



**HAL**  
open science

# NONLINEAR DYNAMICS OF MILLING PROCESS

Anna Carla Araujo, Marcelo Savi

► **To cite this version:**

Anna Carla Araujo, Marcelo Savi. NONLINEAR DYNAMICS OF MILLING PROCESS. X Encontro de Modelagem Computacional, 2007, Nova Friburgo, Brazil. hal-03212339

**HAL Id: hal-03212339**

**<https://hal.science/hal-03212339>**

Submitted on 29 Apr 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



## NONLINEAR DYNAMICS OF MILLING PROCESS

**Anna Carla Araujo** - annaaraujo@cefet-rj.br

CEFET/RJ - Departamento de Engenharia Mecânica

PPEMM - Programa de Pos-Graduação em Engenharia Mecânica e Tecnologia de Materiais

Av. Maracanã, 229 - Bloco E - 5 andar - CEP 20271-110 - Rio de Janeiro - Brasil

**Marcelo Amorim Savi** - savi@mecanica.ufrj.br

Universidade Federal do Rio de Janeiro, COPPE - Departamento de Engenharia Mecânica

Cx.Postal 68.503 - CEP 21941-972 - Rio de Janeiro - Brasil

**Abstract.** *The cutting processes include different manufacturing procedures as milling, turning and drilling. In all of these, the machine tool vibration plays an important role concerning the cutting characteristics. Therefore, its correct understanding is essential in order to improve the workpiece surface quality and avoid tool breakage. Chatter and squeal are some undesirable phenomena related to an improper functioning. This article discusses the nonlinear dynamics associated with the machine tool vibration relative to the workpiece considering a nonsmooth single-degree of freedom system. Milling process is of concern assuming different machining characteristics. The cutting force depends not only on the current position of the tool and the workpiece but also on a delayed value of the displacement as it is a function of the chip thickness. Therefore, the equation of motion is represented by a delay-differential equation that is solved employing the Runge-Kutta method together with a Taylor's series approximation employed to estimate the difference terms. Numerical simulations are carried out showing some situations related to proper and improper functioning during the cutting process.*

**Keywords:** *Milling, Force Prediction, non-linear dynamics, non smooth systems, chatter*

### 1. Introduction

Machining is one of the oldest process for shaping components and due to its versatility and precision achieved through continual innovation, research and development, has become an indispensable process used in manufacturing industry.

The machine tool vibration during the cutting process plays an important role concerning the workpiece surface quality and also the tool durability. Chatter and squeal are some undesirable phenomena related to an improper functioning. During certain cutting conditions, the motions of the workpiece-tool system are characterized by large amplitudes, which are not desirable for obtaining a good surface finish. The undesirable motions, which are often referred to as chatter, can result in wavy surfaces on the workpiece, inaccurate dimensions, and excessive tool wear.

Machining has been modeled by dynamical approach in order to help a proper comprehension of the process (Moon, 1998). For turning operations, Chandiramani & Pothala (2006) analyzed the dynamics of cutting considering a two-degree of freedom (2-dof) and orthogonal cutting modeling to predict chatter. Pratt & Nayfeh (1999) studied the boring bars for turning, which commonly presents chatter problems and experimentally determined modal properties of the tool. Also dedicated to turning, Kalmar-Nagy et al. (2001) showed the existence of a

subcritical Hopf bifurcation in the delay-differential equation used to describe the machining equations of motion. Zhao & Balachandran (2001), Insperger et al. (2003), Gradisek et al. (2005) and Mann et al. (2005) investigated the stability of milling process with single and two-dof systems for different experimental conditions. The mathematical model is represented by delay-differential equations.

Milling process has the peculiarity to have a contact/non-contact behavior due to its geometry and due to tool vibration. This kind of behavior is related to nonsmooth systems that are usually associated with either the friction phenomenon or the discontinuous characteristics as intermittent contacts (Savi et al., 2007). This article represents the milling process by a nonsmooth one-degree of freedom oscillator that is related to cutting and non-cutting behaviors, respectively related to contact and non-contact situations. The force description is done by mechanical approach and is used as an excitation to the system dynamics. Under these assumptions, the equation of motion is represented by a delay-differential equation that is solved employing the Runge-Kutta method together with a Taylor's series approximation employed to estimate the difference terms. Numerical simulations are carried out showing some situations related to proper and improper functioning during the cutting process.

