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Dispersive characteristics of non-linear waves propagating and breaking over a mildly sloping laboratory beach

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Abstract

The dispersive characteristics of unidirectional irregular waves propagating and breaking over a mildly sloping beach are examined using a highly-resolved laboratory dataset. Cross-spectral analyses are used to determine the cross-shore evolution of (single-valued) dominant wavenumber κ and phase velocity c spectra, and lead to the identification of four different regimes of propagation: I - a linear regime where short waves mostly propagate as free components; II - a shoaling regime where non-linear effects at high harmonics are significant but primary components follow the linear wave dispersion relation; III - a shoaling regime near the mean breaking point location, where amplitude dispersion effects at primary components are important; IV - a surf zone regime, where all components propagate slightly faster than non-dispersive shallow water waves. Bispectral analyses performed onshore of the shoaling region show that the presence of forced energy at high harmonics, which originate from non-linear interactions between triads of frequencies, are responsible for the deviations of wavenumber and phase velocity spectral estimates from predictions by the linear dispersion relation, confirming the findings from previous field-based studies. A Boussinesq approximation of the non-linear energy exchanges between triads is then used to quantify the relative amount of forced energy at high harmonics and explain the differences in dispersion properties observed in the shoaling region between broad and narrow-band spectra. Larger relative amounts of forced energy at high frequencies, which suggest more efficient non-linear energy transfers, are found to be associated with larger deviations of dominant κ and c from predictions by the linear dispersion relation.

1. Introduction

Wind-generated surface gravity waves (hereafter short waves) are the principal driver of nearshore dynamics. Close to shore, short waves eventually break and through this process, they enhance the vertical and horizontal mixing of the water column (*e.g.*, Ting and Kirby, 1996; Drazen and Melville, 2009; Clark et al., 2012), drive a setup near the shoreline (*e.g.*, Longuet-Higgins and Stewart, 1964; Stive and Wind, 1982) and control the nearshore circulation at various temporal scales (*e.g.*, Svendsen, 1984; Peregrine and Bokhove, 1998; Bühler and Jacobson, 2001; Bonneton et al., 2010; Castelle et al., 2016). At first order, linear wave theory correctly predicts a number of physical processes associated with the propagation of short waves, such as the refraction or shoaling of directionally spread seas (*e.g.* Longuet-Higgins, 1956; Guza and Thornton, 1980; Elgar et al., 1990, and many others). However, as waves shoal and interact

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11 with a sloping, mobile bed, non-linear processes become dominant. These are responsible for the changes
12 observed in wave shape, from nearly symmetric to more (horizontally) skewed in the shoaling region and
13 more (vertically) asymmetric prior to breaking and in the surf zone (*e.g.* Elgar and Guza, 1985a,b; Doering
14 and Bowen, 1995; Michallet et al., 2011; Rocha et al., 2017). Non-linear effects not only affect the sea
15 surface elevation but also the near-bottom wave orbital velocities and thus play a crucial role in short- and
16 long-term beach morphodynamics (Doering and Bowen, 1986, 1995; Hoefel and Elgar, 2003; Berni et al.,
17 2013; van der Zanden et al., 2017).

18 Weakly non-linear triad interactions (*e.g.* Phillips, 1960; Freilich et al., 1984; Elgar and Guza, 1985a)
19 occurring as short waves propagate landward over a sloping bottom are responsible for these changes in
20 the wave field. The interaction of two primary components of frequencies f_1 and f_2 excite a secondary
21 component f' (either sum $f_1 + f_2$ or difference $f_1 - f_2$), which is bound to the statistically independent
22 primary components. As such, the bound wave component f' does not follow the linear wave dispersion
23 relation (Phillips, 1960; Longuet-Higgins and Stewart, 1962; Freilich et al., 1984). As opposed to non-linear
24 resonant interactions between quadruplets in deep water (Hasselmann, 1962), which require very large
25 distances to be effective, non-linear coupling between triads in nearshore areas are non- or near-resonant
26 and can be very efficient in transforming incident wave spectra over just few typical wavelengths (*e.g.*,
27 see Freilich et al., 1984, and the references therein). As both forced (or 'bound') and free components
28 of directionally spread seas can co-exist in a wave field, there is no longer a unique relation between a
29 frequency and wavenumber (*e.g.* Herbers and Guza, 1994). When forced components dominate over a
30 region of the spectrum, large deviations from predictions by the linear wave dispersion relation can be
31 observed in (single-valued) dominant wavenumber and phase velocity spectra (Thornton and Guza, 1982;
32 Freilich et al., 1984; Elgar and Guza, 1985b). In particular near the breaking point or in the surf zone,
33 most wave components of a typical sea-surface spectrum travel at the speed of non-dispersive shallow-water
34 waves (*e.g.*, see Thornton and Guza, 1982; Elgar and Guza, 1985b; Catalán and Haller, 2008; Tissier et al.,
35 2011), which is due to the dominance of amplitude dispersion effects over frequency ones (Herbers et al.,
36 2002). As noted by Laing (1986), the deviations of measured wave phase speed from predictions by the
37 linear dispersion relation, discussed here for nearshore waves, are quite analogous to those observed in
38 growing seas (*e.g.*, see Ramamonjiarisoa and Coantic, 1976; Mitsuyasu et al., 1979; Crawford et al., 1981;
39 Donelan et al., 1985). In such conditions, growing short-wave fields are dominated by modulated trains
40 of finite amplitude waves to which high-frequency components are bound (Lake and Yuen, 1978; Coantic
41 et al., 1981).

42 In practice, knowledge on the spatial structure of the wave field is generally lacking and the presence of
43 forced energy is therefore difficult to quantify. As forced components at high harmonics are characterised
44 by lower wavenumbers than free components of the same frequency, large errors from depth-inversion algo-
45 rithms based on the linear wave dispersion relation can be expected in regions where non-linear effects are
46 important (*e.g.*, see Holland, 2001; Brodie et al., 2018). The over-predictions of the dominant wavenumbers
47 at high frequencies also explain the commonly reported 'blow-up' when reconstructing the free surface ele-
48 vation from sub-surface pressure measurements with the linear dispersion relation (Bonneton and Lannes,
49 2017; Bonneton et al., 2018; Mouragues et al., 2019; Martins et al., 2020b). This is related to the fact

50 that forced high harmonics are much less attenuated across the vertical than free components of the same
51 frequency (e.g. Herbers and Guza, 1991; Herbers et al., 1992). Nonetheless, most field-based studies on
52 non-linear wave transformation in the shoaling region employed sub-surface hydrodynamic data (whether
53 pressure or orbital wave velocity), not corrected or corrected for depth-attenuation using the linear wave
54 dispersion relation. Field-based studies on wave non-linearity also suffer from other limitations such as a
55 poor spatial resolution and the distance over which waves can be studied. In particular, the cross-shore
56 location where non-linear effects at high harmonics become predominant remains largely unknown.

57 The present paper uses a high-resolution laboratory dataset (GLOBEX, see Ruessink et al., 2013) to
58 study the dispersive properties of irregular waves propagating and breaking over a mildly sloping beach.
59 Besides confirming past findings, the GLOBEX dataset stands out from previously-published field observa-
60 tions for several reasons. The free surface is directly measured with wave gauges and it is highly-resolved in
61 space (several points per wavelength at any stage of propagation). The former aspect removes uncertain-
62 ties currently existing on energy levels at high harmonics as measured in the field by sub-surface pressure
63 sensors, and where the choice of the surface elevation reconstruction method has a strong influence (Bon-
64 neton et al., 2018; Mouragues et al., 2019; Martins et al., 2020b). Furthermore, the experiments considered
65 unidirectional irregular waves, which removes uncertainty about directional effects. In section 2, the high
66 spatial and temporal resolution experimental dataset collected during GLOBEX is briefly presented. Sec-
67 tion 3 introduces the cross-spectral and bispectral analysis techniques and describes the weakly non-linear
68 numerical approach employed here for predicting the cross-shore evolution of energy spectra. In Section 4,
69 the cross-spectral analysis is performed on the surface elevation data from adjacent wave gauges to extract
70 dominant wavenumber spectra $\kappa(f)$, phase velocity spectra $c(f)$ and their evolution across the entire wave
71 flume. From this analysis, we identify four regimes of propagation ranging from a linear up to a surf zone
72 situation, where wavenumber and phase velocity spectra display specific characteristics. In section 5, the
73 bispectral analysis is used to quantify non-linear energy transfers towards harmonics, which play a funda-
74 mental role in the patterns observed at high frequencies in wavenumber and phase velocity spectra. The
75 dominant wavenumber is shown to vary with the amount of forced energy at a particular frequency, with
76 larger deviations from the linear wave dispersion expected for higher forced-to-free energy ratios. Finally,
77 section 6 briefly discusses the results and provides the concluding remarks of this study.

78 **2. Experimental dataset**

79 The Gently sLOping Beach Experiment (GLOBEX) project was performed in a 110-m-long, 1-m-wide,
80 and 1.2-m-high wave flume, located in the Scheldegoet in Delft, the Netherlands (Ruessink et al., 2013).
81 The experiments aimed at collecting high-resolution data of free surface elevation and current velocities
82 in order to study infragravity wave dynamics and short-wave propagation and non-linearities (*e.g.*, see
83 de Bakker et al., 2015; Tissier et al., 2015; Rocha et al., 2017). A combination of 21 capacitance and
84 resistance-type of wave gauges sampling at 128 Hz were deployed along the low-sloping 1:80 concrete beach
85 to measure the free surface elevation (see Fig. 1). 18 of these wave gauges were mounted on movable
86 trolleys, which were repositioned after the repetition of each wave test in order to reach the desired spatial
87 resolution (10 repetitions per test). Similarly, five electromagnetic current meters were fixed to the trolleys

88 and allowed the collection of current velocities at numerous locations across the wave flume (the data from
 89 these current meters is not used here). This spatial resolution makes the GLOBEX dataset unique as it
 90 allows to characterize and quantify non-linearities at various stages of the waves propagation (from a linear
 91 situation up to the surf zone). The wave paddle steering signals included second-order wave generation and
 92 the paddle was equipped with an Active Reflection Compensation system to absorb long waves radiated
 93 from the beach. The water depth at the wave paddle was 0.85 m for all tests.

94 The present study uses free surface elevation measurements from the 70-min-long irregular wave tests
 95 of the A series (A1, A2, A3, see Ruessink et al., 2013). During this series of tests, JONSWAP spectra
 96 were imposed, covering moderate to energetic and broad to narrow-banded sea wave conditions, see table
 97 1. The wave conditions along the flume for these runs are displayed in Fig. 2, through a range of second
 98 and third-order wave parameters. Fig. 2a first shows the root-mean square wave height H_{rms} computed
 99 as $(8\overline{\zeta^2})^{1/2}$ where ζ is the free surface elevation and the overbar denotes the time-averaged operator. Note
 100 that ζ was high-pass filtered using a cutoff frequency at $0.6 f_p$, with f_p the peak wave frequency, so that
 101 the bulk parameters shown in Fig. 2 are computed on the short-wave frequency band only. As conditions
 102 were more energetic during A2, the mean breaking point was located farther seaward than during A1 and
 103 A3. In the inner surf zone, waves were found to be depth-limited during all runs, which is also evidenced by
 104 the near-constant values reached by the non-linearity parameter $\epsilon = H_{rms}/\sqrt{2}h$ in this region of the wave
 105 flume ($\epsilon \sim 0.3$, see Fig. 2b), where h is the mean water depth. Fig. 2c-d show the short-wave skewness S_k
 106 and asymmetry A_s computed respectively as

$$S_k = \frac{\overline{(\zeta - \bar{\zeta})^3}}{(\zeta - \bar{\zeta})^2}{}^{3/2}. \quad (1)$$

$$A_s = -\frac{\overline{(\mathcal{H}(\zeta - \bar{\zeta}))^3}}{(\zeta - \bar{\zeta})^2}{}^{3/2}, \quad (2)$$

107 where $\mathcal{H}\{\cdot\}$ is the Hilbert transform. S_k and A_s are measures of wave asymmetry along the horizontal and
 108 vertical axes respectively and also inform on the energy content at high harmonics (Elgar, 1987; Martins
 109 et al., 2020b; de Wit et al., 2020). For the same imposed wave height, A1 and A3 differed in the shoaling
 110 region, with a more intense shoaling process noted during A3 (Fig. 2a and 2c), as expected for longer
 111 waves. This is also explained by the narrower spectrum imposed during A3, which favoured non-linear
 112 energy transfers to higher harmonics as compared to A1 (de Bakker et al., 2015). Short waves remain
 113 (vertically) symmetric ($A_s \sim 0$) until the location where the largest waves start breaking. Short-wave
 114 asymmetry is maximal in the inner surf zone, where short waves pitch forward and display the commonly
 115 observed sawtooth-like shape. Finally, the Ursell number U_r is shown in Fig. 2e. Here, U_r is defined as the
 116 ratio between the non-linearity ϵ and dispersion $\mu = (\kappa_p h)^2$ parameters, where κ_p is the peak wavenumber
 117 given by the linear wave dispersion relation.

