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Data-driven identification of rotating machines using ARMA deterministic parameter evolution in the angle/time domain

Andreyson Bicudo Jambersi¹ · Samuel da Silva¹ · Jérôme Antoni²

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Abstract

The functional-series angle-/time-varying autoregressive moving-average (AT-FS-ARMA) model was used to model and analyze vibration-based signals from internal combustion engines. This approach is derived from the formulation of the time–angle periodically correlated processes, a relatively new topic in the cyclostationary framework, which has gained attention for modeling of mechanical signals. The AT-FS-ARMA model consists of traditional time-varying FS-ARMA-like models, but with the projection coefficients expanded in terms of the angular variable, dependent on time. Therefore, the method has the advantage of considering the angle periodicities often present in vibration-based signals from rotating and reciprocating machines. The performance is illustrated by an experimental application of signals measured in a diesel internal combustion engine (ICE) with a constant operating speed. The accuracy of the model is evaluated through the residual sum of squares normalized by the series sum of squares. To illustrate the use of the AT-FS-ARMA for vibration analysis of ICEs, parametric angle–frequency spectrum was estimated and compared to angular-varying pseudo-Wigner–Ville distribution/spectrum. The results showed that AT-FS-ARMA provides a useful complementary tool for analysis.

Keywords System identification · Rotating machines and/or reciprocating machines · Internal combustion diesel engine · Angle-/time-varying ARMA models · Cyclostationarity

1 Introduction

The modeling and analysis of rotating and reciprocating machines using vibration-based signals have been important research topics for several decades. The interest in those machines is justified due to their vital position in the industry in general. These machines, which include pumps, turbines, compressors, internal combustion engines (ICEs), generators, fans, and gearboxes, are used in the mechanical

and power industries. For ICEs, studies are generally related to the development of condition monitoring and diagnosis tools, aiming to increase the durability of the engines for longer life cycles and reduce maintenance costs due to unexpected failure and quality control in assembly lines [18]. Moreover, diesel ICEs are still widely utilized since they are present in vehicles and find broad applications as power generators. These latter applications include high-responsibility applications, i.e., emergency generators in nuclear power plants, which are used to stop the nuclear reactor in unexpected situations [30]. Hence, those applications suggest the necessity for robust methods of system identification for damage detection and fault prediction, which optionally may be used during the operation.

Generally, the methods for modeling and analyzing ICEs are classified into two different approaches: (1) those based on finite element methods and (2) those based only on experimental signals. Both methodologies for monitoring ICEs were widely reviewed in a recent paper by Delvecchio et al. [18]. The vibratory characteristics of signals from ICEs, including the vibration sources, ways of propagation and methods of analysis, are also presented in Refs. [4, 18].

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The experimental signals-based methods depend on signal processing techniques and require feature extraction, which consists of defining specific signal characteristics to monitor and identify the ones related to faults and damages.

The fundamental principle of the experimental signals-based approaches is to establish an association between some deviation in the vibration signal statistics and some healthy-state reference(s). However, due to the complexity of the vibration and acoustic-based signals from the ICEs, analysis and feature extraction are more difficult than for other rotating machines, e.g., gearbox and bearings elements, because the signal processing tools to analyze ICEs are not well consolidated yet [4, 18, 30]. Different signal and system identification approaches have been used for vibration analysis, vibratory source separation and feature(s) extraction. Among those methods are the nonparametric techniques, which include the classical time-frequency representations based on the short-time Fourier transform (STFT), Wigner–Ville distribution (WVD), pseudo-WVD and its variations [14, 20, 30, 45], and wavelet-based [33]. The WVD-based may present cross-terms, and the STFT-based methods present limitations due to the uncertainty principle [15], whereas the wavelet-based methods require an appropriate choice of the wavelet and are sometimes difficult to interpret by a nonspecialist, and its diagnosis is hard to automatize [4].

In addition, the parametric identification and analysis of vibration signals based on autoregressive models has been used in general applications for both stationary [17, 19] and nonstationary [9, 22, 34, 49, 51] scenarios, as well as for time-varying structures [40, 44]. The common models are the AR/ARMA.¹ Those parametric methods require the model structure to be defined by the analyst and the order(s) to be selected. In general, this selection uses measured data and an order selection criterion, by a trial-and-error procedure, which is generally based on the estimation of several candidate models and the evaluation of some fitness criterion [46].

