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On a Shared Scooter Service under Ring Shape, Free-Floating and Prior Booking

Abstract

Shared Scooter Services (S3) have been deployed in many cities to provide public transport to individual users on a free-floating basis. The article brings about an analytical traffic model for such service under ring shape, i.e. postulating there is a ring road along which the scooters are required to park and expected to be driven, in order to avoid detours and increase vehicle productivity as well as local availability to potential users. Prior booking is postulated, supposedly on a web app. Also postulated are the homogeneity in both space and time. With respect to the ring circumference, the daily volume of potential trips, the statistical distributions of trip lengths, walk speeds and ride speeds, as well as the fleet size, operation period and transaction times, analytical formulas are provided for (i) access lengths and times, together with usage probability, (ii) ride lengths and times, (iii) vehicle occupation at the fleet level, (iv) the average number of available vehicles. Traffic equilibrium is shown to exist and to be unique or bi-valued with one fluid state versus one congested state. Its characteristic equation is recast as a fleet sizing rule according to demand volume and target access length, which stand respectively as the quantitative and qualitative service objectives.

Keywords: shared mobility services; free-floating scooters; vehicle occupation; fleet sizing; stochastic model; traffic equilibrium.

1/ Introduction

Background. In recent years, e-scooters have emerged as a convenient mode of travel for many individuals. As a vehicle, an e-scooter is agile and its motor enables for speeds up to 20 km/h and beyond: this is quite efficient in the urban setting. Furthermore, its small size and light weight make it easy to carry out e.g. in a car trunk or in transit vehicles (buses, trains).

The parallel development of connectedness both for vehicles and for people owing to smartphones has given rise to many shared mobility services. Among them, e-scooters sharing systems have been deployed in many big cities worldwide.

To what extent may such e-scooter sharing services contribute to public transport in urban areas? This question carries obvious stakes for mobility planners in territories. Shared e-scooters look environmental friendly compared to private cars, not only in terms of matters utilized and energy consumed but also to the cityscape: their parking requires little space and their traffic is by far less intrusive and dangerous to pedestrians.

Objective. A number of traffic models have been developed to perform traffic studies with much detail in space and time. This paper brings about a simple traffic model for an e-scooter shared mobility service under ring shape. The principle is to channel the users' trips in the territorial space, so as to increase vehicle availability along the ring, to avoid detours, to ease the service logistics (from vehicle maintenance to energy refills, passing by fleet repositioning). By simultaneously reducing the operational costs and increasing vehicle productivity, a ring-shaped service is expected to reach commercial profitability under wider conditions of demand and territory, thereby enabling the planners to expand the spatial outreach of public transport.

In the model, the territorial conditions are represented as the radius of the ring and the riding speed, a daily volume of trips and the average length of them. The fleet size is given and the service is operated in free-floating mode. The service users go to the closest available scooter, which they book in advance using a mobile application. From these assumptions, the model determines the occupation of the vehicle fleet, its availability, the access conditions in terms of walk lengths and walk times, as well as riding times depending on users' ride speeds.

Methodology. Homogeneity is postulated regarding both space and time. Spatial homogeneity is postulated for trip origin and destination endpoints, hence for the positions of available vehicles. Combined to the ring shape, this spatial homogeneity gives rise to circular symmetry. Temporal homogeneity is postulated along a daily period of “active duty”: for instance a 14h period from early morning to evening.

Traffic is modeled in disaggregate and stochastic way. Each service request is identified with its own service user. Trip lengths are modeled as a statistical distribution. Users are modeled in terms of walk speeds and ride speeds, with joint statistical distribution.

By facing the total workload demanded by the users to the fleet size times the period duration, we establish the condition of traffic equilibrium at the system level as a characteristic equation with respect to the number of vacant vehicles. In appendix, we show that the macroscopic equilibrium condition is consistent with the stochastic equilibrium of a two-state Markov model at the disaggregate level of one scooter (Available vs. Busy states). All in all, given Q the daily trip volume and N the fleet size, the model determines the average walk length say ℓ , which indicates service quality. The relationship can be expressed equivalently as a function $\hat{\ell}(N, Q)$ or a function $\hat{N}(Q, \ell)$, which is useful for planning purposes.

