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Study on obstacle avoidance for fractional artificial potential fields

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Abstract: One of the main problems related to path planning is to find a method which will effectively allow the robot or vehicle to avoid obstacles, provided that these obstacles can be static or dynamic. One of the most interesting methods for path planning is the use of the artificial potential fields in order to create a representation of the environment as proposed by Khatib and Ge & Cui. These approaches enable handling online situations, which is desirable when facing uncertain obstacles appearing in the path. Three propositions are provided in the repulsive potential fields to avoid acceleration oscillations occurring while the ego-vehicle enters the limit boundaries of an obstacle. The advantages and limitations of the proposed methods will be explored. The performance of the different propositions will be compared by using criteria such as length and energy in a simple scenario.

Keywords: Path planning, Reactive path planning, Potential Field, Fractional potential field, Poisson's equation.

1. INTRODUCTION

Nowadays, developments on autonomous vehicle have become a noticeable reality in the society, whether they are aerial, underwater or ground ones. In any case, path planning for any autonomous vehicle is one of the most important tasks to be successfully accomplished. One of the most active developments concerns drones. Many potential applications exist mainly related to agriculture, security and medical tool transportation.

A fairly popular method for path planning is the Artificial Potential Field Method (see Khatib (1986)). It basically consists in treating the space as a potential field in which the vehicle is guided through a virtual force. Several modifications for the repulsive and attractive forces expressions have been proposed in order to take into account the kinematics of both the obstacle and the vehicle (see Ge and Cui (2002)). The attractive potential has been re-interpreted as a control loop (see Metoui et al. (2009)) and control analysis has been presented regarding the fractional attractive field (see Melchior et al. (2018)). One proposition for the repulsive field was developed by using the Weyl potential (see Poty (2006)). A more detailed and optimized version of the method was recently introduced by combining the use of artificial potential fields and genetic algorithms (Receveur et al. (2019)).

Despite the popularity of the Artificial Potential Field method, the repulsive potential field lacks continuity in its outer boundary (such as the Weyl fractional repulsive field) and may produce oscillations when approaching an obstacle, thus increasing the energy consumption. Moreover, local minima also are problematic, as the vehicle may be "locked" in a place other than the target.

In order to reduce these drawbacks, alternative expressions for the repulsive potential field are proposed. An approach based on corrective polynomials will be explored to eliminate the oscillation problem. An alternative and more compact expression for the repulsive field will be derived from Poisson's equation. The addition of a tangential force component to the repulsive force field will also be explored.

Section 2 will present a brief review of the state of art. The two main types of path planning approaches (local and global) will be presented and compared. Section 3 will present the contributions on fractional potential fields in order to avoid acceleration oscillations when entering the frontier limits of the obstacles. Finally, Section 4 will present simulations of all the proposed ideas with a simple 2D scenario. The results of the simulations will be discussed and the different propositions will be compared.

2. STATE OF THE ART

The path planning problem can be divided into two different types: global and local approaches, the main difference relying on the knowledge of the environment.

2.1 Global approaches

A global approach of path planning is mainly characterized by the assumption that the whole environment is known *a priori*. Not only is the location of the goal known, but the locations and dimensions of all obstacles at stake are assumed known. These allow the use of optimization in order to minimize different criteria such as length, energy, danger or other factors that can be taken into account. However, the calculations can be complex and heavy if the vehicle is supposed to be in an online situation. Most

of the time, these methods will first use a tool in order to describe the environment, such as a visibility graph or a Voronoi diagram and then apply an algorithm over the description provided by the graphs. The most well-known algorithms used for this purpose are the Dijkstra and A-star ones (see Nilsson (1969)). Some other approaches lack the use of optimization tools, but are still considered as global methods because of the *a priori* knowledge of the environment. This situation is not a problem if the scenario exactly happens as predicted. However, in real life situations, it is well-known that pedestrians and drivers may change their speed and/or directions, and therefore, the proposed path is no more suited.