## 2. Machining Model

Machining process is very complex because it involves different related phenomena. During cutting processes, for example, it is common to have temperature variations that induce dramatic changes in expected behaviors. Although all these complexity, some simple models may be used in order to obtain useful information concerning tool and workpiece behaviors as dynamic tool prediction.

The milling process is of concern in this article and its modeling is done by considering that it is a full immersed milling in the  $y$  direction and, therefore, the tool vibration is neglected in this direction. Hence, a single-degree of freedom system is assumed as shown in Figure 1, a schematic picture of the cutting process presenting the tool and the workpiece. Basically, the model considers a mass-spring-dashpot system representing the tool. Linear behavior is assumed for these elements and, therefore, they are respectively represented by parameters  $m$ ,  $k$  and  $c$ . The workpiece, on the other hand, is represented by a spring-dashpot, represented by parameters  $k_s$  and  $c_s$ . The distance between the tool and the workpiece is represented by a negative gap  $g$  as the tool is engaged into the workpiece.

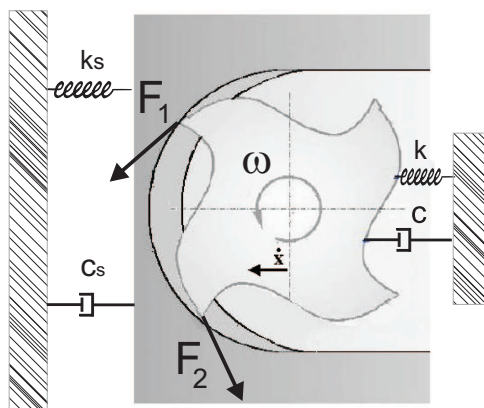


Figure 1: Dynamic of an End Milling Tool

This oscillator represents the end milling tool and its center has a displacement  $x$  that is associated with the sum of the predicted position plus the distance around the center due to the tool vibration (Moon, 1998). Under these assumptions, the milling process can be represented by the following equation:

$$m\ddot{x} + c\dot{x} + kx = 0, \text{ without contact} \quad (1)$$

$$m\ddot{x} + (c + c_s)\dot{x} + (k + k_s)x = -F_x, \text{ with contact} \quad (2)$$

where  $F_x$  represents the cutting force that may be properly described, as discussed in the next section.

## 2.1 Cutting Force Prediction

The prediction of the cutting force in machining has attracted the attention of many researchers that proposed different forms of modeling: analytical, numerical, experimental and hybrid approaches (Ehmann et al., 1997). Mechanical (Kline et al., 1982) and empirical approaches are certainly the most usual modeling employed in order to estimate cutting forces during milling. In this paper, we consider the prediction of forces in the end milling that involves instantaneous cutting forces modeled by mechanical approach with a dynamical deflection included in the force model.

The instantaneous differential cutting force for a single tooth  $\mathbf{f}_i$  with no helix angle is written based on Martelotti equation (Tlustý & MacNeil, 1975):

$$\mathbf{f}_i(t) = \mathbf{K}_c t_c b \quad (3)$$

where  $t_c$  is the uncut chip thickness,  $b$  is the depth of cut and  $\mathbf{K}_c$  is a vector with the specific cutting force coefficients.