Periodic time-varying parametric identification has been applied to analyze ICEs under constant speed regime of operation [13, 24, 39]. König et al. [25] employed periodic time-varying AR filters to model and predict the cylinder pressure signals in ICEs using vibration-based signals under constant rotation speed regime. The nonstationary parametric approaches use time-varying parameters in their models, in which temporal dependence may be classified as unstructured, stochastic or deterministic evolution, accordingly to how the parameters can evolve through time. The deterministic parameter evolution (DPE) approach is particularly interesting for vibration analysis [34]. This class of methods

imposes a deterministic structure for the time-varying coefficients. The time-varying parameters are expanded into constant coefficients of projection into functional subspaces (FS), whose evolution occurs in terms of deterministic and preselected time-varying functions. Through an appropriate selection of the FS, those models achieve high parsimony (i.e., their representation requires a limited number of parameters) and can track slow or fast dynamics [44], especially when orthogonal functions are selected. For vibration and speech analysis, trigonometric functions have often been used to compose the FS [9, 22, 23, 34]. Due to the use of functional subspaces, those models are often referred to as FS-TAR/TARMA.

It is possible to compute parametric spectral estimation, providing spectra with potential high resolution, avoiding cross-terms, and offering more details than obtained using traditional nonparametric approaches [26, 38]. However, the quality of the obtained spectrum is affected by model underfitting or overfitting. For example, for an all-poles (AR-part only) model spectral analysis, the model order should be the “optimum” one, i.e., the one that provides the best mean square compromise between bias and variance in the spectral estimates. If the selected order is too low, it may cause the resonances in data to be unresolved. On the other hand, orders too high may be less noise-sensitive [31] and are susceptible to produce spurious peaks (numerical instabilities), which results in a significant variance in the spectral estimates [38, 50]. More recently, parametric identification methods have been developed for vibration analysis using both vibration data and environmental and operation parameters to evaluate the effects of the latter at the extracted features vibration response and include uncertainty into model [10, 12, 48]. Also, recursive DPE-based methods have been investigated with prominent applications for online identification [28, 29].

In addition, the cyclostationary framework has been used for modeling, identification, analysis and fault detection on rotating machines [7, 36, 37]. The cyclostationary techniques are useful in describing signals carrying (hidden-)periodicities and, therefore, are well suited for rotating machinery applications. Based on this framework, Antoni and others proposed the angle-/time-correlated models, which are an extension of the cyclostationary modeling under nonstationary speed conditions [1, 8]. Those models assume that when varying speed or considerable speed fluctuations are present, the time periodicity is not verified anymore, but an angular periodicity may be held. This angular periodicity of time-varying mechanical signals is then used to provide formal descriptions of those processes. Moreover, a tutorial on cyclostationarity focusing on mechanical applications is presented in Ref. [5], while a survey on cyclostationary signals and applications may be found in Ref. [32]. By the assumption of cyclostationarity, vibration characterization

¹ AR: autoregressive. ARMA: autoregressive moving average.

of ICEs has been performed in the past using a Wigner–Ville distribution at the angle–frequency plane. Those results allow one to relate the energy content of a signal at each angle instant [6].

This paper’s main objective is to present a functional-series angle/time-varying autoregressive moving-average (AT-FS-ARMA) model for parametric identification of rotating machines and illustrate the application of this model using vibration data measured from accelerometers on a four-stroke diesel ICE operating at a constant speed. The method provides parametric spectra that may be used for characterizing the vibration signals in terms of the angular instants.

This paper is organized as follows. First, the AT-FS-ARMA model is introduced in Sect. 2, and some fundamental aspects of its formulation are discussed. The results and discussion are presented in Sect. 3, where the method is illustrated using real data from a four-cylinder four-stroke diesel ICE. For that, some essential characteristics of the signals and the preprocessing of the data are discussed. The model structure is selected using the BIC criterion, and the AT-FS-ARMA model is estimated and validated. A comparison between the smoothed pseudo-WV distribution and spectrum and the parametric estimated ones using the AT-FS-ARMA is presented. Finally, the conclusions are summarized in Sect. 4.