Harmonic Potential Fields The harmonic potential field method (see Garrido et al. (2010)) belongs to the global approach branch. This technique derives from Khatib's original use of potential fields for path planning in which obstacles and the goal are modeled as boundary conditions and the Laplace equation of electrostatics is used:

$$\nabla^2 U = 0. \quad (1)$$

In this approach the whole potential U is found over the totality of the free space and the virtual force related to this potential will guide the vehicle to the goal. Its main advantage is the lack of local minima. Still, limitations exist as it is impossible to distinguish different types of obstacles as they will all be mostly treated as a Dirichlet boundary condition ($U = 1$ on the boundary) or as a Neumann one ($\frac{\partial U}{\partial n} = 0$ where n stands for a unitary vector in the normal direction outside of the boundary). Using Dirichlet conditions can create extremely weak force field, and if the computing precision is insufficient, the gradient of the potential may vanish in some parts of the space. Furthermore, using Neumann conditions will generate paths that will be tangential to the obstacles, allowing the vehicle to come extremely close to the obstacles.

Bi-harmonic Equation Another approach inspired by the properties of partial differential equations is based on the theory of elasticity and the bi-harmonic equation (see Guys (2014)). In this case, the environment is modeled as a plate (in the 2D case) and the goal and obstacles represent pressure variations in the plate. The virtual stress derived from solving the bi-harmonic equation guides the vehicle. Simulations of the paths generated by this method proved to be highly unsuccessful, as the field vanishes in several zones as a consequence of the mathematical theory behind this technique (and not because of a technical limitation such as the Dirichlet condition in the harmonic field method) and a later step of adjustment is required in order to fully generate a path. The equations to be solved require heavier mathematical power, as a 25 point-stencil is used in order to calculate the bi-harmonic operator of a function in 2D, opposed to the 5 or 9 point stencil used to calculate a Laplacian (see Saudi and Sulaiman (2012)).

2.2 Local approaches

For local approaches, environment full knowledge being not available, the vehicle position is calculated at each iteration in order to get it closer to the goal. From the available embedded sensors, its main advantage is its implementation easiness in real-time situations.

Bug Algorithms Perhaps the oldest local path planning method are the *Bug* algorithms. They are based on insect motion that tends to closely surround the obstacles in front of them instead of deviating their paths as soon as they see an obstacle. The most known algorithms of this type are the so called *Bug1* and *Bug2*. In both of them, a straight line is drawn between the vehicle and the goal at the beginning of the operation. The vehicle will follow this path until crossing an obstacle, going around it. In figure 1, a path generated by *Bug2* is shown. Its main drawback is that it can generate paths that are extremely long and way too far below the optimal solution.

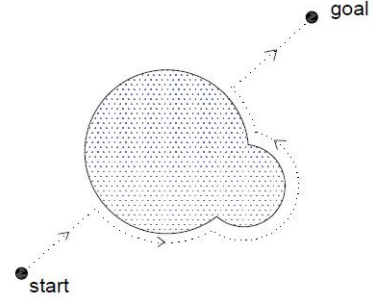


Fig. 1. Path generated by using the *Bug2* algorithm

Artificial Potential Fields One of the most used method based on a local approach is the artificial potential field method (see Khatib (1986)). The idea is to model the goal as an attractive potential U_{att} that will guide the vehicle towards it and to model the obstacles as repulsive potentials U_{rep} which will make the vehicle go away from them. The global potential will be the sum of the attractive and repulsive potentials. A gradient descent method is later used in order to calculate the direction in which the robot should move forward at each step, such as:

$$\vec{F} = -\nabla(U_{att} + U_{rep}) \quad (2)$$

$$x_{i+1} = x_i + \alpha \frac{\vec{F}_i}{|\vec{F}_i|} \quad (3)$$

where α is the step in the descent and x_i is the robot position at the i -th iteration.

