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## DYNAMICAL ANALYSIS OF AN END MILLING PROCESS

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**Abstract.** *The cutting processes include different manufacturing procedures as milling, turning and drilling. In all of these processes, the machine tool vibration plays an important role concerning the cutting characteristics. Therefore, its correct understanding is essential in order to improve the workpiece surface quality and avoid tool breakage. Chatter and squeal are some undesirable phenomena related to an improper functioning. This article analyses the nonlinear dynamics associated with the end milling tool and its vibration relative to the work piece considering a nonsmooth two-degree of freedom system. End milling process is of concern assuming different machining characteristics. In this work, the cutting force is considered as a composition of contact/non-contact of the tool and the workpiece with stick-slip behavior guided by friction force and the prescribed velocity of the tool holder. Numerical simulations are carried out showing some situations related to proper and improper functioning during the cutting process.*

**Keywords:** *End milling, force prediction, nonlinear dynamics, nonsmooth systems, chatter, machining.*

### 1. INTRODUCTION

Machining process is associated with a complex dynamics that involves the coupling of different phenomena. During cutting processes, for example, it is common to have temperature variations that induce dramatic changes in expected behaviors. Although all these complexity, some simple models may be used in order to obtain useful information concerning tool and workpiece behaviors as dynamic tool prediction.

The machine tool vibration during the cutting process plays an important role concerning the workpiece surface quality and also the tool durability. Chatter and squeal are some undesirable phenomena related to an improper functioning. During certain cutting conditions, the motions of the workpiece-tool system are characterized by large amplitudes, which are not desirable for obtaining a good surface finish. The undesirable motions, which are often referred to as chatter, can result in wavy surfaces on the workpiece, inaccurate dimensions, and excessive tool wear.

The analysis of machining using a dynamical approach has been done by different research efforts. The first works in this way is due to Tobias (1965), Merritt (1965) and Tlustý (Altintas, 2000) that investigate the chatter on the process. Their research provided instability diagrams to inform the safe parameters to avoid chatter. After them, a new approach has been developed using nonlinear dynamic tools as bifurcation analysis to provide better comprehension of the process (Moon, 1978).

Another references treated machining operations by dynamical approach. For turning operations, Chandiramani and Pothala (2006) analyzed the dynamics of cutting considering a two-degree of freedom system (2-dof) and orthogonal cutting modeling to predict chatter. Pratt and Nayfeh (1999) studied the boring bars for turning, which commonly presents chatter problems and experimentally determined modal properties of the tool. Also dedicated to turning, Kalmar-Nagy, Stjepan and Moon (2001) showed the existence of a subcritical Hopf bifurcation in the delay-differential equation used to describe the machining equations of motion. The stability of the milling process was investigated by Zhao and Balachandran (2001), Insperger *et al* (2003) and Gradisek *et al* (2005) by considering single and two degree of freedom systems for different experimental conditions. The mathematical model is represented by delay-differential equations.

Milling process has the peculiarity to have a contact/non-contact behavior due to its geometry and due to tool vibration. This kind of behavior is related to nonsmooth systems that are usually associated with either the friction phenomenon or the discontinuous characteristics as intermittent contacts (Savi *et al*, 2007).

This article investigates the milling process representing the end milling tool by a mass-spring-dashpot system and the contact with the workpiece by a nonsmooth contact/non-contact system. The process of cutting is related to a stick-slip behavior that defines whether the chip is being removed from the workpiece. The tool holder displacement is prescribed and the local force that allows the slip motion is related to the workpiece shear stress. Under these assumptions, the equation of motion is represented by a differential equation that is solved employing the Runge-Kutta method. Numerical simulations are carried out showing some situations related to proper and improper functioning during the cutting process.

### 2. DYNAMICAL MODEL

The milling process is of concern by assuming that it is a full immersed milling in the  $x$  direction. Figure 1(a) shows the general view of the tool and the workpiece, where it is identified the prescribed displacement,  $X_h$  and the tool tip

displacement,  $X_t$ . This process is modeled by assuming a nonsmooth system that is composed by a primary system that represents the tool and a secondary system, representing the workpiece. The primary system consists of a linear spring-dashpot-mass oscillator with parameters  $m$ ,  $k$ ,  $c$  and displacement  $X_t$ , and presents a gap that separates itself from the secondary system that represents the work-piece.

