# Hurricanes as unstable fixed points of the tropical atmospheric dynamics

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(Dated: 30 March 2022)

Hurricanes — and more broadly tropical cyclones — are high-impact weather phenomena whose adverse socio-economic and ecosystem impacts affect a considerable part of the global population. Despite having a reasonably robust meteorological understanding of tropical cyclones, their simulation in numerical models remains challenging. Consequently, future changes in the frequency of occurrence and intensity of tropical cyclones are still debated. Here, we diagnose possible reasons for the poor representation of tropical cyclones in numerical models, by considering the cyclones as chaotic dynamical systems. We follow 197 tropical cyclones which occurred between 2010 and 2020 in the North Atlantic using the HURDAT2 and ERA5 datasets. We measure the cyclones' instantaneous number of active degrees of freedom (local dimension) and the persistence of their sea-level pressure and potential vorticity fields. During the most intense phases of the cyclones, and specifically when cyclones reach hurricane strength, there is a collapse of degrees of freedom and an increase in persistence, hinting to the existence of an unstable fixed point of the dynamics. Hurricanes may thus be interpreted as unstable fixed points of rotational energy, and their evolution is well-captured by the potential vorticity map of the cyclone eye. In analogy with high-dissipation intermittent events in turbulent flows, this suggests strategies to improve numerical simulations of intense tropical cyclones, and specifically the need for adaptive parametrisation schemes.

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## I. LEAD PARAGRAPH

**Tropical cyclones are both high-impact weather events and challenging phenomena from the point of view of numerical modelling. While their lifecycle is relatively well understood, there are still difficulties in the representation of their dynamics in weather and climate models, and in drawing robust conclusions on how different climate conditions may affect their frequency of occurrence and intensity. Here, we consider tropical cyclones as chaotic dynamical systems. We show that the formation of particularly intense cyclones, termed hurricanes in the North Atlantic, coincides with a reduction of the phase space of the atmospheric dynamics to a low-dimensional object, where few rotational kinetic degrees of freedom dominate the dynamics. This behavior, also encountered in laboratory turbulent flows near strongly dissipative structures, is typical of unstable fixed points of high-dimensional dynamical systems. This analogy suggests the need for adaptive parameterisations to integrate the governing equations when simulating intense tropical cyclones in numerical climate models.**

## II. INTRODUCTION

Tropical cyclones are high-impact extreme weather events. For example, they are the costliest natural disaster category in the United States[1,2], with the damage related to hurricane Katrina (2005) alone amounting to about 1% of the gross domestic product of the country[2]. Trends in the frequency of occurrence and intensity of tropical cyclones are difficult to discern in observations because of their relative rarity and of the brevity of highly spatially and temporally resolved datasets, which rely on satellite observations[3]. Projections of future climates indicate an increase in the intensity of tropical cyclones in the North Atlantic sector, albeit only with medium confidence[4] as reproducing the dynamics of the most severe events is difficult even in the most advanced global or regional climate models[5]. Indeed, while mid-latitude synoptic dynamics mostly originate from the chaotic structure of the motions associated with baroclinic instability[6,7], tropical cyclones are characterized by a rapid organization of convectively unstable flows whose dynamics is turbulent and highly sensitive to boundary conditions[8]. To understand the reasons for the poor representation of tropical cyclones in numerical models, we adopt a dynamical system methodology which represents the cyclones as states of a chaotic, high-dimensional system. We specifically compute two metrics reflecting instantaneous properties of the cyclones, namely persistence and local dimension. Local dimension is a proxy for the system's number of active degrees of freedom, and can be linked to the system's predictability[9–11]. Persistence provides information about the dominant time scale of the dynamics. Both metrics may easily be applied to large datasets, such as climate reanalyses. They have recently provided insights on a number of geophysical phenomena, including transitions between transient metastable states of the mid-latitude atmosphere[9,12], palaeoclimate attractors[13,14], slow earthquake dynamics[15] and changes in mid-latitude atmospheric predictability under global warming[16].

All these applications have taken an Eulerian point-of-view, focusing on a fixed spatio-temporal domain. Here, we provide the first application of the two metrics from a (semi)-Lagrangian perspective, by computing the persistence and local dimension of tropical cyclones which we track in space and time. This approach is particularly suited to study the complex behavior of convectively unstable flow systems[17,18]. Our aim is to understand whether tropical cyclones — and especially the most intense ones — have an underlying structure similar to a generic point of the phase space or whether their dynamics has peculiar specificities. The first case would imply that numerical parametrizations developed for generic tropical climate states should work well when applied to small-scale features of tropical cyclones. The second case would imply that cyclones are unstable fixed points of the phase space, thus leading to the conclusion that parametrizations designed for generic climate states will not work properly. Indeed, fixed points have different time scales and phase-space directions with respect to a generic point, and thus call for a tailored treatment.

In the rest of the study, we compute the persistence and local dimension of tropical cyclones, and use these to outline a strategy to improve their numerical simulation.

