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1 Dynamical footprints of Hurricanes in the Tropical Dynamics

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18 Hurricanes — and more broadly tropical cyclones — are high-impact weather phenomena
19 whose adverse socio-economic and ecosystem impacts affect a considerable part of the
20 global population. Despite our reasonably robust meteorological understanding of tropical
21 cyclones, we still face outstanding challenges for their numerical simulations. Conse-
22 quently, future changes in the frequency of occurrence and intensity of tropical cyclones
23 are still debated. Here, we diagnose possible reasons for the poor representation of tropical
24 cyclones in numerical models, by considering the cyclones as chaotic dynamical systems.
25 We follow 197 tropical cyclones which occurred between 2010 and 2020 in the North At-
26 lantic using the HURDAT2 and ERA5 datasets. We measure the cyclones instantaneous
27 number of active degrees of freedom (local dimension) and the persistence of their sea-
28 level pressure and potential vorticity fields. During the most intense phases of the cyclones,
29 and specifically when cyclones reach hurricane strength, there is a collapse of degrees of
30 freedom and an increase in persistence. The large dependence of hurricanes dynamical
31 characteristics on intensity suggests the need for adaptive parametrisation schemes which
32 take into account the dependence of the cyclone’s phase, in analogy with high-dissipation
33 intermittent events in turbulent flows.

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34 I. LEAD PARAGRAPH

35 **Tropical cyclones are both high-impact weather events and challenging phenomena from**
36 **the point of view of numerical modelling. While their lifecycle is relatively well understood,**
37 **there are still difficulties in the representation of their dynamics in weather and climate mod-**
38 **els, and in drawing robust conclusions on how different climate conditions may affect their**
39 **frequency of occurrence and intensity. Here, we consider tropical cyclones as chaotic dynam-**
40 **ical systems. We show that the formation of particularly intense cyclones, termed hurricanes**
41 **in the North Atlantic, coincides with a reduction of the phase space of the atmospheric dy-**
42 **namics to a low-dimensional and persistent object, where few rotational kinetic degrees of**
43 **freedom dominate the dynamics. This suggests the need for adaptive parameterisations to**
44 **integrate the governing equations when simulating intense tropical cyclones in numerical**
45 **climate models.**

46 II. INTRODUCTION

47 Tropical cyclones are high-impact extreme weather events. For example, they are the costli-
48 est natural disaster category in the United States^{1,2}, with the damage related to hurricane Katrina
49 (2005) alone amounting to about 1% of the gross domestic product of the country². Trends in
50 the frequency of occurrence and intensity of tropical cyclones are difficult to discern in observa-
51 tions because of their relative rarity and of the brevity of highly spatially and temporally resolved
52 datasets, which rely on satellite observations³. Projections of future climates indicate an increase
53 in the intensity of tropical cyclones in the North Atlantic sector, albeit only with medium confi-
54 dence⁴. Indeed, reproducing the dynamics of the most severe events is difficult even in the most
55 advanced global or regional climate models⁵. For example, while mid-latitude synoptic dynamics
56 mostly originate from the chaotic structure of the motions associated with baroclinic instability^{6,7},
57 tropical cyclones are characterized by a rapid organization of convectively unstable flows whose
58 dynamics is turbulent and highly sensitive to boundary conditions⁸. To understand the reasons
59 for the poor representation of tropical cyclones in numerical models, we adopt a dynamical sys-
60 tem methodology which represents the cyclones as states of a chaotic, high-dimensional system.
61 We specifically compute two metrics reflecting instantaneous properties of the cyclones, namely
62 persistence and local dimension. Local dimension is a proxy for the system's number of active

63 degrees of freedom, and can be linked to the system’s predictability^{9–11}. Persistence provides in-
64 formation about the dominant time scale of the dynamics. Both metrics may easily be applied
65 to large datasets, such as climate reanalyses. They have recently provided insights on a number
66 of geophysical phenomena, including transitions between transient metastable states of the mid-
67 latitude atmosphere^{9,12}, palaeoclimate attractors^{13,14}, slow earthquake dynamics¹⁵ and changes in
68 mid-latitude atmospheric predictability under global warming¹⁶.

69 All these applications have taken an Eulerian point-of-view, focusing on a fixed spatio-temporal
70 domain. Here, we provide the first application of the two metrics from a (semi)-Lagrangian per-
71 spective, by computing the persistence and local dimension of tropical cyclones which we track in
72 space and time. This approach is particularly suited to study the complex behavior of convectively
73 unstable flow systems (see, e.g., ¹⁷ and chapter 12 in¹⁸). After putting the tropical cyclones in
74 the dynamical system framework, we may investigate whether they act as generic points of the
75 phase space or whether their dynamics exhibits a peculiar behavior. In the first case, the numerical
76 parametrizations developed for generic tropical climate states should work well when applied to
77 small-scale features of tropical cyclones. In the second case, cyclones dynamical properties are
78 dependent on their intensity or dynamical phase (tropical, subtropical, extratropical). This suggests
79 that leading parametrizations designed for generic tropical convection will not work properly when
80 cyclones are present in a domain.

81 In the rest of the study, we compute the persistence and local dimension of tropical cyclones,
82 and use these to outline a strategy to improve their numerical simulation.

83 **III. OBSERVABLES FOR CYCLONE DYNAMICS**

84 The historical cyclone data are the "best track data" from the Atlantic HURDAT2 database¹⁹,
85 developed by the National Hurricane Center. This database provides, amongst other variables,
86 the location of tropical cyclones, their maximum winds, central pressure and categorisation. The
87 values are obtained as a post-storm analysis of all available data, collected both remotely and in-
88 situ. We specifically consider separately hurricanes (HU), tropical storms (TS) and post-tropical
89 cyclones associated with an extratropical transition (EX). We further use instantaneous potential
90 vorticity (PV) at 500 hPa and sea-level pressure (SLP) data from ECMWF’s ERA5 reanalysis²⁰.
91 For both datasets we make use of 6-hourly data, and additionally data at the time when the HUR-
92 DAT2 database displays a cyclone landfall; the ERA5 data is retrieved at a horizontal resolution

93 of 0.25° .

