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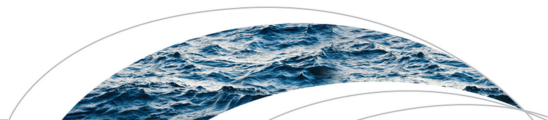
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## RESEARCH ARTICLE

10.1002/2014WR015431

### Key Points:

- A novel nonparametric approach for climate extremes is proposed
- The method is fast and flexible
- Simulations and a real application show the potentiality of the method

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# A fast nonparametric spatio-temporal regression scheme for generalized Pareto distributed heavy precipitation

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**Abstract** Analyzing the behavior of heavy precipitation, high temperatures, and extremes of other environmental variables has become an important research topic both for hydrologists and climatologists. Extreme value theory provides a well-developed mathematical foundation to statistically model excesses above a high threshold. Practitioners often assume that those excesses approximately follow a generalized Pareto distribution. To infer the two parameters of this distribution, a variety of estimations has been proposed and studied. Among them, maximum likelihood estimation offers an elegant way to include covariates, but imposing an explicit form on the parameters dependence. When analyzing large data sets, this procedure can be too slow and sometimes produce aberrant values due to optimization problems. To overcome these drawbacks, a method based on probability weighted moments and Kernel regression is proposed, tested, and applied to a Swiss daily precipitation data set. The method is implemented as a freely available R package.

## 1. Introduction

Weather and climate extremes affect societies and ecosystems and often induce fatalities and large material losses. To reduce the impact of those events and to improve risk assessment studies, it is important to obtain accurate statistical features of extremes such as 50 year return levels [e.g., Toreti et al., 2010; Schindler et al., 2012; Kharin et al., 2013; Papalexioiu and Koutsoyiannis, 2013; Serinaldi and Kilsby, 2014]. Extreme value theory (EVT) [e.g., Embrechts et al., 1997; Beirlant et al., 2004; de Haan and Ferreira, 2006] provides a solid mathematical framework for studying hydrological and climatological extremes [e.g., Katz et al., 2002]. In the identically and independently distributed (i.i.d.) setup and under mild conditions [e.g., Coles, 2001], this theory states that the probability distribution of excesses over a high threshold should follow the generalized Pareto (GP) distribution. This means that the survival (tail) function of excesses [Pickands, 1975] can be modeled by

$$\bar{G}_{\sigma,\xi}(y) = 1 - G_{\sigma,\xi}(y) = \begin{cases} \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi} & \text{if } \xi \neq 0, \\ \exp\left(-\frac{y}{\sigma}\right) & \text{if } \xi = 0. \end{cases} \quad (1)$$

where  $\sigma > 0$  and  $y \geq 0$  when  $\xi \geq 0$ , and  $y \in [0, -\sigma/\xi[$  when  $\xi < 0$ .  $\sigma$  and  $\xi$  are called the scale and shape parameters, respectively. The shape parameter plays an important role because it determines the tail-type (i.e., heavy, exponential, or with a finite upper right bound). To estimate the two GP parameters in the i.i.d. hypothesis, several methods have been developed and studied during the last 30 years [e.g., Hosking, 1990; Cooley et al., 2007; Zhang, 2007; Caeiro and Ivette Gomes, 2011] besides the maximum likelihood (ML) [Smith, 1985]. In hydrology, Greenwood et al. [1979] and Landwehr et al. [1979] introduced the so-called probability weighted moments (PWM). This method has been very popular in hydrology [e.g., Hosking et al., 1985] because it is conceptually simple, easy to implement, fast to run, and it has good performance for most distributions encountered in geosciences. To broaden its domain of validity, i.e., allowing heavier tails, Diebolt et al. [2008] introduced the generalized probability weighted moments (GPWM) for the GP distribution.