## III. OBSERVABLES FOR CYCLONE DYNAMICS

The cyclone historical data are the "best track data" from the Atlantic HURDAT2 database[19], developed by the National Hurricane Center. This database provides, amongst other variables, the location of tropical cyclones, their maximum winds, central pressure and categorisation. The values are obtained as a post-storm analysis of all available data, collected both remotely and in-situ. We specifically consider separately hurricanes (HU), tropical storms (TS) and post-tropical cyclones associated with an extratropical transition (EX). We further use instantaneous potential vorticity (PV) at 500 hPa and sea-level pressure (SLP) data from ECMWF's ERA5 reanalysis[20].

For both datasets we make use of 6-hourly data; the ERA5 data is retrieved at a horizontal resolution of 0.25°.

Our analysis includes all tropical cyclones classified in HURDAT2 from 2010 to 2020 included. We use semi-Lagrangian observables, i.e. we select a horizontal domain around the tropical cycloe location, of size $\sim$1200 $\times$ 1200 km (41 $\times$ 41 grid points in ERA5). The choice of SLP is motivated by its widespread use in hurricane tracking[21] and the fact that it is a first approximation of the horizontal velocity streamfunction. The PV is often used in the study of tropical cyclones and relates to their intensification and symmetry structure[22,23], and takes explicitly into account the strength of the cyclones' warm core. Indeed, PV may be viewed as a metric of latent heat release and therefore of the intensity of the diabatic processes taking place in the tropical cyclones (formation of clouds, solid and liquid precipitation) [24,25]. We specifically select mid-level PV, following for example[26,27]. As control parameter we chose the maximum winds from HURDAT2, since this quantity can be directly connected to the economic loss caused by tropical cyclones[28].

## IV. A DYNAMICAL SYSTEMS VIEW OF TROPICAL CYCLONES

We follow tropical cyclones in phase space as states of a chaotic, high-dimensional dynamical system. Each instantaneous state of the cyclone, as represented by a given atmospheric variable, corresponds to a point in a reduced phase space (namely a special Poincaré section). We sample these states at discrete points $i$, determined by the temporal resolution of the HURDAT2 data, that is every 6h or whenever the HURDAT2 database displays a cyclone landfall. Our aim is to diagnose the dynamical properties of the instantaneous (in time) and local (in phase-space) states of the cyclone, as represented by the chosen atmospheric variable and geographical domain (physical space in Fig. 1). To do so, we leverage two metrics issuing from the combination of extreme value theory with Poincaré recurrences[29–31]. We consider the ensemble $\{X_i\}$, which in our analysis are SLP or PV maps of all timesteps for all tropical cyclones in our dataset, always centred on the cyclones' location. We further consider a state of interest $\zeta$, which would correspond to a single SLP or PV map drawn from this dataset. We then define logarithmic returns as:

$$g(X_i, \zeta) = -\log[\mathrm{dist}(X_i, \zeta)] \tag{1}$$

Here, *dist* is the Euclidean distance between pairs of SLP or PV maps, but more generally it can be any distance function between two vectors which tends to zero as the two vectors increasingly resemble each other. We thus have a time series $g$ of logarithmic returns which is large at times when $X_i$ is close to $\zeta$.

We next define exceedances as $u(\zeta) = g(X_i, \zeta) - s(q, \zeta)\ \forall\ g(X_i, \zeta_x) > s(q, \zeta)$, where $s(q, \zeta)$ is a high threshold corresponding to the q$th$ quantile of $g(X_i, \zeta)$. These are effectively the previously-mentioned Poincaré recurrences, for the chosen state $\zeta$ (phase space in Fig. 1). The Freitas-Freitas-Todd theorem[29,30] states that the cumulative probability distribution $F(u, \zeta)$ is approximated by the exponential member of the Generalised Pareto Distribution. We thus have that:

$$F(u, \zeta) \simeq \exp\left[-\vartheta(\zeta)\frac{u(\zeta)}{\sigma(\zeta)}\right] \tag{2}$$

The parameters $s(q, \zeta)$, namely the threshold, and $\sigma$, namely the scale parameter of the Generalised Pareto Distribution, depend on the chosen state $\zeta$, while $\vartheta$ is the so-called extremal index, namely a measure of clustering[32]. We estimate it here using the Süveges Estimator[33].

From the above, we can define two dynamical systems metrics: local dimension ($d$) and persistence ($\theta^{-1}$). The local dimension is given by $d(\zeta) = 1/\sigma(\zeta)$, with $0 \leq d \leq +\infty$. When $X_i$ contains all the variables of the system, the estimation of $d$ based on extreme value theory has a number of advantages over traditional methods (e.g. the box counting algorithm[34]). First, it does not require to estimate the volume of different sets in scale-space: the selection of $s(q)$ based on the quantile provides a selection of different thresholds $s$ which depends on the recurrence rate around the point $\zeta$. Moreover, it does not require the a-priori selection of the maximum embedding dimension, as the observable $g$ is always a univariate time-series. Even when $X_i$ does not contain all variables of the system, the estimation of $d$ through extreme value theory is still a powerful tool to compare different states of high-dimensional chaotic systems[35].