This method generates radial fields, which are the main cause of local minima in which the vehicle can become trapped. It is also impossible to distinguish the different obstacles as in the original formulation of the method, only the distance to the obstacle is taken into account in order to estimate its contribution to the path generation.

Ge and Cui Potential Field Another main drawback to the artificial potential field method is that it doesn't take into account the dynamics of the environment. The speed of neither the obstacles nor the goal is used. The use of the speed was introduced as a modification of the potential field method (see Ge and Cui (2002)). The main interest of this contribution was the modification of the attractive potential definition:

$$\vec{F}_{att} = \alpha_p(\vec{X}_{goal} - \vec{X}_{ego}) + \alpha_v(\vec{V}_{goal} - \vec{V}_{ego}) \quad (4)$$

where \vec{X} stands for position and \vec{V} for speed, and subscripts *goal* for the target and *ego* for the ego vehicle.

Control-loop interpretation of the Artificial Potential Field

This type of force is linear and can easily be analyzed in the Laplace domain, as a controller. A complete analysis and control design was developed at the IMS laboratory (see Melchior et al. (2018)) to guarantee a robust tracking of the paths generated by this type of potential.

The attractive force, being described in terms of position and speed error can be interpreted as a lead-phase controller (see Poty (2006)). A novel controller design has been proposed in Receveur et al. (2019).

Vortex Fields Other variations of the artificial potential field include the use of vortex fields (see De Medio et al. (1991)) in which the mathematical formulation tries to reduce the presence of local minima and uses inverse tangent function, but it is not a widely spread method in literature.

Limit cycle Technique Another local approach technique which is interesting is the limit cycle technique (see Aalbers (2013)). This method is based upon the phenomena of limit cycles that is widely studied when analyzing nonlinear systems. A pseudo phase portrait is established around the obstacle. The variables x_1 and x_2 used to establish the phase portrait do not represent position and speed, but x and y positions in the 2D case.

Optic Flow A type of technique that is still not fully understood relies on the use of the optic flow concept in order to guide a vehicle and avoid obstacles (see Serres and Ruffier (2017)). The main interest nowadays relies on a better development of the theory and a deeper understanding of the way in which animals use the optic flow (mostly flies) in order to guide themselves.

2.3 State of art synthesis

The state of art reveals that in spite of the possibility of easily optimizing the paths, methods relying on a global approach are not easy to implement when dealing with an uncertain environment in which unexpected obstacles may appear or move. The information required for the implementation of these techniques will most certainly not be fully available and the calculation time required in order to generate a path is also limiting. This is the main reason why the use of a local approach is more well-suited for an uncertain environment exploration. The artificial potential field offers the possibility of being easy to use in a real-time application and has a mathematical background that is well-known and possible to explore and modify easily.

3. ANALYSIS AND CONTRIBUTIONS

3.1 The Weyl fractional repulsive potential field

The artificial potential field method as it was originally created, generates its paths by taking into account the distance to the obstacle. However, in real life cases, the nature of an obstacle may vary and it would be desirable to be able to distinguish different types of obstacles. A new definition based on the Weyl potential (see Melchior et al. (2001)) allows to establish a flexible potential field

form. The mathematical definition is given within a range (distance to the obstacle) going from r_{min} to r_{max} :

$$U_{rep}(r) = \frac{r^{n-2} - r_{max}^{n-2}}{r_{min}^{n-2} - r_{max}^{n-2}}, \quad n \neq 2. \quad (5)$$

The variable n is used to measure the degree of danger of the obstacle and can vary continuously. This modifies the shape of the potential around the obstacle, which is better illustrated by figure 2. A convex shape is obtained for $1 < n < 3$, and a concave one is obtained above 3. A semi-empirical method to estimate n in terms of the obstacles is presented in Receveur et al. (2019).