118 3. Methods

119 3.1. Computation of wavenumber and phase velocity spectra

120 Cross-spectral analysis between adjacent wave gauges is used to compute the dominant wavenumber
 121 and phase velocity spectra across the wave flume. As this approach provides phase differences (or delay)

122 between two signals in the frequency domain (*e.g.*, see Ochi, 1998), it has been successfully used in the past
 123 to study, in both the laboratory and field, the dispersive properties of ocean waves propagating in deep
 124 (*e.g.*, see Ramamonjiarisoa and Coantic, 1976; Mitsuyasu et al., 1979) and intermediate to shallow waters
 125 (Thornton and Guza, 1982; Freilich et al., 1984; Elgar and Guza, 1985b).

126 Let C_{x_1, x_2} denote the cross-spectrum computed with surface elevation timeseries from two gauges
 127 located at positions x_1 and x_2 . The coherence $\text{coh}(f)$ and phase $\phi(f)$ spectra computed between x_1 and x_2
 128 are then respectively given by

$$\text{coh}_{x_1, x_2}(f) = \left[\frac{C_{x_1, x_2}(f)C_{x_1, x_2}^*(f)}{C_{x_1, x_1}(f)C_{x_2, x_2}(f)} \right]^{1/2} \quad (3)$$

$$\phi_{x_1, x_2}(f) = \arctan \left[\frac{\text{Im}\{C_{x_1, x_2}(f)\}}{\text{Re}\{C_{x_1, x_2}(f)\}} \right], \quad (4)$$

129 where $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ are the real and imaginary parts of the cross-spectra respectively and $*$ denotes the
 130 complex conjugate. As shown in Fig. 3a with an example of cross-spectral analysis performed during A2 in
 131 the deepest region of the flume ($x \sim 10$ m), the phase spectra $\phi(f)$ provides a phase lag per frequency that
 132 is bounded between $-\pi/2$ and $\pi/2$. The time delay (in sec) per frequency is obtained from the unwrapped
 133 phase ϕ^{unw} which, in the case of progressive waves propagating in one dimension, is easily retrieved from
 134 the phase jumps (*e.g.*, see Fig. 3a). The wavenumber $\kappa(f)$ and (cross-shore) phase velocity $c(f)$ spectra
 135 are then readily computed as

$$\kappa(f) = \phi_{x_1, x_2}^{unw}(f) / \Delta x \quad (5)$$

$$c(f) = 2\pi f \Delta x / \phi_{x_1, x_2}^{unw}(f), \quad (6)$$

136 where Δx is the spacing between the two wave gauges. As in Herbers et al. (2002), κ refers to the
 137 single-valued wavenumber modulus, representative of a mixed sea-state composed of both free and forced
 138 components (*i.e.*, for a given angular frequency $\omega = 2\pi f$, energy in the (ω, k) space is spread across several
 139 wavenumbers k). In practice, κ and c provide estimates at $x = (x_1 + x_2)/2$ of the dominant wavenumber (in
 140 an energy-averaged sense) and the corresponding propagation speed respectively, as shown by the analysis
 141 on synthetic data in Appendix A. This aspect is also discussed in Section 5. Cross-spectra were computed
 142 using Welch's method and 63 Hann-windowed records of 128 seconds, which were overlapping by 50%. This
 143 resulted in each spectral estimate having approximately 70 equivalent degrees of freedom and a spectral
 144 resolution of 0.008 Hz.

145 As observed by many authors in the past, the coherence spectra at high harmonics were found to be
 146 quite sensitive to the spacing between the two adjacent wave gauges. This is especially true in the deepest
 147 parts of the wave flume, where energy levels at these frequencies are quite low and the spacing between
 148 the two gauges can represent several wavelengths. An example is provided in Fig. 3b, which shows the
 149 coherence spectra computed over the flat section of the flume during A2 with $\Delta x = 0.93L_p$, L_p being the
 150 peak wavelength given by linear wave theory. The coherence remains very high ($\text{coh} > 0.95$) at frequencies
 151 between 0.6 and 1.5 f_p , which explain over 86% of the short-wave variance at this location. However, valleys
 152 in the coherence can be observed around 2.5 f_p for this particular spacing configuration and the coherence
 153 weakens quickly after 2.8 f_p (Fig. 3b). Since using a different spacing slightly shifts the coherence 'valleys',
 154 several combinations of wave gauges and spacing were used to obtain spectral estimates at a single location

155 with coherence higher than typically 0.5 at all frequencies (similar idea as that used by Herbers et al.,
 156 2002, with their field observations). After the removal of data associated with a coherence inferior to 0.5
 157 ($\text{coh}^2 \gtrsim 0.25$ in Fig. 3b), spectral estimates were ensemble-averaged. When non-linearities are strong,
 158 such as near the breaking point or in the surf zone, the coherence remains high up to $4-5 f_p$, as long as
 159 Δx is taken sufficiently small (typically $0.2-0.4 L_p$ in those regions). In such cases the ensemble-averaging
 160 procedure is not really necessary but it was used all along the flume for consistency in the analysis.

161 3.2. Computation of bispectra

162 The power bispectrum of discretely sampled data corresponds to a representation in the frequency
 163 domain of its third-order cumulant or moment. As it provides information on the strength of the phase
 164 coupling between triads of frequencies f_1 , f_2 and $f_1 + f_2$, the bispectrum is a useful and powerful tool for
 165 studying non-linear phenomena in ocean waves (Hasselmann et al., 1963; Elgar and Guza, 1985a). Here,
 166 we use the definition given by Kim and Powers (1979):

$$B(f_1, f_2) = \mathcal{E} [A(f_1) A(f_2) A^*(f_1 + f_2)], \quad (7)$$

167 where $A(f)$ are the complex Fourier coefficients and \mathcal{E} is an expected, or ensemble-average, value. Similar
 168 to the cross-spectrum phase, the biphase is obtained from the bispectrum as

$$\beta(f_1, f_2) = \arctan \left[\frac{\text{Im}\{B(f_1, f_2)\}}{\text{Re}\{B(f_1, f_2)\}} \right]. \quad (8)$$

169 It is particularly insightful to recast the biphase as a function of the Fourier coefficients phases $\theta(f)$:
 170 $\beta(f_1, f_2) = \theta(f_1) + \theta(f_2) - \theta(f_1 + f_2)$ (*e.g.*, see Kim et al., 1980; Elgar and Guza, 1985a). The amount of
 171 energy transfer between near-resonant components depends on their relative phases, whose information is
 172 contained in the biphase (Hasselmann et al., 1963; Kim et al., 1980).

173 Bispectra were computed on the free surface elevation signals down-sampled to 16 Hz by averaging
 174 estimates from 126 Hann-windowed records of 128 seconds, which were overlapping by 75%. Statistical
 175 stability was increased by merging estimates over three frequencies (*e.g.*, see Elgar and Guza, 1985a) yielding
 176 approximately 205 equivalent degrees of freedom, and a spectral resolution of 0.023 Hz. A validation of the
 177 bispectra computation is provided in Appendix B, which compares surface elevation third-order moments
 178 computed across the whole wave flume via bispectral (Elgar and Guza, 1985a; Elgar, 1987) and statistical
 179 definitions (Eq. 1 and 2).

180 3.3. Non-linear energy transfers

181 In the present study, we are principally interested in the growth of high harmonics due to non-linear
 182 energy exchanges between triads and the effect of these forced components on the dispersive properties of
 183 shoaling waves. To quantify the relative importance of forced energy at high harmonics, we represent these
 184 exchanges with a Boussinesq approximation in a spectral energy balance equation, following a similar ap-
 185 proach as developed in Herbers et al. (2000). Such Boussinesq approximation has already demonstrated its
 186 potential to predict, in a very computationally efficient manner, the cross-shore evolution of spectral com-
 187 ponents in the nearshore (Herbers and Burton, 1997; Norheim et al., 1998; Herbers et al., 2000; de Bakker
 188 et al., 2015; Padilla and Alsina, 2017). Note that the Boussinesq approximation (detailed below) restricts
 189 the present modelling approach to A2 and A3 since conditions during A1 are too dispersive.

190 For unidirectional waves propagating shoreward on an alongshore-uniform beach and assuming a weak
 191 reflection at the shoreline, a balance between the cross-shore gradient of the energy flux spectrum $F(f)$,
 192 a source term $S_{nl}(f)$ quantifying the non-linear energy exchanges between triads, and a dissipation term
 193 $S_{dis}(f)$ reads (*e.g.*, Eq. 1 of Herbers et al., 2000)

$$\frac{\partial F(f)}{\partial x} = S_{nl}(f) + S_{dis}(f). \quad (9)$$

194 Breaking processes are ignored so that the dissipation term S_{dis} reduces to the energy losses by bottom
 195 friction S_{fr} , here simply modelled after Thornton and Guza (1983):

$$S_{fr}(f) = \rho c_f \frac{1}{6\pi} \left(\frac{2\pi f}{\sinh |k|h} H \right)^3, \quad (10)$$

196 where h is the mean water depth and $|k|$ is the wavenumber modulus obtained from the linear wave
 197 dispersion relation (hereafter denoted κ_L , see Section 4). The friction coefficient c_f was set to 0.0055 after
 198 calibration in the deepest section of the wave flume.