The persistence of the state $\zeta$ is measured via the extremal index $0 < \vartheta(\zeta) < 1$. We define the inverse of the average residence time of trajectories around $\zeta$ as: $\theta(\zeta) = \vartheta(\zeta)/\Delta t$, with $\Delta t$ being the timestep of the underlying data (here 6 hours). Since the extremal index is non-dimensional, $\theta(\zeta)$ has units of frequency. $\theta^{-1}$ is then a measure of persistence. If $\zeta$ is a fixed point of the attractor $\theta(\zeta) = 0$. For a trajectory that leaves the neighborhood of $\zeta$ at the next time iteration, $\theta = 1$. A caveat of our approach is that our dataset is constructed from a sequence of cyclones which is not continuous in space-time. This may introduce a bias in our calculation of $\theta$ if the final state of a cyclone is a recurrence of the initial state of the following cyclone. This is highly unlikely due to the very different nature of the growth versus weakening stages of tropical cyclones. We further note that this does not affect the computation of $d$, which is insensitive to time reshuffling.

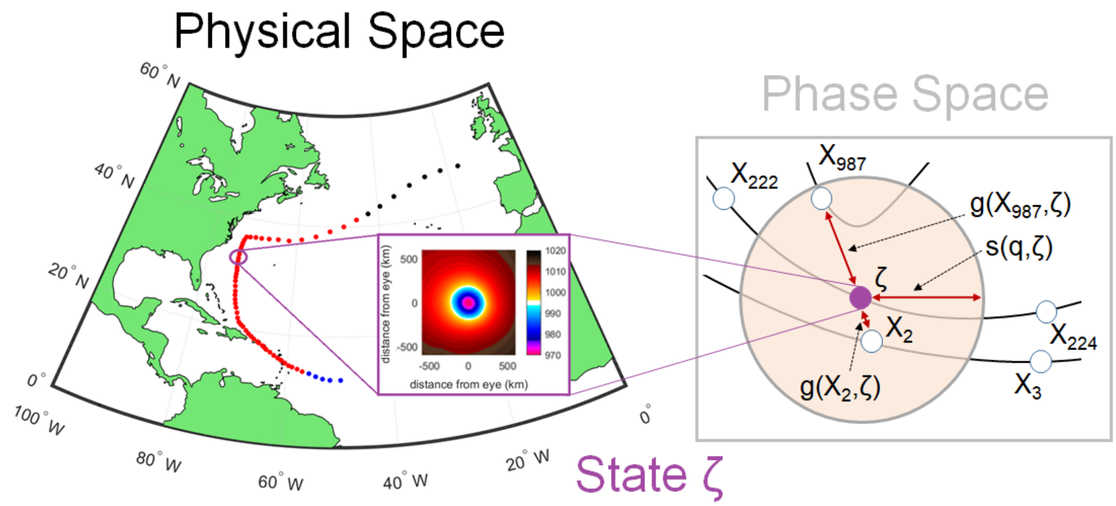


FIG. 1. Schematic of the computation of the dynamical systems metrics for an instantaneous state of a tropical cyclone. We take a snapshot of the cyclone in physical space (black quadrant), in this example a latitude-longitude map of sea-level pressure, which corresponds to state $\zeta$ in our reduced phase space. The right hand side panel shows the discrete sampling of the phase-space at points $X_i$ (white circles). The shaded circle is a 2D representation of the hyper-sphere determined by the high threshold $s(q,\zeta)$, which defines recurrences. The logarithmic distances between measurements defined by $g(X_i,\zeta)$ are marked by double-headed arrows. For all points within the hyper-sphere, $g(X_i,\zeta) > s(q,\zeta)$ holds. In the schematic, only two measurements satisfy this condition (adapted from[14]).

While the derivation of $d$ and $\theta^{-1}$ may seem very abstract, the two metrics can be related to the properties of the tropical cyclones. $d$ is a proxy for the active number of degrees of freedom of the cyclones' instantaneous states. On the other hand, $\theta^{-1}$ measures the persistence of such states and is related to the dominant time scale of the dynamics (the Lyapunov exponent[36]). Both these quantities are known to be connected to the dynamical (Kolmogorov Sinai) entropy since the seminal work of Young[37].

