1. Context: observer for unsteady aerodynamism
2. Physics, data & reduced order model (ROM)
3. Simulation, measurements & data assimilation
4. Reduced order model under location uncertainty
5. Results
PART I

CONTEXT :
OBSERVER FOR UNSTEADY AERODYNAMISM
Estimate and predict unsteady aerodynamics and aeroelasticity -- in real-time -- for better active control loops

- active flutter suppression
- Passengers comfort
- Drag reduction

Estimation and prediction:
- Flow
- Lift, drag
- ...

Controller
Simple model

Observer
Simple model

Plane sensors
MEMS
LIDAR

Plane aerodynamics / aeroelasticity

Which simple model? How to combine model & measurements?
PART II

PHYSICS, DATA & REDUCED ORDER MODEL
**REDUCED ORDER MODEL (ROM)**

Solution of an PDE with the form:

\[ v(x, t, \alpha) \approx \sum_{i=0}^{n} b_i(t) \phi_i(x) \gamma_i(\alpha) \]

<table>
<thead>
<tr>
<th></th>
<th>Full space</th>
<th>Reduced space</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution coordinates</strong></td>
<td>( v_q(x_i, t)q_i )</td>
<td>( (b_i(t))_i )</td>
</tr>
<tr>
<td><strong>Dimension</strong></td>
<td>( M \times d \sim 10^7 )</td>
<td>( n \sim 10 - 100 )</td>
</tr>
</tbody>
</table>

Order of magnitude examples in CFD
POD-GALERKIN

- **Principal Component Analysis (PCA)** on a *dataset* to reduce the dimensionality:

  ![Diagram of PCA process]

  - Off-line simulations → Snapshots \( v(x, t_i) \) → **PCA** → Spatial modes \( \phi_i(x) \)

- **Approximation**:

  \[
  v(x, t) \approx \sum_{i=0}^{n} b_i(t) \phi_i(x)
  \]

- **Projection of the “physics” onto the spatial modes**:

  \[
  \int_{\Omega} dx \ \phi_i(x) \cdot (\text{Physical equation (e.g. Navier-Stokes)})
  \Rightarrow \text{ROM for very fast simulation of temporal modes}
  \]
PART III

SIMULATION, MEASUREMENTS & DATA ASSIMILATION
COMBINING SIMULATIONS AND MEASUREMENTS

- Numerical Simulation (ROM) → erroneous
- Data assimilation (particle filtering with tempering & mutation)
- On-line measurements → incomplete → possibly noisy

More accurate estimation globally in space

\[ p(x|y) \propto p(y|x)p(x) \]

Need for uncertainty / errors quantification → Random dynamics

\[ p(x_{t+1}|x_t) \]

Velocity

- 3 m.s\(^{-1}\)
- 5 m.s\(^{-1}\)

\[ p(x) \]

\[ p(y|x) \]
PART IV

REDUCED ORDER MODELS UNDER LOCATION UNCERTAINTY
**LOCATION UNCERTAINTY MODELS (LUM)**

\[ \nu = \sum_{i=0}^{n} b_i \phi_i + \text{Residual} \sigma \dot{B} \]

- Randomized Navier-Stokes model
  - Good closure
  - Good model error quantification for data assimilation

**LUM**
- Memin, 2014
- Resseguier et al. 2017 a, b, c, d
- Cai et al. 2017
- Chapron et al. 2018
- Yang & Memin 2019

**SALT**
- Holm, 2015
- Holm and Tyranowski, 2016
- Arnaudon et al. 2017
- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al. 2018 a, b
- Cotter and al. 2019

**References**:
- Mikulevicius & Rozovskii, 2004
- Flandoli, 2011
- Cotter and al. 2017
- Resseguier et al. 2019 a, b
SUMMARY

Off-line: Building ROM

- Physics (Navier-Stokes)
- Randomized Physics (LUM)
- DNS code
- Data
- Stochastic ROM

