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Detecting intense hurricanes from low resolution datasets via dynamical indicators

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ABSTRACT

21 Although the life-cycle of hurricanes is well understood, many of the un-
22 derlying physical processes occur at scales below those resolved by global
23 climate models (GCMs), so that projecting future changes in hurricane char-
24 acteristics remains challenging. We assess the capability of dynamical system
25 metrics to identify intense cyclones even in coarse resolution datasets, where
26 wind speed may be not accurately represented. We compute dynamical indi-
27 cators, namely the persistence and number of active degrees of freedom, from
28 the horizontal wind field of 146 tropical cyclones occurred between 2010
29 and 2018 using ERA5 reanalysis data at 0.25° horizontal resolution, and
30 link these to the maximum sustained winds as detected from observational
31 datasets. Our analysis provides a representation of cyclones in phase space
32 and allows to: i) identify different stages of the cyclones' life cycle as distinct
33 regions of the phase space; and ii) locate regions of the phase space associated
34 with intense cyclones (as detected from observations). Specifically, we find
35 that the most intense cyclones are associated with a strong decrease of the
36 instantaneous dimension and an increase in persistence of tropical cyclones.
37 This relation could be used for detection of intense cyclones in comparatively
38 coarse resolution datasets, such as those issued from GCM simulations or
39 century-long reanalyses.

40 **1. Introduction**

41 Tropical cyclones are high-impact extreme weather events. In the United States of America,
42 they are the costliest natural disaster category (Smith and Katz 2013; Grinsted et al. 2019), with
43 the damage related to hurricane Katrina (2005) alone amounting to about 1% of the country's gross
44 domestic product (Grinsted et al. 2019). Trends in the frequency of occurrence or intensity of trop-
45 ical cyclones are difficult to discern in observations, because of the comparative rarity of the events
46 and of the brevity of highly spatially and temporally resolved datasets, which rely on in-situ and
47 satellite observations (Chang and Guo 2007). Projections of future climate indicate an increase in
48 the intensity of hurricanes in the North Atlantic sector, albeit only with medium confidence (Kossin
49 et al. 2017) because of the difficulties in reproducing the dynamics of the most severe hurricanes
50 even in the most advanced global or regional climate models (Roberts et al. 2020b). Indeed, while
51 mid-latitudes synoptic dynamics mostly originates from the chaotic structure of the motions asso-
52 ciated with baroclinic instability (Lorenz 1990; Schubert and Lucarini 2015), tropical cyclones are
53 characterized by a rapid organization of convectively unstable flows whose dynamics is turbulent
54 and highly sensitive to boundary conditions (Muller and Roms 2018).

55 Destructive tropical cyclones often experience rapid intensification, which occurs when a trop-
56 ical cyclone gains dramatically in strength over a short period of time (Sanders 1986). The rapid
57 intensification phenomenon, well explained from a theoretical viewpoint (Klein 2010; Soloviev
58 et al. 2017), results in an enhancement of the destructive potential of the cyclone and in a degraded
59 predictability of its trajectory (Lee et al. 2016). It is often detected via an increment(generally 40
60 Kts) of sustained winds over a period of 24h. It is challenging to detect this phenomenon in global
61 climate models (GCMs), due to the coarse resolution of wind field data. Emanuel (2017) discusses

several wide-reaching implications of the poor representation of hurricanes in GCMs for present and future climates.

Here, we investigate whether it may be possible to obtain a reliable indicator of intense tropical cyclones from relatively coarse gridded atmospheric data. Specifically, we compute two metrics that describe cyclones as states of a chaotic high-dimensional dynamical system, and reflect the persistence and dimension (i.e. the number of active degrees of freedom) of instantaneous states of the cyclones. These metrics have recently provided insights on a number of geophysical phenomena, including transitions between transient metastable states of the mid-latitude atmosphere (Faranda et al. 2017; Hochman et al. 2019), palaeoclimate attractors (Brunetti et al. 2019; Messori and Faranda 2021) slow earthquake dynamics (Gualandi et al. 2020) and changes in mid-latitude atmospheric predictability under global warming (Faranda et al. 2019). All these applications have taken a Eulerian approach, and have thus focused on a fixed spatio-temporal domain, rather than tracking the evolution of specific physical phenomena. Here, we apply for the first time the two metrics in a Lagrangian perspective. A Lagrangian dynamical systems framework is particularly convenient to study the complex behavior of convectively unstable flow systems (Crisanti et al. 1991; Vulpiani 2010), such as tropical convection. As an additional benefit over previous dynamical systems approaches (e.g Wolf et al. 1985; Cao 1997), these metrics can be easily applied to large datasets, such as climate reanalyses or climate models. Their Lagrangian implementation enables to characterize the phase-space structure of tropical cyclones in terms of their dimension and persistence, and observe whether these single out the most intense cyclones. The rationale is that the signature of intense cyclones propagates through scales due to the presence of inverse cascades as suggested by Levich and Tzvetkov (1985) and, more recently, by Faranda et al. (2018). If this were indeed the case, we could then use the region of phase space targeted by intense cyclones in observations to detect the same phenomenon in datasets where we cannot explicitly access the

86 relevant spatial scales. Our study is organized as follows: first, we describe the data, observables
87 for cyclone dynamics and the theoretical bases supporting the computation of the dynamical indi-
88 cators. Then, we show the general characteristics of the phase space of tropical cyclones for the
89 different observables. Finally, we investigate rapid intensification and discuss our results.