However, the main advantage of the ML method over the method-of-moments approaches is its flexibility. It can be easily modified to handle covariates, e.g., by incorporating time varying GP parameters. This feature is important, for instance, to investigate the effects of trends on hydro-climate extremes [e.g., Coles, 2001];

Katz et al., 2002; Kharin et al., 2013]. In practice, numerical problems with the optimization procedure can arise and it is not unfrequent to obtain inaccurate or even aberrant ML return levels, especially for heavy tails. Computational time can also become an important issue for large data sets. In addition, the ML procedure requires a parametric form, a strong a priori knowledge, that may be difficult to know or justify for most applications.

To overcome the aforementioned drawbacks of the ML method and with the aim of providing a fast, simple, and flexible alternative, we propose and study a nonparametric method that keeps the main GPWM/PWM approach advantages. In section 2, both the model and the estimation method are described. Section 3 is focused on simulations and an application of the proposed procedure to daily precipitation observed over Switzerland in the period 2000–2010. Finally, discussion and conclusions are provided in section 4.

## 2. A Nonstationary Regression for GP Parameters

### 2.1. Assumptions

Most EVT applications in climatology and hydrology are based on the assumption that the excesses marginal distributions can be adequately fitted by GP distributions. The shape parameter  $\xi$  and its associated uncertainty are difficult to estimate and  $\xi$  is often assumed to be constant in space and time within a climatologically homogenous region [e.g., Sang and Gelfand, 2009]. While, the scale parameter  $\sigma$  can vary according to some well-chosen geographical and/or atmospheric covariates [e.g., Cooley et al., 2007; Blanchet and Lehning, 2010]. In this paper, we work within the same framework, i.e.,  $\sigma = \sigma(\mathbf{X})$  where  $\mathbf{X}$  denotes the covariate vector. In contrast to the aforementioned studies, no parametric form is imposed on  $\sigma(\mathbf{X})$ . Concerning the dependence structure, we assume that the excesses, conditionally on  $\mathbf{X}$ , are independent. This assumption may not be satisfied in practice, but it is extremely complex to infer a multivariate GP distribution [Rootzén and Tajvidi, 2006; Falk et al., 2010] in high dimensions. Actually, most hydrological studies dealing with a complex dependence structure assumes that a preprocessing step has been applied to fit the excesses marginal laws at each weather station. Then, the marginals are transformed into Fréchet and a multivariate extreme value density is inferred [e.g., Beirlant et al., 2004; Cooley and Sain, 2010; Cooley et al., 2012]. With this respect, our approach can be used to perform the preprocessing step and transforming the marginals into unit Fréchet. Concerning the analysis of maxima, recent studies have taken advantage of complex multivariate max-stable procedures [e.g., Ribatet et al., 2009; Gaume et al., 2013]. In particular, Davison et al. [2012] analyzed summer and winter maxima of 51 weather stations in Switzerland from 1962 to 2008. One drawback of working with maxima is that different or similar extreme weather events are indiscriminately pooled together over the same season.

With regards to the nonparametric EVT approaches used in hydrology, Gardes and Girard [2010] proposed a nearest neighbor procedure to construct estimates of the tail index. They applied their method to extreme French rainfall in the Cévennes-Vivarais region. Compared to this work, we do not impose a positive shape parameter but we just assume GP marginals. Finally, the use of kernel smoothing for extremal quantile regression was investigating by Daouia et al. [2013].

Before recalling the GPWM/PWM method, we need to introduce a few notations. Let  $n$  be the total number of observed excesses and let  $\mathbf{X}_1, \dots, \mathbf{X}_n$  denote the covariates associated with those excesses (e.g., time and space).  $Z$  and  $Y(\mathbf{X}_i)$  are GP distributed random variables with shape parameter  $\xi$  and scale parameters  $\sigma$  and  $\sigma(\mathbf{X}_i)$ , respectively.