The uncut chip thickness  $t_c$  for the end milling can be written as a function of the angle  $\phi_i(t)$ , the angle of each cutting flute  $i$  with respect to the normal direction of the feed per tooth  $s_t$  [mm/rev] (Kline et al., 1982) as:

$$t_{ci}(t) = s_t \sin \phi_i(t) \quad (4)$$

The feed per tooth is related to the feed  $v_f$  [mm/min], the spindle speed  $\omega$  [rpm] and the number of teeth  $N_t$  by:

$$s_t = \frac{v_f}{\omega N_t} \quad (5)$$

By considering  $\phi_1(t)$  the angle for a first flute, defined as the first flute that touches the piece and  $\xi$  the angle between the teeth:

$$\phi_i(t) = \phi_1(t) + \xi(i - 1) = \frac{2\pi\omega}{60}t + \xi(i - 1) \quad (6)$$

If the flute is in contact with the piece,  $\phi_i(t)$  is inside an angle range defined by the initial and final angles of the contact,  $\theta_i$  and  $\theta_f$ , and there is a contribution of this flute for computing the total force. If  $\phi_i(t)$  is not inside that range, the flute force is zero as there is no material to cut (Araujo et al., 2004).

$$\theta_i < \frac{2\pi\omega}{60}t + \xi(i - 1) < \theta_f \implies \mathbf{f}_i(t) \neq 0 \quad (7)$$

By assuming a full immersed milling, where the initial and the final angles are, respectively,  $\theta_i = 0$  and  $\theta_f = \pi$ , the force are given by:

$$\begin{aligned} \mathbf{f}_i(t) &= \mathbf{K}_c b s_t \sin \phi_i(t) & \text{if } \sin \phi_i(t) \geq 0 \\ \mathbf{f}_i(t) &= 0 & \text{if } \sin \phi_i(t) < 0 \end{aligned} \quad (8)$$

The total force is calculated by the sum:

$$\mathbf{F}(t) = \sum_{i=1}^{N_t} \mathbf{f}_i(t) = b s_t \sum_{i=1}^{N_t} \mathbf{K}_c(t) \sin \phi_i(t) \quad \text{whenever } \sin \phi_i(t) \geq 0 \quad (9)$$

The specific cutting force coefficients can be written in tangential and radial components,  $K_t$  and  $K_r$ . These components, for mechanistic models, are supposed to be constant for each tool-workpiece pair (Araujo et al., 2006).

$$\mathbf{K}_c(t) = \begin{vmatrix} K_x(t) \\ K_y(t) \end{vmatrix} = \begin{vmatrix} \cos \phi_i(t) & \sin \phi_i(t) \\ \sin \phi_i(t) & -\cos \phi_i(t) \end{vmatrix} \begin{vmatrix} K_t \\ K_r \end{vmatrix} \quad (10)$$

The total force  $\mathbf{F}(t)$  can be written as a function of  $F_x(t)$  and  $F_y(t)$  components:

$$F_x(t) = b s_t \sum_{i=1}^{N_t} (K_r \sin^2 \phi_i(t) + K_t \sin \phi_i(t) \cos \phi_i(t)) \quad \text{whenever } \sin \phi_i(t) \geq 0 \quad (11)$$

$$F_y(t) = b s_t \sum_{i=1}^{N_t} s_t (K_t \sin^2 \phi_i(t) - K_r \sin \phi_i(t) \cos \phi_i(t)) \quad \text{whenever } \sin \phi_i(t) \geq 0 \quad (12)$$

Notice that this approach does not considers that the vibration changes the cutting force. At this point, a dynamical approach is of concern in order to introduce these forces as excitations of the dynamical system that represents the milling process.

## 2.2 Equations of Motion

The end milling tool position  $x = x(t)$  of the center is represented by the sum of the predicted position plus the distance around the center due to the tool vibration (Moon, 1998).

$$x_c = x_o + x = \frac{s_t \omega t}{60} + x \quad (13)$$

By considering a dynamical point of view, a new parcel should be added (or removed) from the cutting force from Equation 11 as the tool position is changing and there is more (or less) material to cut. Since the tool is vibrating from the beginning of the cutting process, the material machined over (or below) the predicted trajectory changes the instantaneous cutting area for the acting flute. The additional material is computed as  $\Delta x = x(t) - x(t - \tau)$ , where  $\tau$  is the time between the passage of two flutes.