## V. DYNAMICAL PROPERTIES OF TROPICAL CYCLONES: COLLAPSE OF DEGREES OF FREEDOM AND PERSISTENCE IN INTENSE STORMS

Figure 2a, b shows the values of dimension $d$ and inverse persistence $\theta$ computed on SLP and 500 hPa PV, with maximum winds in colours. The two local dimensions show different ranges, with $d_{SLP} < 30$ and $d_{PV}$ attaining higher values. This reflects the richer spatial structure of the PV field at multiple spatial scales, which reflect both convective and larger-scale aspects of the cyclones. SLP instead reflects the synoptic-scale structures ($\sim 10^3$ km). The range of local dimensions found is relatively low compared to the number of grid-points used, which is $41 \times 41$. This means that the majority of the degrees of freedom are frozen when we follow coherent convective phenomena such as tropical cyclones. Moreover, the lag-0 cross-correlation coefficient between $d_{SLP}$ and $d_{PV}$ is 0.23, suggesting that the two variables carry different information. The persistence range is also different for SLP and PV, with $0.1 < \theta_{SLP} < 1$ and $0.3 < \theta_{PV} < 0.8$. In units of time, these values indicate an SLP persistence between 6 and 60 hours and a PV persistence between 7.5 and 20 hours. A timescale of 1–2.5 days is consistent with the synoptic-scale intensification of a cyclone, while timescales of a few hours to a day are consistent with changes in the convective structure of a cyclone. The lag-0 cross-correlation coefficient between $\theta_{SLP}$ and $\theta_{PV}$ is 0.02, even lower than for $d$, again suggesting that the two carry little mutual information.

We now connect the values of $d$ and $\theta$ for SLP and PV to the underlying physics of the storms using the maximum wind speed. For SLP (Figure 2a) we note a strong dependence of $\theta$ on the maximum winds. Low to moderate winds are associated with high $\theta$, while stronger winds correspond to lower $\theta$. A weaker relation holds for $d_{SLP}$ and maximum winds. For PV (Figure 2b), strong winds match low $d$ values and intermediate-to-high $\theta$ values. Thus, SLP suggests that intense cyclones correspond to persistent states, while PV that they display a low local dimension and intermediate-to-low persistence. Looking at the scatterplots and PDFs of the two dynamical systems metrics conditioned on the HURDAT2 cyclone classification (Figure 2c, d), provides a picture consistent with the above. For SLP, HU and EX display a markedly higher persistence than TS. For PV, HU display a lower dimension and lower persistence than both TS and EX. The medians of all PDFs are significantly different at the 1% level under a Wilkoxon ranksum test, except for $d_{SLP}$ for HU and EX (not shown). We interpret these dynamical system properties as follows. When the storms produce strong winds and diabatic phenomena (HU with high PV values and strong precipitation), the convective-scale dynamics collapses to an object with few degrees of freedom (low $d_{PV}$), yet low persistence (high $\theta_{PV}$). Nonetheless, the synoptic-scale HU field is highly persistent (low $\theta_{SLP}$), with values comparable to those of EX. SLP reflects a quasi-symmetrical horizontal cyclonic structure, which for both HU and EX is characteristic of the cyclone over an extended period of time. Weaker TS likely do not have a coherent cyclonic