### 2.2. The Stationary Case

The PWM estimator emerges from the method-of-moments [Greenwood et al., 1979; Landwehr et al., 1979; Hosking and Wallis, 1987]. The PWM method for the GPD is based on the estimation of two moments given by  $\mu_r = \mathbb{E}[Z \bar{G}_{\sigma, \xi}^r(Z)]$ , where  $\bar{G} = 1 - G$  is provided by equation (1) for the GP distribution [for details and a general definition see Diebolt et al., 2007]. It can be shown that

$$\mu_r = \sigma \frac{1}{(1+r)(1+r-\xi)}, \quad (2)$$

for any nonnegative real  $r$ . When  $r = 0$  and  $r = 1$ , Hosking and Wallis [1987] showed, in the i.i.d. case, that the induced estimators of  $\xi$  and  $\sigma$  are asymptotically unbiased for  $\xi < 0.5$ . While using the concept of

U-statistics, *Furrer and Naveau* [2007] derived their distributions for finite sample sizes. An approach based on an adapted U-statistics was also proposed by *Hsing and Wu* [2004] for dealing with the stationary case.

To relax the restriction  $\xi < 0.5$ , *Diebolt et al.* [2007] studied the GPWM with  $r = 1$  and  $r = 1.5$ ,

$$\sigma = \frac{2.5\mu_{1.5}\mu_1}{2\mu_1 - 2.5\mu_{1.5}} \text{ and } \xi = \frac{4\mu_1 - (2.5)^2\mu_{1.5}}{2\mu_1 - 2.5\mu_{1.5}}. \tag{3}$$

In the i.i.d. case, they analyzed a specific class of estimators that are asymptotically unbiased with a normal distribution and a covariance matrix for any  $\xi < 1.5$ . One characteristic of these inference methods is that no optimization scheme is needed. Hence, an immediate advantage of the GPWM/PWM approach over the ML method is its computational simplicity.

### 2.3. General Case With Covariates

When  $\sigma(\mathbf{X})$  varies according to a covariate  $\mathbf{X}$ ,  $\mu_r$  can be extended by defining

$$\mu_r(\mathbf{X}) = \mathbb{E}[Y(\mathbf{X})\bar{G}_{\sigma(\mathbf{X}),\xi}^r(Y(\mathbf{X}))],$$

where  $Y(\mathbf{X})$  follows GP  $(\sigma(\mathbf{X}), \xi)$  distribution. As in (2),

$$\mu_r(\mathbf{X}) = \sigma(\mathbf{X}) \frac{1}{(1+r)(1+r-\xi)}. \tag{4}$$

This can be rewritten as

$$\mu_r(\mathbf{X}) = \sigma(\mathbf{X}) \mathbb{E}[Z\bar{G}_{1,\xi}^r(Z)],$$

where  $Z$  follows GP  $(1, \xi)$  distribution. From (4), we can express  $\xi$  and  $\sigma(\mathbf{X})$  in function of three PWMs, say  $\mu_0(\mathbf{X})$ ,  $\mu_r(\mathbf{X})$ , and  $\mu_s(\mathbf{X})$  with  $0 < r < s$

$$\xi = \frac{(1+s)^2 - (1+r)^2\alpha_{rs}}{(1+s) - (1+r)\alpha_{rs}} \text{ and } \sigma(\mathbf{X}) = \mu_0(\mathbf{X})(1-\xi), \tag{5}$$

with

$$\alpha_{rs} = \frac{\mathbb{E}[Z\bar{G}_{1,\xi}^r(Z)]}{\mathbb{E}[Z\bar{G}_{1,\xi}^s(Z)]}. \tag{6}$$