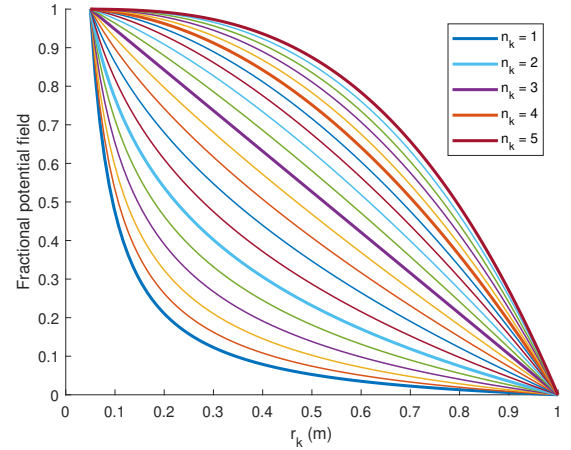


Fig. 2. The fractional repulsive potential shape

The approach developed at IMS laboratory directly uses the force derived from the potential field in the physical sense rather than as a simple guideline for the direction to follow at each iteration. Deriving the force generated by the fractional repulsive potential:

$$|F_{rep}(r)| = (n-2) \frac{r^{n-3}}{r_{min}^{n-2} - r_{max}^{n-2}}, \quad (6)$$

has a magnitude at $r = r_{max}$ of:

$$|F_{rep}(r_{max})| = (n-2) \frac{r_{max}^{n-3}}{r_{min}^{n-2} - r_{max}^{n-2}}. \quad (7)$$

Thus, the force generated by an obstacle is not continuous at $r = r_{max}$, but it presents a force step. This is the source of oscillations in the trajectory generated by using this type of potential and can significantly increase the energy consumption, which is undesirable (see figure 4).

3.2 Contribution with corrective polynomials

One proposed solution in order to keep continuity at the outer boundary r_{max} is to add an additional corrective term. The additional term will provide an attractive force at the outer boundary that will cancel the repulsive one at r_{max} and therefore guarantee the continuity in the boundary. The repulsive potential is then defined by:

$$U_{rep-new}(r) = U_{rep}(r) + U_{pol}(r) \quad (8)$$

from where one draws:

$$F_{rep-new}(r) = F_{rep}(r) + \frac{\partial U_{pol}(r)}{\partial r} \quad (9)$$

with

$$\frac{\partial U_{pol}(r)}{\partial r} = -(n-2) \frac{r_{max}^{n-3}}{r_{min}^{n-2} - r_{max}^{n-2}}; \quad (10)$$

consequently, at $r = r_{max}$:

$$F_{rep-new}(r_{max}) = 0. \quad (11)$$

Two simple corrective terms are proposed: a first and a second order polynomial. These are defined as follows:

$$U_{pol-1}(r) = -(n-2) \frac{r_{max}^{n-3}}{r_{min}^{n-2} - r_{max}^{n-2}} r \quad (12)$$

$$U_{pol-2}(r) = Ar^2 + Br \quad (13)$$

with:

$$\begin{cases} A = -\frac{(n-2)r_{max}^{n-3}}{2\delta(r_{min}^{n-2} - r_{max}^{n-2})} \\ B = -(n-2) \frac{r_{max}^{n-3}}{r_{min}^{n-2} - r_{max}^{n-2}} \left[1 - \frac{r_{max}}{\delta}\right] \end{cases} \quad (14)$$

The first order polynomial will simply provide a constant attractive force that will eliminate the continuity problem. A second order polynomial will provide a force profile with a linear shape. This means that the second order polynomial can be manipulated in order to be attractive only around the boundary but that will be able to quickly provide an additional repulsive force as the vehicle enters more into the danger zone of the obstacle. A variable δ is introduced for the use of this corrective term. This variable will indicate the width of the zone near the boundary of the obstacle for which the additional term provides an attractive force. If the vehicle gets to a distance to the object smaller than $r_{max} - \delta$, the corrective term will also contribute to the repulsion.