94 Our analysis includes all tropical cyclones classified in HURDAT2 from 2010 to 2020 included.
95 We use semi-Lagrangian observables, i.e. we select a horizontal domain around the tropical cyclone
96 location, of size $\sim 1200 \times 1200$ km (41×41 grid points in ERA5). The choice of SLP is motivated
97 by its widespread use in hurricane tracking²¹ and the fact that it is a first approximation of the
98 horizontal velocity streamfunction. The PV is often used in the study of tropical cyclones and
99 relates to their intensification and symmetry structure^{22,23}, and takes explicitly into account the
100 strength of the cyclones warm core. Indeed, PV may be viewed as a metric of latent heat release
101 and therefore of the intensity of the diabatic processes taking place in the tropical cyclones (cloud
102 formation, precipitation)^{24,25}. We specifically select mid-level PV, following for example^{26,27}.
103 As control parameter, we chose the maximum winds from HURDAT2, since this quantity can be
104 directly connected to the economic loss caused by tropical cyclones²⁸.

105 IV. A DYNAMICAL SYSTEMS VIEW OF TROPICAL CYCLONES

106 We follow tropical cyclones in phase space as states of a chaotic, high-dimensional dynamical
107 system. Each instantaneous state of the cyclone, as represented by a given atmospheric variable,
108 corresponds to a point in a reduced phase space (namely a special Poincaré section). We sample
109 these states at discrete points i , determined by the temporal resolution of the HURDAT2 data,
110 that is every 6h or whenever the HURDAT2 database displays a cyclone landfall. Our aim is
111 to diagnose the dynamical properties of the instantaneous (in time) and local (in phase-space)
112 states of the cyclone, as represented by the chosen atmospheric variable and geographical domain
113 (physical space in Fig. 1). To do so, we leverage two metrics issuing from the combination of
114 extreme value theory with Poincaré recurrences^{29–31}. We consider the ensemble $\{X_i\}$, which in
115 our analysis are SLP or PV maps of all timesteps i for all tropical cyclones in our dataset, always
116 centred on the cyclones location. We further consider a state of interest ζ , which would correspond
117 to a single SLP or PV map drawn from this dataset. We then define logarithmic returns as:

$$118 \quad g(X_i, \zeta) = -\log[\text{dist}(X_i, \zeta)] \quad (1)$$

119 Here, "dist" is the Euclidean distance between pairs of SLP or PV maps, but more generally it
120 can be any distance function between two vectors which tends to zero as the two vectors increas-
121 ingly resemble each other. We thus have a time series g of logarithmic returns which is large at

122 times i when X_i is close to ζ .

123 We next define exceedances as $\{u(\zeta) = g(X_i, \zeta) - s(q, \zeta) \forall i : g(X_i, \zeta) > s(q, \zeta)\}$, where
124 $s(q, \zeta)$ is a high threshold corresponding to the q th quantile of $g(X_i, \zeta)$. These are effectively
125 the previously-mentioned Poincaré recurrences, for the chosen state ζ (phase space in Fig. 1).
126 The Freitas-Freitas-Todd theorem^{29,30} states that the cumulative probability distribution $F(u(\zeta))$
127 is approximated by the exponential member of the Generalised Pareto Distribution. We thus have
128 that:

$$129 \quad F(u, \zeta) \simeq \exp \left[-\vartheta(\zeta) \frac{u(\zeta)}{\sigma(\zeta)} \right] \quad (2)$$

130 The parameters u , namely the exceedances, and σ , namely the scale parameter of the Gener-
131 alised Pareto Distribution, depend on the chosen state ζ , while ϑ is the so-called extremal index,
132 namely a measure of clustering³². We estimate it here using the Süveges Estimator³³.

133 From the above, we can define two dynamical systems metrics: local dimension (d) and per-
134 sistence (θ^{-1}). The local dimension is given by $d(\zeta) = 1/\sigma(\zeta)$, with $0 < d \leq +\infty$. When X_i
135 contains all the variables of the system, the estimation of d based on extreme value theory has a
136 number of advantages over traditional methods (e.g. the box counting algorithm³⁴). First, it does
137 not require to estimate the volume of different sets at different scales: the selection of $s(q)$ based
138 on the quantile provides a selection of different thresholds s which depends on the recurrence rate
139 around the point ζ . Moreover, it does not require the a-priori selection of the maximum embedding
140 dimension, as the observable g is always a univariate time-series. Even when X_i does not contain
141 all variables of the system, the estimation of d through extreme value theory is still a powerful tool
142 to compare different states of high-dimensional chaotic systems³⁵.

143 The persistence of the state ζ is measured via the extremal index $0 < \vartheta(\zeta) < 1$. We define the
144 inverse of the average residence time of trajectories around ζ as: $\theta(\zeta) = \vartheta(\zeta)/\Delta t$, with Δt being
145 the timestep of the underlying data (here 6 hours). Since the extremal index is non-dimensional,
146 $\theta(\zeta)$ has units of frequency. θ^{-1} is then a measure of persistence. If ζ is a fixed point of the
147 attractor $\theta(\zeta) = 0$. For a trajectory that leaves the neighborhood of ζ at the next time iteration,
148 $\theta = 1$. A caveat of our approach is that our dataset is constructed from a sequence of cyclones
149 which is not continuous in space-time. This may introduce a bias in our calculation of θ if the final
150 state of a cyclone is a recurrence of the initial state of the following cyclone. This is highly unlikely
151 due to the very different nature of the growth versus weakening stages of tropical cyclones. We
152 further note that this does not affect the computation of d , which is insensitive to time reshuffling.