Article structure. The rest of the article is in seven parts. Section 2 describes the system: territorial conditions, service supply and travel demand. Section 3 introduces the model architecture. After modeling successively Service access in Section 4, User rides in Section 5 and Scooter occupation in Section 6, the model is integrated in Section 7. Lastly, Section 8 concludes and points to theoretical developments.

2/ System description and mathematical notation

Inside an urbanized territory, assume that a roadway circuit is designated as pathway for the shared e-scooters. By assumption, it can be used in either directions and its overall shape is a circular or quasi-circular ring of radius R .

On the demand side, we consider that each day, during a time period of H , there are Q trips that use the shared mobility service, half of which in each traffic direction. The trip origin points are distributed evenly along the ring. From their origin points the trips have spatial lengths L_R with statistical distribution that is identical from all origins. From the assumptions about the trips' origins and lengths, it follows that the destination points are also evenly distributed along the ring.

Among the population of customers, each service user has his own walk speed w and ride speed v : assumedly, these are cruising speeds over the walk and ride legs respectively. Walk speeds are statistically distributed with CDF W and ride speeds with CDF S . Statistical independence is assumed between them and also with the generation of trips.

By assumption, the service is operated under free-floating mode all along the ring.

Coming to service quality, the user spends a ride time t_R that depends on the trip length L_R and his own ride speed, v :

$$t_R = L_R/v. \quad (1)$$

Complementarily, there is a time to get to an available scooter, denoted t_A , with subscript A for Access. Trip planning, including booking a scooter at a given place, un-parking it at the origin and parking it at the destination, takes a transaction time of t_T . Then, to the user the overall trip time adds up to

$$t_{\text{ART}} := t_A + t_R + t_T. \quad (2)$$

On the supply side, there are N scooters constituting the service fleet, supposedly all in operational condition. At any instant, each of them has a probability p_O of being occupied, which means Busy for riding as well as during the access of the user that has booked it, and during the transaction operations. Thus, the trip time t_{ART} is also the resource occupation time for the scooter used for riding.

At any instant, the average number of vacant scooters, say k , satisfies that

$$k = N \cdot (1 - p_O). \quad (3)$$

From the postulates of spatial homogeneity for demand trips, we deduce that each available scooter has its position distributed evenly along the ring, and that the respective positions of the k available scooters are statistically independent.

3/ Model architecture

Among the model components, the following parts are exogenous:

- on the demand side, trip volume Q , trip length distribution $CDF(L_R)$ with mean \bar{L}_R , walk speed CDF W and ride speed CDF S .
- on the supply side, fleet size N , operating period H , transaction time t_T ,
- as well as ring radius R that pertains to the territory as it is involved both with the demand and the supply of the shared mobility service.

From these exogenous conditions, the model enables us to derive the following endogenous components:

- on the demand side, ride time t_R , access length L_A and time t_A , trip time t_{ART} .
- on the supply side, average number of vacant scooters k and occupation probability p_O .

The model architecture involves four sub-models:

- (i) the Access model that relates L_A to k , and then t_A to L_A and W ;
- (ii) the Ride model that relates t_R to L_R and S ;
- (iii) the Occupation model that relates p_O to t_{ART} , H , N and Q ;
- (iv) the Traffic equilibrium model balances the state variable k : it ensures that its value is consistent throughout the chain of influences along the previous three models.

Figure 1 exhibits the model architecture as a graph system of components and dependences. Each model component is represented by a box, i.e. a graph node. Each straightforward influence of one model component onto another is indicated by a link between their respective nodes, with orientation according to causality. The components and links are organized by

sub-model: in other words, each sub-model consists in a sub-system of dependences. The figure shows that the traffic equilibrium model acts as a feedback to the other three sub-models.