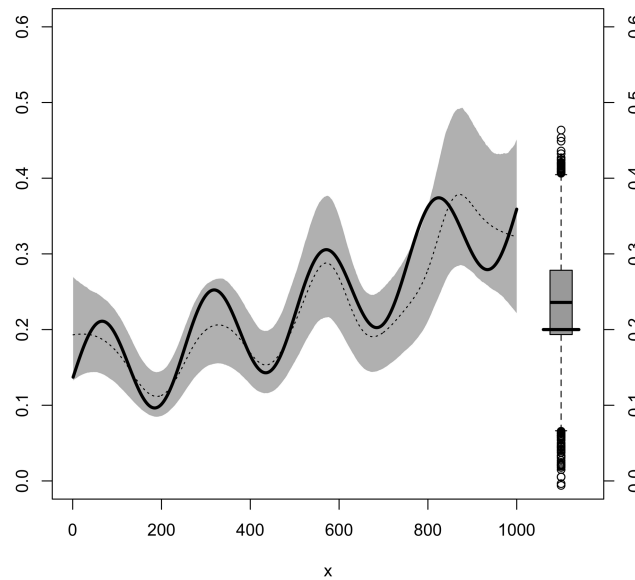
It is important to emphasize that we have changed the system of equation (3) by a new one described by (5) and (6). Our new system allows us to express  $\xi$  as a function of  $\alpha_{rs}$ , the latter does not depend on  $\mathbf{X}$ . Now the only variables depending on  $\mathbf{X}$  are  $\sigma(\mathbf{X})$  and  $\mu_0(\mathbf{X})$ . In other words, by introducing  $\alpha_{rs}$  we have clearly decoupled the fact that  $\xi$  and its associated PWMs do not depend on  $\mathbf{X}$ , but  $\sigma(\mathbf{X})$  and  $\mu_0(\mathbf{X})$  do. This remark leads to a very simple inference scheme. Suppose that  $\hat{\mu}_0(\mathbf{X})$  and  $\hat{\alpha}$  represent any estimators for  $\mu_0(\mathbf{X})$  and  $\alpha_{rs}$  with  $r = 1$  and  $s = 2$ , then from (5)  $\xi$  and  $\sigma(\mathbf{X})$  are inferred with

$$\hat{\xi} = \frac{9-4\hat{\alpha}}{3-2\hat{\alpha}} \text{ and } \hat{\sigma}(\mathbf{X}) = \hat{\mu}_0(\mathbf{X})(1-\hat{\xi}). \tag{7}$$

Concerning  $\hat{\mu}_0(\mathbf{X})$ , we simply implement a classical kernel regression approach [*Nadaraya, 1964; Watson, 1964*]. Let  $K$  be a weighting Kernel, e.g., a zero-mean Gaussian density function, we set

$$\hat{\mu}_0(\mathbf{X}) = \frac{1}{\sum_{i=1}^n K(\mathbf{X}-\mathbf{X}_i)} \sum_{i=1}^n Y(\mathbf{X}_i)K(\mathbf{X}-\mathbf{X}_i). \tag{8}$$

To estimate  $\alpha_{rs}$ , we face a small hurdle. The realizations of the i.i.d. random variables  $Z_i = Y(\mathbf{X}_i)/\sigma(\mathbf{X}_i)$  are unobservable because the  $\sigma(\mathbf{X}_i)$ 's are unknown. Fortunately,  $\alpha_{rs}$  is a ratio and consequently, dividing  $Z_i$  by  $\sigma(\mathbf{X})$  or by  $\mu_0(\mathbf{X})$  keeps it unchanged. Having previously estimated  $\hat{\mu}_0(\mathbf{X})$  by a Kernel



**Figure 1.** For a  $GPD(\sigma(x), \xi)$ , the solid black line represents the true scale parameter  $\sigma(x)$  in function of  $x$  ( $x$  axis). The shape parameter is constant and equals to 0.2 (right axis). From one realization, the boxplot and the gray 90% confidence intervals represent the estimated shape and scale (left axis) obtained by resampling, respectively.

smoother, the unobserved  $Z_i$ 's can be replaced by their estimated renormalized version  $Z'_i = Y(\mathbf{X}_i) / \hat{\mu}_0(\mathbf{X}_i)$ . At this stage, one can simply use his/her favorite inference PWM method to estimate  $\mathbb{E}[Z' \overline{G}'_{1,\xi}(Z')]$  for  $r=1, 2$ . In this exercise, they are inferred by using the U-statistic approach of *Furrer and Naveau [2007]* and applying a triweight kernel [e.g., *Härdle, 1991*]. Thus, the developed inference scheme is very similar to the classical PWM method. The two new ingredients are the use of Kernel regression for estimating  $\hat{\mu}_0(\mathbf{X})$  and the new system of equations to be solved. The proposed approach (hereafter, Kernel-PWM) can be summarized by the following steps:

1. Compute  $\hat{\mu}_0(\mathbf{x})$ , via equation (8);
2. calculate the vector

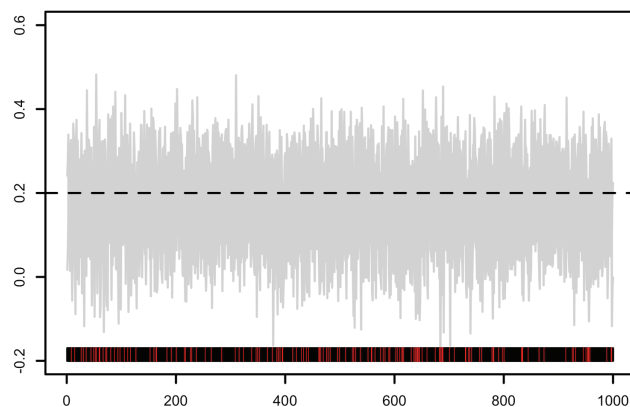
$$\{z'_i\} = \{y(\mathbf{x}_i) / \hat{\mu}_0(\mathbf{x}_i)\};$$

3. compute the two PWMs  $r=1, 2$  from the sample  $\{z'_i\}$  and then set  $\hat{\alpha}$  as the ratio of those PWMs;
4. derive  $\hat{\sigma}(\mathbf{x})$  and  $\hat{\xi}$  from equation (7).

To get confidence intervals, new samples can be generated under the estimated values and the proposed fast four-step algorithm repeated to explore the sampling variability. From a theoretical point of view, the statistical properties of  $\hat{\mu}_0(\mathbf{X})$  can be directly obtained from the classical Kernel regression literature [*Wand and Jones, 1995*]. As for any kernel-based approach, the choice of the bandwidth remains a delicate task. Still, classical approaches for bandwidth selection can be used in our procedure, but caution is necessary when the shape parameter is large.

### 3. Analysis of Nonstationary Excesses

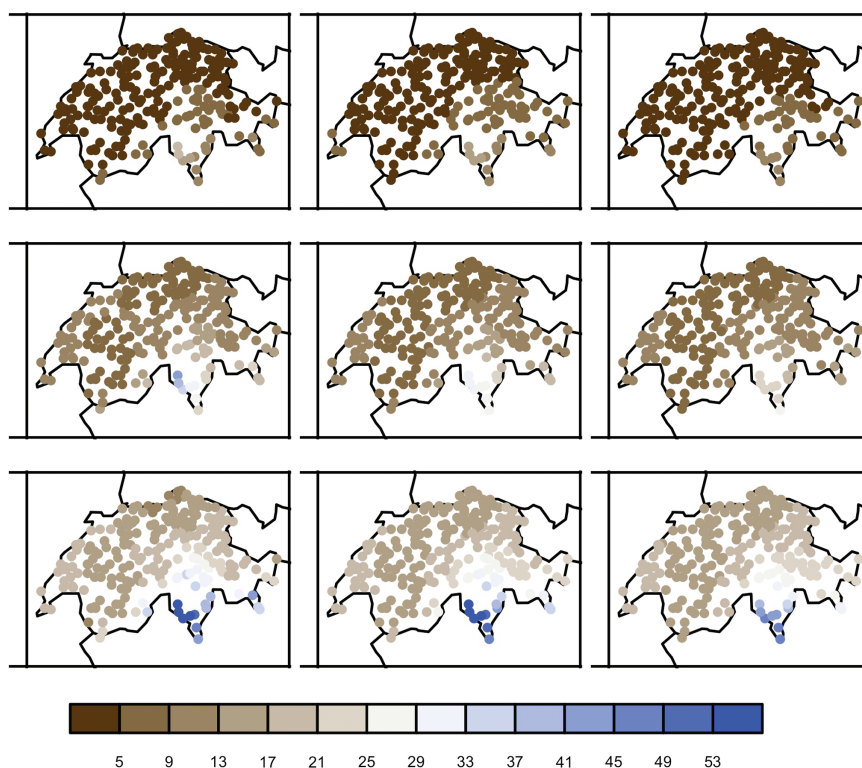
To illustrate the proposed procedure, we simulate one nonstationary sequence  $(Y(x_1), \dots, Y(x_m))$  of length  $m = 1000$  distributed according to  $GP(\sigma(x), \xi)$ , where  $\xi = 0.2$  and  $\sigma(x)$  is the combination of a periodic and an exponential signal, see the solid black line ( $x$  axis) in Figure 1. The inferred  $\hat{\sigma}(x)$  reproduces reasonably well the true behavior of  $\sigma(x)$  and the gray 90% confidence interval contains the true value. Some boundary effects (especially with higher values of the scale parameter) affect the estimation. The true shape parameter is clearly within the inferred boxplot. To validate this first example, we repeat this experiment 1000 times. Figure 2 displays the shape parameter and the 90% confidence interval for each simulation. The vertical red lines on the  $x$  axis correspond to the coverage probability occurrences and, as expected for a 90% confidence level for a thousand replicas, around 100 (precisely 136) false positive have been detected. To apply our method to a real data set, we focus on extreme precipitation in Switzerland recorded at 220 stations from 2001 to 2010 in autumn. Heavy precipitation is defined as being above the 90% quantile at each location. We assume that these excesses follow a GP distribution with a constant shape parameter. This latter hypothesis has been checked by fitting individually each station and testing if these values were significantly different from a countrywide parameter. The spatially varying scale parameter estimates are displayed in Figure 3. The top, middle, and bottom rows correspond to the 5%, 50%, and 95% values, respectively. To assess the influence of the bandwidth, the columns represent three different bandwidths, 0.3, 0.5, and 0.7, respectively. Basically, the results are robust with respect to these three bandwidth choices. We recognize the classical spatial pattern of heavy rainfall in Switzerland with larger values in Ticino,