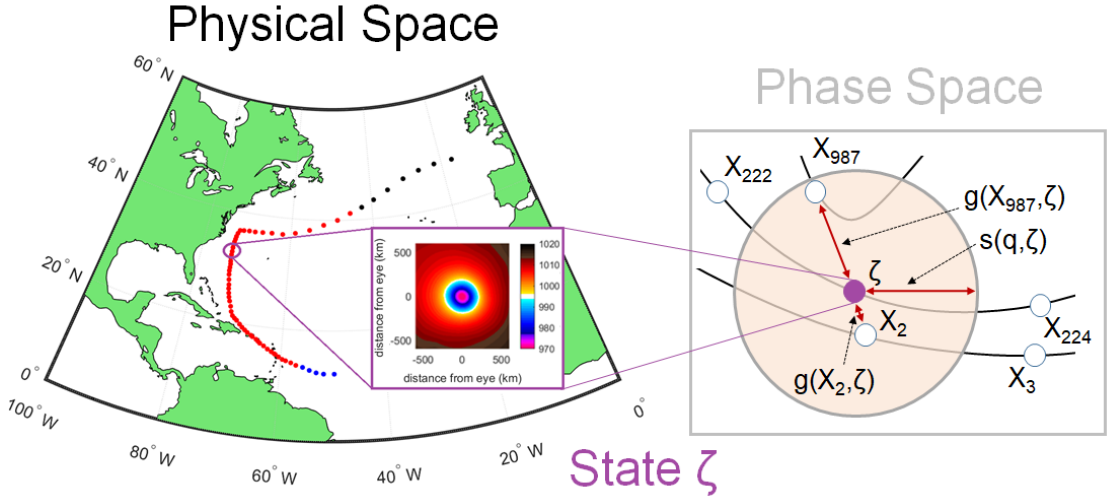


FIG. 1. Schematic of the computation of the dynamical systems metrics for an instantaneous state of a tropical cyclone. We take a snapshot of the cyclone in physical space (black quadrant), in this example a latitude-longitude map of sea-level pressure, which corresponds to state ζ in our reduced phase space. The right hand side panel shows the discrete sampling of the phase-space at points X_i (white circles). The shaded circle is a 2D representation of the hyper-sphere determined by the high threshold $s(q, \zeta)$, which defines recurrences. The logarithmic distances between measurements defined by $g(X_i, \zeta)$ are marked by double-headed arrows. For all points within the hyper-sphere, $g(X_i, \zeta) > s(q, \zeta)$ holds. In the schematic, only two measurements satisfy this condition (adapted from¹⁴).

153 While the derivation of d and θ^{-1} may seem very abstract, the two metrics can be related to
 154 the properties of the tropical cyclones. d is a proxy for the active number of degrees of freedom
 155 of the cyclones instantaneous states. On the other hand, θ^{-1} measures the persistence of such
 156 states and is related to the dominant time scale of the dynamics (the Lyapunov exponent³⁶). Both
 157 these quantities are known to be connected to the dynamical (Kolmogorov Sinai) entropy since the
 158 seminal work of Young³⁷.

159 **V. DYNAMICAL PROPERTIES OF TROPICAL CYCLONES: COLLAPSE OF**
 160 **DEGREES OF FREEDOM AND INCREASE IN PERSISTENCE IN INTENSE STORMS**

161 Before focusing on the analysis of the dynamics of tropical cyclones specifically, we assess
 162 the peculiarity of their dynamical footprints when compared with a box of the tropical Atlantic

163 ocean. We use ERA5 6h data for SLP and PV covering the period 2017-2021 and considered the
 164 squared horizontal domain spanning $10N < \text{Latitude} < 20N$ $-50W < \text{Longitude} < 40W$. This region is
 165 chosen because it is located in an area where there are tropical features (such as disorganized and
 166 organized convection, tropical waves, ...) but not strong trade winds, that is synoptic structures
 167 are not advected either to the east or to the west. These latitudes were mostly avoided by the
 168 traders in their trips to the Americas because of the absence of winds³⁸. We disregarded the choice
 169 of other tropical regions as they may be affected by stronger trade winds, or may include a wrong
 170 proportion of sub-tropical or extra-tropical features.

171 The results are shown in Figure 2. For SLP (Fig. 2a), that is the non cyclonic states do not
 172 feature any particular structure and they are characterized by non-persistent behavior ($\theta \lesssim 1$) and
 173 a range of dimensions similar to those of the tropical cyclones. For PV, at a first glance, there is no
 174 clear separation between control box and tropical cyclones of the distributions on the basis of the
 175 analysis of the diagrams (Fig. 2b) with d and θ spanning a similar range of values. On the other
 176 hand, the analysis of the violin plots presented in Fig. 2c-f) show that the distributions are different.
 177 To quantify this difference we apply a two-sided Cramer-von Mises test at the 0.05 significance
 178 level³⁹. The p-values found (virtually 0) imply that the null hypothesis that the two samples come
 179 from the same distribution can be rejected hinting to a statistically significant difference.

180 The previous analysis shows that the distribution of dynamical properties of tropical cyclones
 181 is significantly different from the one of a control box of the tropical Atlantic ocean. However,
 182 a fixed box control experiment, although in a region without strong average mean flow, is dif-
 183 ferent from a region which moves following cyclones centers. The HURDAT2 database used in
 184 this study divides cyclones timesteps in different categories which span tropical, subtropical and
 185 extra-tropical objects with different intensities. The first step to understand whether the dynamical
 186 indicators exhibit a dependence on cyclone intensities is therefore to compute averages of d and
 187 θ conditioned to HURDAT2 cyclones status. Results are shown in Table 1. For tropical (pink)
 188 cyclones status, we remark a monotonic decrease of the average d_{SLP} and θ_{SLP} from the inten-
 189 sity, supporting the idea that intense cyclones yield low dimensional and persistent states of the
 190 dynamics when tracked by SLP observable. When looking at 500 hPa PV dynamical indicators,
 191 there is a non-monotonic relationship between intensity and d_{PV} while θ_{PV} increases monotonon-
 192 ically with the intensity. For subtropical or extratropical cyclone status (turquoise in Table 1) we
 193 obtain similar results as tropical status for d_{SLP} , θ_{SLP} and θ_{PV} while d_{PV} monotonically increases.
 194 Finally we also report for completeness the other categories included in HURDAT2 in Table 1

195 (mustard) including Disturbances and Non-tropical lows. This first analysis suggests that there is
196 a dependence of the dynamical indicators from cyclones intensities and type.