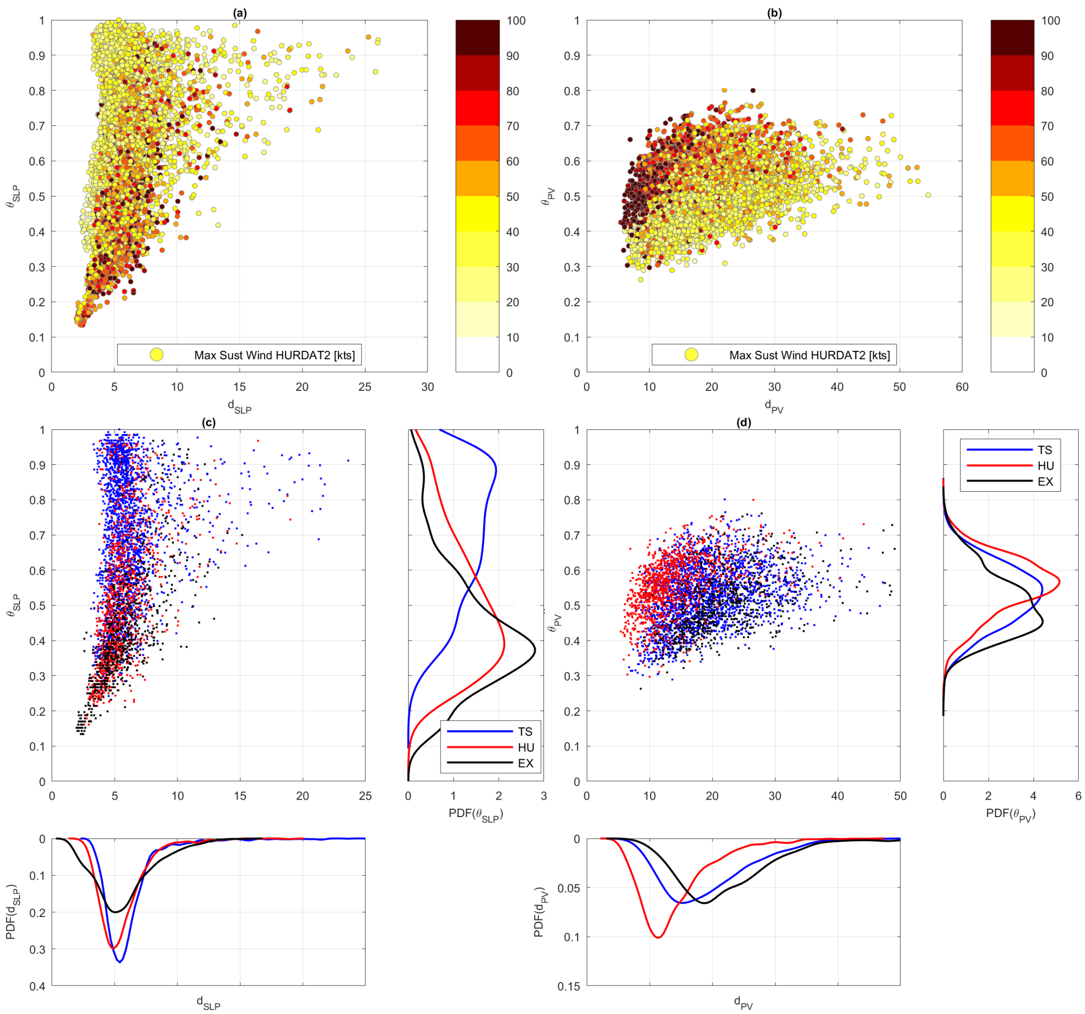


FIG. 2. Dimension $d$ and inverse persistence $\theta$ of tropical cyclones, calculated on sea-level pressure (SLP; a,c) and 500 hPa potential vorticity (PV; b, d). The colourscales show maximum wind (a, b) and cyclone classification (c,d, see legend). Side panels show the corresponding PDFs. TS: Tropical Storm; HU: Hurricane; EX: Extratropical cyclones.

core throughout their life cycle, as reflected in the high values of $\theta_{SLP}$. In the dynamical systems framework, the SLP and PV properties of the hurricanes may be interpreted as the signature of an unstable fixed point in the underlying phase-space, i.e. a state of the dynamics where the temporal and spatial scales are deformed. However, the different relationships between the dynamical indicators of SLP and PV with intense hurricanes make it difficult to understand the nature of the unstable fixed point (saddle or spiral type).

The mean SLP and PV footprints of the system are qualitatively similar across all three cy-

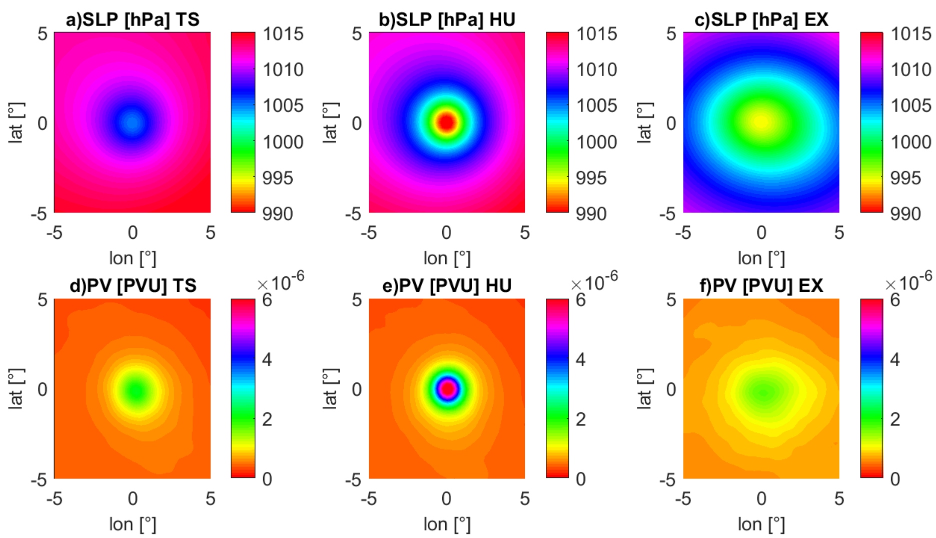


FIG. 3. Average sea-level pressure (SLP, hPa, a–c) and 500 hPa potential vorticity (PV, PVU, d–f) maps conditioned on cyclone classification (TS: Tropical Storm, a,d; HU: Hurricanes, b, e; EX: Extratropical cyclones, c,f).

clone categories (Fig. 3), although EX show a larger spatial scale than both TS and HU. In all three cases, the structures are roughly axisymmetric, showing that the EX cyclones included in HURDAT2 still retain tropical-like characteristics. Clearer differences emerge when looking at the standard deviation of the SLP and PV maps, computed at each gridpoint over all maps included in our analysis (Fig. 4). Here, HU and TS show qualitatively similar, axisymmetric structures, while EX show a clear meridional asymmetry in SLP and a less marked zonal asymmetry in PV. Notwithstanding the broad similarity in mean structure between three cyclone categories, the dynamical systems metrics are nonetheless able to differentiate their characteristics. This suggest that they sample from the systems' dynamic variability and other subtle differences that do not emerge from the composite maps, such as the evolution of the system's mean structure during the different phases of its lifecylce.