**Figure 2.** Estimated shape parameter (y axis) from 1000 replicas (x axis) based on the setup described in Figure 1. The vertical red lines correspond to the samples outside of the estimated 90% coverage probability. As expected for 1000 replicas, around 100 false positive (red lines) occurrences are detected.

especially in the southwestern part of this area, and lower values over the northern part. With respect to the shape parameter, its estimate is around 0.12 with a 90% confidence interval of  $[-.67, .58]$ . The confidence range is large because our database only covers 10 years. Still, our inferred shape parameter corresponds to estimates obtained by MeteoSwiss [Umbricht *et al.*, 2013] for longer time periods and by Papalexiou and Koutsogiannis [2013, see their Figure 13]. In a sense, we have traded time with space and, even with a short time length, our estimates are consistent with past studies. One advantage of a Kernel-based approach is the possibil-

ity to extrapolate return levels. For our Swiss example, we first need to infer threshold values at unobserved locations. This is undertaken by universal kriging [e.g., Wackernagel, 2003] using elevation as external drift. The 50 year return levels map in Figure 4 emphasizes the complex spatial structure in Ticino and reveals the dryer belt in the south-west. This feature is not highlighted in Figure 4.10 of the recent MeteoSwiss report [Umbricht *et al.*, 2013] because of the lack of spatial coverage in this region. This requires further investigation [see also Huser and Davison, 2014].



**Figure 3.** Inferred scale parameter obtained from heavy precipitation (i.e., threshold at the 90% quantile of wet days) recorded at 220 stations in Switzerland from 2001 to 2010 in autumn. The top, middle, and bottom rows correspond to the 5%, median, and 95% values, respectively. The columns from the left represent three different bandwidths, 0.3, 0.5, and 0.7, respectively.