The workpiece system has weightless slider connected to a linear spring-dashpot system with parameters  $k_s$ ,  $c_s$  that can present progressive motion when the acting force exceeds  $F_t$ , here represented by a dry friction, and displacement  $X_p$ , the tool-work piece contact position.

Similarly to the stick-slip phenomena reported by Marian (2001), the progressive motion  $X_c$  occurs when the force acting on the slider exceeds the threshold of the dry friction force, considering that there is no cutting fluid. Figure 1(b) presents a schematic picture of the cutting process system related to the milling.

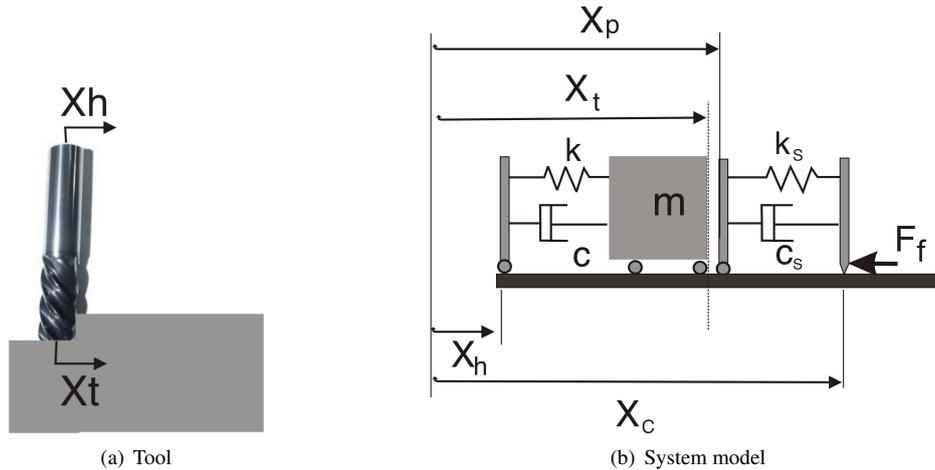


Figure 1. Tool and system modeling

As the system may operate in stick-slip phases and contact-non contact situations, its dynamic behavior is nonlinear and its equations should consider each case. The system dynamics may be understood as a three stage motion as represented in Figure 2: non-contact, contact in stick and contact in slip. The non-contact stage may be understood as a two independent systems representing the situation where the tool is not in contact with the workpiece. Figure 2(a) shows that the displacements  $X_t$  and  $X_h$  are not connected with  $X_c$  and  $X_p$  by the mass.

An auxiliary displacement  $g$  is used to locate the work-piece position related to the tool position, this variable remains equal to  $X_t$  (as in Figure 2a) in non-contact case and in stick phase, as it will be shown on Figure 2. The displacement  $g$  changes only when the slider slips simulating the chip removal.

As long as  $g$  is greater than  $X_t$ , the system remains separate. When there is no more gap between  $X_t$  and  $X_p$ , the systems are in contact and the dynamic changes, as shown in Figure 2(b), for stick stage, and Figure 2(c) for slip stage. In stick stage the slider displacement does not change and the system is restricted to a fixed boundary. In slip stage this boundary is free and  $X_c$  displacement changes. The equations of motion are formulated by considering each kind of response separately.

## 2.1 Non-contact Stage

In non-contact stage, when  $X_t < g$ , the secondary system has no movement and its displacements does not change  $\dot{X}_p = \dot{X}_c$  and the primary system is described by the dynamic equation of the single degree of freedom, as follows:

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = 0 \quad (1)$$

The force acting on the tool tip is calculated by:

$$F_t = -k(X_t - X_h) - c(\dot{X}_t - \dot{X}_h) \quad (2)$$

In order to rewrite the equations 2 for either contact and non-contact situations, the Heaviside function  $H(\cdot)$  is included. Note that the damping coefficient when the tool retracts from the workpiece ( $\dot{X}_t < 0$ ) is not considered.

$$F_t = -k(X_t - X_h) - c(\dot{X}_t - \dot{X}_h) - H[X_t - g] \left( k_s(X_t - X_c) + H[\dot{X}_t]c_s\dot{X}_t \right) \quad (3)$$

The difference between chip displacement  $X_c$  and work-piece displacement  $X_p$  when the secondary system is free is constant and it is defined as  $\delta$ , used to calculate the chip movement  $X_p$  on slip phase.