In the following Sections, we will introduce in turn the Access model, the Ride model and the Occupation model, before turning to the Traffic equilibrium model. Thus, the order of presentation matches the logical thread in Figure 1.

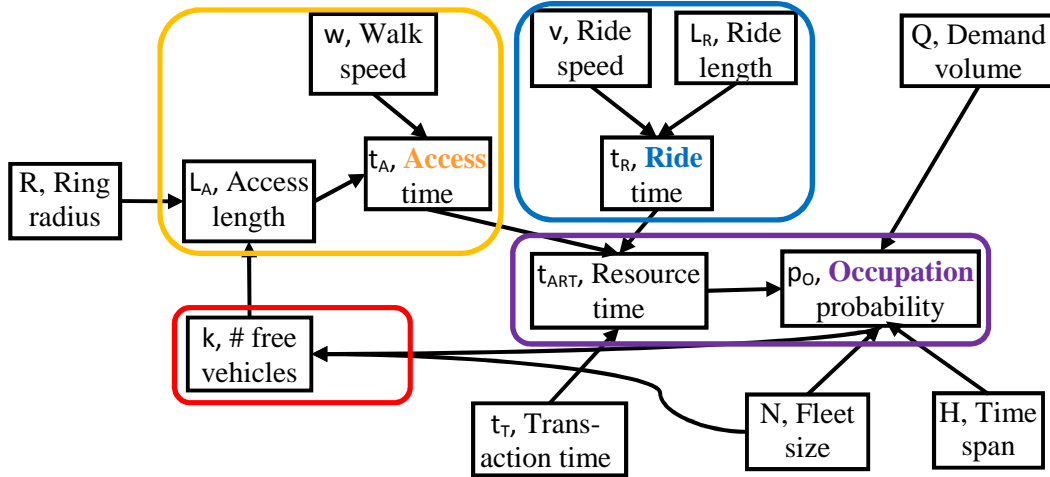


Fig. 1: Traffic Model Layout (adapted from Leurent, 2019a).

4/ The Access model

Let us consider a service customer that is planning his trip by looking for an available scooter (typically via a mobile app). We assume he will select the available scooter closest to his initial position, so as to minimize the walk length and walk time, which is presumably more tedious than the ride time.

Each available scooter is positioned at a point distributed evenly along the ring: thus its distance from the trip origin point is distributed uniform between 0 and πR . As the k available scooters are statistically independent, the access length is the minimum of k i.i.d. random variables $U[0, \pi R]$. Its CDF is thus, denoting $F_1(x) := \min\{x/\pi R, 1\}$ for $x \geq 0$:

$$F_{L_A}(x) := \Pr\{L_A \leq x\} = 1 - (1 - F_1(x))^k. \quad (4)$$

Proof. For any scooter $i \in \{1, \dots, k\}$, the length L_i from the customer origin point has CDF F_1 . The minimum L_A of k independent lengths so distributed satisfies that

$$\Pr\{L_A > x\} = \Pr\{L_i > x : \forall i\} = \prod_{i=1}^k 1 - F_1(x) = (1 - F_1(x))^k.$$

Coming to the access time t_A , from the definition of cruising walk speeds w it holds that

$$t_A = L_A/w. \quad (5)$$

It can safely be assumed that L_A and w are independently distributed. Then, the average access time satisfies that

$$\bar{t}_A := E[t_A] = E[L_A] \cdot E[1/w]. \quad (6)$$

Instance. Taking $w \approx \text{LN}(\mu_w, \sigma_w)$, then $1/w \approx \text{LN}(-\mu_w, \sigma_w)$ and $E[1/w] = \exp(-\mu_w + \sigma_w^2/2) = \exp(\sigma_w^2)/E[w]$. From the properties of LN laws, $\exp(\sigma_w^2) = 1 + \gamma_w^2$ with respect to the relative dispersion $\gamma_w := \text{SD}[w]/E[w]$. The relation $E[1/w] \approx (1 + \gamma_w^2)/E[w]$ holds as a good approximation for any positive random variable with not so large relative dispersion.