90 **2. Data and Methods**

91 *a. Representation of tropical cyclones in ERA5 data*

92 We follow 146 Atlantic tropical cyclones which occurred between 2010 and 2018 and base
93 the computation of the two dynamical systems metrics on hourly-mean data from the ERA5
94 reanalysis (Hersbach et al. 2020) sampled every 6h, and additionally whenever the HURDAT2
95 database (Landsea and Franklin 2013) displays a cyclone landfall entry. Tracks and classification
96 of the cyclones are also taken from the HURDAT2 database (Landsea and Franklin 2013). HUR-
97 DAT2 is the most complete database of tropical cyclone tracks for the Atlantic basin since 1950,
98 providing 6-hourly sampling of position and speed of the eye, and maximum sustained winds at a
99 5 kts resolution. No wind speed adjustment has been made, contrary to what is done by Emanuel
100 et al. (2005).

101 There is a large discrepancy between the maximum sustained winds reported in HURDAT2 and
102 the closest quantity that we can derive from ERA5, namely the maximum horizontal wind at 1000
103 hPa over one hour, shown in Figure 1. It is important to remark that, while ERA5 wind speed is
104 averaged over one hour, tropical cyclone intensity is usually defined using the one-minute average.
105 This reflects on the large discrepancy between the highest wind values found in ERA5 (≈ 80 kts)
106 and in HURDAT2 (≈ 150 kts). This affects the capability of ERA5 to both accurately assign
107 each hurricane to the appropriate Saffir-Simpson category and to detect rapid intensification. In

particular, for cyclone intensity estimation, this implies that we are not able to separate hurricanes between category 2 and 5. Concerning rapid intensification, HURDAT2 points to 47 out of the 146 considered cyclones, while ERA5 detects none. This analysis suggests that ERA5 shares the difficulties of current state-of-the-art GCMs when it comes to representing the intensity of tropical cyclones Kim et al. (2018). We also underline that the ERA5 horizontal resolution used here is of the same order of magnitude as that of the HighResMIP (High Resolution Model Intercomparison Project, Haarsma et al. (2016)) and PRIMAVERA models, which are amongst the current best tools to study climate change impacts on tropical cyclones (Roberts et al. 2020a).

b. Observables of cyclone dynamics

We adopt a Lagrangian framework, i.e. we consider a domain of size $\sim 1200 \text{ km} \times 1200 \text{ km}$ (41×41 grid points), centred on the eye of each cyclone. In the vertical dimension, we consider 9 pressure levels from 1000 hPa to 200 hPa. As observables, we chose sea-level pressure (SLP) maps, horizontal velocity at 1000 hPa (VH), vertical velocity (W) and the 3D kinetic energy field computed at each grid point as half of the quadratic sum of the velocity components. We relate these quantities to the maximum sustained winds derived from the HURDAT2 database, a quantity that can be directly connected to the economic loss caused by tropical cyclones (Zhai and Jiang 2014).

c. A dynamical systems view of tropical cyclones

We follow tropical cyclones in phase space as states of a chaotic high-dimensional dynamical system. Each instantaneous state of the cyclone, as represented by a given atmospheric variable, corresponds to a point along the trajectory representing the evolution of the system, which we sample at discrete intervals determined by the temporal resolution of our data. For example, we

may have 6-hourly SLP latitude-longitude maps which, at every time step, are centred on the cyclone's eye. Our aim is to diagnose the dynamical properties of the instantaneous (in time) and local (in phase-space) states of the cyclone, as represented by the chosen atmospheric variable and chosen geographical domain (physical space in Fig. 2). To do so, we leverage two metrics issuing from the combination of extreme value theory with Poincaré recurrences (Freitas et al. 2010; Lucarini et al. 2012, 2016)

We first consider a long trajectory $x(t)$, which in our example would be given by x =SLP maps of all time steps for all tropical cyclones in our dataset, always centred on their eye. We thus effectively construct a Lagrangian observable. We further consider a state of interest ζ_x , which would correspond to a single SLP map drawn from this dataset (state ζ in Fig. 2). We then define logarithmic returns as:

$$g(x(t), \zeta_x) = -\log[\text{dist}(x(t), \zeta_x)] \quad (1)$$

Here, *dist* is the Euclidean distance between two SLP maps, but it can be any distance function between two vectors which tends to zero as the two vectors increasingly resemble each other. We thus obtain a time series of logarithmic returns g , which takes large values at times when x closely resembles ζ_x .