$$\tau = \frac{60 \xi}{2\pi\omega} \quad (14)$$

Under this assumption, the force can be rewritten adding the terms  $x = x(t)$  and  $x_\tau = x(t - \tau)$  to the feed per tooth parcel:

$$F_x(t) = b \sum_{i=1}^{N_t} (s_t + \Delta x) (K_r \sin \phi_i(t) + K_t \cos \phi_i(t)) \quad (15)$$

Moreover, by considering the idea of the nonsmooth oscillator, the force is zero when there is no flute in contact with the workpiece and also, while in vibration, when the tool position is less than feed per tooth, the negative gap between the piece and the workpiece.

$$\begin{aligned} F_x(t) &= b \sum_{i=1}^{N_t} (s_t + \Delta x) (K_r \sin \phi_i(t) + K_t \cos \phi_i(t)) & \text{if } x \geq -s_t \\ F_x(t) &= 0 & \text{if } x < -s_t \end{aligned} \quad (16)$$

It should be highlighted that the cutting process is related to a maximum force,  $F_{max}$ , and if this level is achieved its value is suddenly reduced and instantaneously released.

Therefore, employing this forces as the excitation of the nonsmooth oscillator that represents the cutting process, the system dynamics may be represented as follows:

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \text{if } x < -s_t \quad (17)$$

$$m\ddot{x} + (c + c_s)\dot{x} + (k + k_s)x = -F_x = -\bar{K}(s_t + \Delta x) \quad \text{if } x > -s_t \quad (18)$$

These equations represents different behavior for the system with and without contact. When the tool has contact with the workpiece, the damping and the stiffness are a sum of the workpiece and tool. On the other hand, when the tool lost the contact with the workpiece, the damping and stiffness are only calculated by considering the tool vibration as a bending beam.

Note that  $\bar{K}$  is a constant for this rotation angle  $\phi_i$ , and it is not dependent from  $x$ :

$$\bar{K} = \sum_{i=1}^n (K_t b \sin^2(\phi_i) + K_r b \sin(\phi_i) \cos(\phi_i)) \quad \text{whenever } \sin(\phi_i) \geq 0 \quad (19)$$

### 2.3 Numerical Procedure

Numerical simulations are carried out considering the fourth order Runge-Kutta method, however, in order to deal with the difference term in the right hand side, it is considered two different situations. The first one is when  $t > \tau$  and therefore, the  $x_\tau = x(t - \tau)$  has been already computed and the equation of motion may be rewritten as follows:

$$\ddot{x} + \frac{(c + c_s)}{m} \dot{x} + \frac{(k + k_s + \bar{K})}{m} x = -\frac{\bar{K}(s_t - x_\tau)}{m} \quad (20)$$

This system may be written in terms of a first order system as follows,

$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -p_1 & -p_2 \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ p_3 \end{pmatrix} \quad (21)$$

where:

$$p_1 = \frac{(c + c_s)}{m}; \quad p_2 = \frac{(k + k_s + \bar{K})}{m}; \quad p_3 = \frac{-\bar{K}(s_t - x_\tau)}{m}; \quad (22)$$

For time instants where  $t < \tau$ , it is necessary to estimate the values of  $x_\tau$  that is done by employing a Taylors series expansion Cunningham (1954):

$$x - x_\tau = \tau \dot{x} - \frac{\tau^2}{2} \ddot{x} \quad (23)$$

Under this assumption, the equation of motion become:

$$m\ddot{x} + (c + c_s)\dot{x} + (k + k_s)x = -\bar{K}(s_t + \tau \dot{x} - \frac{\tau^2}{2} \ddot{x}) \quad (24)$$

And this equation can be written as a first order system as:

$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -p_5/p_4 & -p_6/p_4 \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ p_7/p_4 \end{pmatrix} \quad (25)$$

where

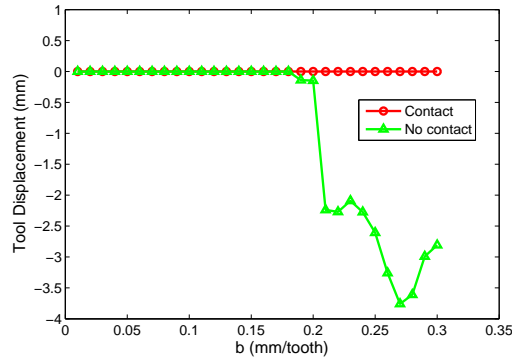
$$p_4 = 1 - \frac{\bar{K}\tau^2}{2m}; \quad p_5 = \frac{(c + c_s + \bar{K}\tau)}{m}; \quad p_6 = \frac{(k + k_s)}{m}; \quad p_7 = \frac{-\bar{K}s_t}{m} \quad (26)$$

### 3. Numerical Simulations

In order to show the capabilities of the proposed model, numerical simulations are carried out considering an end milling tool with diameter  $d = 19.05\text{mm}$ , four flutes and no helix angle. The tool characteristics are represented by the mass, the stiffness and the damping coefficients:  $m = 0.6\text{kg}$ ,  $k = 2000\text{N/mm}$  and  $c = 0.004\text{Ns/mm}$ , respectively. The workpiece properties are represented by the stiffness ( $k_s = 2 \cdot 10^7 \text{ N/mm}$ ) and the damping ( $c_s = 0.005\text{Ns/mm}$ ), values taken from Mann et al. (2005). The maximum force that the chip does not brake is taken as  $F_{max} = 2000\text{N}$ .

Our analysis starts by considering the influence of depth of cut  $b$  on the system dynamics. Therefore, it is assumed different values of this parameter while the others are fixed. Besides the other presented parameters, it is assumed that  $s_t = 0.15\text{mm}$  and  $\omega = 200\text{rpm}$ .

Figure 2 shows the maximum displacement in situation with and without contact. Notice that, for small values of  $s_t$ , system is always in contact, presenting a proper functioning. Nevertheless, the increase in this value tend to produce situations where the system starts to work presenting both behaviors. This curve may be useful to identify regions of stable, regular motions, associated with desirable functioning.

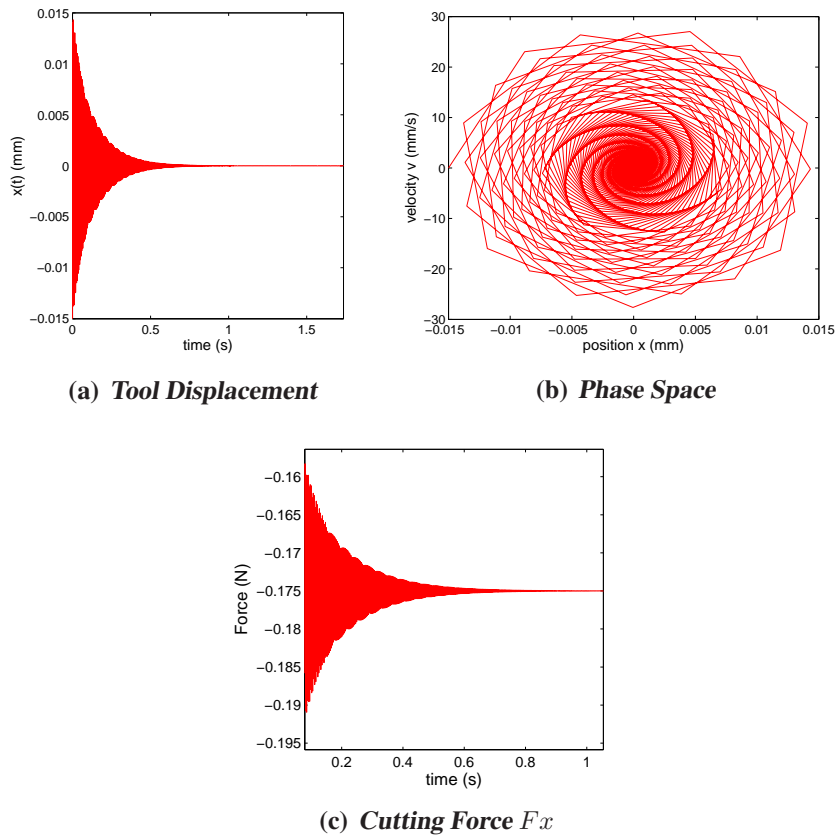


**Figure 2:** Maximum tool displacement in steady-state when  $s_t = 0.15\text{mm}$ ,  $\omega = 200\text{rpm}$  and  $b \in [0.01, 0.30] \text{ mm}$