199 Assuming that the wave field is weakly non-linear, weakly dispersive, and that these effects are of similar
 200 order ($U_r \sim O(1)$, *i.e.* Boussinesq regime), the non-linear source term S_{nl} can be approximated with an
 201 integral of the bispectrum as follows (Herbers and Burton, 1997; Norheim et al., 1998; Herbers et al., 2000):

$$S_{nl}(f) = \frac{3\pi f}{h} \int_{-\infty}^{\infty} \text{Im}\{B^*(f', f - f')\} df'. \quad (11)$$

202 Eq. 11 differs from the expression of Herbers et al. (2000) (their Eq. 2) in several points: the conjugate
 203 of B is taken in order to be consistent with their definition of the bispectrum (conjugate of the present
 204 definition); S_{nl} is here defined as a function of f (not ω) and the definition with the full integral is kept
 205 (symmetric properties of the bispectrum are not used to decompose it). Previous studies proposed evolution
 206 equations for the energy and bispectra (*e.g.*, the stochastic model of Herbers and Burton, 1997) to simulate
 207 the propagation and transformation of directional seas. As in Herbers et al. (2000), we take advantage of
 208 the spatial resolution of the present dataset and directly evaluate S_{nl} and S_{fr} at each available cross-shore
 209 location using trapezoidal rules for approximating the integrals. Besides removing uncertainties associated
 210 with its cross-shore prediction, using measured bispectra has for advantage to relax the limitations of the
 211 stochastic Boussinesq model of Herbers and Burton (1997) (such as the distance over which bispectra can
 212 be propagated, see Freilich et al., 1984). In order to be consistent with the original equation derived by
 213 Herbers and Burton (1997, their Eq. 22a), the energy in Eq. 9 is assumed to propagate at the shallow
 214 water wave speed so that $F(f) = \rho g E(f) \sqrt{gh}$, with ρ the water density and g the acceleration of gravity.
 215 After these considerations, Eq. 9 simplifies to

$$\frac{\partial E(f) \sqrt{gh(x)}}{\partial x} = \frac{1}{\rho g} (S_{nl}(f) + S_{fr}(f)). \quad (12)$$

216 Integrating this equation (in space) between the location of the first gauge x_0 and any location x_1 prior
 217 to the mean breaking point location yields the following expression for the energy density spectrum at x_1
 218 $E(x_1, f)$:

$$E(x_1, f) = E(x_0, f) \frac{\sqrt{gh(x_0)}}{\sqrt{gh(x_1)}} + \frac{1}{\rho g \sqrt{gh(x_1)}} \int_{x_0}^{x_1} (S_{nl} + S_{fr}) dx. \quad (13)$$

219 Finally, the ratio $\int_{x_0}^{x_1} S_{nl} dx / F(x_1)$, which represents the energy flux received (or lost) via non-linear
 220 coupling between x_0 and x_1 over the total energy flux at x_1 , is used in the following as an approximation
 221 of the relative amount of forced energy at x_1 . Outside the surf zone, where dissipative processes dominate,
 222 this estimation was found to be more reliable than the bicoherence, which is often used as a proxy for such
 223 an estimation but lacks general consensus upon its definition, see Appendix C for more details.

224 4. Wavenumber and phase velocity spectra of shoaling and surf zone waves

225 In this section, we present and describe the main results from the cross-spectral and bispectral analyses.
 226 The computation of dominant wavenumber κ and phase velocity c spectra for varying degrees of non-
 227 linearity resulted in the identification of four different regimes of propagation, which broadly consist of: a
 228 linear regime (stage I), a shoaling regime relatively far from the mean breaking point (stage II), a shoaling
 229 regime near the mean breaking point (stage III) and a surf zone regime (stage IV). The Ursell number U_r
 230 (Fig. 2e) was used as a reference to define these regimes of propagation, which are characterized by similar
 231 patterns - between all wave tests considered here - in wavenumber and phase velocity spectra. For all
 232 stages, $\kappa(f)$ and $c(f)$ spectra are compared to predictions from the linear wave dispersion relation, which
 233 links the spatial and temporal information of a linear wave field:

$$\omega^2 = g|k| \tanh(|k|h). \quad (14)$$

234 In the following, the subscript 'L' is used throughout the manuscript to refer to κ or c values that are solution
 235 to Eq. 14 (*i.e.* κ_L and $c_L = \omega/\kappa_L$). For conciseness, the propagation as linear waves is discussed using
 236 the results from the most non-linear case (A2), already shown in Fig. 3. The other stages of propagation
 237 focus on the differences between broad and narrow spectra, *i.e.* between A2 and A3. Note that κ and c
 238 characteristics during A1 are very similar to those obtained during A2 for similar U_r numbers. Finally,
 239 given the relatively high spatial resolution, frequency-wavenumber power spectra $P(\omega, k)$ (*e.g.*, see Redor
 240 et al., 2019) were computed using 2D Fourier analysis at four different cross-shore sections corresponding
 241 to stages I, II, III and IV, by averaging over 63 Hann-windowed time-records of 128 seconds overlapping by
 242 50% (Fig. 4). The chosen length of the cross-shore section (width of the data window for computing the
 243 2D Fast-Fourier transform) is a best compromise for reaching a proper resolution in wavenumber k and still
 244 assuming little change in wave type over the section. The 2D Fourier analysis shown in Fig. 4 provides a
 245 qualitative information on energy spreading in (ω, k) space and is particularly useful to illustrate what the
 246 single-valued dominant wavenumber represents in a mixed sea-state. Fig. 4 will be used in the present and
 247 following sections to analyse the link between non-linear energy exchanges and the dispersive properties of
 248 the wave field.

249 4.1. Propagation as free, linear waves (I)

250 Fig. 3 shows for the most non-linear case A2 an almost perfect match for $f > 0.6 f_p$ between the
 251 measured wave phases, wavenumbers and phase velocity spectral estimates with the predictions from the
 252 linear wave dispersion relation (Fig. 3a, 3c and 3d respectively). The decrease of coherence at high
 253 frequencies (Fig. 3b) can be explained by the relatively low energy content in the spectrum tail ($> 86\%$ of
 254 the variance is contained between $0.6 f_p$ and $1.5 f_p$ during A2). It also provides an explanation for the slight

255 deviations of measured κ and c from predictions by the linear wave dispersion relation (Fig. 3c-d). However,
 256 the 2D Fourier analysis performed over the flat section of the flume (Fig. 4a-b) reveals the presence of
 257 energy along the dispersion relation for bound high-frequency components (*e.g.*, see at $f > 2.5f_p$ for A2
 258 and around $2f_p$ for A3), suggesting that energy transfers towards high harmonics already occur in this
 259 region. Nonetheless, the energy is mainly distributed along the linear dispersion relation, which indicates
 260 that short-wave components were mostly propagating as free waves in the deepest regions of the wave flume
 261 ($U_r \sim O(0.01 - 0.1)$).

262 Data points in the infragravity frequency band ($f < 0.6f_p$) correspond to wave components generated
 263 via difference interactions and that are bound to short-wave groups. As such, bound infragravity waves
 264 propagate at a speed close to the corresponding group velocity, which is lower than their intrinsic phase
 265 velocity. This is confirmed in Fig. 3d with a good correspondence between measured c values with short-
 266 wave groups propagation speed (c_{env} , shown as the orange line), estimated via cross-spectral analysis of
 267 short-wave groups envelop. The latter was computed following Janssen et al. (2003), as the absolute value
 268 of the analytical signal of ζ : $|\zeta + i\mathcal{H}\{\zeta\}|$ (no low-pass filtering is needed for the present application). Note
 269 that the discrepancies observed at $f \in [0.3f_p; 0.6f_p]$ can be explained by the lower levels of energy at these
 270 frequencies: $E(f) \sim O(10^{-5}) \text{ m}^2/\text{Hz}$ as opposed to $E(f) \sim O(10^{-4}) \text{ m}^2/\text{Hz}$ around $f = 0.1f_p$.

271 4.2. Shoaling of weakly non-linear waves (II)

272 At higher Ursell number ($U_r \sim 0.3$), wave non-linearities become significant and non-linear energy
 273 transfers to high harmonics promote the dominance of bound high harmonics (Fig. 4c-d). This is observed
 274 in the considerable deviation of dominant wavenumbers and phase velocity spectral estimates from the
 275 predictions of the linear dispersion relation for $f > 1.5f_p$ (see Fig. 5c-f). During A3, the phase velocity
 276 values at $2f_p$ and $3f_p$ are equal to that of the peak frequency (difference $< 1\%$, $c(f_p) = 1.95 \text{ m/s}$), while
 277 during A2 they lie between the value at f_p and the predictions of the linear dispersion relation ($\sim 7\%$
 278 difference from $c(f_p) = 2.18 \text{ m/s}$). At the second harmonic $2f_p$, most of the non-linear energy transfers
 279 occur via sum interactions of components around f_p (self-self interactions) as evidenced by the positive
 280 imaginary part of B^* around (f_p, f_p) , see Fig. 5g and 5h for tests A2 and A3 respectively. Although less
 281 evident in the bispectra, the normalised bispectra shown in Fig. C1 indicate that strong coupling involving
 282 f_p and higher harmonics also exist in the shoaling region. These are particularly strong around $(2f_p, f_p)$
 283 for A3, which explains the significant growth of the third harmonic $3f_p$ observed for that test (Fig. 5b).
 284 Consistent with field observations (*e.g.* Elgar and Guza, 1985a; Norheim et al., 1998), relatively strong non-
 285 linear exchanges by sum interactions are observed during A2, despite the broader spectrum conditions (note
 286 the scale difference between the A2 and A3 cases). Short-wave frequencies distant from f_p (*e.g.* $\sim 1.5f_p$)
 287 show a strong coupling with f_p , which is the consequence of a broader spectrum for A2 compared to A3.
 288 A striking result in this regime of propagation is the fact that, despite non-linearities becoming important
 289 ($\epsilon \sim 0.12 - 0.2$, $S_k \sim 1$), the principal wave components follow the linear wave dispersion relation. The
 290 spatial structure of the wave field for $f \in [0.6f_p; 1.5f_p]$ is therefore well-described by the linear wave theory
 291 (Fig. 5c and 5d). However, this is not the case for $f > 1.5f_p$, where the overestimation of wavenumbers by
 292 the linear dispersion relation increases with f (up to a factor 2.5 at $3.5f_p$). As the biphase at both (f_p, f_p)
 293 and $(f_p, 2f_p)$ are close to 0, this regime of propagation is consistent with Stokes-like non-linearities (*e.g.*,

294 see Elgar and Guza, 1985a).

295 In the infragravity band, wave components are still bound to short-wave groups as indicated by the
296 good match between spectral estimates of phase velocity and the propagation speed of short-wave groups.
297 Energy transfers towards the infragravity band concentrate at frequencies around $0.1-0.2f_p$ (Fig. 5a and
298 5b) and principally originate from strong difference interactions which transfer energy from f_p towards that
299 infragravity frequency and components at frequency slightly lower than f_p (see the negative imaginary part
300 of B^* along the f_p anti-diagonal, Fig. 5g and 5h).

301 4.3. Shoaling of non-linear waves in the vicinity of the breaking point (III)

302 The next regime analysed in Fig. 6 corresponds to non-linear waves approaching the mean breaking
303 point location ($U_r \sim 0.7$). At this Ursell number, wavenumber and phase velocity spectra become less
304 frequency-dependent at high frequencies. This can be explained by the fact that most of the energy at
305 those frequencies is bound to primary components in the range $[0.6f_p; 1.5f_p]$, and that c values display
306 little variations at these frequencies ($c/c_L(f_p)$ is within 5% of $c(f_p)/c_L(f_p)$). As a result, wavenumbers at
307 high frequencies are relatively well described by a simple dispersion relation given by $\kappa(f) = 2\pi f/c(f_p)$
308 (Fig. 6c-d). Note that at these depths ($\kappa_p h \sim 0.57$ and 0.47 for A2 and A3 respectively), this approximately
309 corresponds to the shallow water dispersion relation $\kappa_{sw}(f) = 2\pi f/\sqrt{gh}$, which explains why most of the
310 energy is spread around this dispersion relation in Fig 4e-f. Non-linear amplitude effects become increasingly
311 important ($\epsilon = 0.25$ and $S_k = 1.4-1.5$) and deviations between $c(f_p)$ and predictions by the linear dispersion
312 relation are observed (Fig. 6e-f). As noted and observed by Herbers et al. (2002), this is the result of non-
313 linear interactions, which alter the wavenumber of all three wave components involved. Indeed, bound high
314 harmonics contribute to an increase in skewness and height of principal wave crests, which has for effect
315 to enhance their propagation speed. Variations in the strength of non-linear interactions thus explain the
316 varying magnitude of the deviations of $c(f_p)$ from predictions by the linear dispersion relation as observed
317 in both field and laboratory settings (Thornton and Guza, 1982; Herbers et al., 2002; Tissier et al., 2011).
318 For this range of Ursell number, overestimations of κ by the linear dispersion relation remain close to a
319 factor 2 at $f = 3.5f_p$.