197 To go more in depth, Figure 3a, b shows the values of dimension d and inverse persistence θ
198 computed on SLP and 500 hPa PV, with maximum winds in colours. The two local dimensions
199 show different ranges, with $d_{SLP} < 30$ and d_{PV} attaining higher values. This reflects the fact that
200 the PV dynamics involve multiple spatial scales, which reflect several underlying phenomena com-
201 ing from convective and larger-scale aspects of cyclones and tropical dynamics, e.g. atmospheric
202 waves⁴⁰. SLP, on the other hand, reflects the synoptic-scale structures ($\sim 10^3$ km). The range of
203 local dimensions found is relatively low compared to the number of grid-points used, which is 41
204 $\times 41$. This means that the majority of the degrees of freedom are frozen when we follow coherent
205 convective phenomena such as tropical cyclones. Moreover, lag-0 cross correlation coefficient
206 between d_{SLP} and d_{PV} is 0.23, suggesting that the two variables carry different information. The
207 persistence range is also different for SLP and PV, with $0.1 < \theta_{SLP} < 1$ and $0.3 < \theta_{PV} < 0.8$. In
208 units of time, these values indicate an SLP persistence between 6 and 60 hours and a PV persis-
209 tence between 7.5 and 20 hours. A timescale of 1–2.5 days is consistent with the synoptic-scale
210 intensification of a cyclone, while timescales of a few hours to a day are consistent with changes
211 in the convective structure of a cyclone. The lag-0 cross correlation coefficient coefficient between
212 θ_{SLP} and θ_{PV} is 0.02, even lower than for d , again suggesting that the two carry different informa-
213 tion.

214
215 We now connect the values of d and θ for SLP and PV to the underlying physics of the storms
216 using the maximum wind speed. For SLP (Figure 3a) we note a strong dependence of θ on
217 the maximum winds. Low to moderate winds are associated with high θ , while stronger winds
218 correspond to lower θ . A weaker relation holds for d_{SLP} and maximum winds. For PV (Figure 3b),
219 strong winds match low d values and intermediate-to-high θ values. Thus, SLP suggests that
220 intense cyclones correspond to persistent states, while PV that they display a low local dimension
221 and intermediate-to-low persistence. Looking at the scatterplots and PDFs of the two dynamical
222 systems metrics conditioned on the HURDAT2 cyclone classification (Figure 3c, d), provides a
223 picture consistent with the above. For SLP, HU and EX display a markedly higher persistence
224 than TS. For PV, HU display a lower dimension and lower persistence than both TS and EX. The
225 medians of all PDFs are significantly different at the 1% level under a Wilcoxon rank sum test,
226 except for d_{SLP} for HU and EX (not shown). We interpret these dynamical system properties

Hurdat Entry	Short Description	d_{SLP}	θ_{SLP}	d_{PV}	θ_{PV}
	Control Box	5.4 ± 1.5	0.98 ± 0.02	17 ± 9	0.48 ± 0.07
WV	Tropical Waves	7.4 ± 2.2	0.87 ± 0.07	14 ± 5	0.47 ± 0.06
TD	Tropical Depressions (<34 kts)	6.3 ± 3	0.76 ± 0.16	19 ± 7	0.49 ± 0.08
TS	Tropical Storms (34-63 kts)	6.1 ± 2.3	0.7 ± 0.17	19 ± 7	0.53 ± 0.08
HU	Hurricanes (>64 kts)	5.6 ± 1.8	0.51 ± 0.19	14 ± 5	0.55 ± 0.08
SD	Subtropical Depressions (<34 kts)	6.2 ± 1.8	0.64 ± 0.18	17 ± 8	0.43 ± 0.07
SS	Subtropical Storms (>34 kts)	5.7 ± 1.3	0.54 ± 0.15	19 ± 6	0.47 ± 0.07
EX	Extratropical Cyclones	5.8 ± 2.2	0.43 ± 0.18	22 ± 7	0.50 ± 0.08
DB	Disturbances	5.5 ± 1.9	0.70 ± 0.15	18 ± 9	0.48 ± 0.08
LO	Non-tropical Lows	5.9 ± 2.6	0.71 ± 0.19	18 ± 7	0.46 ± 0.07

TABLE I. Average values of the dynamical indicators per class of HURDAT2 entries and for the control box. The error is given in terms of one standard deviation of the mean. Pink: tropical synoptic objects, turquoise: subtropical or extratropical objects, moustard: none of the previous categories, fluo green: control box values.

227 as follows. When the storms produce strong winds and diabatic phenomena (HU with high PV
228 values and strong precipitation), the convective-scale dynamics collapses to an object with few
229 degrees of freedom (low d_{PV}), yet low persistence (high θ_{PV}). Nonetheless, the synoptic-scale
230 HU field is highly persistent (low θ_{SLP}), with values comparable to those of EX. SLP reflects a
231 quasi-symmetrical horizontal cyclonic structure, which for both HU and EX is characteristic of
232 the cyclone over an extended period of time. Weaker TS likely do not have a coherent cyclonic
233 core throughout their life cycle, as reflected in the high values of θ_{SLP} .

234 The mean SLP and PV footprints of the system are qualitatively similar across all three cy-
235 clone categories (Fig. 4), although EX show a larger spatial scale than both TS and HU. In all
236 three cases, the structures are roughly axisymmetric, showing that the EX cyclones included in
237 HURDAT2 still retain tropical-like characteristics. Clearer differences emerge when looking at the
238 standard deviation of the SLP and PV maps, computed at each gridpoint over all maps included
239 in our analysis (Fig. 5). Here, HU and TS show qualitatively similar, axisymmetric structures,
240 while EX show a clear meridional asymmetry in SLP and a less marked zonal asymmetry in PV.
241 Notwithstanding the broad similarity in mean structure between three cyclone categories, the dy-

242 dynamical systems metrics are nonetheless able to differentiate their characteristics. This suggests
243 that they sample from the systems dynamic variability and other subtle differences that do not
244 emerge from the composite maps, such as the evolution of the system mean structure during the
245 different phases of its lifecycle.