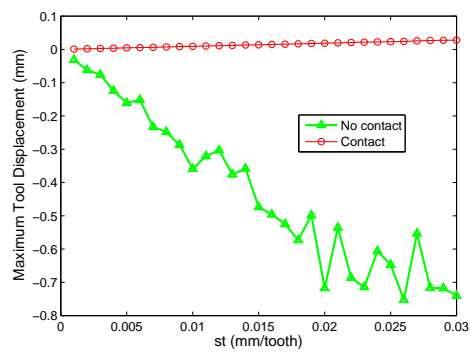
Different system responses are now in focus, considering what can be considered as proper and improper functioning. Figure 3 shows steady-state displacement time history and phase space plots for  $s_t = 0.15\text{mm}$ ,  $b = 0.01\text{mm}$  and  $\omega = 200 \text{ rpm}$ . Notice that non-contact behavior is always related to large displacements and therefore, are undesirable for machining purposes.

At this point, the same analysis is done considering  $b = 0.5 \text{ mm}$  and  $\omega = 200 \text{ rpm}$  fixed, varying the feed per tooth  $s_t$ . As expected, the same qualitative results are obtained and, the increase in feed values tends to cause improper functioning.

Figure 4 shows the maximum displacement in situation with and without contact. Once again, this curve may be useful to identify regions of stable, regular motions, associated with desirable functioning.



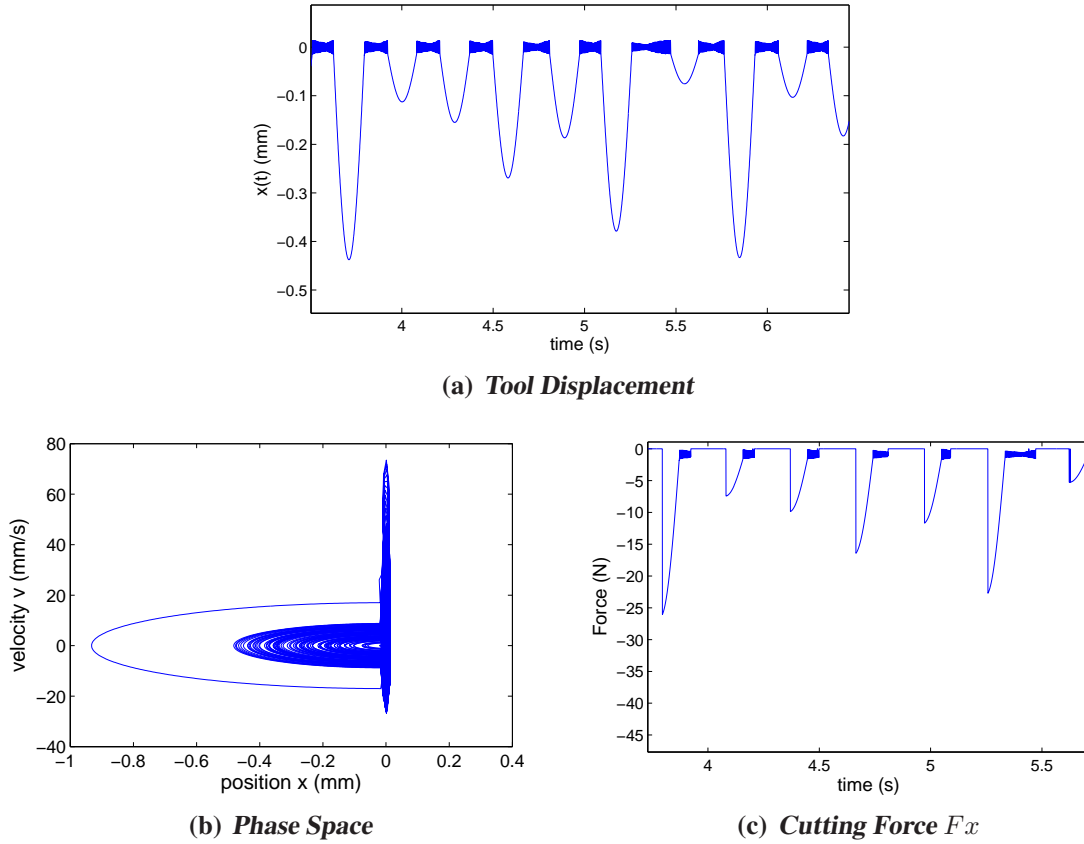
**Figure 3:** Numerical response for  $s_t = 0.15\text{mm}$ ,  $b = 0.01\text{mm}$  and  $\omega = 200\text{ rpm}$