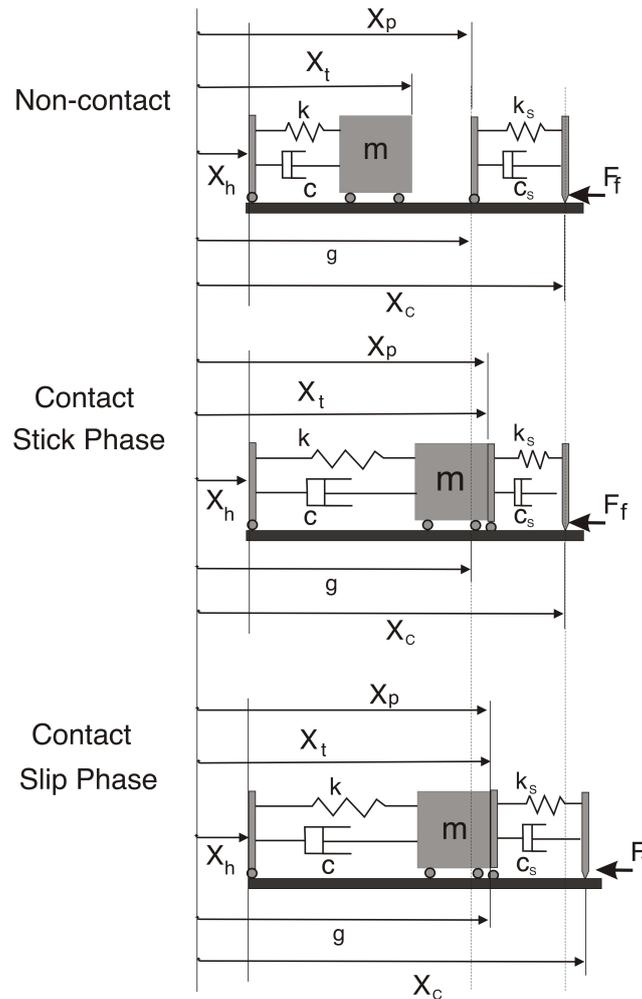


Figure 2. Contact / Non-contact situations

## 2.2 Contact - Stick Stage

The chip, represented by the slider, is not removed in stick stage. This case occurs when the friction force  $F_f$  is less than a maximum friction force  $F_{fmax}$  supported by the workpiece material, which can be considered as a function of the shear area and the maximum shear stress.

$$F_{fmax} = A_s \tau_s \quad (4)$$

The friction force is calculated, in stick phase, by:

$$F_f = H[X_t - g] \left( k_s(X_t - X_c) + H[\dot{X}_t] c_s \dot{X}_t \right) \quad (5)$$

## 2.3 Contact - Slip Stage

During slip stage, when the chip is being removed, the maximum friction force is achieved. The chip position  $X_c$  follows the workpiece displacement,  $X_c = X_p + \delta$ , and  $g$  assumes the work-piece position  $g = X_p$ . The friction force is, in slip phase, the value of maximum friction force  $F_f = F_{fmax}$ .

The force applied in the tool tip in slip phase cannot be calculated by equation (4) due to the difference between  $k_s(X_t - X_c) + H[\dot{X}_t] c_s \dot{X}_t$  and  $F_{fmax}$ . The force  $F_t$  in slip phase is:

$$F_t = -k(X_t - X_h) - c(\dot{X}_t - \dot{X}_h) - F_{fmax} \quad (6)$$

## 2.4 Equations of motion

Under these assumptions, the system dynamics may be represented by a nonsmooth system as follows:

$$m(\ddot{X}_t - \ddot{X}_h) + k(X_t - X_h) + c(\dot{X}_t - \dot{X}_h) = \mathbf{F} \quad (7)$$

