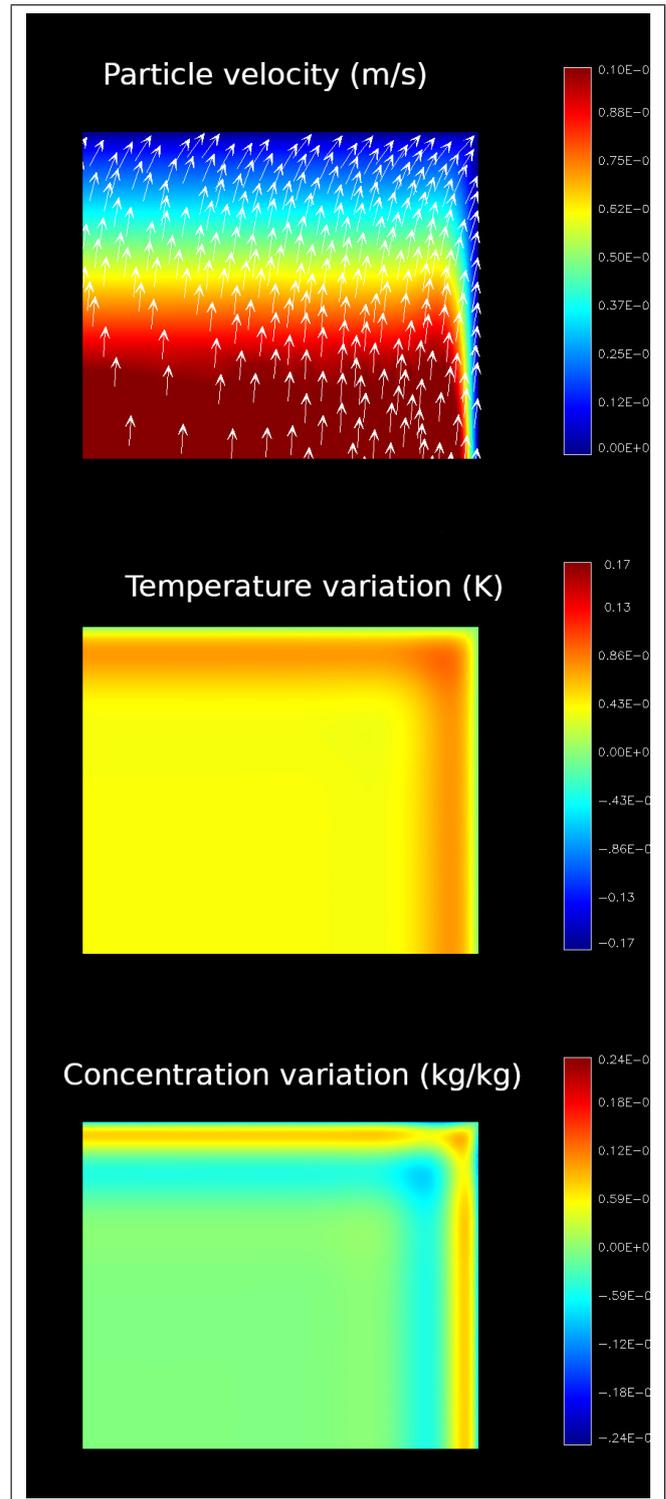
A next step will be to integrate in the formulation boundary conditions that take into account the effects of liquid film coating the wall and resulting concentration flows that can occur during an acoustic cycle when one of the mixture's component is a saturated vapour [7].

#### 5. REFERENCES

- [1] L. D. Landau and E. M. Lifshitz, *Fluid mechanics*. Pergamon Press, 1982.
- [2] Y. Rocard, "Sur l'amortissement des ondes sonores dans un milieu gazeux homogène," *Journal de Physique et Le Radium*, vol. 1, pp. 426–437, 1930.
- [3] G. W. Swift and P. S. Spoor, "Thermal diffusion and mixture separation in the acoustic boundary layer," *J. Acoust. Soc. Am.*, vol. 106, pp. 1794–1800, Oct. 1999.
- [4] D. A. Geller and G. W. Swift, "Saturation of thermoacoustic mixture separation," *J. Acoust. Soc. Am.*, vol. 111, pp. 1675–1684, 2002.
- [5] R. M. Gavioso, D. Madonna Ripa, C. Guianvarc'h, G. Benedetto, P. A. Giuliano Albo, R. Cuccaro, L. Pitre, and D. Truong, "Shell Perturbations of an Acoustic Thermometer Determined from Speed of Sound in Gas Mixtures," *Int. J. Thermophys.*, vol. 31, pp. 1739–1748, Sept. 2010.
- [6] C. Guianvarc'h and M. Bruneau, "Acoustic fields in binary gas mixtures: Mutual diffusion effects throughout and beyond the boundary layers," *J. Acoust. Soc. Am.*, vol. 131, pp. 4252–4262, June 2012.
- [7] C. Guianvarc'h, M. Bruneau, and R. M. Gavioso, "Acoustics and precondensation phenomena in gas-vapor saturated mixtures," *Phys. Rev. E*, vol. 89, p. 023208, Feb. 2014.
- [8] N. Joly, M. Bruneau, and R. Bossart, "Coupled Equations for Particle Velocity and Temperature Variation as the Fundamental Formulation of Linear Acoustics in Thermo-Viscous Fluids at Rest," *Acta Acust. united Ac.*, vol. 92, p. 8, 2006.
- [9] N. Joly, "Finite Element Modeling of Thermoviscous Acoustics on Adapted Anisotropic Meshes: Implementation of the Particle Velocity and Temperature Variation Formulation," *Acta Acust. united Ac.*, vol. 96, pp. 102–114, 2010.
- [10] S. Chapman and T. G. Cowling, *The mathematical theory of non-uniform gases*. Cambridge University Press, 1970.
- [11] R. A. Aziz, "A highly accurate interatomic potential for argon," *J. Chem. Phys.*, vol. 99, p. 4518, 1993.
- [12] J. B. Mehl, "Ab initio properties of gaseous helium," *C. R. Physique*, vol. 10, pp. 859–865, Nov. 2009.
- [13] J. J. Hurly, J. W. Schmidt, S. J. Boyes, and M. R. Moldover, "Virial Equation of State of Helium, Xenon, and Helium-Xenon Mixtures from Speed-of-Sound and Burnett PpT Measurements," *Int. J. Thermophys.*, vol. 18, pp. 579–634, 1997.
- [14] M. L. Martin, R. D. Trengrove, K. R. Harris, and P. J. Dunlop, "Excess second virial coefficients of some dilute gas mixtures," *Aust. J. Chem.*, vol. 35, p. 1525, 1982.
- [15] E. A. Mason and H. Ubisch, "Thermal conductivity of rare gas mixtures," *Phys. Fluids*, vol. 3, p. 355, 1960.
- [16] F. W. Giacobbe, "Estimation of Prandtl numbers in binary mixtures of helium and other noble gases," *J. Acoust. Soc. Am.*, vol. 96, pp. 3568–3580, 1994.
- [17] K. P. Srivastava, "Mutual diffusion of binary mixtures of helium, argon and xenon at different temperatures," *Physica*, vol. 25, pp. 571–578, 1959.



**Figure 1.** Particle velocity, temperature and concentration variations associated to a 1-D acoustic wave propagating in a He-Ar mixture, in a direction normal to a rigid wall located at  $u = 0$ .



**Figure 2.** Amplitudes of the particle velocity, temperature variation and concentration variation fields in a He-Ar mixture in a corner of a cylindrical duct with rigid walls (3 mm from lateral wall and 2.5 mm from top wall).