2 Parametric modeling in the time–angle domain

A traditional time-varying ARMA (TARMA) model is given by [22]:

$$x(t_n) = - \underbrace{\sum_{i=1}^{n_a} a_i(t_n)x(t_{n-i})}_{\text{AR-part}} + \underbrace{\sum_{i=1}^{n_c} c_i(t_n)e(t_{n-i})}_{\text{MA-part}} + e(t_n), \quad e(t_n) \sim \text{NID}(0, \sigma_e^2(t_n)), \tag{1}$$

and is presented as a generalization of the time-invariant ARMA model of the nonstationary case. Here, t_n stands for the discrete-time variable, $x(t_n)$ is a N -sample scalar and time-dependent stochastic process and $a_i(t_n)$ denotes $i = 1, \dots, n_a$ and $c_i(t_n)$ for $i = 1, \dots, n_c$ are the time-varying AR and MA i th coefficients; $e(t_n)$ is an unobservable uncorrelated white-Gaussian nonstationary innovation process, with zero mean and variance $\sigma_e^2(t_n)$, which is also time varying, contrary to the traditional stationary ARMA model. The values n_a and n_c address the AR/MA-part orders, and NID stands for normally independently distributed.

One common strategy to represent time-dependent dynamics imposes a deterministic parameter evolution (DPE) for the coefficients $a_i(t_n)$ and $c_i(t_n)$, which are assumed as a set of known/preselected deterministic time functions belonging to a functional subspace (FS). Due to that, the time-varying models with DPE structure are also known as FS-TARMA. Those models are capable of tracking fast time evolution by choosing an orthogonal basis for the FS. The FS-TARMA models have been properly used for conditioning monitoring of diesel engines [13], time-varying structure identification [11, 40] and fault diagnosis [9, 44].

In contrast to the FS-TARMA models, the proposed AT-FS-ARMA approach expands the model coefficients in an angle-function functional basis belonging to a functional subspace, where the angular dependency is directly related to the instantaneous position of a reference shaft instead of time. This model can be used to identify vibration-based signals from rotating and reciprocating machines and is derived from a new framework, where the signals from machines are modeled as angle-/time-cyclostationary processes. The signal periodicity is described in terms of the angular variable, whereas the signal dynamics stills represented varying through time. Broader discussions on cyclostationarity in mechanical systems, as well as its definition, statistics, and examples of systems with such behavior, were given by Antoni [5], while more details about the angle/time modeling are presented in references [2, 3]. The AT-FS-ARMA model is a significant original contribution of this paper.

Let us define the instantaneous angular shaft displacement variable, which depends on the discrete-time instant $\phi_n = \phi(t_n)$. Hence, an AT-FS-ARMA model with parameters dependent on both time and angle is proposed as:

$$x(t_n) = - \sum_{i=1}^{n_a} a_i(\phi_n)x(t_{n-i}) + \sum_{i=1}^{n_c} c_i(\phi_n)e(t_{n-i}) + e(t_n), \tag{2}$$

where

$$a_i(\phi_n) = a_i(\phi_n + \Phi), \quad c_i(\phi_n) = c_i(\phi_n + \Phi), \quad \sigma_e^2(\phi_n) = \sigma_e^2(\phi_n + \Phi), \tag{3}$$

are functions dependent on both time and angle that are angle periodic with a period equal to Φ . The angular periodicity also indicates that the innovation variance $\sigma_e^2(\phi_n)$ is cyclic with a period related to a shaft/rotating component position. This period is in general equal to $\Phi = k2\pi$, where $k \in \mathbb{Z}$. Usually, the period is $\Phi = 2\pi$ for rotating machines,

and it is equal to $\Phi = 4\pi$ for four-stroke diesel engines. Assuming the DPE-based structure for the AT-FS-ARMA model, the parameters for the AR part $a_i(\phi_n)$ and MA part $c_i(\phi_n)$ as well as the innovation variance $\sigma_e^2(\phi_n)$ are expanded into angle-dependent functional basis in the form:

$$\begin{aligned} a_i(\phi_n) &\triangleq \sum_{k=1}^{p_a} \alpha_{i,k} G_{b_a(k)}(\phi_n), \quad c_i(\phi_n) \triangleq \sum_{k=1}^{p_c} \gamma_{i,k} G_{b_c(k)}(\phi_n), \\ \sigma_e^2(\phi_n) &\triangleq \sum_{k=1}^{p_s} s_{i,k} G_{b_s(k)}(\phi_n), \end{aligned} \tag{4}$$

and substituting Eq. (4) into Eq. (2), we have a closed-form formulation of the AT-FS-ARMA model as:

$$\begin{aligned} x(t_n) &= - \sum_{i,k} \alpha_{i,k} \underbrace{G_{b_a(k)}(\phi_n)}_{z_k(\phi_{n-i})} x(t_{n-i}) + \sum_{i,k} \gamma_{i,k} \underbrace{G_{b_c(k)}(\phi_n)}_{y_k(\phi_{n-i})} e(t_{n-i}) + e(t_n), \\ e(t_n) &\sim \text{NID}(0, \sigma_e^2(\phi_n)), \end{aligned} \tag{5}$$

Then, the AT-FS-ARMA model can be expressed as:

$$x(t_n) = - \sum_{i,k} \alpha_{i,k} z_k(\phi_{n-i}) + \sum_{i,k} \gamma_{i,k} y_k(\phi_{n-i}) + e(t_n), \quad e(t_n) \sim \text{NID}(0, \sigma_e^2(\phi_n)). \tag{6}$$

The angle-/time-dependent FS-ARMA model is described in terms of $n_a(p_a + 1) + n_c(p_c + 1) + p_s$ coefficients. As previously mentioned, the DPE-based method requires the preselection of a proper family of basis functions, which in general may assume in *a priori* knowledge about the signal and previous analyst experience. Vibration signals from reciprocating/rotating machines suggest angular periodicity of the sources of vibrations linked to a reference shaft. Therefore, an angle-periodic basis is an appropriate choice to express this dependency.

Thus, to express the angle/time periodicities, a functional subspace with an angular-period equals to $\Phi = 4\pi$ composed by Fourier series expansions angular coefficients is selected [8]:

$$\begin{cases} G_1(\phi_n) = 1 \\ G_{2k}(\phi_n) = \sin\left(\frac{k\phi_n}{2}\right), & k > 1 \\ G_{2k+1}(\phi_n) = \cos\left(\frac{k\phi_n}{2}\right), & k > 1 \end{cases} \tag{7}$$

where $G_{b(k)}$ for $k = 1, \dots, \max(p_a, p_c)$ are the angle-/time-dependent functional subspaces and k are the indices of the subspace basis functions. The angle-dependent Fourier series used as basis functions in Eq. (7) are achieved through a general model that originated in the angle-/time-cyclostationary modeling of signals [1, 8]. The model is estimated using a ‘‘Relaxed’’ multistage weighted least-squares (RMS-WLS) method to compute the model coefficients of projections and the innovation variance [41, 43].

3 Results and discussion

The experimental data were measured from a four-stroke four-cylinder IC diesel engine from a passenger car. The experiments have been carried out with the transmission at the neutral position. The vibration signals were saved at a constant speed regime of operation using one accelerometer positioned on the engine block close to a cylinder. Additionally, an encoder provides an one-top-per-revolution signal. All data were acquired at the sampling frequency of $f_s = 20.48$ kHz.

A complete cycle of the four-stroke engine has four stages: (1) admission, (2) compression, (3) ignition, and (4) expansion/exhaust. The dependence between the thermodynamic cycles and the crankshaft position suggests that the vibration signals are angle periodic at every two turns of the crankshaft ($\Phi = 4\pi$), which contains four combustions. The sources of the vibrations in IC engines are mostly due to the movement of the moving parts, the tilting of pistons, the distributor and injection systems,

and forces caused by the pressure variations inside the chambers. The combination of all those vibration sources results in a complex signal for analysis. More details on the vibration characteristics of IC diesel engines can be found in Chapter 3 of the thesis by Antoni [4].