246

247 VI. DYNAMICAL SYSTEMS METRICS AND RAPID INTENSIFICATION

248 We now investigate whether the same dynamical systems framework can be used to investigate
249 rapid intensification. Rapid intensification occurs when a tropical cyclone gains strength dramati-
250 cally in a short period of time⁴¹. This phenomenon, difficult to explain from a theoretical point of
251 view^{42,43}, results in an enhancement of the destructiveness potential of the cyclone and in a lower
252 predictability of its trajectory⁴⁴. Rapid intensification is usually quantified using the increment Δv
253 of maximum winds over 24h. According to this definition, a cyclone is rapidly intensifying (resp.
254 weakening) when $\Delta v > 35$ kts (resp. $\Delta v < -35$ kts). In phase space, rapid changes of the dynamics
255 correspond to approaching unstable regions of the attractor^{45,46}. Our working hypothesis is that
256 variations in the dynamical systems metrics may be able to track these transitions. Figures 6 and
257 7 show the values of (a) Δd and (b) $\Delta \theta$ associated with the rapid intensification or weakening of
258 the cyclones. The Δ are again computed over a period of 24 hours. Lateral panels show the PDFs
259 of Δd and (b) $\Delta \theta$ conditioned on the rapid weakening or intensification. In both Figures 6 and 7
260 the medians of all PDFs for rapid weakening or intensification are significantly different at the 1%
261 level under a Wilcoxon rank sum test, except for $\Delta \theta_{PV}$. Rapid intensification is associated with a
262 clear decrease of θ_{SLP} and a weak decrease of d_{PV} . In other words, there is a large coherence of
263 the dynamics of the cyclones tracked by the increased persistence of the SLP. This is accompanied
264 by a decrease of the degrees of freedom in PV. The rapid weakening displays instead a decreased
265 SLP persistence and a marked increase in d_{PV} .

266 VII. IMPLICATIONS OF THE RESULTS FOR THE NUMERICAL SIMULATION OF 267 HURRICANES

268 We now discuss our results in the framework of the dynamical systems theory established for
269 the indicators of persistence and dimensionality. From this viewpoint, high persistence and low

270 dimensional states are found at unstable fixed points of the dynamics. The link between unstable
271 fixed points and persistence has been established in theorem 4.2.7 in³¹. The theorem states that
272 the extremal index θ is smaller than 1 at periodic points and that its value depends on the degree
273 of periodicity. The physical implication of the theorem is that the more stable the state, the closer
274 the value of θ to 0. The limiting case, $\theta = 0$ corresponds to an infinite cluster length, that is the
275 dynamics never leave the state, namely an equilibrium fixed point. If instead the value is close to
276 0 but not zero, the system sticks around the state for a long time, but it will eventually leave. This
277 is a property of fixed points that have at least one unstable direction through which the system can
278 leave the neighborhood. There is no formal theorem on the connection between a low dimension
279 and fixed points, but an argument based on synchronization in³⁵. In this study the local dimension
280 is computed for spatially extended systems: Coupled Lattice Maps (CLMs). These dynamical
281 systems are characterized by a coupling by adjacent sites. In the limit for extreme coupling, the
282 CLMs have a fixed point where all the dynamics is synchronized and $d = 1$ (one particle is in
283 the state of all the others). In real systems where perfect coupling does not exist, the states of
284 low dimension d also correspond to synchronized states. For both the cases, to the best of our
285 knowledge, it has not been proved that having a low θ and d is a sufficient condition for unstable
286 fixed points. Furthermore the phase space that we use in our study is rather unusual: i) we do not
287 consider the full set of variables but only two observables that project the dynamics of the cyclones
288 on a special low dimensional subset, ii) the domain is moving and it is centered on the eye of the
289 storm; yet it is not a Lagrangian phase space, because we only follow the eye and not each single
290 fluid parcel.

291 Besides the exact mathematical meaning of our results, they are useful to highlight some prac-
292 tical aspects related to the simulation of these objects in climate and weather models. The large
293 spread of the dynamical properties obtained in tropical cyclones and the strong dependency on
294 the intensity suggests that a parametrization independent on cyclone intensity may fail to resolve
295 their dynamics, especially for intense cyclones. Parameterizations are devised for typical states
296 of tropical dynamics (isolated thunderstorms), but not specifically for the organized states of the
297 most intense tropical cyclones. Hurricanes, i.e. intense tropical cycones, would then be analogous
298 to dissipative singularities of turbulent flows⁴⁷, or *black holes* of the atmospheric dynamics⁴⁸. In
299 these cases, the physics is far from that of the average states of the system, such that adaptive
300 scaling laws and targeted parametrizations are needed. Thus, the computation of the dynamical
301 systems metrics could support the development of hurricane-specific parameterizations.

302 As a caveat, we underline that our semi-Lagrangian approach does not allow to relate the
303 present results to the predictability of the trajectories of the tropical cyclones examined in this
304 study, unlike the Eulerian approach applied to extra-tropical motions in ^{9–11}. Furthermore, we
305 have used the ERA5 dataset, which has a fair but not highly-resolved representation of the con-
306 vective scales of hurricane dynamics.

307 To conclude, we have shown that the physical characteristics of tropical cyclones may be
308 understood in terms of dynamical systems metrics, which are capable of singling out peculiar
309 states of the dynamics. Our results support the idea that cyclones can be understood as being
310 reached along specific directions of the dynamics, consistent with instanton theory⁴⁹ and the no-
311 tion of melancholia states⁵⁰. This perspective opens intriguing possibilities, including the use
312 of importance sampling algorithms⁵¹ to select simulations which approach the hurricanes states
313 as detected from the dimension–persistence analysis in the phase space. For example, in⁵² we
314 propose a methodology, based on dimension and persistence metrics, to reconstruct the statistics
315 of cyclone intensities in coarse-resolution datasets, where maximum wind speed and minimum
316 sea-level pressure may not be accurately represented. We conclude that the dynamical systems
317 metrics outlined here could help to address several open problems in representing the climatology
318 of cyclone dynamics and provide strategies for their parametrization and their characterization in
319 climate simulations.

320

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331 IX. DATA AVAILABILITY

332 ERA5 data are available on the C3S Climate Data Store on regular latitude-longitude grids
333 at $0.25^\circ \times 0.25^\circ$ resolution at <https://cds.climate.copernicus.eu/#!/home>, accessed on
334 2022-02-23

335

336 HURDAT2 is a database provided by NOAA and freely available at https://www.aoml.noaa.gov/hrd/hurdat/Data_Storm.html, accessed on 2022-02-23