## VI. DYNAMICAL SYSTEMS METRICS AND RAPID INTENSIFICATION

We now investigate whether the same dynamical systems framework can be used to investigate rapid intensification. Rapid intensification occurs when a tropical cyclone gains strength dramati-

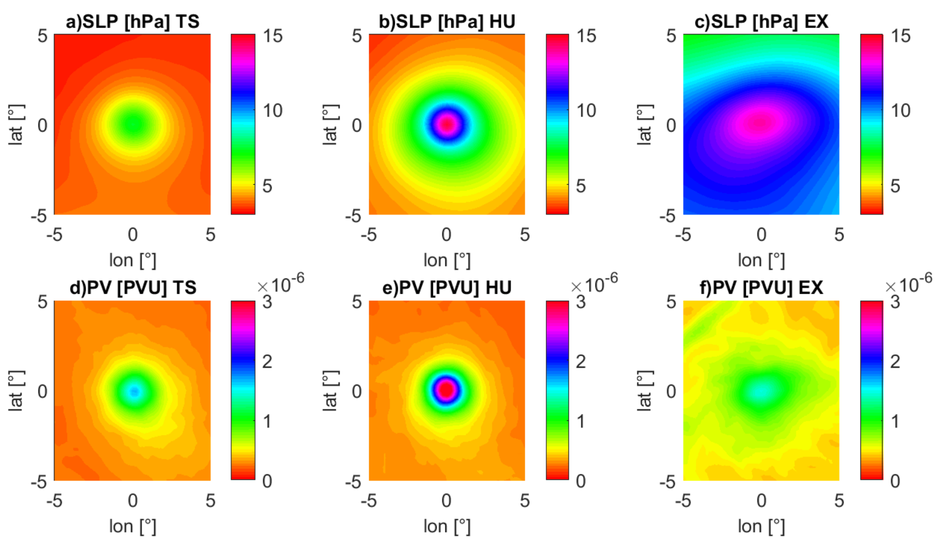


FIG. 4. Same as in Fig. 3, but for the standard deviation of the maps.

cally in a short period of time[38]. This phenomenon, difficult to explain from a theoretical point of view[39,40], results in an enhancement of the destructiveness potential of the cyclone and in a lower predictability of its trajectory[41]. Rapid intensification is usually quantified using the increment $\Delta v$ of maximum winds over 24h. According to this definition, a cyclone is rapidly intensifying (resp. weakening) when $\Delta v > 35$ kts (resp. $\Delta v < -35$ kts). In phase space, rapid changes of the dynamics correspond to approaching unstable fixed points or event to tipping to other basin of attraction[42,43]. Our working hypothesis is that variations in the dynamical systems metrics may be able to track these transitions. Figures 5 and 6 show the values of (a) $\Delta d$ and (b) $\Delta\theta$ associated with the rapid intensification or weakening of the cyclones. The $\Delta$ are again computed over a period of 24 hours. Lateral panels show the PDFs of $\Delta d$ and (b) $\Delta\theta$ conditioned on the rapid weakening or intensification. In both Figures 5 and 6 the medians of all PDFs for rapid weakening or intensification are significantly different at the 1% level under a Wilkoxon ranksum test, except for $\Delta\theta_{PV}$. Rapid intensification is associated with a clear decrease of $\theta_{SLP}$ and a weak decrease of $d_{PV}$. In other words, there is a large coherence of the dynamics of the cyclones tracked by the increased persistence of the SLP. This hints to the fact that dynamics approaches an unstable fixed point[44]. This is accompanied by a decrease of the degrees of freedom in PV, again consistent with approaching an unstable fixed point of the dynamics. The rapid weakening displays instead a decreased SLP persistence and a marked increase in $d_{PV}$. We interpret this as a departure from the neighbourhood of a fixed point towards the main basin of attraction of the tropical atmospheric dynamics.

## VII. IMPLICATIONS OF THE RESULTS FOR THE NUMERICAL SIMULATION OF HURRICANES

From a dynamical systems viewpoint, high persistence and low dimensional states are found at unstable fixed points of the dynamics. Properties such as the local entropy, persistence and number of active degrees of freedom are greatly affected in the proximity of unstable fixed points, coinciding with deformation of the typical spatial and temporal lengths of the dynamics. This phase-space phenomenon is reminiscent of what is observed in physical space, when approaching singularities of turbulent dynamics with well-identified front-like or spiral-like coherent structures accompanying a point of very strong dissipation[45,46]. Although dynamics at fixed points can be fully resolved when having a perfect model of the underlying dynamics, unstable fixed points are by nature fragile to noise or approximation in the sense that any perturbation will escape following unstable directions. This may explain from a dynamical systems viewpoint why it is so difficult to obtain an adequate representation of intense tropical cyclones in climate models. Parameterisations are devised for typical states of tropical dynamics (disorganized storms), but not specifically for the organized states of the most intense tropical cyclones. Hurricanes would then be analogous to dissipative singularities of turbulent flows[45], or *black holes* of the atmospheric dynamics[46]. In these cases, the physics is far removed from that of the average states of the system, such that adaptive scaling laws and targeted parametrizations are needed. Thus, the computation of the dynamical systems metrics could support the development of hurricane-specific parameterizations.