3.3 Contribution with tangential component

A major drawback regarding the use of potential fields is the possibility of local minima in which the vehicle can be trapped. A proposition is made regarding the addition of a tangential force in order to avoid these situations:

$$\vec{F}_{rep-total} = \vec{F}_{rad}(r) + \vec{F}_{tang}(r) \quad (15)$$

in which \vec{F}_{rad} and \vec{F}_{tang} respectively are the radial and tangential components of the field. It should be noted that propositions with tangential path planning already exist (see Zhou et al. (2018)), but they involve the use of global optimization techniques.

3.4 Potential derived from Poisson's equation

Another way to derive a potential field that is able to take into account differences between obstacles and at the same time to eliminate the continuity problem is to create a local variation of the harmonic potential field method. This can be done by defining a limited zone around the obstacle and deriving an explicit solution for a field around it.

Let us define a punctual obstacle, delimited by a danger zone in $[r_{min}, r_{max}]$, to which a repulsive field U_{rep} is obtained around it. Using the Laplace equation (1) allow us to impose the continuity of the force at the outer

boundary by applying a Neumann boundary condition, such as:

$$\left. \frac{\partial U(r)}{\partial r} \right|_{r=r_{max}} = 0. \quad (16)$$

The main inconvenient by using the Laplace equation (1) in such a situation is that it will generate a constant potential, which will produce no force and isn't useful for path planning. However, it is still possible to use Poisson's equation in order to solve this issue, as this equation will consider a virtual charge density distributed over the danger zone of the obstacle. A Neumann condition is used on the outer boundary to eliminate the continuity issue and a Dirichlet condition can be used to make the potential zero right at this boundary. The derived potential is expressed by:

$$U_{poisson}(r) = \rho \left[\frac{r^2 - r_{max}^2}{4} + \frac{r_{max}^2}{2} (\ln r_{max} - \ln r) \right] \quad (17)$$

with ρ standing for a parameter that will be linked to the danger degree notion introduced by the fractional Weyl potential.

4. SIMULATION RESULTS

4.1 Simulation scenario and criteria used

All of the different propositions presented in the previous section were tested in 2D in a zone near a static obstacle. The model used for the vehicle in order to generate the reference is a punctual mass of $M = 1kg$. The simulation parameters used to test the polynomials and the tangential component are presented in table 1.

Table 1. Simulation parameters

x_{goal}	y_{goal}	x_{init}	y_{init}	r_{min}	r_{max}	n
1.5 m	5 m	1 m	-1 m	0.9 m	1.5 m	1.5

The simulations were performed by implementing a block diagram as shown in figure 3. The position of the goal is the set-point of the control loop. The position error is then used in order to generate the attractive force by using the block $C(s)$, which contains the definition of the attractive potential. The repulsive force due to the obstacles is introduced into the loop as a disturbance. The relationship between the vehicle position and the force acting on him is calculated by using a punctual mass model.

The main criteria used to evaluate the generated path are the total length of the path J_{long}

$$J_{long} = \sum_i \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \quad (18)$$

and the energy required to go through the path J_{energy}

$$J_{energy} = m \int \mathbf{a} \cdot \mathbf{v} dt. \quad (19)$$

These two criteria will be calculated and analyzed for all the simulations in order to compare them.

4.2 Corrective polynomials and tangential component

The acceleration profiles of corrective polynomials and tangential field methods are presented in figure 4, and the performance criteria are presented in table 2.