The counting of the tachometer pulses and the instantaneous angular speed were both estimated by the encoder signal. Then, the vibration signal was initially preprocessed to include an integer number of the thermodynamic cycles by windowing it into an even number of tachometer pulses. The preprocessing of the accelerometer signal also includes mean and trend removal. The angular displacement of the crankshaft is approximated through a discrete-time integral of the instantaneous angular speed signal. The measured vibration response signal is originally 15 s and shown in Fig. 1. This signal structure presents high-amplitude peaks produced by the explosions, followed by strong transients until the energy produced by the impacts dissipates through the engine block. A zoom in a segment of the signal may allow one to see the angular periodicity of the signal representing the function of the number of engine’s cycles (as shown in Figs. 2 and 3 where the tachometer signal is also represented).

Figure 4 illustrates the relationship between the crankshaft cycles and the measured vibration response.

A nonparametric spectrogram was also estimated for an initial analysis. Figure 5 corresponds to the nonparametric PSD obtained through STFT using a Gaussian moving

Fig. 1 Vibration response signal for the steady-state condition at a nominal speed of 1500 RPM

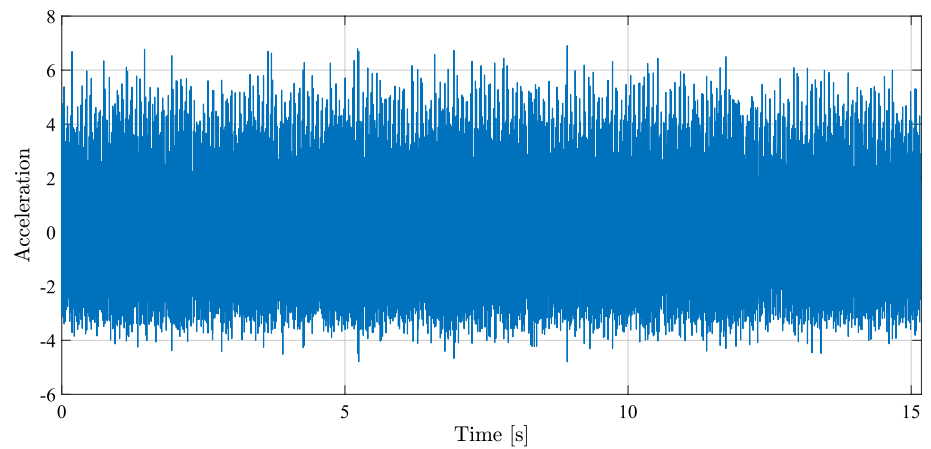


Fig. 2 A zoom at a small length of the vibration response signal at a nominal speed of 1500 RPM

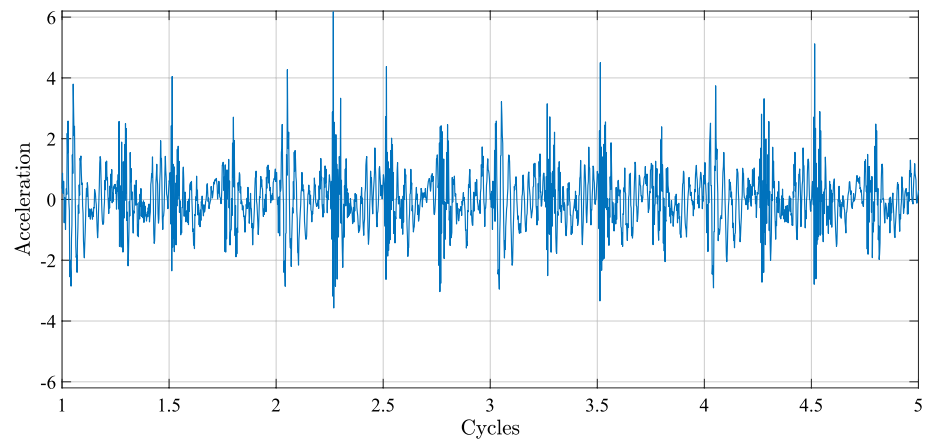
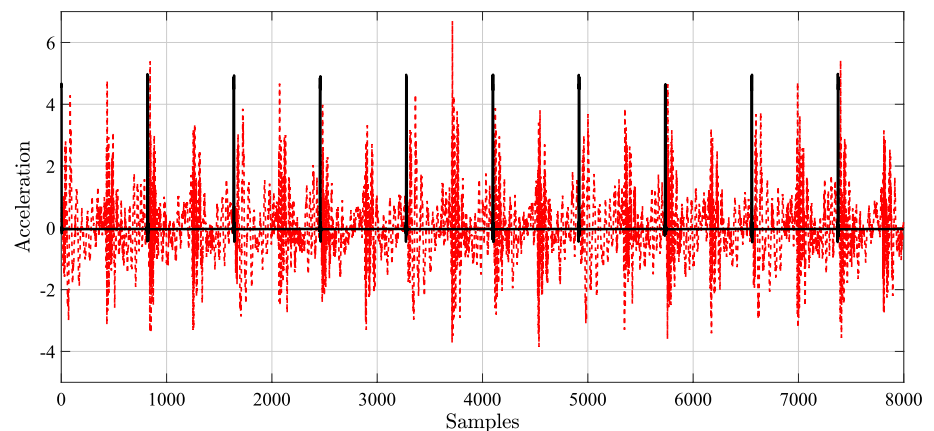