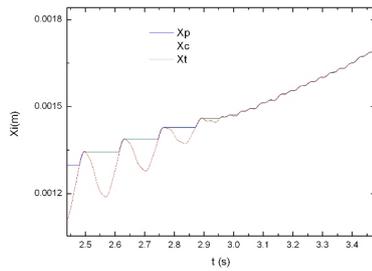
where

$$\mathbf{F} = \begin{cases} F_f & \text{if } F_f < F_{fmax} \\ F_{fmax} & \text{elseif} \end{cases} \quad (8)$$

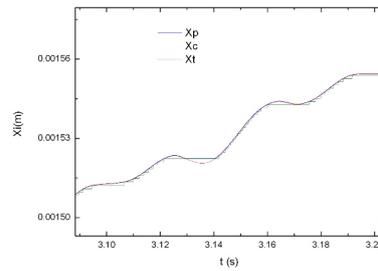
and

$$F_f = H[X_t - g] \left( k_s(X_t - X_c) + H[\dot{X}_t]c_s\dot{X}_t \right) \quad (10)$$

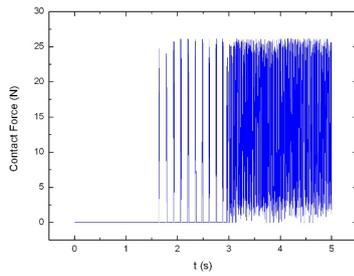
### 3. NUMERICAL SIMULATIONS



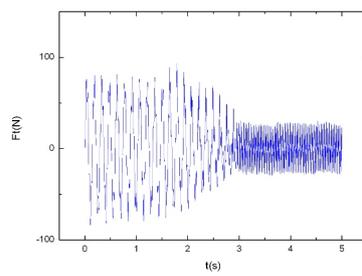
(a) Displacement



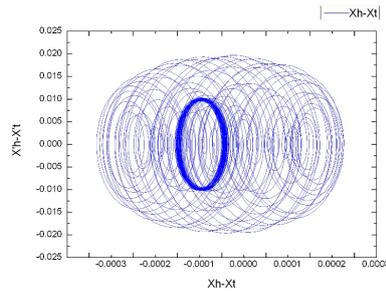
(b) Displacement - focused window 0.1 s



(c) Contact force



(d) Tool force



(e) Phase space

Figure 3.  $v_f = 0.5\text{mm/s}$ ,  $\tau_s = 100\text{MPa}$  and  $w = 1800\text{rpm}$

Numerical simulations are carried out in order to evaluate the model capability to describe milling process. The governing equations are solved by considering the fourth order Runge-Kutta method with time steps less than  $5 \cdot 10^{-4}\text{s}$ . The end milling tool parameters as diameter, run-out, damping (Mann, 2005) and system parameters are presented in Table 1. The depth of cut was  $10\text{mm}$  and the distance between the tool holder and the workpiece in  $t = 0$  is  $1\text{mm}$ . Moreover, it is assumed that the workpiece material is under plastic behavior, then the damping coefficient for the workpiece is defined as 10 times the damping tool coefficient ( $c_s = 10c$ ). Note that for  $\dot{X}_t < 0$ , when the tool moves back from the workpiece,  $c_s$  vanishes.

The machine tool is represented by the tool holder and it has a prescribed displacement  $X_h$  defined by the feed velocity  $v_f$ , the spindle speed  $w$  and the run-out distance  $\rho$  between the tool axis and the spindle speed axis.

$$X_h(t) = v_f t + \rho \sin(wt) \quad (11)$$

The milling tool is considered as a flexible body and linear elastic behavior is assumed for the end milling tool material and, therefore, the tool parameters  $m$ ,  $k$  and  $c$  are calculated by considering solid mechanics principles from the tool geometry and material properties.

The stiffness of the primary system is defined by considering the tool as a circular beam in bending. Under this assumption, there is a relation given by:

$$k = \frac{3E_t\pi d^4}{64L^3} \quad (12)$$

where tool material Young modulus  $E_t$  is considered as 200 GPa,  $d$  is tool diameter and  $L$  is the free tool length.

The workpiece stiffness is evaluated by assuming that the workpiece is a compression beam, therefore:

$$k_s = \frac{E_c dt}{10L} \quad (13)$$

where workpiece material Young modulus  $E_c$  is considered as 200 GPa and  $t$  is the depth of cut.