On-line: Simulation & data assimilation

- Data assimilation (particle filtering)
- Measurements
- Stochastic ROM
- Temporal modes $b_i$
- Flow $v = \sum_{i=0}^{n} b_i \phi_i$
PART V

RESULTS:
UQ &
FAST OBSERVER
OF THE FLOW
NUMERICAL RESULTS: 3D WAKE AT REYNOLDS 300

Reference:
PCA-projection of the “true” simulation (10^7-dof DNS)
(Optimal from 8-dof linear decomposition)

Our method:
POD-Galerkin with Navier-Stokes under location uncertainty (LUM)

State-of-art:
POD-Galerkin with Navier-Stokes + optimally tuned eddy viscosity & additive noise

Reduced order models with 8 degrees of freedom
- only 1 PIV spatial point (local, blurred and noisy measure) is assimilated 10 times by vortex shedding cycle
CONCLUSION
CONCLUSION

- Reduced order model (ROM): for very fast and robust CFD ($10^7 \rightarrow 6$ degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LUM
- Data assimilation: to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LUM
- First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

NEXT STEPS

- Real measurements (PIV, Mems, ...)
- Increasing the degrees of freedom ($n$)
- Parametric ROM
- Increasing Reynolds (reduced DNS $\rightarrow$ reduced LES $\rightarrow$ reduced DDES)
MODEL UNDER LOCATION UNCERTAINTY, THE TRACER ADVECTION EXAMPLE

\[
\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \Theta \hat{\mathbf{B}} \cdot \nabla \Theta = 0
\]

Large scales:
- \( \boldsymbol{w} \)

Small scales:
- \( \sigma \hat{\mathbf{B}} \)

Variance tensor:
- \( a = a(x, x) = \lim_{\tau \to 0} \frac{1}{\tau^2} \mathbb{E} \left[ \sigma dB (\sigma dB)^T \right] \)

Drift correction

Multiplicative random forcing

Balanced energy exchanges

\[
\mathbf{D} = \frac{1}{2} \mathbf{a} \nabla \Theta
\]
REduced Models Under Location Uncertainty: Galerkin Projection Gives SDES for Resolved Modes

\[ \int_{\Omega} \phi_i \cdot \text{(stochastic Navier-Stokes)} \]

\[
\frac{d b_i}{dt} = F_i(b) + (\alpha_{i,j} \dot{B}_t)^T b + \theta_{i,j} \dot{B}_t
\]

Large scales:
\[ \sigma \dot{B} \]
Small scales:
\[ \sigma \dot{B} \]
Variance tensor:
\[
\frac{\partial}{\partial t} = F(b) + \{\sigma, \dot{b}\} + \{\alpha, \dot{\theta}\}
\]

2\text{nd order polynomial: coefficients given by physics,}
\[
(\phi_j)_j \quad \text{and} \quad a = \sigma \dot{B} (\sigma \dot{B})^T \tau
\]

Correlations to estimate
\[ \alpha_{pi,\alpha_{qj}}, \alpha_{pi,\theta_j}, \theta_i, \theta_j. \]
NUMERICAL RESULTS:
3D WAKE AT REYNOLDS 300

Reference:
PCA-projection of the “true” simulation (10^7-dof DNS) (Optimal from 2-dof linear decomposition)

Our method:
POD-Galerkin with Navier-Stokes under location uncertainty (LUM)

State-of-art:
POD-Galerkin with Navier-Stokes + optimally tuned eddy viscosity & additive noise

Reduced order models with 2 degrees of freedom
- only 1 PIV point (local, blurred and noisy measure) is assimilated 10 times by vortex shedding cycle
NUMERICAL RESULTS: 3D WAKE AT REYNOLDS 300

Reduced order models with $n$ degrees of freedom
> only 1 PIV spatial point (local, blurred and noisy measure)
is assimilated 10 times by vortex shedding cycle

$n=2$  
$n=4$  
$n=8$