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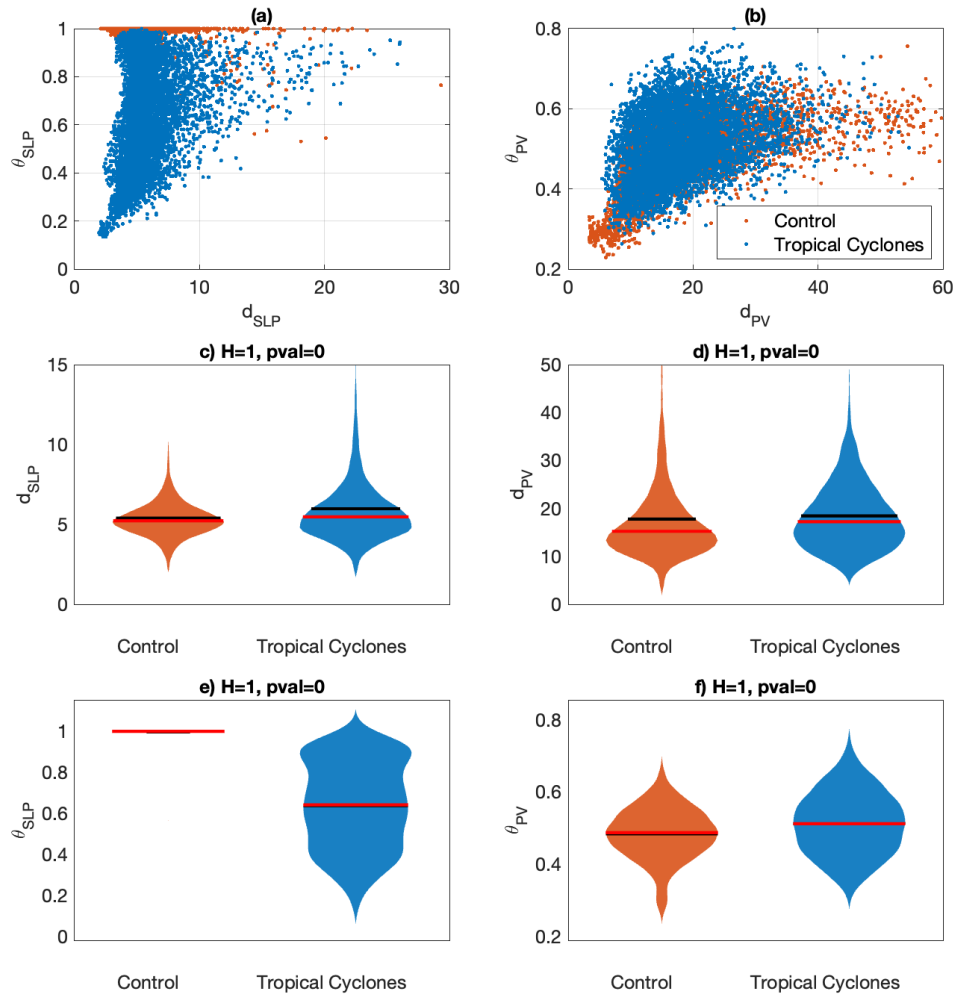


FIG. 2. Dimension d and inverse persistence θ for 6h hourly 2017-2021 ERA5 datasets on a control box $[10N < \text{lat} < 20N \ -50W < \text{lon} < 40W]$ (orange) and for the semilagrangian ERA5 data for tropical cyclones timesteps and center on the cyclones eye coordinates from HURDAT2 database (blue), calculated on sea-level pressure (SLP; a,c,e) and 500 hPa potential vorticity (PV; b, d,f). Panels (a,b) show the dimension-persistence diagrams; panels (c-f) show the violin plots (fatness of the patched area corresponds to the probability density) for the different dataset with the mean (red bars) and the black (medians). Note that the violin plots for the Control box in panel e) are not visible because all values are very close to 1.

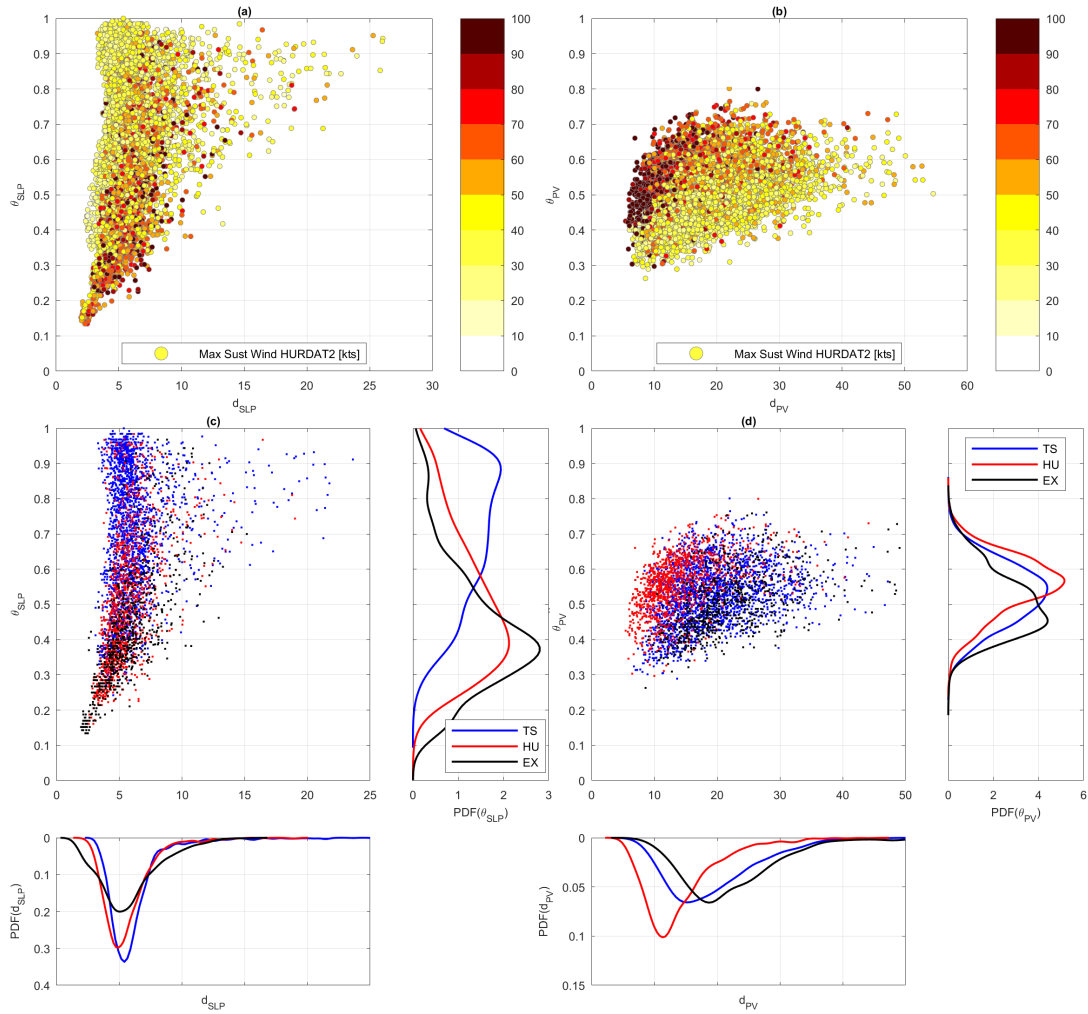


FIG. 3. Dimension d and inverse persistence θ of tropical cyclones, calculated on sea-level pressure (SLP; a,c) and 500 hPa potential vorticity (PV; b, d). The colourscales show maximum wind (a, b) and cyclone classification (c,d, see legend). Side panels show the corresponding PDFs. TS: Tropical Storm; HU: Hurricane; EX: Extratropical cyclones.