Fig. 3 Segment of the vibration response signal (-) and its relation with the tachometer signal (→) for the steady-state condition at a nominal speed of 1500 RPM (color figure online)



window with 2048 data points with 200 advance samples and 2048 DFT points. In the PSD, the darker the color, the lower the energy content at this frequency range, as indicated by the color bar. Also, from Fig. 5, the energy of the signal decreases considerably for frequencies above 4.5 kHz. This information may be helpful to determine whether the signals can be down-sampled before using the parametric identification method.

After the range of frequency with high energy content is established, the next step is to select the model structure. This procedure requires the selection of the model orders and basis dimensions for the AT-FS-ARMA(n_a, n_c) $_{[p_a, p_c, p_s]}$. This may be achieved using Akaike's information criterion (AIC), Bayesian information criterion (BIC), by evaluation of the mean squared error (MSE) or using other model order selection criteria [34, 35]. Several conditions

Fig. 4 Segment of the crankshaft position signal (mapped from 0 to 4π) - - and its relation with the tachometer signal (-) for the steady-state condition at a nominal speed of 1500 RPM (color figure online)

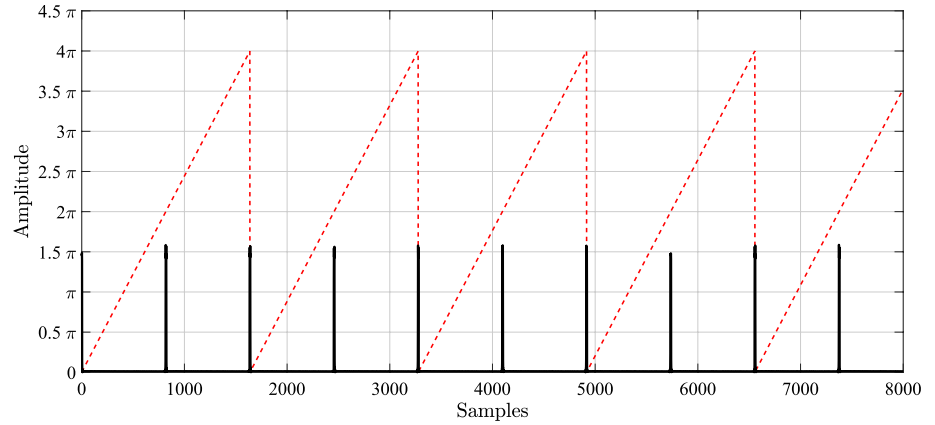
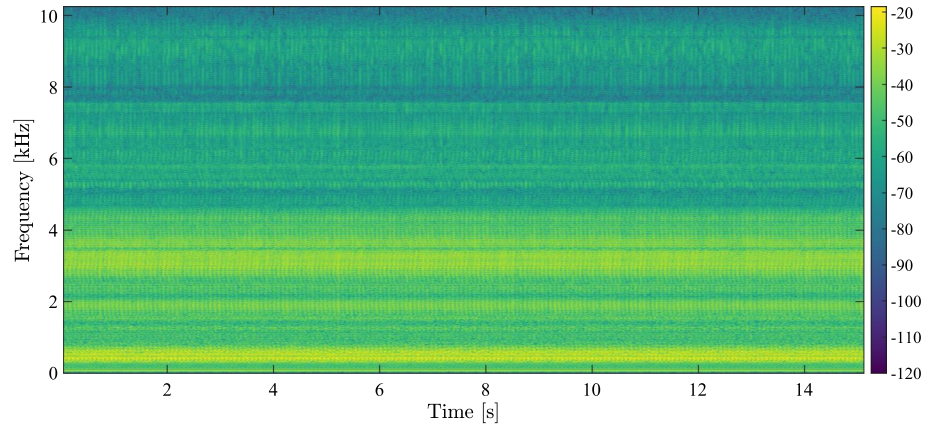