We next define exceedances as $u(\zeta_x) = \{t, g(x(t), \zeta_x) > s(q, \zeta_x)\}$, where $s(q, \zeta_x)$ is a high threshold corresponding to the q th quantile of $g(x(t), \zeta_x)$. These are effectively the previously-mentioned Poincaré recurrences, for the chosen state ζ_x (phase space in Fig. 2). The Freitas-Freitas-Todd theorem (Freitas et al. 2010; Lucarini et al. 2012) states that the cumulative probability distribution $F(u, \zeta)$ converges to the exponential member of the Generalised Pareto Distribution. We thus have that:

$$F(u, \zeta_x) \simeq \exp \left[-\vartheta(\zeta_x) \frac{u(\zeta_x)}{\sigma(\zeta_x)} \right] \quad (2)$$

The parameters u and σ depend on the chosen state ζ_x , while ϑ is the so-called extremal index, namely a measure of clustering of points near ζ_x (Moloney et al. 2019). We estimate it here using the Süveges Estimator (Süveges 2007).

From the above, we can define two dynamical systems metrics: local dimension (d) and persistence (θ^{-1}). The local dimension is given by $d(\zeta_x) = 1/\sigma(\zeta_x)$, with $0 < d < +\infty$. The persistence by $\theta^{-1}(\zeta_x) = \Delta t / \vartheta(\zeta_x)$, where Δt is the time step of the data being analysed and $0 \leq \theta \leq 1$.

While the derivation of d and θ^{-1} may seem very abstract, the two metrics can be related intuitively to the physical properties of the tropical cyclones. d is a proxy for the active number of degrees of freedom of the cyclones' instantaneous states, while θ^{-1} measures the persistence of such states.

3. Dynamical properties of tropical cyclones and their connection to sustained winds

We now turn to analysing the properties of tropical cyclones in phase space for the different observables described in Sect. 2 against the maximum sustained winds. We begin with SLP and KE. SLP is widely used in cyclone tracking (Elsner 2003), and provides a first approximation of the horizontal velocity stream function. KE is relevant to the study of cyclones because of both its direct connection with the wind speed and its link with the phases of rapid intensification/decay of the cyclones (Krishnamurti et al. 2005).

Figure 3 shows the values of dimension d and inverse persistence θ for SLP (a) and KE (b) against the maximum sustained wind. The values of d are bounded by $d < 35$ for both the observables, despite the kinetic energy being derived from the 3D wind field, while SLP is 2D. The range of local dimensions found is relatively low compared to the number of grid-points used, which is

172 order of 1600 for SLP and 16000 for KE. This means that the majority of the degrees of freedom
 173 are frozen when we follow coherent convective phenomena such as tropical cyclones. Moreover,
 174 although the range of values spanned by the local dimensions is similar for KE and SLP, the lag-0
 175 correlation coefficient between d_{SLP} and d_{KE} is relatively low, at 0.2. The persistence range is
 176 different for SLP and KE, with $0.1 < \theta_{SLP} < 1$ and $0.1 < \theta_{KE} < 0.6$. If we convert these values
 177 to hours, we get that the SLP persistence range is between 1 and 10 hours and the KE persistence
 178 ranges between 2 and 10 hours. Despite the different range of values, the lag-0 correlation between
 179 θ_{SLP} and θ_{KE} is 0.4, higher than for d , suggesting that the two carry more mutual information.

180 Maximum sustained winds are closely connected to the values of d and θ for SLP and KE. For
 181 SLP (Figure 3a) we note a strong dependence of θ on the maximum sustained winds. Low-to-
 182 moderate winds are associated with non-persistent states of the cyclones, while stronger winds
 183 typically point to more persistent states. High but not extreme wind speed values (60 to 85 kts)
 184 are concentrated on particularly low values of θ_{SLP} . For large values of d_{SLP} , maximum sustained
 185 winds are typically weak. An even clearer relation emerges between d_{KE} , θ_{KE} and the maximum
 186 sustained winds (Figure 3b). There, the strongest winds are associated with the lowest values of d
 187 and θ .

188 Fig. 3c,d) shows the distributions of d and θ conditional on the HURDAT2 classification of the
 189 cyclones; the least organised states (tropical storms, blue) are characterised by low persistence and
 190 relatively high dimension. On the opposite end, low dimensional and highly persistent states cor-
 191 respond to Hurricanes (red) or extratropical transitions of tropical cyclones (black). As a caveat,
 192 we note that the most intense events somewhat deviate from the modal shifts between the distribu-
 193 tions. For example, the d_{KE} distribution for Tropical Storms (blue line in the lower panel of Fig.
 194 3d) shows a much higher mode than that for Hurricanes (red line).