As a caveat, we underline that our semi-Lagrangian approach does not allow to relate the present results to the predictability of the trajectories of the tropical cyclones examined in this study, unlike the Eulerian approach applied to extra-tropical motions in [9–11]. Furthermore, here we have used the ERA5 dataset which has a fair but not highly-resolved representation of the convective scales of hurricane dynamics.

To conclude, we have shown that the physical characteristics of tropical cyclones may be understood in terms of dynamical systems metrics, which are capable of singling out peculiar states of the dynamics. Our results support the idea that cyclones can be understood as being reached along specific directions of the dynamics, consistent with instanton theory[47] and the notion of melancholia states[48]. This perspective opens intriguing possibilities, including the use of importance sampling algorithms[49] to select simulations which approach the hurricanes' fixed points as detected from the dimension–persistence analysis in the phase space.

## VIII. ACKNOWLEDGMENTS

The authors acknowledge the support of the INSU-CNRS-LEFE-MANU grant (project DINCLIC), as well as the grant ANR-19-ERC7-0003 (BOREAS). This work has received support from the European Union's Horizon 2020 research and innovation programme (Grant agreement No. 101003469, XAIDA) and from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant agreement No. 948309, CENÆ project). B. Dubrulle was partly supported by the ANR, project EXPLOIT (grant agreement No. ANR-16-CE06-0006-01).

## IX. DATA AVAILABILITY

ERA5 data are available on the C3S Climate Data Store on regular latitude-longitude grids at 0.25° x 0.25° resolution at `https://cds.climate.copernicus.eu/#!/home`, accessed on 2022-02-23

HURDAT2 is a database provided by NOAA and freely available at `https://www.aoml.noaa.gov/hrd/hurdat/Data_Storm.html`, accessed on 2022-02-23