The feed per tooth for a two flute milling tool is written in equation 13 as  $z$  is the number of teeth.

$$f_t = \frac{v_f}{wz} = \frac{v_f}{2w} \quad (14)$$

The dry friction is evaluated from the shear strength calculated by the shear area as a function of the depth of cut  $t$  and approximated by:

$$A_s = \int_0^\pi t f_t \sin\phi d\phi \quad (15)$$

Table 1. Tool, workpiece and machine tool properties.

Diameter ( $d$ )	Height ( $L$ )	Tool Damp. ( $c$ )	Elastic Mod. ( $E_t$ )	Run-out ( $\rho$ )	Spindle Speed ( $w$ )	Depth of cut ( $t$ )
10 mm	100 mm	20 N s/m	200 GPa	0.05 mm	1800 rpm	10 mm

The system dynamics is investigated by considering different operational conditions of the milling process. The idea is to represent different aspects related to proper and improper functioning. In order to describe these conditions, different machine tool velocities and workpiece material are used.

Initially, let us consider a condition that represents the ideal functioning during the milling process. The feed velocity and the shear stress limit used for this example was  $v_f = 0.5\text{mm/s}$  and  $\tau_s = 100\text{MPa}$ .

Figures 3(a) and 3(b) presents the displacement time history of different parts of the system, namely the tool, the work-piece and the chip. Figure 3 also show the contact force (3c), the tool force (3(d)) and the phase space of the response (3e). It can be noted the moment that the tool holder makes the tool to touch the workpiece between 2 and 3s. After that the contact force varies from zero until 25N, the force that the tool cuts the workpiece. Note that, to achieve this maximum contact force, the force transferred by the tool holder to the tool is approximately 100N.

The uncut chip thickness is not considered as in Martellotti equation (Araujo *et al*, 2006) . The chip thickness is given by the simulation as the displacement steps that the work-piece material let the tool cut. In order to improve the surface quality, the chip steps must be as smaller as possible. Also, the tool cannot touch and leave the work-piece with large displacements, because it causes impacts on the tool.

The increase of system velocities can change the dynamical response of the system and this can be associated with surface quality and operational conditions. In order to show this kind of behavior, different spindle speeds and feed velocities are now considered. Under these conditions, the response is changed. Another important parameter that may cause different dynamical response is related to the material work-piece property, represented by the shear stress limit and its impact on the stick-slip behavior. As when the shear stress limit is achieved, the chip is removed, for a softer material, the velocities can be higher without an impact on chip thickness.

Figure 4(a) and 5(a) presents the time history of the displacements on the system with  $0.1\text{mm/s}$  and  $2\text{mm/s}$  respectively. It can be noted that for higher feed velocities the chip formation is smoother and the movement described by the tool holder is reflected on the chip movement. Figures 4 and 5 also show the contact force (4(b) and 5(b)), the tool force (4(c) and 5(c)) and the phase space of the response (4(d) and 5(d)).

Figure 6(a) and 7(a) presents the displacement time history on the system with  $200\text{MPa}$  and  $500\text{MPa}$ , respectively. It can be noted that for higher stress limits the steps are very high and the surface quality is as expected. The contact forces, shown on Figures 6(b) and 7(b), are higher for harder materials and the tool forces has an strong transient behavior in Figure 7(b) that with softer materials, it is not present. Phase spaces of the response are presented on Figures 6(d) and 7(d).