**Figure 4:** Maximum displacement in steady-state when  $\omega = 200\text{rpm}$ ,  $b = 0.5\text{mm}$  and  $s_t \in [0.001, 0.030]\text{ mm}$



Figure 5 shows steady-state displacement time history, phase space and force  $F_x$  when  $s_t = 0.016\text{mm}$ ,  $b = 0.5\text{mm}$  and  $\omega = 200\text{ rpm}$ . Notice that non-contact behavior is always related to large displacements and therefore, are undesirable for machining purposes.



**Figure 5:** Numerical response for  $s_t = 0.016\text{mm}$ ,  $b = 0.5\text{mm}$  and  $\omega = 200\text{ rpm}$

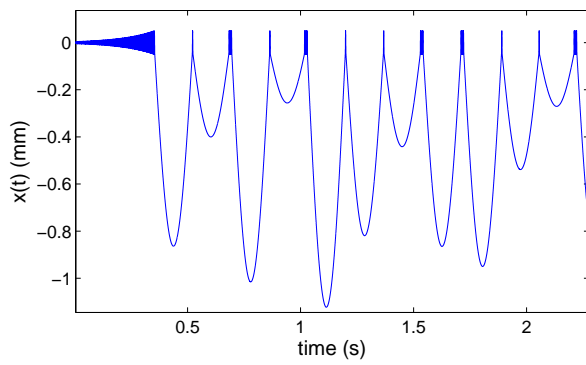
In order to observe a critical machining situation, it is considered a simulation with  $s_t = 0.05\text{mm}$ ,  $b = 0.15\text{mm}$  and  $\omega = 1500\text{ rpm}$  (Figure 5). This Figure presents the steady-state tool displacement, phase space plots and force  $F_x$ , showing a contact/non-contact response with great amplitudes variations.

#### 4. Conclusions

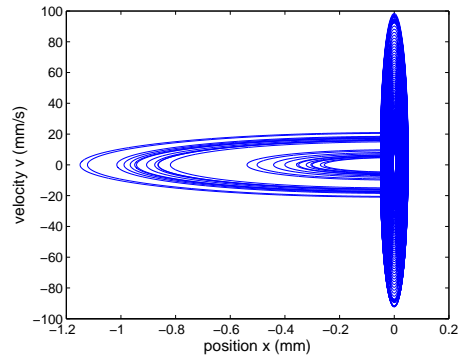
The analysis of milling process is proposed by a dynamical approach. A nonsmooth single-degree of freedom oscillator is proposed in order to model this cutting process. The forces involved during cutting is modeled by considering a mechanical approach and are used as an excitation of the dynamical system. The mathematical model is represented by a nonsmooth delay-differential equations that is numerically solved by the Runge-Kutta method together with a method based in Taylors series in order to estimated delayed displacements. Numerical simulations are carried out considering the influence of depth of cut and feed per tooth in the system dynamics. Results show that non-contact vibrations are related to improper functioning and can be used as a criterion to define functioning conditions.