5/ The Ride model

So far we have defined L_R as the trip length: the subscript R for Ride suggests that it is equivalent to the ride length. Yet the access conditions influence the effective ride lengths, say L'_R , in two respects: first, depending on the direction of walking with respect to that of riding, the effective ride length will be

$$L'_R = L_R \pm L_A. \quad (7)$$

Second, in an even deeper way, it may occur that the Access distance be greater than the trip length. In other words, the distribution of trip lengths is shaped by service availability in space. In reality, it would not occur that $L_R < L_A$ for a trip using the service.

Here we shall keep to the assumption of an exogenous distribution of Ride lengths, of given average \bar{L}_R . Neglecting the case for negative values, this is consistent with (7) as the net contribution of the $\pm L_A$ part is null.

Coming to the ride time, from the definition of ride speeds v it holds that

$$t_R = L'_R/v. \quad (8)$$

It can safely be assumed that L'_R and v are independently distributed. Then the average ride time satisfies that

$$\bar{t}_R := E[t_R] = E[L'_R] \cdot E[1/v]. \quad (9)$$

6/ The Occupation model at the fleet level

Given an exogenous transaction time of t_T , and postulating that the scooter is booked by the user from his origin point and instant, then it is assigned to the customer during not only the ride time but also the access time and the transaction time. The total service time $t_{ART} := t_A + t_R + t_T$ applies to both the service user and the scooter as a resource.

At the service level, during one day of operations, the total resource occupation time amounts to

$$\mathcal{R} = Q \cdot \bar{t}_{ART}. \quad (10)$$

To make this workload feasible in the time period H , the fleet size N must satisfy that

$$N \cdot H \geq Q \cdot \bar{t}_{ART}. \quad (11)$$

Under this requirement, the proportion of occupation in resource time amounts to

$$p_O := \frac{Q \cdot \bar{t}_{ART}}{N \cdot H}. \quad (12)$$

From our postulates of homogeneity in time and space, this proportion can be taken as the probability of occupation for any scooter at any time, in the absence of any other information (such as the particular assignment of the scooter to some user request).

7/ Model integration

We have chained up the influences between the state variables, from an exogenous number k of available scooters in the Access model, to the Occupation probability p_O in the Occupation model, passing by the formation of access time t_A and ride time t_R . To close the model it remains to link the k variable to the occupation probability, thereby making k endogenous.

From the occupation probability applied to the fleet size N , on average there are $N \cdot p_0$ occupied vehicles. The average number of available vehicles is then $k = N \cdot (1 - p_0)$, hence

$$k = N - Q \cdot \bar{t}_{\text{ART}}/H. \quad (13)$$

As \bar{t}_R stems from \bar{L}_R in a straightforward way, we keep its notation along with t_T in \bar{t}_{RT} . As for \bar{t}_A , we put it as $\bar{t}_A = t_A^0/(k+1)$ with $t_A^0 := \pi R \cdot E[1/w]$.

By doing so, we have assimilated an integer-valued and instantaneous k involved in t_A , with an average k that is positive and real-valued. Of course it is an approximation which enables us to obtain the following characteristic equation:

$$k = N - (\bar{t}_{\text{RT}} + \frac{t_A^0}{k+1})Q/H. \quad (14)$$

This is a second-degree equation in k . We restate it as follows, letting $' := N - Q \cdot \frac{\bar{t}_{\text{RT}}}{H} - 1$:

$$k(k+1) = (N'+1)(k+1) - Q \cdot t_A^0/H. \quad (15)$$

Then, $k^2 - N'k - (N'+1 - Q \cdot \frac{t_A^0}{H}) = 0$

$$(k - N'/2)^2 = N'+1 - Q \cdot \frac{t_A^0}{H} + (N'/2)^2$$

$$k^* = \frac{N'}{2} + \varepsilon \sqrt{N'+1 - Q \cdot \frac{t_A^0}{H} + (N'/2)^2} \text{ for } \varepsilon \in \{-1, +1\}. \quad (16)$$