195 We test for significant differences in storm dynamics separately for d_{SLP} , d_{KE} , θ_{SLP} and θ_{KE}
 196 by comparing the three densities for the cases of tropical storms, hurricanes, and extratropical
 197 transitions under the null hypothesis that they have the same distribution (side and bottom panels
 198 of 3c,d)). We choose the significance level $\alpha = 0.05$. In particular, we apply a Kruskal-Wallis
 199 test to determine if there are significant differences in location and a k -sample Anderson-Darling
 200 test for more general shape differences among the three distributions. Then, we perform pairwise
 201 Wilcoxon signed rank tests to determine which distribution is stochastically dominant compared to
 202 the others. Finally, we also repeat the Anderson-Darling test in a pairwise fashion to determine if
 203 there are differences in scale between distributions that do not show significant location shifts. All
 204 the grouped Anderson-Darling and Kruskal-Wallis tests reject H_0 (all p-values $< 2.2 \times 10^{-16}$),
 205 suggesting that at least one distribution is significantly different from the others in all cases, possi-
 206 bly due to a location shift. The pairwise Wilcoxon tests suggest that both d_{SLP} and d_{KE} for tropical
 207 storms are larger than for hurricanes and extratropical transitioning storms; moreover, d_{KE} for
 208 hurricanes is also larger than for extratropical storms. The pairwise Anderson-Darling test points
 209 to a difference in scale between hurricanes and extratropical storms even in d_{SLP} . All tests are
 210 significant for θ , showing the lowest persistence in the tropical storm stage and the highest during
 211 extratropical transitions (see Table 1).

212 Figure 4 shows the same analysis as in Figure 3 but for VH and W. VH is the closest quantity to
 213 the maximum sustained winds and W may be related to convection. d_{VH} and θ_{VH} show a strong
 214 dependence on the maximum sustained winds (Fig. 4a), with the extreme values all located in
 215 correspondence of minima of the dynamical indicators. For W, the distribution of θ exhibits two
 216 modes, resulting in the partial separation of two clouds of points in the in d - θ scatterplot (Fig. 4b).
 217 The largest sustained winds are found in correspondence with the d and θ minima of the cluster
 218 corresponding to larger θ . The analysis conditioned to different phases of the cyclones (Figure 4

c,d) shows that VH is possibly the observable that separates most clearly hurricanes from tropical storms, as the modes are distinct in both d_{VH} and θ_{VH} , whereas for W the separation is less evident.

As for Figure 3 we have tested the statistical significance of the differences among the distributions of d_{VH} , d_W , θ_{VH} and θ_W . The grouped Anderson-Darling and Kruskal-Wallis tests reject H_0 at all levels (all p-values $< 2.2 \times 10^{-16}$), suggesting at least one distribution being significantly different from the others. The pairwise Wilcoxon tests (Table 1) suggest that d_{VH} is shifted towards significantly larger values for tropical storms than for hurricanes, while d_W is centered around lower values for hurricanes. However, while the distributions of d_W for hurricanes and extratropical storms do not differ significantly in terms of location shift, the distribution of d_W for tropical storms is stochastically dominant respect to the case of extratropical storms, despite a visually small difference in the figure. Similar results hold for the persistence: θ_{VH} displays significantly larger values for tropical storms, while the distributions of hurricanes and extratropical storms are not statistically different. In the case of θ_W , the distribution for the hurricanes is shifted towards significantly lower values, while no difference is found between tropical and extratropical storms.

We remark that the dynamical systems indicators obtained using 1000 hPa horizontal velocity (Figure 3, panel c) provide the best separation between TS and HU/EX storms, with higher velocity values concentrated at low values of both indicators. We parameterize the relationship between maximum sustained wind v_{max} and d_{VH}, θ_{VH} with a multiple linear regression $v_{max,i} = \alpha + \beta_d d_{VH,i} + \beta_\theta \theta_{VH,i} + \varepsilon_i$, where i denotes the observation and ε_i is a Gaussian white noise sequence. Both dynamical systems indicators have a significant explanatory effect on v_{max} (p-values $< 3 \times 10^{-16}$), with $\beta_d = -1.14$, and $\beta_\theta = -72.16$. The persistence coefficient is two orders of magnitude larger (in absolute value) than the one of the dimension, despite there being only roughly one order of magnitude difference in the scales of the two variables, pointing

243 to a larger effect of θ_{VH} . The goodness of fit is $R^2 = 0.22$. This indicates that a large fraction
244 of v_{max} variability cannot be explained by the two indicators alone, and/or the presence of strong
245 nonlinearities that are not caught by linear regression.

246 In the dynamical systems framework, a collapse of degrees of freedom and an increase of per-
247 sistence are indicative of approaching unstable fixed points, i.e. special states of the dynamics
248 where temporal and spatial scales are deformed. An example is the behavior encountered in the
249 Lorenz 1963 attractor (Lorenz 1963) near the unstable fixed points corresponding to the centres
250 of the wings of the butterfly-shaped attractor, where both d and θ attain low values [Figure A1
251 in Faranda et al. (2017)]. We discuss the implications of this result for the numerical modelling of
252 tropical cyclones in Sect. 6.