#### Acknowledgements

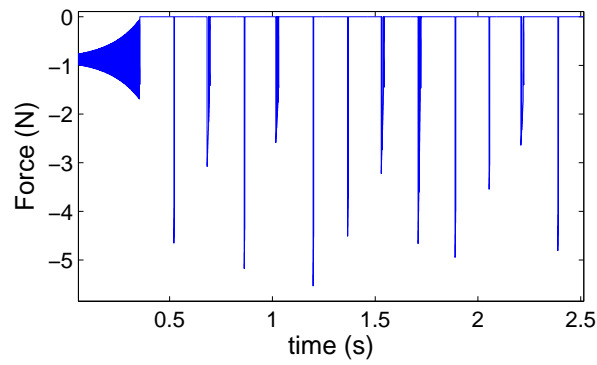
The authors acknowledge the support of Brazilian Research Councils CNPq and FAPERJ.



(a) *Tool Displacement*



(b) *Phase Space*



(c) *Cutting Force  $F_x$*

**Figure 6:** Numerical response for  $s_t = 0.05\text{mm}$ ,  $b = 0.15\text{mm}$  and  $\omega = 1500\text{ rpm}$

## REFERENCES

- Araujo, A. C., Silveira, L., J., & Kapoor, S. G., 2004. Force prediction in thread milling. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. XXVI, n. 1, pp. 82–88.
- Araujo, A. C., Silveira, L., J., Kapoor, S. G., Jun, M., & Devor, R., 2006. A model for thread milling cutting forces. *International Journal Of Machine Tools And Manufacture*, vol. 46, n. 11, pp. 1170–1170.
- Chandiramani, N. K. & Pothala, T., 2006. Dynamics of 2 dof regenerative chatter during turning. *Journal of Sound and Vibration*, vol. 290, pp. 448–464.
- Cunningham, W. J., 1954. A non-linear differential-difference equation of growth. *Mathematics*, vol. 40, pp. 708–713.
- Ehmann, K. F., Kapoor, S. G., DeVor, R., & Lazoglu, I., 1997. Machining process modeling: a review. *Journal of Manufacturing Science and Engineering*, vol. 119, pp. 655–663.
- Gradisek, J., Kalveram, M., Insperger, T., Weinert, K., Stepan, G., Govekar, E., & Grabec, I., 2005. On stability prediction for milling. *International Journal Of Machine Tools And Manufacture*, vol. 45, pp. 769–781.
- Insperger, T., Mann, B., Stepan, G., & Bayly, P., 2003. Stability of up-milling and down-milling, part 1: alternative analytical methods. *International Journal Of Machine Tools And Manufacture*, vol. 43, pp. 25–34.
- Kalmar-Nagy, T., Stepan, G., & Moon, F., 2001. Subcritical hysteresis bifurcation in the delay equation model for machine tool vibrations. *Nonlinear Dynamics*, vol. 26, pp. 121–142.
- Kline, W., DeVor, R., & Lindberg, J., 1982. The prediction of cutting forces in end milling with application to cornering cuts. *Int. Mach. Tool. Des. Res.*, vol. 22, pp. 7–22.
- Mann, B., Garg, N., Young, K., & Helvey, A., 2005. Milling bifurcation from structural asymmetry and nonlinear regeneration. *Nonlinear Dynamics*, vol. 42, pp. 319–337.
- Moon, F., 1998. *Dynamics and Chaos in Manufacturing Processes*. J. Wiley, New York.
- Pratt, J. R. & Nayfeh, A. H., 1999. Design and modelling for chatter control. *Nonlinear Dynamics*, vol. 19, pp. 49–69.
- Savi, M., Divenyi, S., Franca, L., & Weber, H., 2007. Numerical and experimental investigations of the nonlinear dynamics and chaos in non-smooth systems. *Journal of Sound and Vibration*, vol. 301, n. 1-2, pp. 59–73.
- Tlustý, J. & MacNeil, J., 1975. Dynamics of cutting in end milling. *Annals of CIRP*, vol. 24, n. 1, pp. 213–221.
- Zhao, M. X. & Balachandran, B., 2001. Dynamics and stability of milling process. *International Journal of Solids and Structures*, vol. 38, pp. 2233–2248.