320 In these regions of the wave flume, surface elevation bispectra are still dominated by real values (note the
321 x2 in imaginary values, Fig. 6g-h), typical of (horizontally) skewed and nearly symmetric waves ($A_s \sim 0$, see
322 Fig. 2c-d and also Masuda and Kuo, 1981; Elgar and Guza, 1985a). The strong coupling between first and
323 second harmonics, which for instance explains the energy peak at $3f_p$ in A3 (Fig. 6b) is here evident in the
324 imaginary part of the bispectra at $(f_p, 2f_p)$ (see also the bicoherence in Fig. C1). Infragravity frequencies
325 between $0.1-0.2f_p$ keep receiving energy via relatively strong difference interactions. As a consequence,
326 prior to short-wave breaking infragravity waves are still bound to short-wave groups during both tests (see
327 Fig. 6e-f, and also de Bakker et al., 2015).

328 4.4. Surf zone waves (IV)

329 In the surf zone (U_r taken at 3.2), wavenumber and phase velocity spectra are frequency-independent
330 (Fig. 7e-f). The large differences observed between the measured phase velocity at peak frequency and that
331 predicted with the linear dispersion relation demonstrate the dominance of amplitude effects over dispersive

332 ones (Herbers et al., 2002). All components travel at a speed slightly larger than that of shallow water
 333 waves ($\sim 10 - 20\%$ depending on the wave test), which is consistent with many past field and laboratory
 334 observations of wave dispersion in the surf zone (Inman et al., 1971; Svendsen et al., 1978; Stive, 1980;
 335 Thornton and Guza, 1982, and many others). For both A2 and A3 tests, the relation $c = \sqrt{gh(1 + \epsilon)}$ is
 336 shown in Fig. 7e-f (see also Fig 4g-h, for the corresponding frequency-wavenumber surface elevation power
 337 spectra). This formula is asymptotic in shallow water to the non-linear dispersion relation given by Hedges
 338 (1976) with the adjustment of Booij (1981) (see also Catalán and Haller, 2008, for an assessment of these
 339 relations for depth-inversion applications). In the inner surf zone, where $\epsilon \sim 0.3$ during all tests (Fig. 2b),
 340 this yields $c \sim 1.14\sqrt{gh}$, a relation also found in the field by Tissier et al. (2011). This relation appears
 341 as an upper bound, in an average sense (*e.g.* over several wave groups), for the propagation speed of wave
 342 components in the inner surf zone.

343 In the short-wave frequency range, bispectra display relatively weak and nearly equal real and imaginary
 344 parts, characteristics of asymmetric (pitched forward) broken waves, with biphases approaching $-\pi/2$ (Elgar
 345 and Guza, 1985a; Doering and Bowen, 1987): for instance during A3, $\beta(f_p, f_p) \sim \beta(f_p, 2f_p) \sim -\pi/3$. Since
 346 these values were still close to 0 during stage III, this indicates that bound high-harmonic components (here
 347 only shown for $2f_p$ and $3f_p$) slowly drift out of phase from the principal component starting from the mean
 348 breaking point location. This is in contrast with short-wave group-forced bound infragravity waves that
 349 slowly drift out of phase with the short-wave envelope in the shoaling region (*e.g.*, see Elgar and Guza,
 350 1985a). Since this process occurs over long distances, the differences in phase velocities between f_p , $2f_p$
 351 and $3f_p$ are not sufficiently large to be observed in the present dataset. Finally, in the inner surf zone,
 352 intense and complex energy transfers occur within the infragravity band and also between infragravity and
 353 short-wave components. For more information on these processes during the GLOBEX experiments, the
 354 reader is referred to the study of de Bakker et al. (2015).

355 5. Role of non-linear energy transfers on κ and c

356 Most differences observed in wavenumber and phase velocity spectra between broad and narrow-banded
 357 wave conditions concentrate in the shoaling region (stages II and III). In stage II, c values at $2f_p$ and $3f_p$
 358 during A2 lie between $c(f_p)$ and the values predicted by linear wave theory while $c(f_p)$, $c(2f_p)$ and $c(3f_p)$ are
 359 all equal during A3 (Fig. 5e-f). As these differences between broad versus narrow-band spectra are likely
 360 explained by the relative importance of forced energy at those frequencies (see the cross-spectral analysis
 361 performed on synthetic data and discussed in Appendix A and also Herbers and Guza, 1992; Herbers et al.,
 362 2000), we analyse non-linear energy transfers to $2f_p$ and $3f_p$ in more details here.

363 Wave amplitudes at f_p , $2f_p$ and $3f_p$ (a_{f_p} , a_{2f_p} and a_{3f_p} respectively) computed from energy spectra
 364 modelled with Eq. 13 are compared against observations in Fig. 8. Overall, the Boussinesq approach
 365 of Herbers and Burton (1997) for the non-linear energy transfers between triads accurately predicts the
 366 growth of second and third harmonics across the shoaling zone for both broad and narrow-band wave tests.
 367 The cross-shore evolution of a_{f_p} , a_{2f_p} and a_{3f_p} are very well described up to the mean breaking point, with
 368 mean absolute percentage errors (MAPE) lower than 5% and 13% for a_{2f_p} and a_{3f_p} respectively. With
 369 respect to the amount of (free) energy imposed at the paddle, the narrow-banded conditions during A3

370 promote more efficient energy transfers, with growths of a_{2f_p} and a_{3f_p} by a factor 3 and 10 respectively,
 371 while these factors are only between 1.5 and 2 during A2.

372 A closer look at the cross-shore evolution of the source terms (Fig. 9) shows that friction effects are
 373 negligible at high frequencies but not around the peak frequency where the energy dissipated through
 374 bottom friction is of similar order than the energy lost via non-linear coupling. During A3 (Fig. 9b), the
 375 steady increase of $S_{nl}(2f_p)$ indicates a gradual growth of a_{2f_p} in the shoaling region (see also Fig. 8b). This
 376 growth is most intense right before the mean breaking point location, around the location corresponding
 377 to stage III. Computations of the ratio $\int_{x_0}^{x_1} S_{nl} dx / F(x_1)$ at the location x_1 corresponding to stage II (resp.
 378 III) suggest that approximately 70% (resp. 80%) of the energy at $2f_p$ is forced during A3. At $3f_p$, these
 379 estimations range between 90% and 100% during stage II and III respectively. Although the resolution in
 380 wavenumbers does not allow a clear separation of forced and free energy at $2f_p$, the frequency-wavenumber
 381 power spectra shown in Fig. 4d (II) and 4f (III) corroborate these numbers and clearly indicate that most of
 382 the energy at $3f_p$ is forced. In contrast, non-linear energy exchanges between triads display a more complex
 383 picture for the broader conditions of A2. $S_{nl}(2f_p)$ oscillates around 0 so that when spatially integrated,
 384 non-linear energy exchanges explain only 20% (30%) and 30% (50%) of the total energy at $2f_p$ and $3f_p$
 385 respectively during stage II (III). These numbers are quite consistent with the bicoherence values shown
 386 in Appendix C. For both wave tests, b^2 values are found to vary little between stage II and III: for A2,
 387 $b^2(f_p, f_p) \sim 0.3$ and $b^2(f_p, 2f_p) \sim 0.2$, while these values oscillate between 0.55 and 0.65 for A3. $b^2(f_p, f_p)$
 388 and $b^2(f_p, 2f_p)$ can be considered as crude estimates of the relative amount of forced energy at $2f_p$ and $3f_p$
 389 respectively (Kim and Powers, 1979).

390 The effect of varying relative amounts of forced energy not only explain the differences observed in κ and
 391 c spectra at high frequencies between A2 and A3 but also their variation across frequencies. During A3 for
 392 instance, non-linear energy transfers in the short-wave frequency band were predominantly towards $2f_p$ and
 393 $3f_p$ (Fig. 5h and 6h). At these harmonics, energy is predominantly forced ($> 70\%$ around $2f_p$ and $> 90\%$
 394 around $3f_p$), thus, the dominant κ and corresponding c values are directly related to the values at the peak
 395 frequency. In contrast, frequencies located in valleys between harmonics (*i.e.* $1.5f_p$ and $2.5f_p$) receive very
 396 little energy through the coupling of triads. This eventually leads to the patterns in c observed in Fig. 5f,
 397 which are very similar to those obtained by Crawford et al. (1981) with their model for modulated wave
 398 trains. During A2, lower relative amounts of forced energy are found at high harmonics and dominant
 399 wavenumbers lie between $\kappa(f_p)$ and the intrinsic value at that frequency predicted by the linear dispersion
 400 relation. Furthermore, due to the broader spectrum conditions, this forced energy is more spread across
 401 frequencies, which explains the weaker variations of κ and c across frequencies as compared to A3 (*e.g.*, see
 402 Fig. 5e-f).

403 The present study considers unidirectional wave fields transforming across a mildly sloping beach. A
 404 typical field situation where incident wave spectra exhibit varying degrees of directional spreading is unclear
 405 as far as non-linear energy transfers are concerned. According to Boussinesq theory (*e.g.*, Herbers and
 406 Burton, 1997), directional spreading has only a weak influence on the efficiency of non-linear energy transfers
 407 by sum interactions, as opposed to difference interactions. For instance, these authors predicted a weaker
 408 growth of high-harmonic bound wave by 10-20% for very large directional spreads (60°). For more realistic

409 spreading angles in the nearshore, de Wit et al. (2020) also recently noted variations of approximately 10%.
 410 Although small, such a decrease in efficiency of non-linear energy transfer towards high harmonics due to
 411 an increasing directional spreading might lead to lower relative amounts of forced energy, subsequently
 412 leading to slightly less important deviations of κ and c spectral estimates from predictions by the linear
 413 wave dispersion relation (see also Appendix A). The larger spectral bandwidth generally associated with
 414 wave spectra exhibiting a large directional spreading is, however, likely to be the principal reason explaining
 415 weaker couplings between triads and thus weaker relative amounts of forced energy at high harmonics.

416 6. Discussion and concluding remarks

417 Cross-spectral and bispectral analyses were employed on a highly-resolved surface elevation dataset
 418 to study the dispersive properties of waves shoaling and breaking over a mildly sloping beach. For all
 419 wave tests considered here, four regimes of propagation (I to IV) with specific characteristics in dominant
 420 wavenumber and phase velocity spectra could be defined using a local Ursell number. Stage II ($U_r \sim 0.3$) is
 421 particularly interesting as it simultaneously shows significant non-linear effects at high harmonics (typically
 422 $f > 1.5f_p$), which are evidenced by the large deviations of κ and c spectra from predictions by the linear
 423 wave dispersion relation, and a clearly linear spatial field for the primary components. Although less
 424 energy was imposed during A3 as compared to A2, the narrow-banded conditions promoted relatively more
 425 important non-linear energy transfers towards high harmonics such that phase velocity at $2f_p$ and $3f_p$ were
 426 found equal to those at the peak frequency. For A2, with a broader spectrum imposed, forced energy at
 427 high frequencies was found in relatively smaller proportion and c spectral estimates lied between values at
 428 the peak frequency and that given by the linear dispersion relation. Closer to the mean breaking point
 429 (stage III), non-linear energy transfers were found more intense, which, together with the less dispersive
 430 conditions compared to stage II explain that κ and c spectra become less frequency dependent. In the surf
 431 zone (stage IV), wavenumber and phase velocity spectra essentially showed the same characteristics for all
 432 tests and a modified shallow water dispersion that accounts for non-linearity ($c = \sqrt{gh(1 + \epsilon)}$) accurately
 433 describes the dispersive properties of the wave field. Some of the results presented here for shoaling and
 434 breaking waves confirm previous findings obtained in the field (Thornton and Guza, 1982; Elgar and Guza,
 435 1985b; Herbers et al., 2002, among others), yet the unprecedented level of accuracy obtained with the
 436 GLOBEX dataset allowed to highlight and quantify the effect of varying levels of forced energy at high
 437 harmonics on the dispersive properties of the wave field.