There are two solutions to this equation:

- the high root $k_+ = \frac{N'}{2} + \sqrt{N'+1 - Q \cdot \frac{t_A^0}{H} + (N'/2)^2}$,
- the low root $k_- = \frac{N'}{2} - \sqrt{N'+1 - Q \cdot \frac{t_A^0}{H} + (N'/2)^2}$.

The high root pertains to a fluid system in which many vehicles are available, whereas the low root is associated to a congested system with few available vehicles and long access times that contribute to vehicle occupation. To the consumers, the fluid state is preferable as it provides shorter access times.

Beyond the ambiguity between the two equilibrium states, the major outcome is the ‘‘fleet sizing rule’’ contained in (14): to the service operator, the main objective of quality of service is to ensure a target access time \hat{t}_A , which is equivalent to ensure a target access distance \hat{L}_A . This involves available scooters in average number of $\hat{k} := \frac{t_A^0}{\hat{t}_A} - 1$. In turn, (14) indicates the target fleet size,

$$N_{\text{hat}} = k_{\text{hat}} + Q \cdot (t_{\text{RT}} + t_{\text{A0}}/k_{\text{hat}} + 1)/H \quad (17).$$

$$\hat{N} := \hat{k} + \frac{Q}{H} \left(\bar{t}_{\text{RT}} + \frac{t_A^0}{\hat{k}+1} \right) = \frac{t_A^0}{\hat{t}_A} - 1 + \frac{Q}{H} (\bar{t}_{\text{RT}} + \hat{t}_A) . \quad (17)$$

This fleet sizing rule involves Q and \hat{t}_A in a straightforward way.

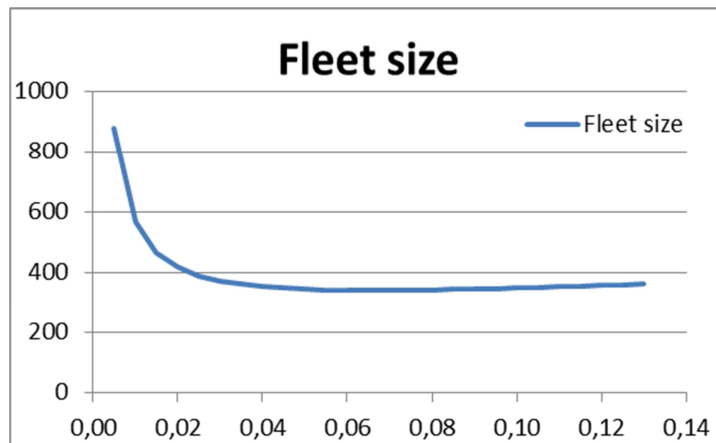


Fig. 2. Fleet size according to target access time.

8/ Conclusion

We have provided an analytical model for a shared scooter service (S3) under ring form and free-floating. Owing to postulates of homogeneity in space and time, we established simple formulas for the average access length and time, ride time, number of available vehicles and probability of occupation. Given the fleet size and the demand volume, the traffic equilibrium is characterized by a second-degree equation with respect to the number of available vehicles. Moreover, the characteristic equation constitutes a fleet sizing rule for a service operator targeting both the demand volume (a quantitative objective of service) and the access time (a qualitative objective).

The S3 traffic model is a physical and technical model. It stands as the scooter counterpart of the ring-shaped taxi service model (Leurent, 2019a) and the ring-shaped Shuttle service model C3 (Leurent, 2019b). It may represent a ring-shaped bike-sharing system as well, if operated in free-floating mode.