253 **4. Dynamical Indicators and Rapid intensification**

254 In the previous section, we have established that the dynamical indicators provide a clear sepa-
255 ration of the different meteorological phases of tropical cyclones. Now we focus on the analysis
256 on the transitions from tropical storms (TS) to hurricanes (HU). Our goal is to understand whether
257 we can use the dynamical indicators to discriminate the transitions associated with rapidly intensi-
258 fying cyclones from the others, and whether the dimension and persistence analysis could be used
259 to detect rapid intensification in a statistical sense in coarse-resolution climate datasets. Rapid in-
260 tensification is usually quantified using the increment Δv of maximum sustained winds over 24h.
261 Several numerical thresholds to define rapid intensification exist (Knaff et al. 2020). Here, we use
262 thresholds of $\Delta v > 50$ Kts or $\Delta v > 40$ Kts, namely the highest thresholds proposed by (Knaff et al.
263 2020).

264 In phase space, rapid changes of the dynamics correspond to transitions through different basins
265 of the attractor (Ghil et al. 2008; Dijkstra 2013). The variation of the dynamical systems indicators

266 can track these transitions because of their direct connection with the underlying entropy of the
267 system: the local dimension d is an instantaneous measure of the active degrees of freedom,
268 and the persistence θ is related to the dominant time scale of the dynamics, i.e. the Lyapunov
269 exponent (Faranda and Vauti 2018). Both these quantities are known to be connected to the
270 dynamical (Kolmogorov-Sinai) entropy, since the seminal work of Young (1982).

271 Results relating the dynamical system metrics to rapid intensification are presented in Figure 5,
272 for the four observables used in this study: SLP (a), KE (b), VH (c) and W (d). Median values
273 are represented by magenta stars (40 KtS) and crosses (50 KtS); ellipses represent one standard
274 deviation from the median. Blue (red) large circles represent the median d for all tropical storms
275 (hurricanes). The green crosses (ellipses) represent the median (standard deviation) of the values
276 of the dimension and persistence for all the tropical storm to hurricane transitions. The results
277 show that transitions associated to rapid intensification target a special region of the d, θ space
278 for KE and for VH. They correspond to lower-dimensional and higher-persistence phases of the
279 dynamics of the cyclones.

280 The decrease in local dimension and inverse persistence implies a decrease of the dynamical
281 (Kolmogorov-Sinai (Young 1982; Latora and Baranger 1999)) entropy of the cyclones, namely a
282 reduction of the kinetic degrees of freedom, as discussed in the previous section. Specifically, the
283 Kolmogorov-Sinai entropy is linked to the Lyapunov exponents (van Beijeren and Dorfman 1995),
284 whose number of positive values is related to d and whose leading value is linked to θ (Faranda
285 and Vauti 2018). It is counterintuitive that large dissipating systems such as rapidly intensifying
286 cyclones yield low entropy states. However, one can observe that the tendency to the order in
287 the dynamical entropy is balanced by enhanced thermodynamic entropy leading to convection and
288 diabatic phenomena.

Finally, we remark that the relationship between rapid intensification and decrease in d_{KE}, d_{VH} and θ_{KE}, θ_{VH} is not systematic: the number of cyclones visiting the magenta balls in Figure 5 is larger than those experiencing rapid intensification, i.e. one cannot use the values of d and θ to identify in a deterministic sense cases of rapid intensification.

5. Implications of the results for the numerical simulation of tropical cyclones

The computation of the dynamical systems indicators for tropical cyclones has shown that there is a strong relationship between such indicators computed for coarse atmospheric fields and the maximum sustained winds taken from observations. In the most intense phase of the cyclones, large maximum sustained winds are associated to low values of both d and θ , in particular if these are estimated on the horizontal velocity field. From a dynamical system viewpoint, the explanation is that air parcels behave almost identically, with rotational degrees of freedom oriented along the global axis of the storm. This behavior is symptomatic of highly persistent, low-dimensional states found at unstable fixed points of the dynamics. These results imply that intense cyclones are characterized by different dynamical properties (entropy, stability) from those not accompanied by large maximum sustained winds and that d and θ give a very good degree of discrimination from the other, less intense, tropical cyclones. Similarly, transitions from tropical storms to hurricanes of rapidly intensifying cyclones primarily reside in particular regions of the d, θ space. This is particularly evident for the metrics computed on horizontal velocity (VH), but to a good degree also on 3D kinetic energy (KE). Vertical velocity (W) provides a less-clear separation, likely because, while the horizontal wind is driven by the cyclone structure and dynamics, the vertical velocities are linked to smaller structures such as convective cells not directly resolved in ERA5.