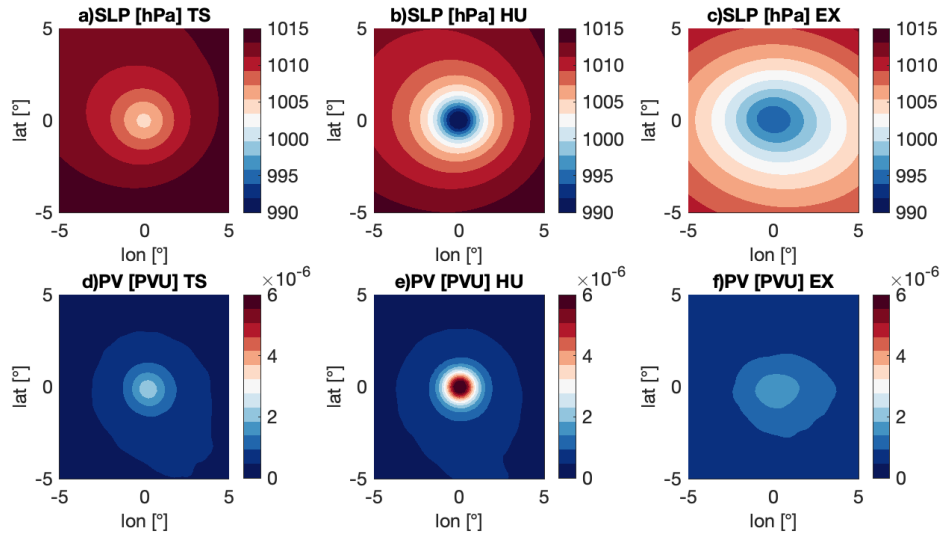


FIG. 4. Average sea-level pressure (SLP, hPa, a–c) and 500 hPa potential vorticity (PV, PVU, d–f) maps conditioned on cyclone classification (TS: Tropical Storm, a,d; HU: Hurricanes, b, e; EX: Extratropical cyclones, c,f).

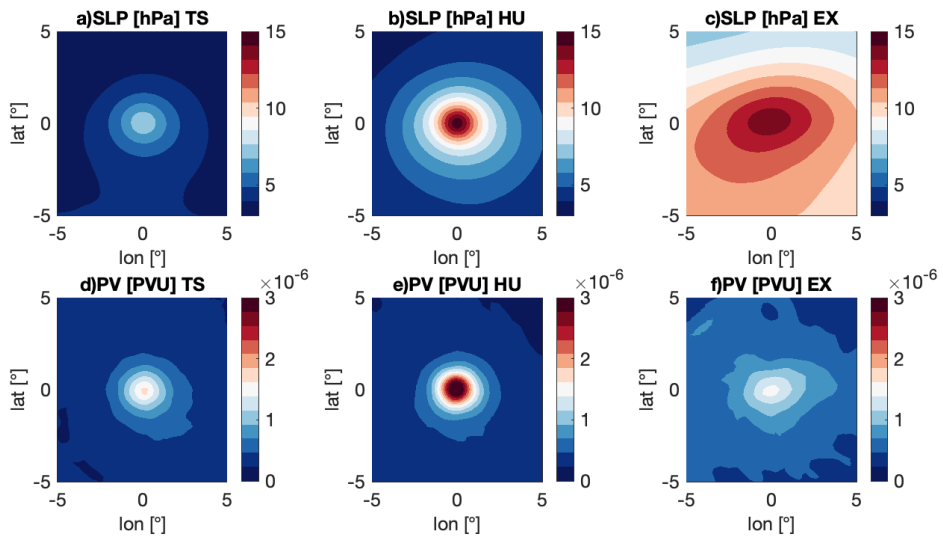


FIG. 5. Same as in Fig. 4, but for the standard deviation of the maps.