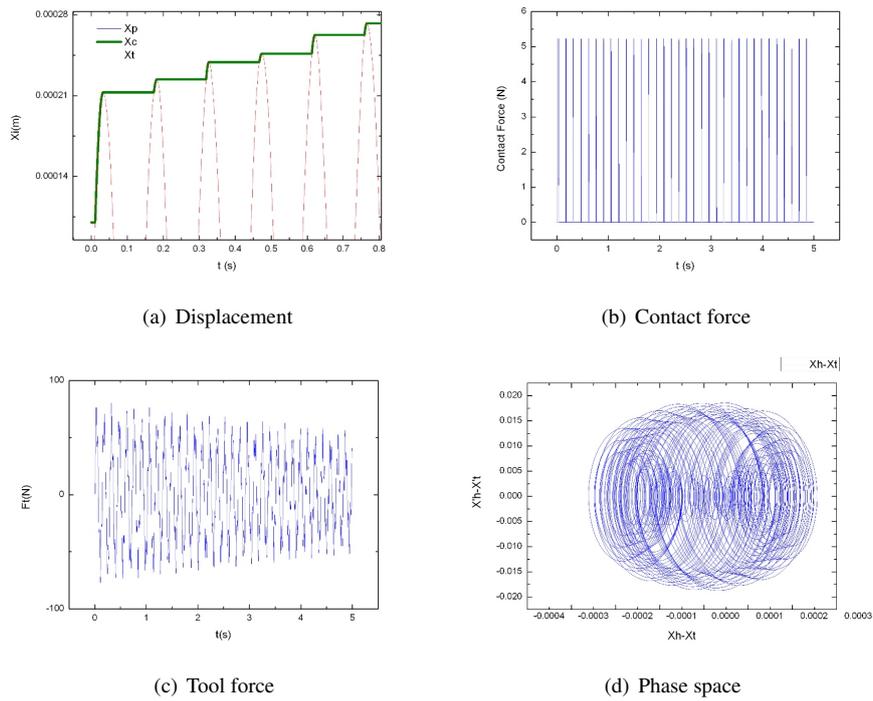


Figure 4.  $v_f = 0.1 \text{ mm/s}$ ,  $\tau_s = 100\text{MPa}$  and  $w = 1800\text{rpm}$

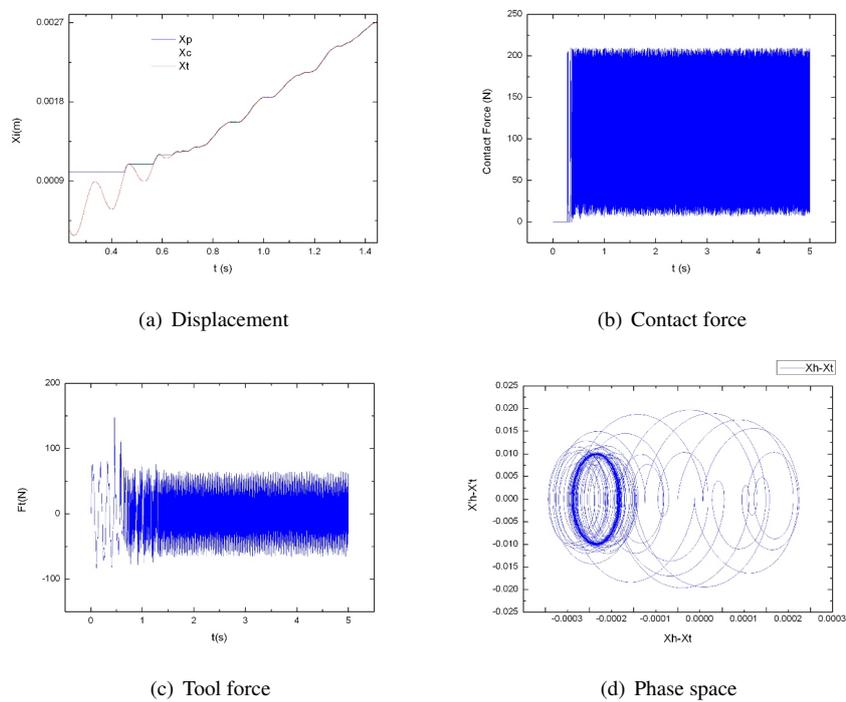


Figure 5.  $v_f = 2\text{mm/s}$ ,  $\tau_s = 100\text{MPa}$  and  $w = 1800\text{rpm}$

Figure 8(a) and 9(a) presents the time history of the displacements on the system with  $3000\text{rpm}$  and  $6000\text{rpm}$  respectively. Both spindle speeds conducted to chip formation that had no smooth displacements. Figures 8 and 9 also show the contact force (8(b) and 9(b)), the tool force (8(c) and 9(c)) and the phase space of the response (8(d) and 9(d)).