438 The detailed analysis of the different regimes of propagation highlighted here indirectly provides an
 439 assessment of the validity of the linear wave dispersion relation at several locations in the nearshore and for
 440 a range of wave conditions. Two specific regions in the short-wave frequency band can be defined: primary
 441 components ($f \in [0.6f_p, 1.5f_p]$) and high harmonics ($f > 1.5f_p$). For the primary components, deviations
 442 from the linear wave dispersion relation are due to non-linear amplitude effects and concentrate in a region
 443 near the mean breaking point and in the surf zone. These effects can cause deviations of κ and c from
 444 predictions by the linear wave dispersion of the order of 10-20%. At high frequencies, deviations are much
 445 larger ($O(100\%)$) and also occur much farther seaward. For instance, stage II corresponds to $\kappa_p h = 0.71$,
 446 0.70 and 0.62 for wave tests A1, A2 and A3 respectively. The fact that such deviations from predictions

447 by the linear dispersion relation are found in κ and c spectra so far seaward of the surf zone for different
448 wave conditions is expected to have strong implications for a wide range of applications, two of which are
449 briefly discussed below.

450 Depth-inversion algorithms such as *cBathy* (Holman et al., 2013) estimate the water depth by applying
451 the linear wave dispersion relation to the most coherent pairs of wave frequency and wavenumber extracted
452 from timeseries of video images (Stockdon and Holman, 2000; Plant et al., 2008). However, due to the
453 working principles of video cameras, video data can be more coherent at high harmonics and frequencies
454 greater than $1.5 f_p$ are often picked to invert the water depth (Stockdon and Holman, 2000; Bergsma and
455 Almar, 2018). In such cases, large deviations of the dominant wave phase velocity from linear wave theory
456 are expected, which likely explains why Brodie et al. (2018) noted an overestimation in the detected phase
457 velocities by 20-100% up to 50 m seaward of the mean breaking point location. In the absence of knowledge
458 on the relative amounts of forced energy at high harmonics, pairs of frequencies and wavenumbers around
459 the peak frequency should be preferred in regions where non-linearities are expected. Similarly, the large
460 overestimation of κ by the linear dispersion relation causes the widely reported blow-up at high harmonics
461 when correcting sub-surface pressure signals for depth attenuation (Bonneton and Lannes, 2017; Mouragues
462 et al., 2019; Martins et al., 2020b). The use of a cutoff frequency prevents for instance the accurate
463 description of third-order parameters and wave height distributions (*e.g.*, see Martins et al., 2020a). In
464 shallow water ($\mu \lesssim 0.3$), a weakly dispersive formula recently developed allows for an accurate correction of
465 the energy levels at high harmonics both seaward of (Bonneton et al., 2018; Mouragues et al., 2019) and in
466 the surf zone (Martins et al., 2020b). However in deeper water, a non-linear fully dispersive reconstruction
467 requires knowledge on the spatial structure of the wave field, which is generally lacking.

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478 **Appendix A. Sensitivity analysis of the cross-spectral technique to the presence of forced** 479 **energy**

480 The situation where both free and forced components exist in a wave field is not entirely clear as far
481 as the cross-spectral analysis is concerned. Some studies consider the cross-spectral estimates to be biased
482 towards bound harmonics (*e.g.*, see Lake and Yuen, 1978; Thornton and Guza, 1982), arguing that the
483 travelling distance between two gauges is reduced for bound components compared to free ones. This is

484 evidently exacerbated when using sub-surface pressure sensors or current meters since bound harmonics
 485 are much less attenuated and will dominate the spectrum at some depth.

486 To analyse the sensitivity of the cross-spectral analysis to different relative levels of forced energy, some
 487 tests are performed here on synthetic data. Starting at a position x_1 , a surface elevation timeseries following
 488 a JONSWAP spectrum with similar parameters to the A3 test is generated (Fig. A1). A Gaussian-shaped
 489 perturbation around $2f_p$ is added to represent forced harmonics: for any f around $2f_p$, the forced harmonic
 490 wavenumber is set at $\kappa_{forced} = 2\kappa(f/2)$, so that we have $c_{forced}(f) = c(f/2)$. For the phases of forced
 491 harmonics, we choose to impose $\theta_{forced}(f) = \theta(f/2)$ for simplicity. The maximum ratio of forced to free
 492 harmonic amplitudes $a_{2f_p, forced}/a_{2f_p, free}$ is set to 6, which corresponds to the maximum ratio of energy
 493 $E(f_p)/E(2f_p)$ of approximately 0.4 that was observed in the flume outside the surf zone. Each wave
 494 component is then propagated using its corresponding wavenumber to a second location x_2 .

495 The results from the cross-spectral analysis performed on the synthetic timeseries at x_1 and x_2 are pro-
 496 vided in Fig. A2 as the wavenumber and phase speed at $2f_p$ shown as a function of $a_{2f_p, forced}/a_{2f_p, free}$.
 497 In the absence of forced components ($a_{2f_p, forced}/a_{2f_p, free} = 0$), components around $2f_p$ follow the dis-
 498 persion relation given by linear wave theory, *i.e.* $\kappa(2f_p) = \kappa_L(2f_p)$ and $c(2f_p) = c_L(2f_p)$. Interestingly,
 499 when forced and free components are found in equal proportion ($a_{2f_p, forced}/a_{2f_p, free} = 1$), the relation
 500 $\kappa(2f_p) \sim (\kappa_L(2f_p) + 2\kappa_L(f_p))/2$ is verified. For higher ratio of forced to free energy, $\kappa(2f_p)$ and $c(2f_p)$
 501 values rapidly converge towards the values corresponding to the peak frequency. Overall, these results
 502 suggest that the cross-spectral analysis is not biased towards forced components, but simply provides a
 503 dominant wavenumber (or averaged in terms of energy) and its corresponding phase velocity.

504 **Appendix B. Validation of the bispectrum computations using the skewness and asymmetry** 505 **test**

506 The wave skewness and asymmetry are third-order moments characterizing the wave shape. These
 507 parameters are directly related to the energy content at high frequencies of the spectra and bispectra
 508 (Elgar, 1987). The fact that the statistical (see Eq. 1 and 2) and the bispectrum-based definitions are
 509 theoretically equivalent provide a means to validate the bispectrum's calculations.

510 Due to its symmetry properties, the bispectrum can be uniquely defined in a single octant in the
 511 frequency-space (Hasselmann et al., 1963; Elgar and Guza, 1985a). If we denote by f_N the Nyquist fre-
 512 quency, the octant with positive frequencies is bounded by the vertices at $(0,0)$, $(f_{N/2}, f_{N/2})$ and $(f_N, 0)$.
 513 The wave skewness and asymmetry are defined by the sum of the real and imaginary parts of the bispec-
 514 trum over this octant respectively, normalized by the cube of the free surface elevation standard deviation
 515 (Elgar, 1987; Elgar and Guza, 1985a). This reads:

$$S_k = \left[12 \sum_n \sum_l \text{Re}\{B(f_n, f_l)\} + 6 \sum_n \text{Re}\{B(f_n, f_n)\} \right] / (\overline{(\zeta - \bar{\zeta})^2})^{3/2} \quad (\text{B.1})$$

$$A_s = \left[12 \sum_n \sum_l \text{Im}\{B(f_n, f_l)\} + 6 \sum_n \text{Im}\{B(f_n, f_n)\} \right] / (\overline{(\zeta - \bar{\zeta})^2})^{3/2} \quad (\text{B.2})$$

516 with $n > l$ and $n + l < N$.

517 The cross-comparison between statistical and bispectrum-based definitions of the wave skewness and
 518 asymmetry computed for A3 is provided in Fig. B1 and shows a perfect match between both definitions,

519 thus validating the present computations of the bispectrum. Note that for these comparisons, blocks were
 520 not tapered and rectangular windows were used for computing the Fast Fourier Transforms. Using any
 521 other types of windows resulted in small differences between the two definitions of both third-order wave
 522 parameters.

523 **Appendix C. Normalised bispectra (bicoherence)**

524 We generally seek a normalisation of the bispectrum so that it takes 1 as value when there is a full
 525 coupling between the three components involved and 0 when there is none. In the case of surface gravity
 526 waves, this would correspond to a situation with the only presence of forced or free energy respectively.
 527 At present, there does not seem to be a consensus in the literature on what definition of the bicoherence
 528 should be used to robustly quantify or at least characterize the relative dominance of forced energy (*e.g.*
 529 Haubrich, 1965; Kim and Powers, 1979; Elgar and Guza, 1985a; Hinich and Wolinsky, 2005; de Bakker
 530 et al., 2015, and many others). The normalisation proposed by Haubrich (1965) is frequently used for
 531 studying non-linearities in the nearshore area but was shown to lead to values greater than 2 (*e.g.* Elgar
 532 and Guza, 1985a). In the present study, this was also the case, but only when merging across frequencies
 533 was performed beforehand. Nonetheless, a slightly different normalisation that was proposed in Hagihira
 534 et al. (2001) is used in the present case as it seems the most appropriate normalisation (see their Appendix):

$$b(f_1, f_2) = \frac{|B(f_1, f_2)|}{\mathcal{E}[|A(f_1)| \cdot |A(f_2)| \cdot |A^*(f_1 + f_2)|]} \quad (\text{C.1})$$

535 As explained in Hagihira et al. (2001), the bicoherence is 1 only when phase angles of all triple products
 536 are equal.

537 Squared bicoherence values b^2 for both II and III regimes of propagation are shown in Fig. C1. As
 538 frequency merging is inappropriate with this definition of the bicoherence (due to the expected value in the
 539 denominator), b values were averaged over a 3x3 square in the frequency space.

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Test	H_s [m]	f_p [Hz]	γ
A1	0.10	0.633	3.3
A2	0.20	0.444	3.3
A3	0.10	0.444	20

Table 1: Significant wave height (H_s) and peak wave frequency (f_p) for the three tests considered in this study. The peak enhancement γ of the JONSWAP spectra characterizes its spectral bandwidth. A value of 3.3 corresponds to broad spectra while a narrow-band spectra is imposed with $\gamma = 20$.

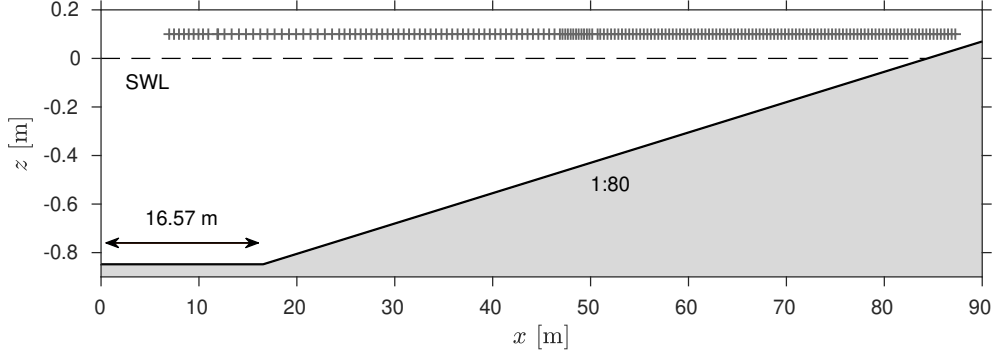


Figure 1: Elevation z of the 1:80 concrete beach against the cross-shore distance x in the Scheldegoot flume during the GLOBEX project. The wave paddle is located at $x = 0$ m. The grey '+' symbols show the position of the wave gauges across the wave flume.