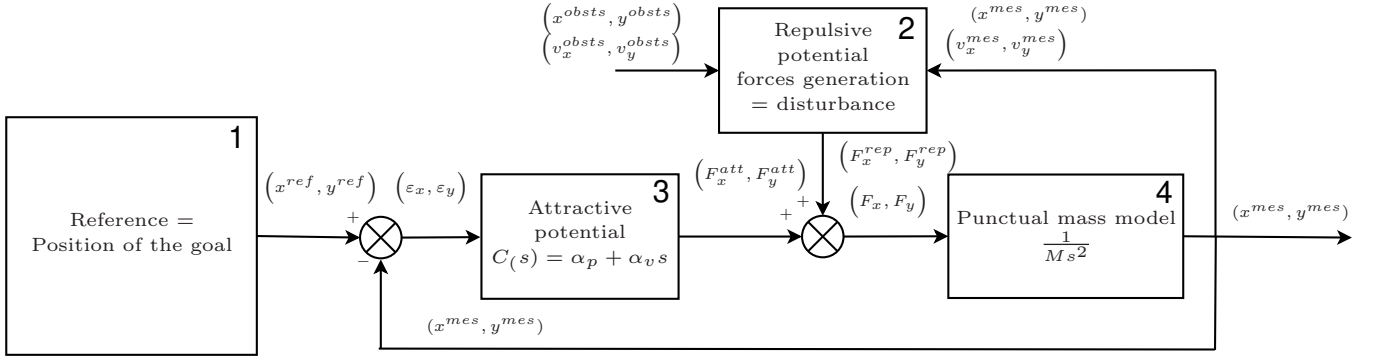


Fig. 3. Control loop with attractive and repulsive PF acting as a controller and a disturbance

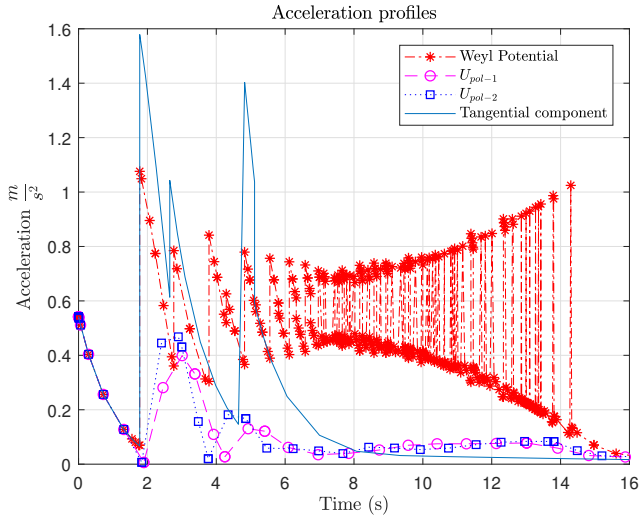


Fig. 4. Acceleration profiles with the correction terms and the tangential component

Table 2. Corrective polynomials and tangential component

	J_{long}	J_{energy}
Weyl Potential	7.22 m	0.50 J
U_{pol-1}	7.07 m	0.32 J
U_{pol-2}	7.15 m	0.33 J
Tangential component	7.19 m	1.16 J

As it can be observed, both polynomial methods provide an important reduction regarding the energy consumption and a slight reduction in the total length of the path. Considering that the second order polynomial can be designed in order to reinforce the original repulsive field, it may be considered as a safer option with no drawbacks as compared to the first order polynomial. The use of the tangential component severely increases the magnitude of the force used to generate the path, which explains why this technique requires a heavy and undesirable energy consumption.

The acceleration profiles show that the use of the correction terms generates lower amplitudes on the whole path when compared to the highly oscillating profile of the uncorrected case (the classic Weyl repulsion definition). Variations in acceleration also prove to be less important with the tangential component addition, but the maximum

amplitudes are higher than those required by the uncorrected case, which explains its heavy energy consumption.

4.3 Simulations using Poisson's potential

For the simulations using Poisson's potential, the main interest is to study the influence of the new scale factor ρ on the generated paths: one using the force field in a purely radial direction and the other in a purely tangential direction (no addition of two components in this case). Both of the paths generated are shown in figures 5 and 6.