These theoretical considerations have a number of concrete implications for current research on hurricanes. Current GCMs — and even reanalysis products — struggle in reproducing maximum

sustained winds comparable to those observed (see Fig. 1). Our study offers a way of mapping intense cyclones in d, θ space and measure, in climate change model experiments or century-long reanalysis products, shifts towards smaller or larger values of the dynamical indicators. This, in turn, may provide a strategy for studying changes in hurricane intensity driven by anthropogenic forcing. Indeed, while d and θ may not be used to provide a deterministic indication of cyclone intensification, they do provide a robust statistical indication. Furthermore, the fact that the dynamics of intense hurricanes approaches that of fixed points of high-dimensional dynamical systems may explain why it is so difficult to adequately represent rapidly intensifying tropical cyclones and hurricanes in numerical models. Parameterizations are devised for typical states of tropical dynamics (disorganized storms) but not specifically for the organized states of tropical cyclones. As a caveat, we underline that our Lagrangian approach does not allow to relate the present results to the predictability of the trajectories of the tropical cyclones, unlike the Eulerian approach applied to extratropical motions in (Faranda et al. 2017; Messori et al. 2017).

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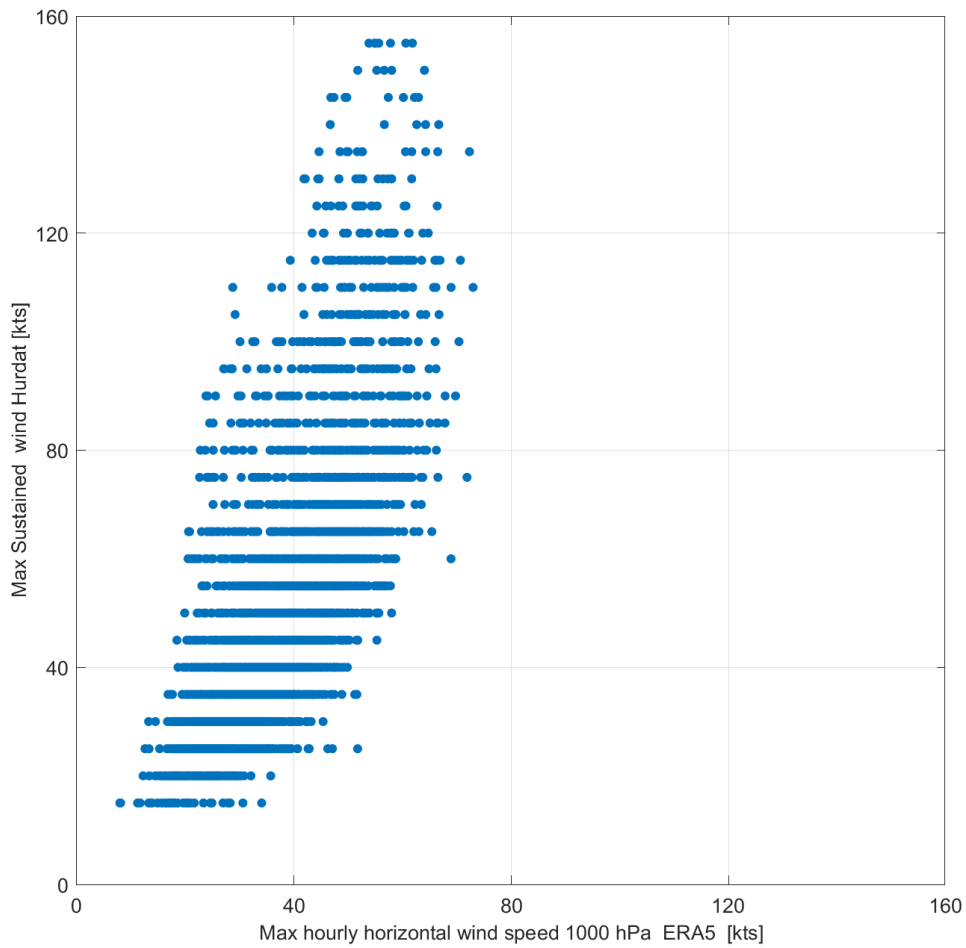
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439		Darling test general shape differences.	23

Wilcoxon paired tests, Figure 3															
d_{SLP}	TS	HU	EX	d_{KE}	TS	HU	EX	θ_{SLP}	TS	HU	EX	θ_{KE}	TS	HU	EX
TS	–	***	***	TS	–	***	***	TS	–	***	***	TS	–	***	***
HU	–	–	n.s.	HU	–	–	***	HU	–	–	***	HU	–	–	n.s.
Wilcoxon paired tests, Figure 4															
d_{VH}	TS	HU	EX	d_W	TS	HU	EX	θ_{VH}	TS	HU	EX	θ_W	TS	HU	EX
TS	–	***	***	TS	–	***	***	TS	–	***	***	TS	–	***	***
HU	–	–	n.s.	HU	–	–	***	HU	–	–	n.s.	HU	–	–	***
Anderson-Darling paired tests, Figure 3															
d_{SLP}	TS	HU	EX	d_{KE}	TS	HU	EX	θ_{SLP}	TS	HU	EX	θ_{KE}	TS	HU	EX
TS	–	***	***	TS	–	***	***	TS	–	***	***	TS	–	***	***
HU	–	–	***	HU	–	–	***	HU	–	–	***	HU	–	–	***
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TS	–	***	***	TS	–	***	***	TS	–	***	***	TS	–	***	***
HU	–	–	***	HU	–	–	***	HU	–	–	***	HU	–	–	***