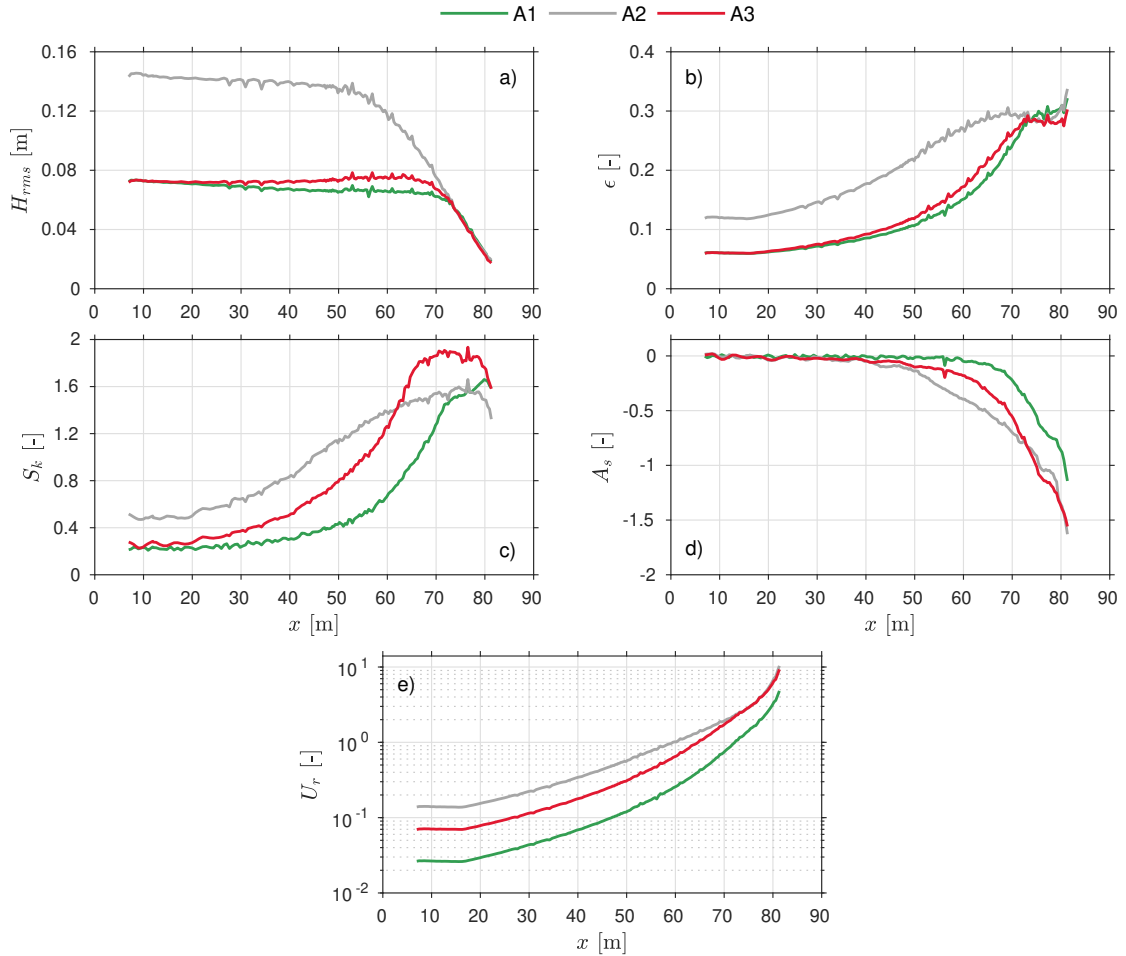


Figure 2: Second and third-order short-wave parameters during tests A1, A2 and A3: a) root-mean square wave height H_{rms} computed as $(8\zeta^2)^{1/2}$; b) corresponding wave amplitude to water depth ratio $\epsilon = H_{rms}/\sqrt{2}h$; c) wave skewness S_k ; d) wave asymmetry A_s and e) Ursell number U_r computed as ϵ/μ . Note that these are short-wave parameters, computed using the high-pass filtered surface elevation signal (frequency cutoff at $0.6f_p$).

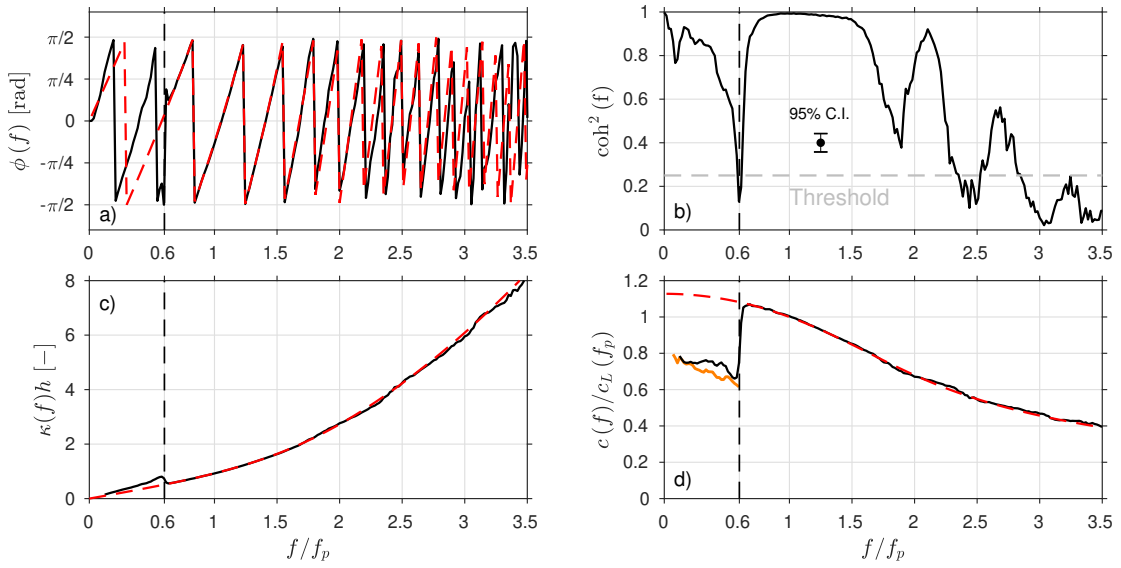


Figure 3: Phase (a), coherence-squared (b), dimensionless wavenumber (c) and phase velocity (d) spectra computed at $x = 10.65$ m for case A2. The spacing between the adjacent gauges is $\Delta x = 0.93L_p$, with L_p the peak wavelength given by the linear wave dispersion relation (Eq. 14). The phase velocity spectrum is normalised by $c_L(f_p)$, the phase velocity predicted by linear wave theory at the peak frequency. The red dashed lines in panels a, c and d correspond to the values given by the linear wave dispersion relation. The separation between infragravity and short wave frequencies ($0.6 f_p$) is shown as dashed black line. In panel b, the gray dashed line corresponds to the coherence-squared threshold used for computing ensemble average spectral estimated of κ and c . In panel d, the orange line corresponds to the short-wave envelope propagation speed.

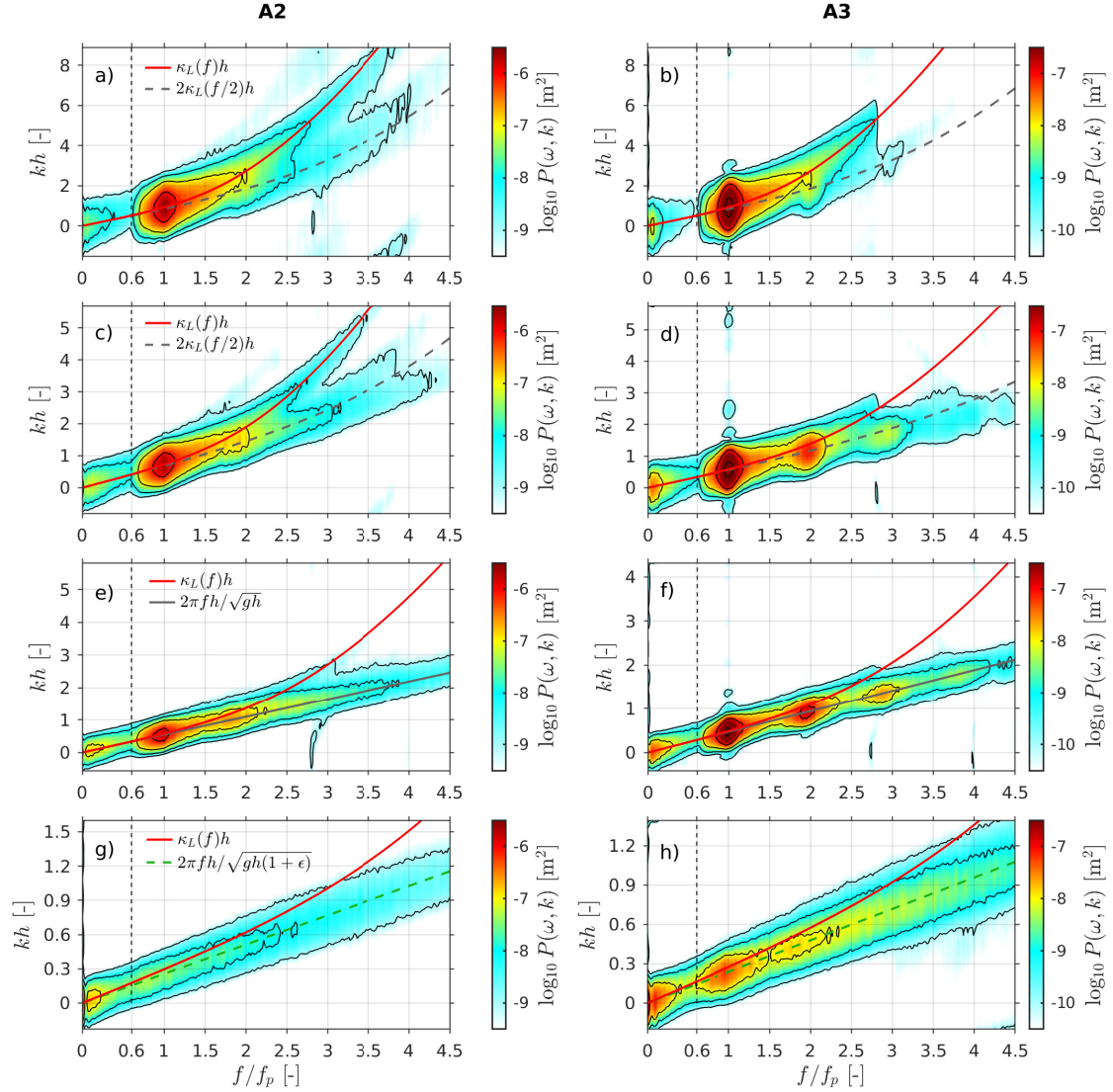


Figure 4: Frequency-wavenumber surface elevation power spectra $P(\omega, k)$ computed for A2 (left) and A3 (right) at the four stages of propagation described in the paper (I: panels a-b; II: panels c-d; III: panels e-f and IV: panels g-h). For A2, these power spectra were computed at I: $x = 9.8 \pm 2.8$ m; II: $x = 38.9 \pm 3.6$ m; III: $x = 54.5 \pm 3.5$ m and IV: $x = 76 \pm 2.4$ m. For A3, the cross-shore locations were I: $x = 9.8 \pm 2.8$ m; II: $x = 53.4 \pm 2.4$ m; III: $x = 62.3 \pm 3.5$ m and IV: $x = 77 \pm 2.4$ m. Each black contour line correspond to a power of 10. The separation between infragravity and short-wave frequencies ($0.6 f_p$) is shown as the dashed black line. In all panels, the red curves correspond to the linear wave dispersion relation. In panels a-d, the dashed grey curves correspond to the dispersion relation for the second harmonic, and was computed based on the assumption that it travels at a similar speed as its principal component. In panels e-f, the grey curve corresponds to the shallow water wave dispersion relation ($c = \sqrt{gh}$) while the dashed green line in panels g-h refer to the modified one ($c = \sqrt{gh(1 + \epsilon)}$).