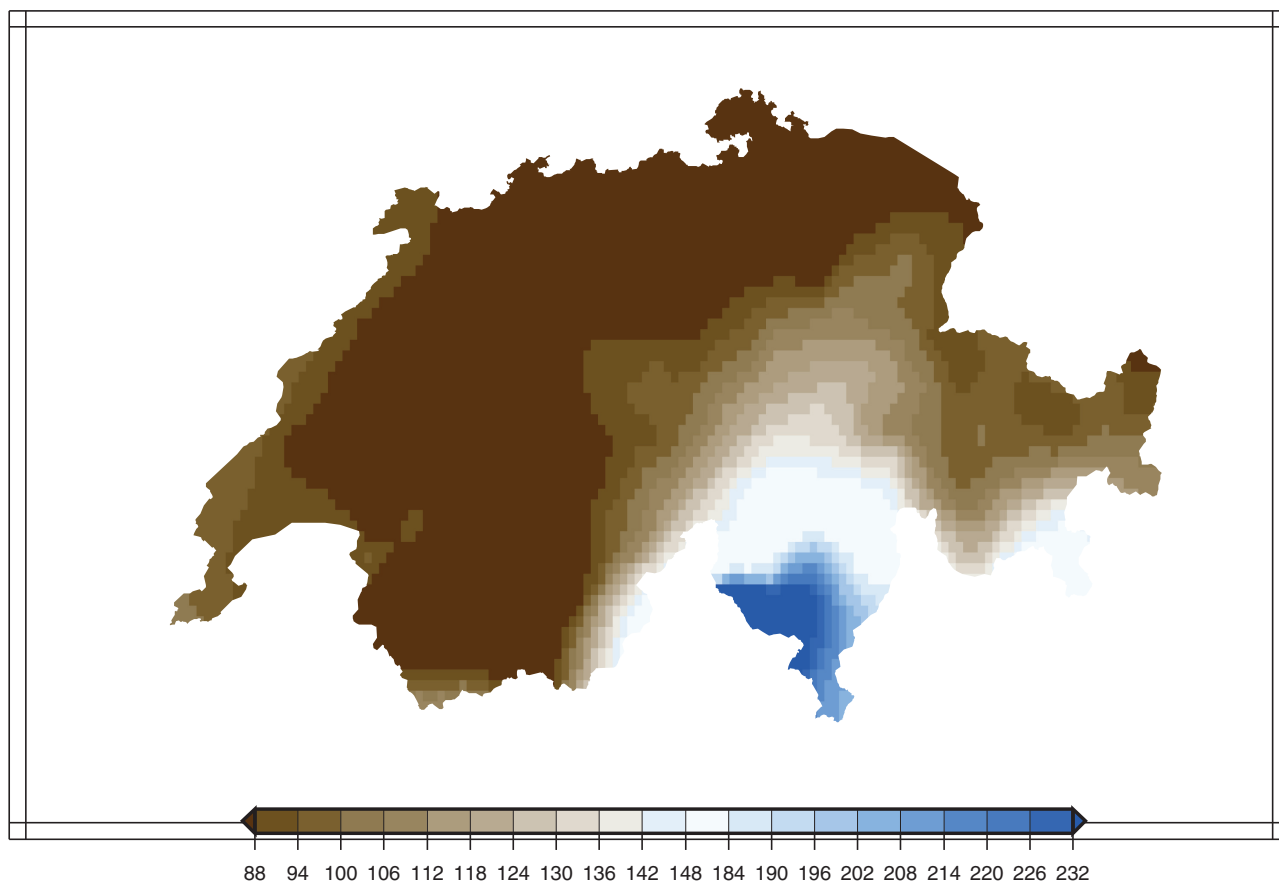


Figure 4. Inferred 50 year return levels in mm for heavy precipitation in Switzerland, see Figure 3.

#### 4. Discussion and Conclusions

A fast, simple, and flexible method based on probability weighted moments and kernel regression has been proposed to model covariate-dependent extremes. It is computationally inexpensive and can be applied to very large data sets. It does not assume any a priori behavior of the scale parameter, but it assumes a constant shape parameter. We tested our approach on simulations and heavy precipitation in Switzerland. The Swiss case study highlights the applicability of the method and its potentiality. Our results are coherent with recent studies. Finally, the method is freely available as an R package that can be requested by email.

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