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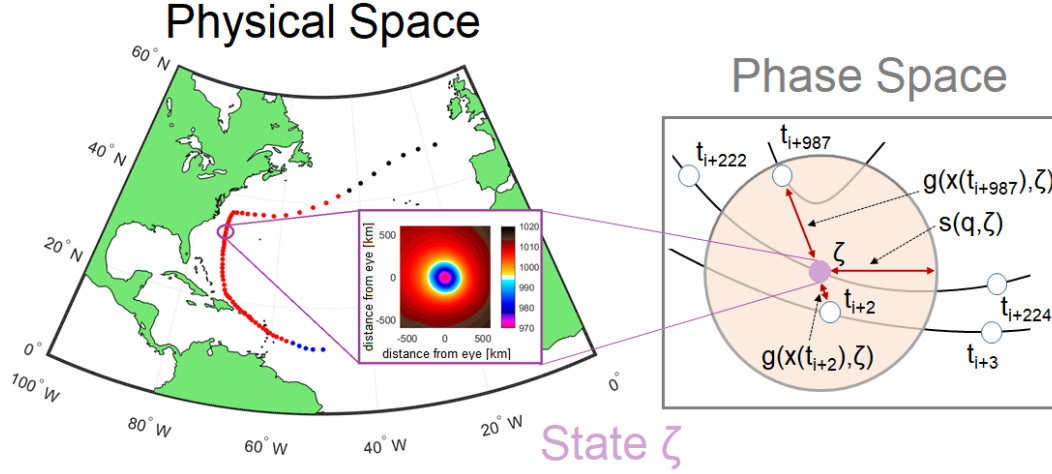


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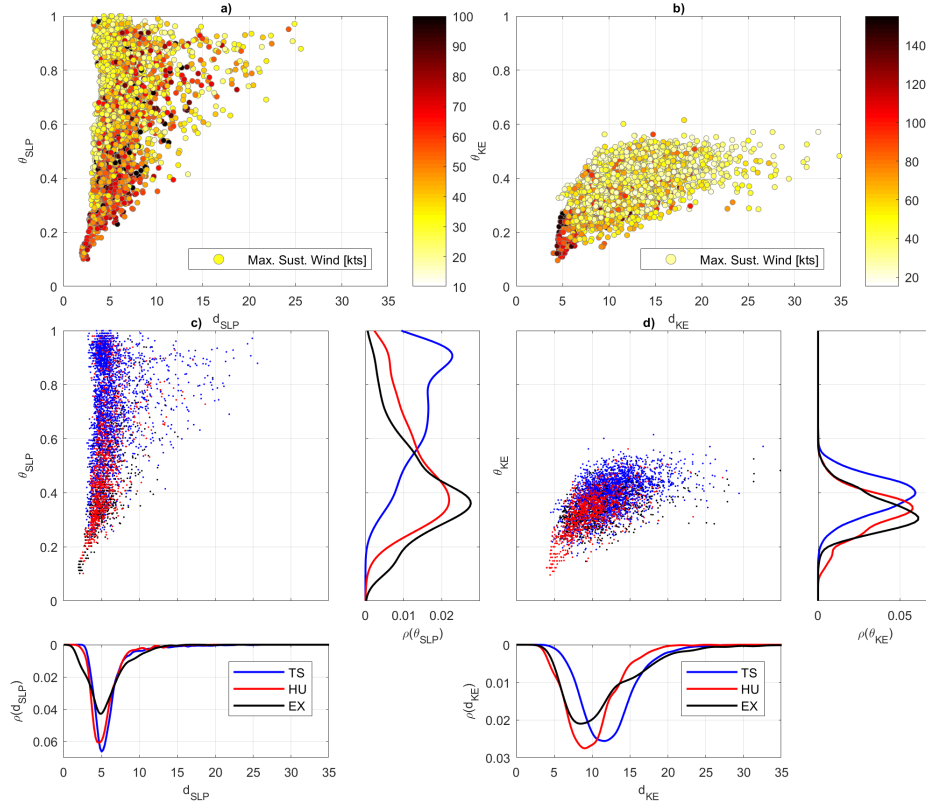