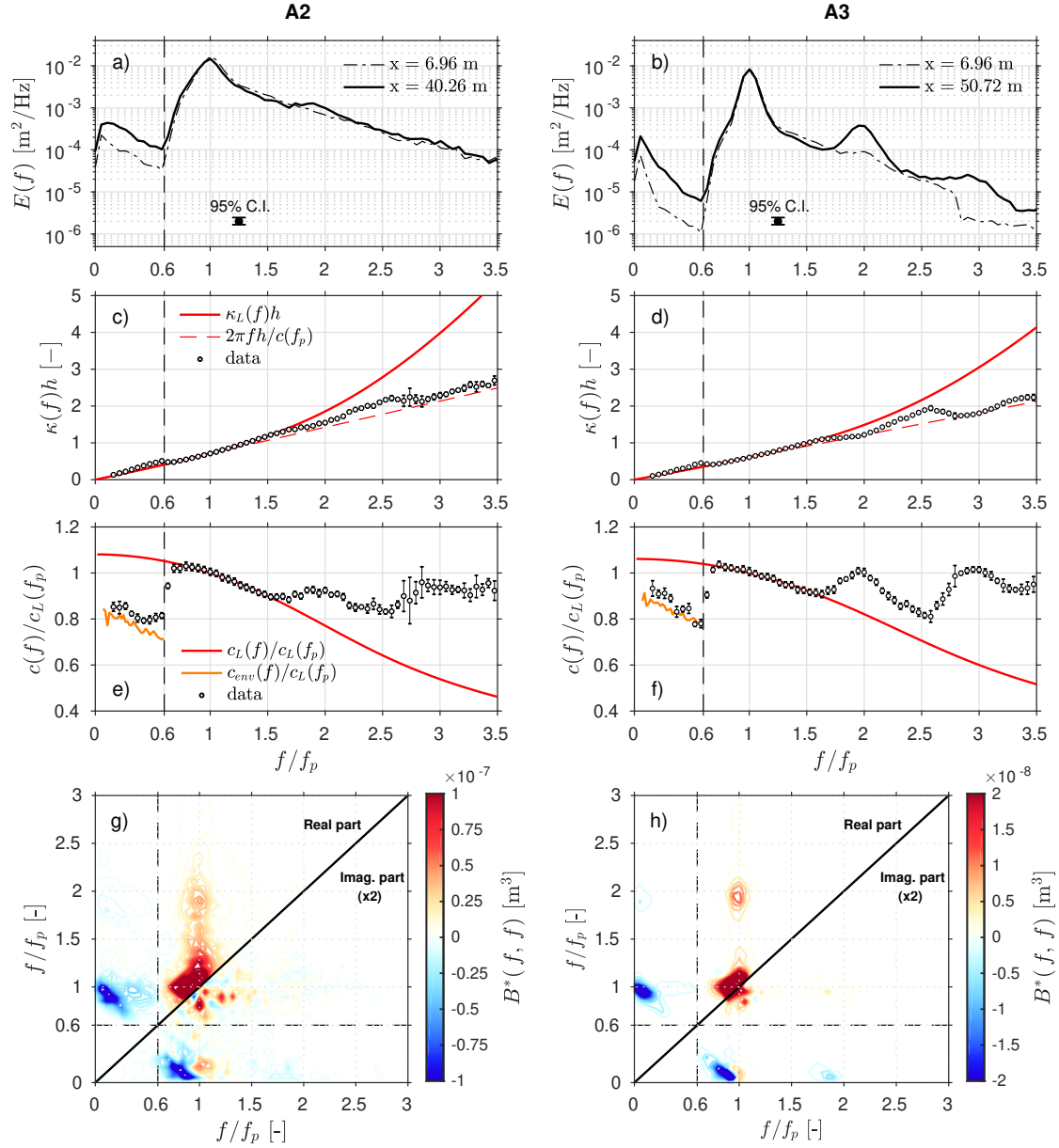


Figure 5: From top to bottom: surface elevation energy density spectra, dimensionless wavenumber spectra, phase velocity spectra normalised by $c_L(f_p)$ and bispectra computed at $U_r \sim 0.3$ (stage II) for wave tests A2 (left panels) and A3 (right panels). The cross-shore location corresponding to this stage is indicated in panels a and b (*cf* figure 2e). The wavenumber and phase velocity spectra were computed with five spacing configurations: each point corresponds to the ensemble-averaged value and the error bars correspond to the standard deviation. For readability, only one on three data points are shown. Red lines in panels c-f correspond to values given by the linear wave dispersion relation. The separation between infragravity and short-wave frequencies ($0.6 f_p$) is shown as the dashed black line. In panels c-d, the red dashed line corresponds to $\kappa(f) = 2\pi f/c(f_p)$. In panels e-f, the orange line corresponds to the short-wave envelop propagation speed denoted c_{env} .

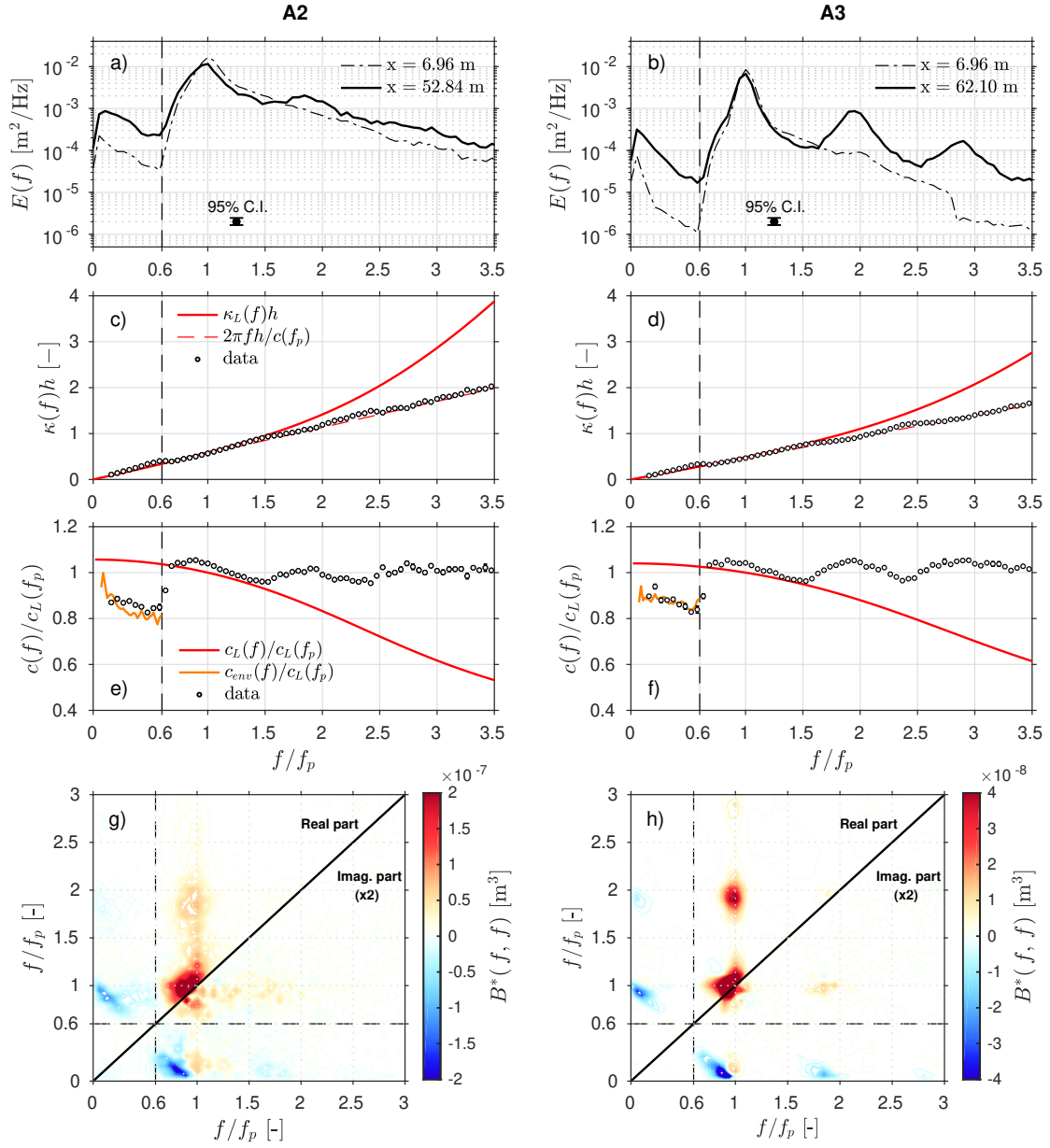


Figure 6: Similar to figure 5, but with $U_r \sim 0.7$ (stage III).

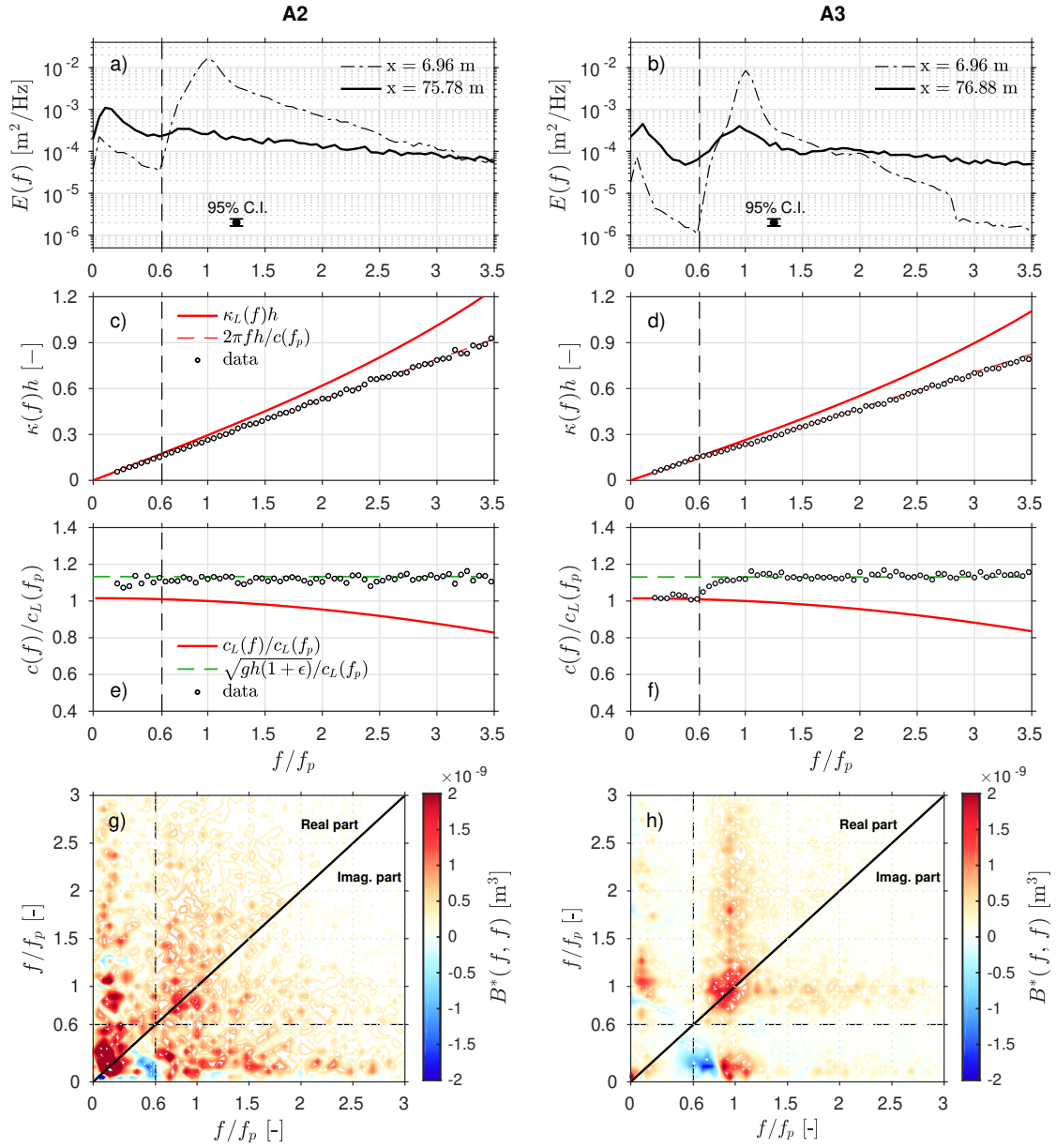


Figure 7: Similar to figure 5, but with $U_r \sim 3.2$ (stage IV). Additionally, the green dashed lines in panels e) and f) correspond to the modified shallow water dispersion relation $c = \sqrt{gh(1+\epsilon)}$.