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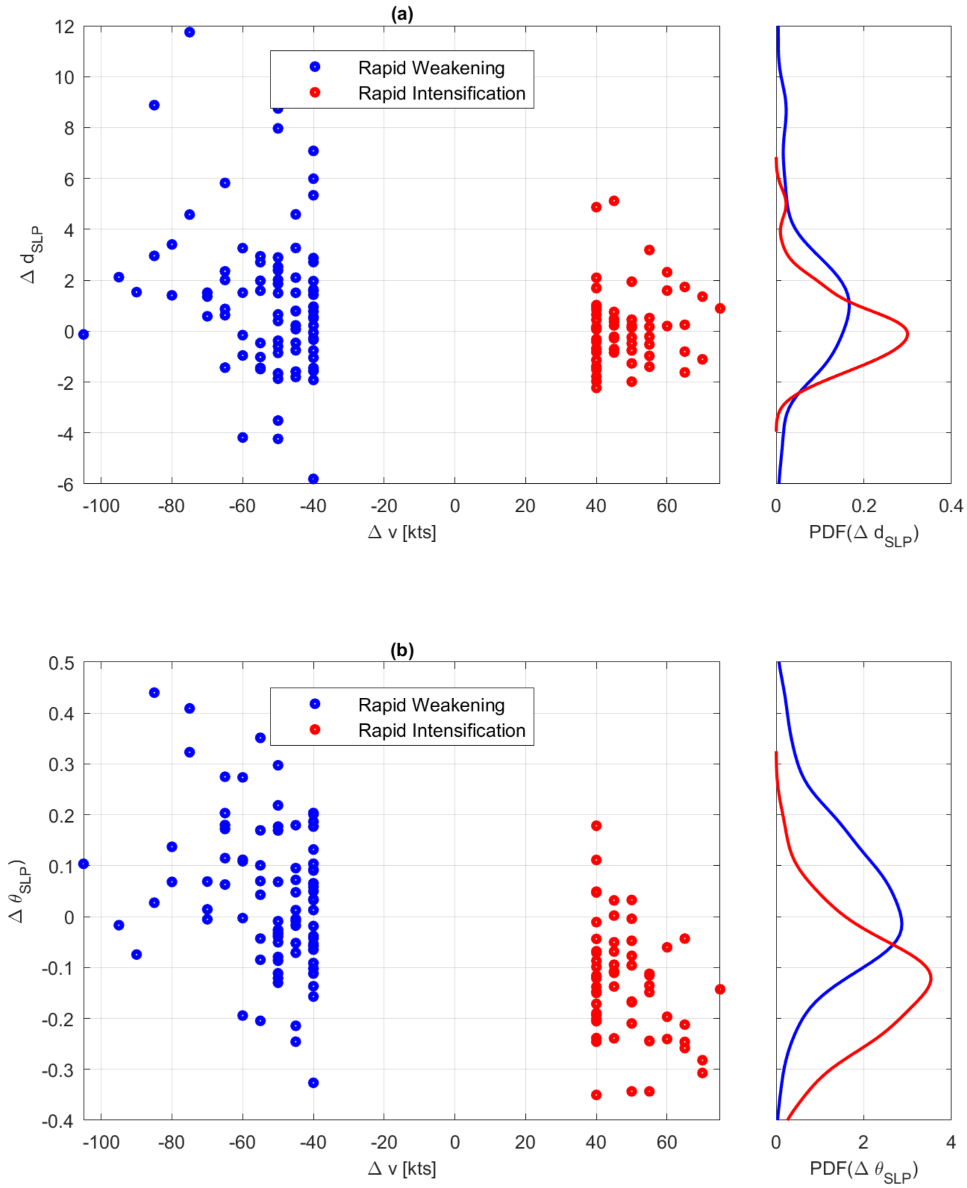
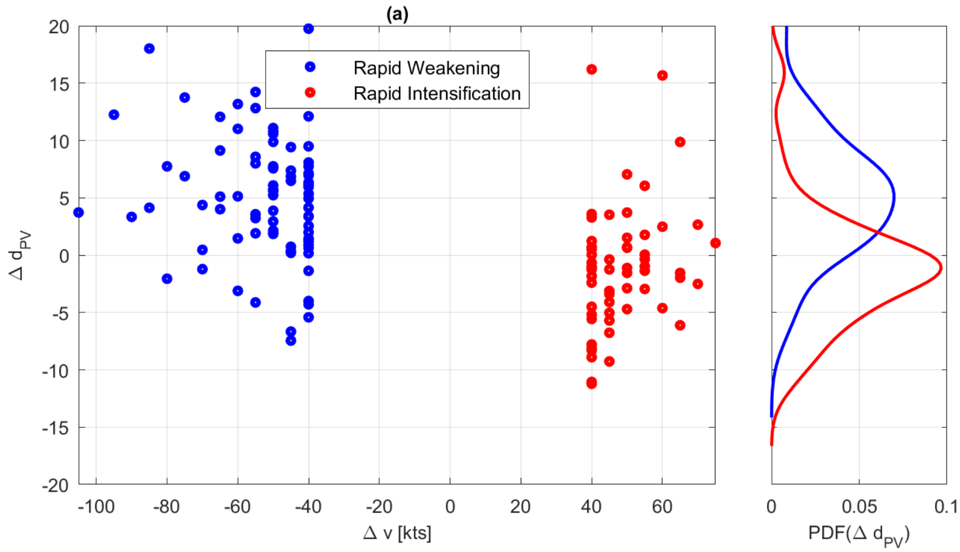



FIG. 5. 24h variation ($\Delta$) of the dimension $d$ (a) and of the inverse persistence $\theta$ (b) computed on SLP versus the 24h variation of maximum winds $v$ for rapidly intensifying (blue) and rapidly weakening (red) cyclones. The side panel shows the corresponding PDFs.



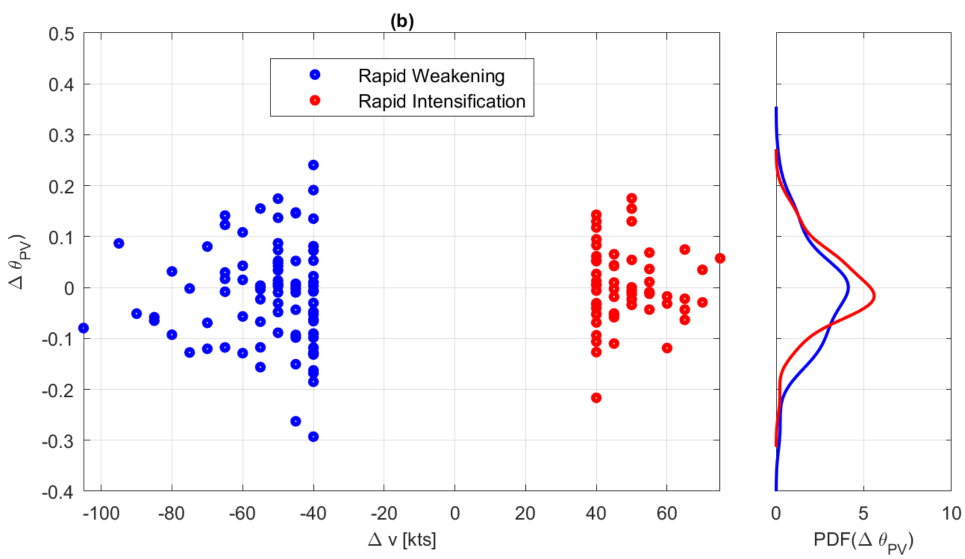




FIG. 6. Same as Fig. 5, but for $d$ and $\theta$ computed on PV at 500 hPa.