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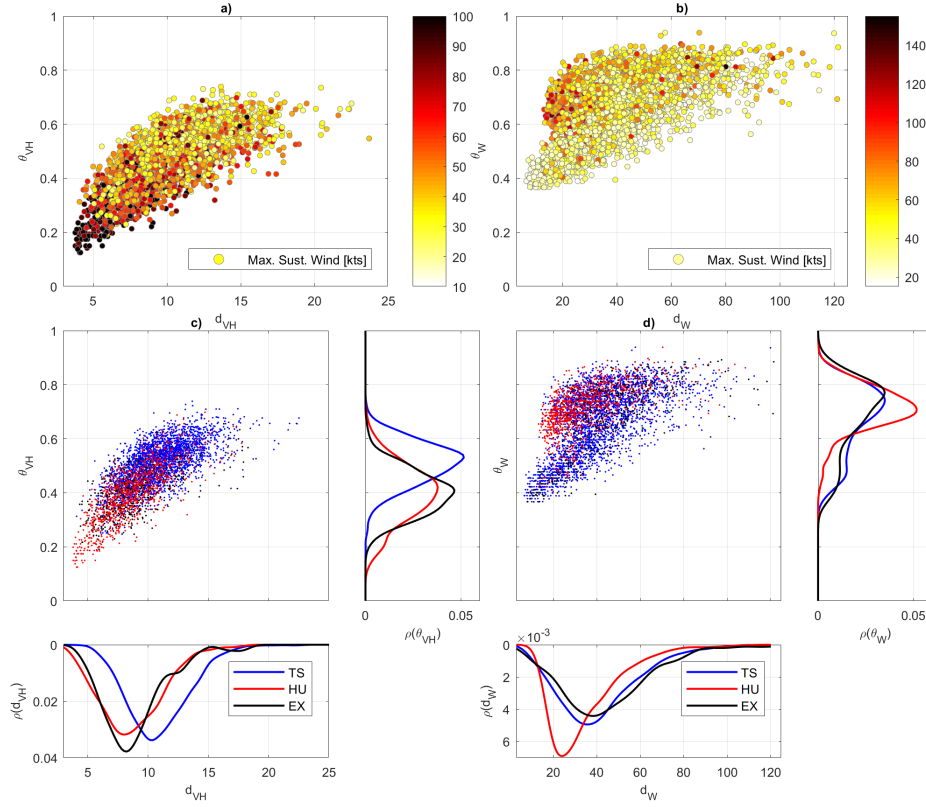


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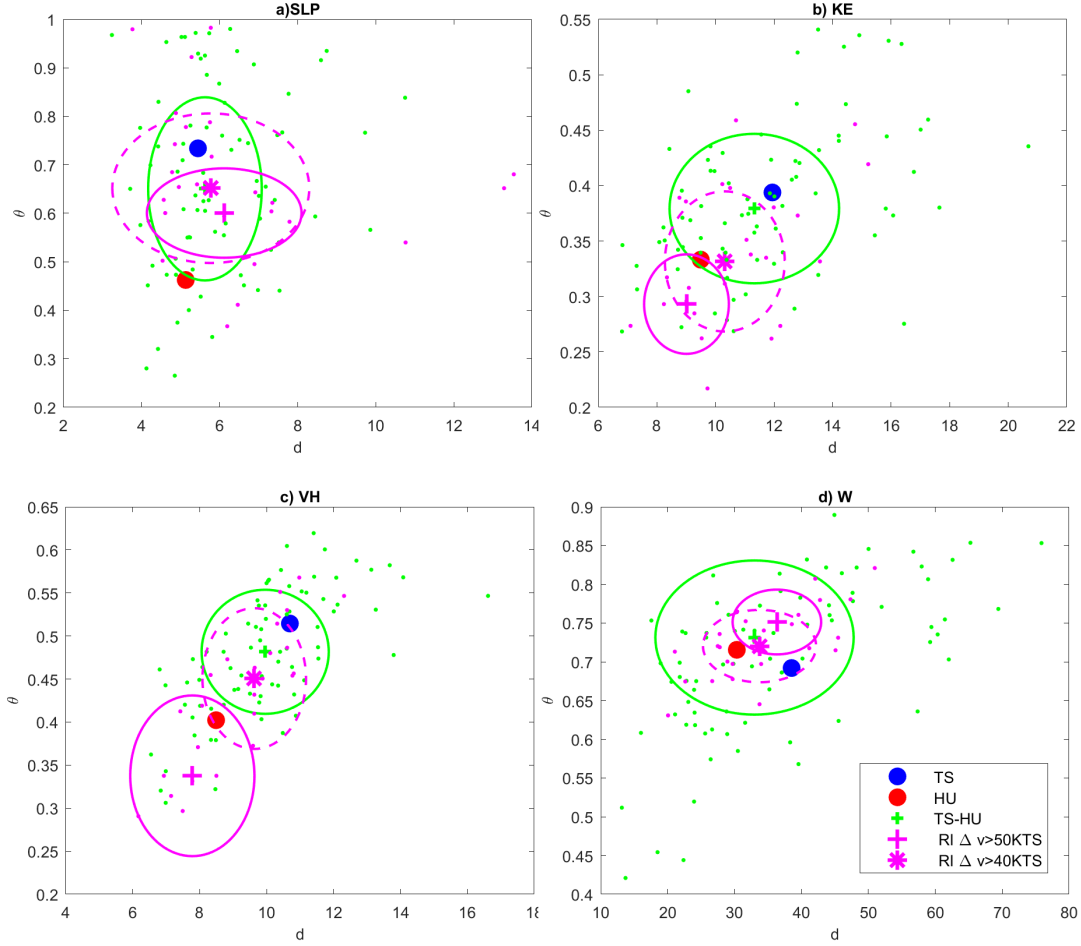


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