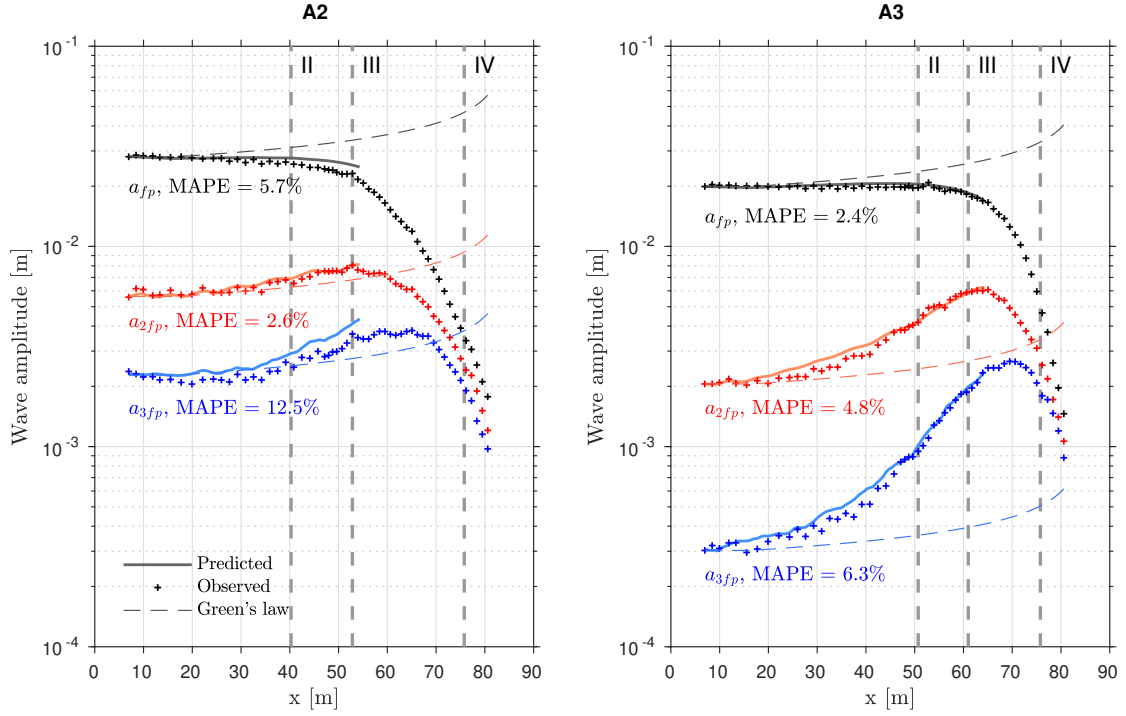


Figure 8: Comparison between observed and predicted (Eq. 13) wave amplitudes at f_p , $2f_p$ and $3f_p$ (a_{f_p} , a_{2f_p} and a_{3f_p} respectively). The cross-shore locations corresponding to the regimes of propagation discussed in section 4 are indicated by the vertical dashed lines for both wave tests. Since the present modelling approach neglects wave breaking-induced energy dissipation, modelled amplitudes are shown only until the approximate mean breaking point location (slightly after stage III). The amplitude growth predicted with Green's law is also shown.

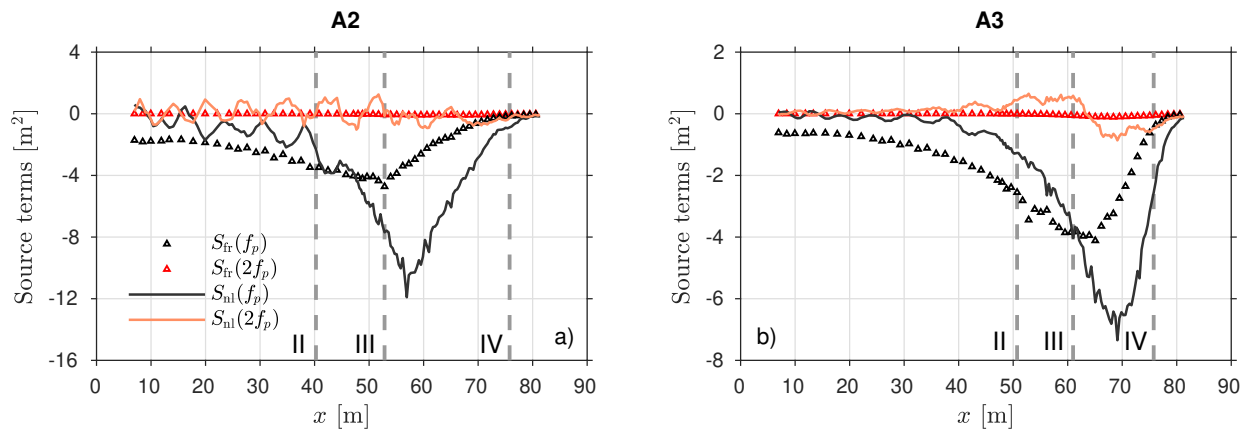


Figure 9: Cross-shore variation of the source terms for the non-linear energy exchanges between triads and for the bottom friction, used in Eq. 13. Only values at the peak frequency (f_p) and the second harmonic ($2f_p$) are shown. The cross-shore locations corresponding to the regimes of propagation discussed in section 4 are indicated by the vertical dashed lines for both wave tests.

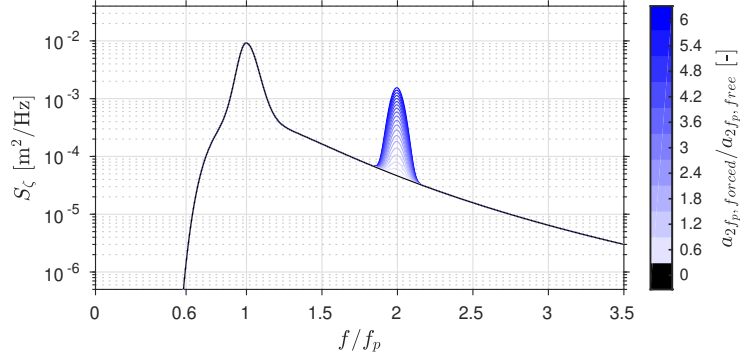


Figure A1: Energy density spectra imposed at a fictive x_1 position, with different ratio of forced to free second harmonic amplitude. In the absence of forced components, the forcing corresponds to a JONSWAP spectrum with parameter similar to A3 (see Table 1).

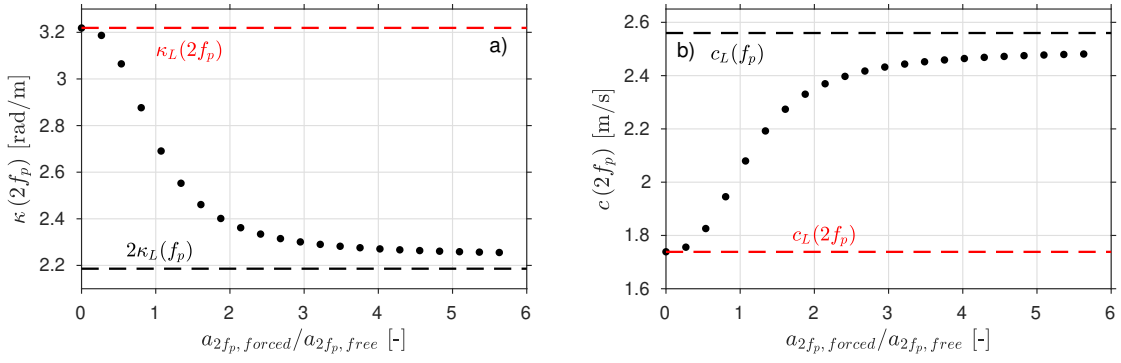


Figure A2: Wavenumber (a) and phase velocity (b) spectra computed on the synthetic surface elevation timeseries for varying ratio of forced to free second harmonic amplitudes.

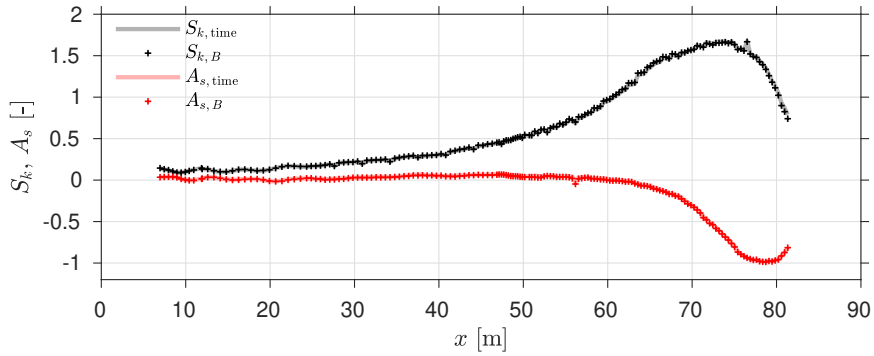


Figure B1: Cross-comparison of third-order wave parameters during A3 computed using the statistical (Eq. 1-2, 'time' subscript) and the bispectrum (Eq. B.1-B.2, 'B' subscript) definitions. No separation between infragravity and short wave frequency bands or between incoming and outgoing wave fields was performed for this comparison.

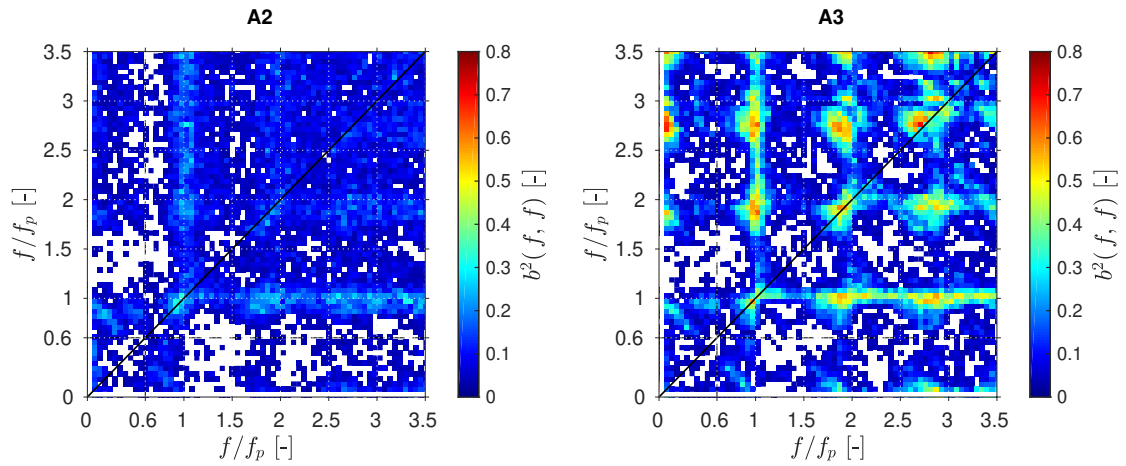


Figure C1: Squared bicoherence b^2 computed using the definition of Hagihira et al. (2001) for wave test A2 (left) and A3 (right). The octant above the 1:1 diagonal corresponds to the bicoherence for stage II while stage III is that under the diagonal. Due to the symmetric properties of the bispectrum, only one octant is shown for each stage. The separation between infragravity and short-wave frequencies ($0.6 f_p$) is shown as the dashed black line. Only bicoherence values greater than the 95% significance level as defined by Haubrich (1965) ($b_{95\%}^2 \geq 6/\text{d.o.f.}$, with d.o.f. the equivalent number of degrees of freedom) are shown.