The economic theory of the S3 model can be developed along the same path as that for the taxi and C3 models, all the more so as the characteristic equation is very close to that of the taxi service. Yet service production for a free-floating S3 is specific as it involves charging operations and vehicle maintenance, which we did not address in detail for the other modes (since vehicle driving, be it human or robotic, enables for offline maintenance and energy refill).

References

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Appendix: a two-state Markov chain model

A shared scooter may be modeled as a stochastic system taking one out of two states A/B in alternation: state A means no current user hence Available to any newcomer, whereas state B stands for Busy, i.e. Occupied by a user. The two states communicate with each other, i.e.

there are transitions from each state to the other one. Figure 3 depicts the two states and the stochastic transitions between them, with respective transition rates of λ' and μ .

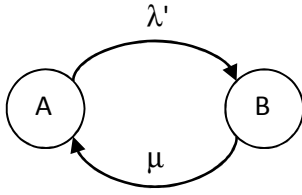


Fig. 3. States and Transitions.

Transition rate μ is the ending rate of an occupation under way. From the duration τ , it is the ratio between the probability $\Pr\{t_{\text{ART}} \leq \tau + \delta\tau \mid t_{\text{ART}} > \tau\}$ that such occupation will end during an elementary interval $[0, \delta\tau]$, and its incremental duration $\delta\tau$. Making the crude approximation that the random variable of vehicule occupation, t_{ART} , is distributed exponential, then the exponential parameter is $1/\bar{t}_{\text{ART}}$. Thus, the transition rate satisfies

$$\begin{aligned} \mu &:= \lim_{\delta\tau \rightarrow 0} \frac{\Pr\{t_{\text{ART}} \leq \tau + \delta\tau \mid t_{\text{ART}} > \tau\}}{\delta\tau} = \lim_{\delta\tau \rightarrow 0} \frac{\Pr\{t_{\text{ART}} \leq \tau + \delta\tau \cap t_{\text{ART}} > \tau\}}{\Pr\{t_{\text{ART}} > \tau\} \cdot \delta\tau} \\ &= \frac{\text{PDF}(t_{\text{ART}})}{1 - \text{CDF}(t_{\text{ART}})} = \frac{\exp(-\frac{\tau}{\bar{t}_{\text{ART}}})/\bar{t}_{\text{ART}}}{\exp(-\frac{\tau}{\bar{t}_{\text{ART}}})} = 1/\bar{t}_{\text{ART}}. \end{aligned}$$

$$\mu = 1/\bar{t}_{\text{ART}}. \quad (18)$$

As for the transition rate from State A (Available) to State B (Busy), it comes from the overall rate of customers arrivals per time unit, Q/H , divided by the number of available vehicles, k :

$$\lambda' = Q/(H \cdot k). \quad (19)$$

The system is in stochastic equilibrium with stationary probability distribution $[p_A, p_B]$ such that $p_A, p_B \geq 0$ and $p_A + p_B = 1$, and the probability flow is conserved in each state:

$$p_A \cdot \lambda' = p_B \cdot \mu. \quad (20)$$

Put in words, the probability flow going out of State A per time unit is equal to that coming in it from State B. Substituting $1 - p_A$ for p_B , the balance condition becomes that

$$p_A \cdot \lambda' = (1 - p_A)\mu \text{ hence } p_A = \mu/(\mu + \lambda') = 1/(1 + \lambda'/\mu) = 1/(1 + Q \cdot \bar{t}_{\text{ART}}/(H \cdot k))$$

At every instant, it also holds that $p_A = k/N$.

Substituting, the balance equation becomes an equation in k :

$$\frac{k}{N} \cdot \left(1 + Q \cdot \frac{\bar{t}_{\text{ART}}}{H \cdot k}\right) = 1 \text{ i.e. } N = k + \bar{t}_{\text{ART}}Q/H.$$

Now, as $\bar{t}_{\text{ART}} = \bar{t}_{\text{RT}} + \frac{t_A^0}{k+1}$, the balance condition of the Markov model is exactly the characteristic equation of the traffic model.