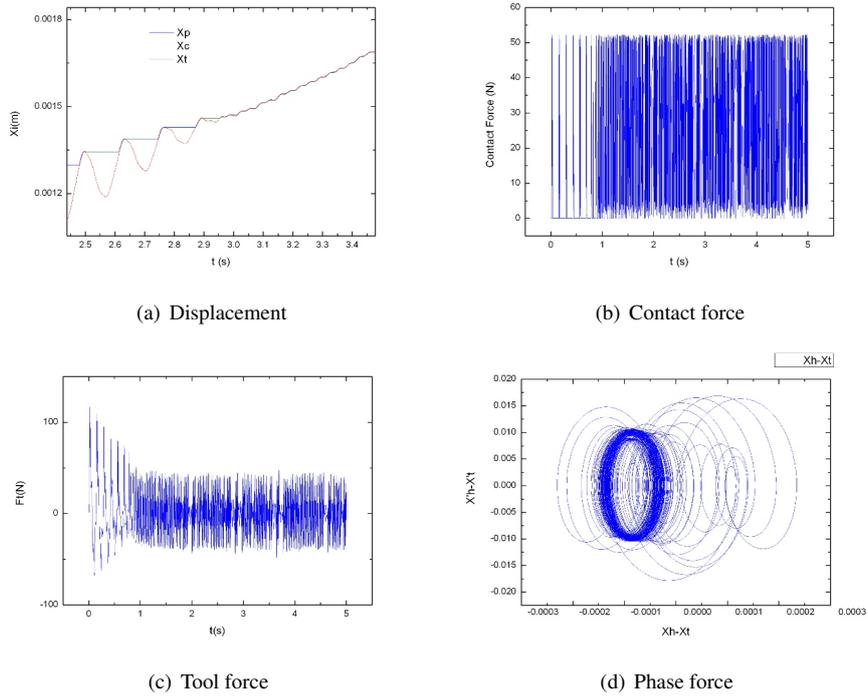


Figure 6.  $v_f = 0.5\text{mm/s}$  and  $\tau_s = 200\text{MPa}$  and  $w = 1800\text{rpm}$

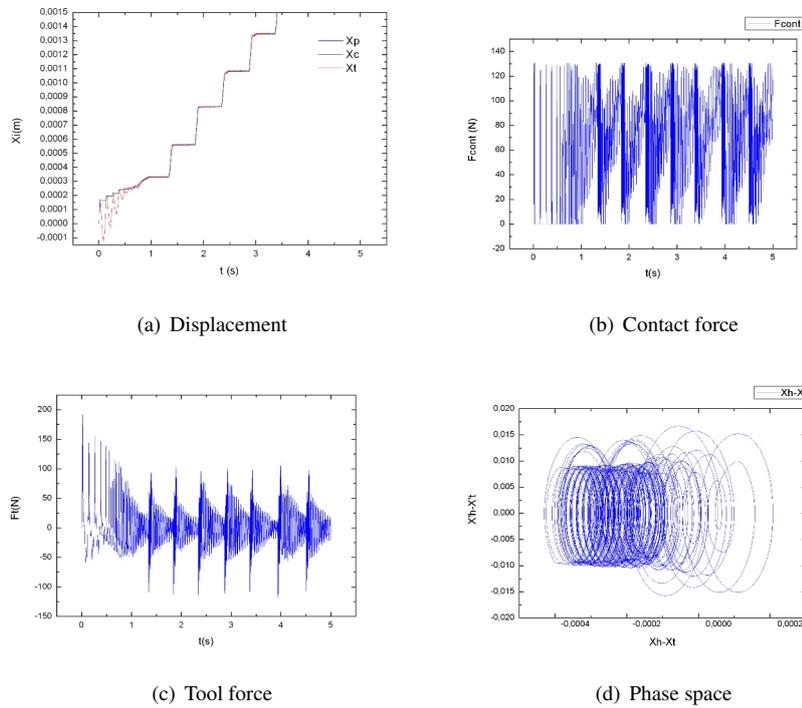


Figure 7.  $v_f = 0.5\text{mm/s}$ ,  $\tau_s = 500\text{MPa}$  and  $w = 1800\text{rpm}$

#### 4. CONCLUSIONS

This article deals with a dynamical analysis of the milling process. A nonsmooth system is used for the mathematical description of the system dynamics. Numerical simulations show some operational conditions presenting situations related to proper and improper functioning. The general aspects of the milling process is captured by the proposed model that can be used to identify improper system response.