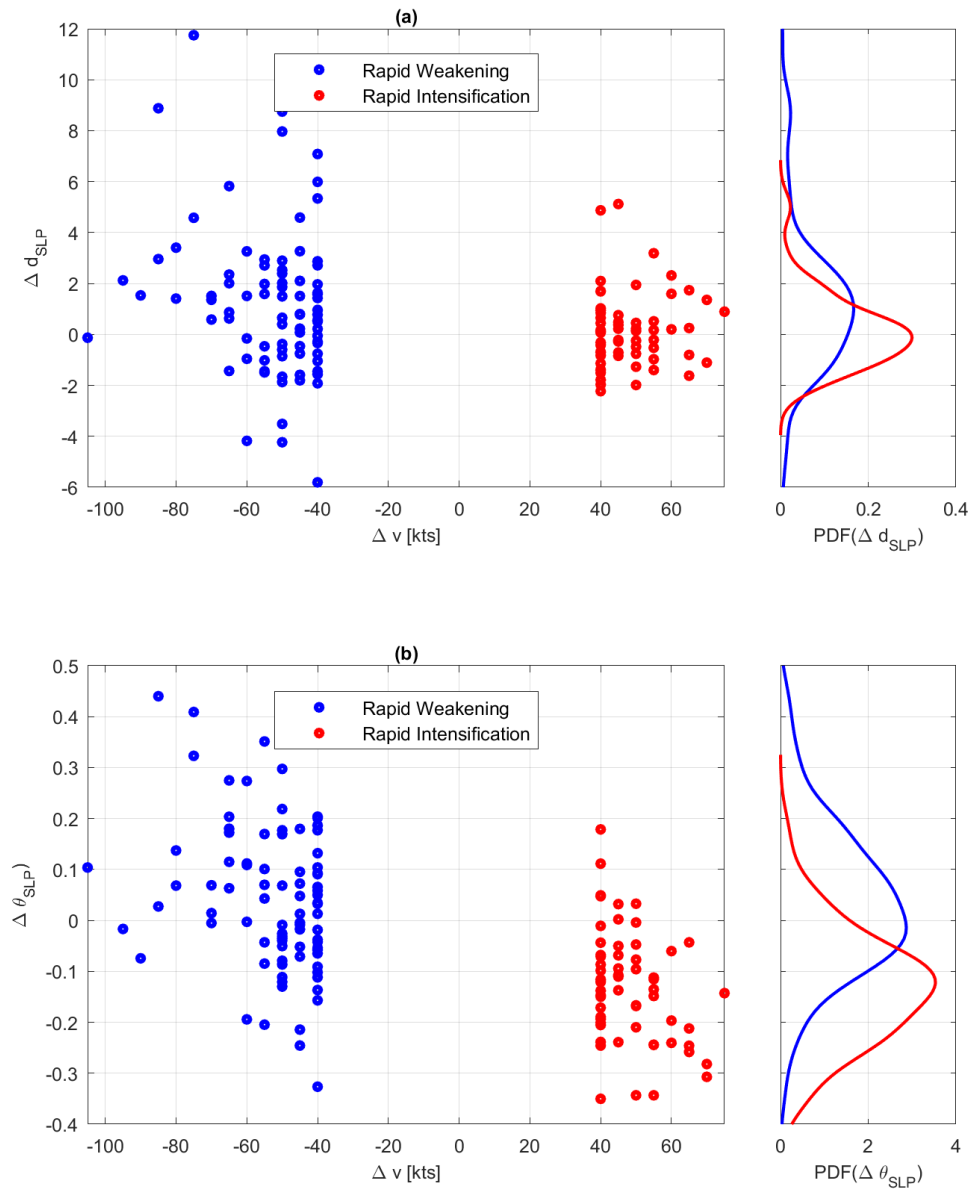


FIG. 6. 24h variation (Δ) of the dimension d (a) and of the inverse persistence θ (b) computed on SLP versus the 24h variation of maximum winds v for rapidly intensifying (blue) and rapidly weakening (red) cyclones. The side panel shows the corresponding PDFs.

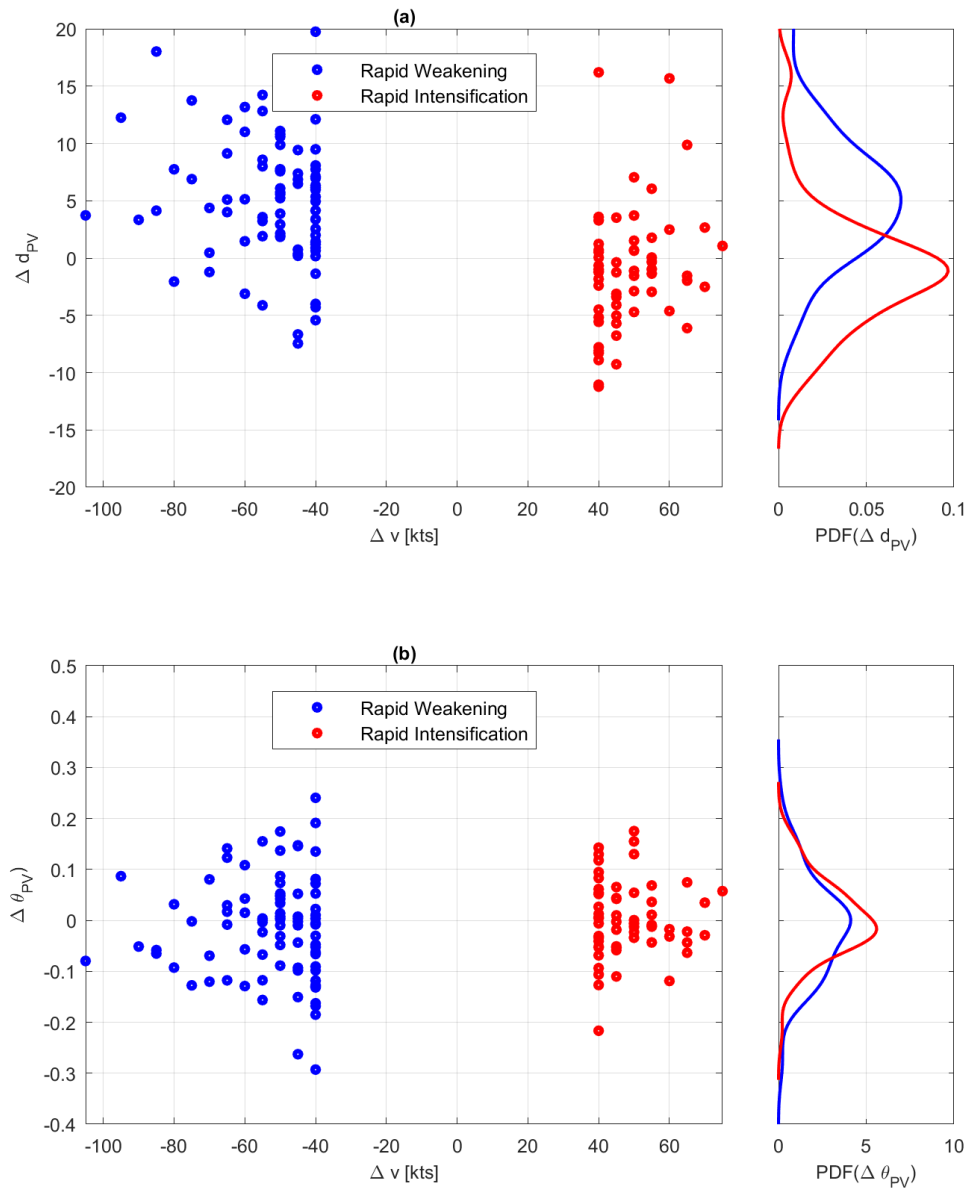


FIG. 7. Same as Fig. 6, but for d and θ computed on PV at 500 hPa.