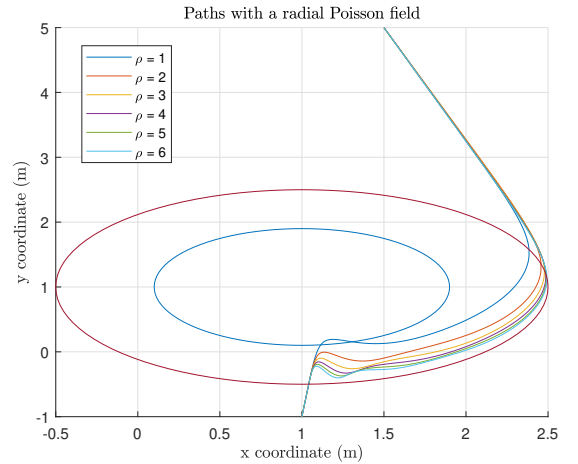


Fig. 5. Trajectories generated by using a radial Poisson field

As can be observed, the use of a radial field generates paths that can easily stay at a safe distance from the obstacle with slight increment of the ρ parameter. However, the paths generated require violent changes of direction that may not be easily achievable by a real vehicle. The paths generated with a tangential field are geometrically smoother, but are critically closer to the obstacle. A cautious choice of the danger parameter may permit the generation of a path without abrupt deviations and a reasonable distance to the obstacle. In the tangential case, an important increase in the ρ parameter is required in order to avoid a collision. In the simulations, it is observed that in the radial case a value around $\rho = 2$ is enough to prevent a collision, whereas this value goes up to $\rho = 4$ for the tangential case with a close distance to the obstacle.

The length and energy criteria for these simulations are presented in the table 3.

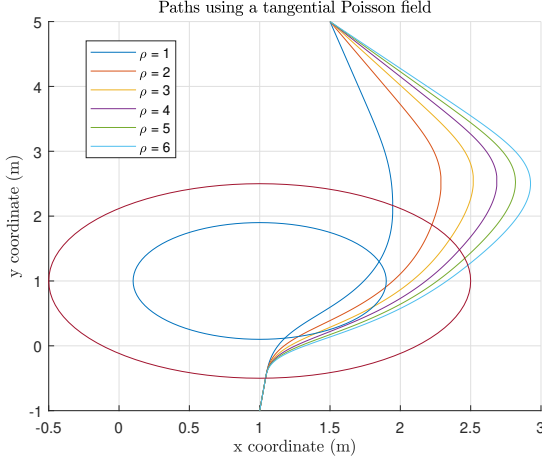


Fig. 6. Trajectories generated by using a tangential Poisson field

Table 3. Length and energy criteria by using a Poisson field

	Radial Poisson Field		Tangential Poisson Field	
ρ	$J_{long-rad}$	$J_{energy-rad}$	$J_{long-tang}$	$J_{energy-tang}$
2	7.05 m	0.31 J	X	X
3	7.11 m	0.32 J	X	X
4	7.14 m	0.33 J	6.83 m	2.11 J
5	7.16 m	0.33 J	6.96 m	2.39 J
6	7.18 m	0.34 J	7.07 m	2.64 J

As it can be seen, the impact of the parameter ρ doesn't heavily impact the energy consumption nor the length of the path as it changes. A slight increase in energy consumption is seen and the same is observed in the path length. The results with the tangential case differ greatly from the radial one. Even if there is a slight reduction in path lengths, the energy consumption is significantly high. The increase in energy consumption is notorious when increasing the danger parameter and it goes up to more than eight times the energy value computed for the first order polynomial (see table 2).

5. CONCLUSION

Three contributions are proposed to reduce acceleration oscillations in the APF as originally proposed by Weyl and Ge & Cui potential field definitions. The use of corrective polynomials proved to be an effective way to avoid the presence of oscillations in the paths generated and therefore an important reduction in the energy consumption was seen in the simulations. To avoid local minima, tangential force fields are proposed, which have the main drawback of a high energy demand. The Poisson force field may be an elegant and alternative way of formulating a repulsion force field, but fails to solve the problem of the heavy energy consumption that comes with the use of tangential fields. Further exploration can be done by taking into account more complex models for the vehicles instead of the simplified punctual mass used here in order to study the possibility of real vehicles to follow the different references generated by these fields.

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