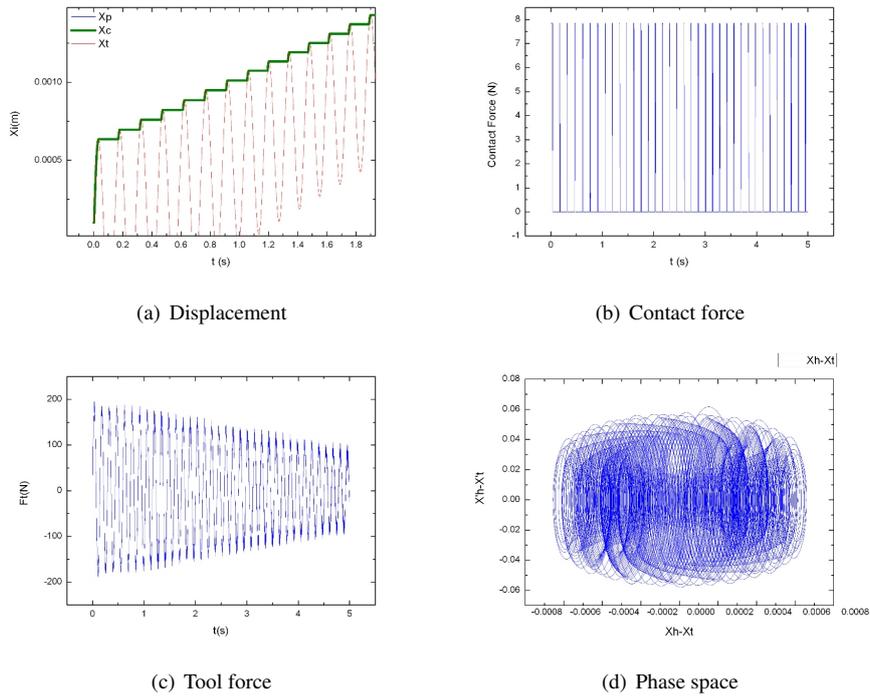


Figure 8.  $v_f = 0.5\text{mm/s}$ ,  $\tau_s = 100\text{MPa}$  and  $w = 3000\text{rpm}$

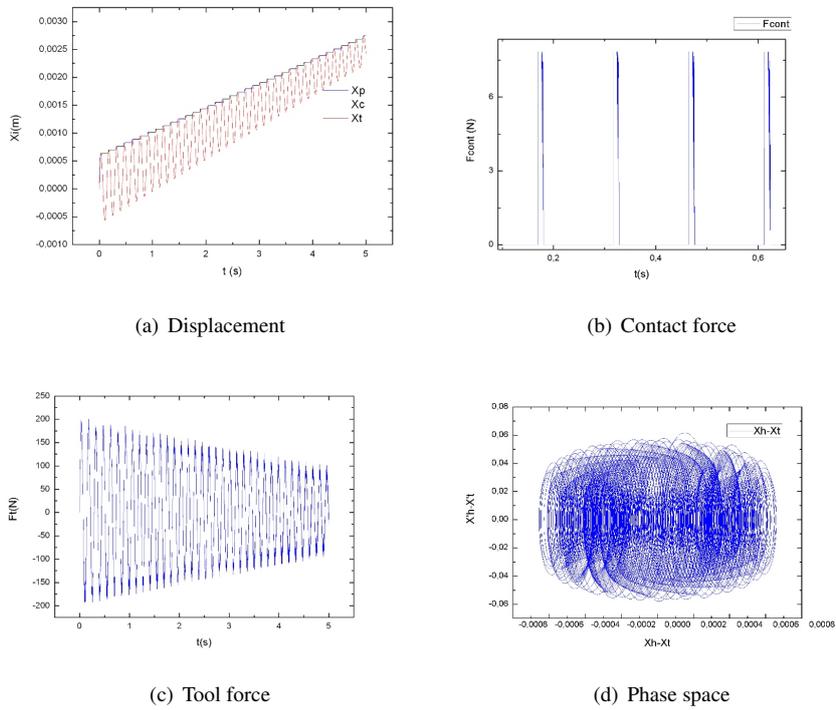


Figure 9.  $v_f = 0.5\text{mm/s}$ ,  $\tau_s = 100\text{MPa}$  and  $w = 6000\text{rpm}$

## 5. ACKNOWLEDGEMENTS

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