Fig. 5 Two-dimensional non-parametric PSD obtained via STFT, using a Gaussian moving window with 2048 data points with 200 advance samples and 2048 DFT points. The darker the color, the lower the energy content (color figure online)



are then examined by varying the data length and down-sampling the signals, which are recommended because the original sampling frequency is too high and the data length is too long. Complete and high-dimensional subspaces are defined to check the sensibility of varying those dimensions.

As aforementioned, the original signal length is too long, and taking into account that it presents periodicity, only a segment of the vibration signal is selected for analysis. This may be achieved by dividing the signal into M segments. This selected M th segment was then truncated to include an integer number of thermodynamic cycles. Therefore, a new number of N samples are obtained. The signal is also resampled into a new sampling frequency f_{sn} using a polyphase anti-aliasing filter. Again, Fig. 5 demonstrates that the high energy content of the signal occurs below 4.5 kHz, so the new sampling frequency was established as at least $f_{sn} \geq 10$ kHz to include information below at least 5 kHz and satisfy the Shannon sampling theorem.

The structure of the model is chosen through the selection of the orders n_a and n_c and the dimension of the functional subspaces p_a and p_c . In this paper, the selection is based on the BIC criterion [34]. For that, several AT-FS-ARMA(n,n)

candidate models are estimated for $n = 1, \dots, 30$ (extended to 50 for situations where the initial evaluation was considered inconclusive). The selected AR-part order n_a is the one that minimizes the BIC value. Then, the same procedure is used to optimize the MA-part order n_c , but now considering $n_c = 1, \dots, n_a$, where the selected n_c order will again be the one that minimizes the BIC. The removal of FS excessive dimensions procedure is not performed in this work [42]. This procedure described above was repeated for different signal lengths $N_{n's}$ and sampling frequency f_{sn} . All the bases were adopted as containing the same dimension $p = p_a = p_c = p_s$.

Table 1 provides the obtained results, where $d = (n_a p_a + n_c p_c)$ indicates the number of invariant parameters that describe the resultant model. This table shows that the combination of parameters from Test #6 provided the lowest BIC value. Therefore, the AT-FS-ARMA(16, 3)_[25,25,25] model was selected as the one that would best represent the signal. For Test #6, the sampling frequency is reduced to 3/4 of its original value, and the resulting signal contains 14,605 samples and is approximately 1/15 of the original duration. The BIC values

Table 1 AT-FS-ARMA model identification: BIC minimum values obtained for candidate models

Model	f_{sn} (Hz)	N_n (samples)	p	n_a	n_c	d	min(BIC value ($\times 10^3$))
# 1	10,240	9737	25	13	1	350	3.5231
# 2	10,240	14,651	25	26	7	825	3.2290
# 3	10,240	9737	15	25	8	495	3.3879
# 4	10,240	9737	50	12	1	650	4.2239
# 5	15,360	14,605	15	18	6	345	-0.5253
# 6	15,360	14,605	25	16	3	475	-1.1384
# 7	10,240	9737	15	18	6	525	3.7280

Fig. 6 AT-FS-ARMA model identification: **a** BIC values of AT-FS-ARMA(n, n)_[25,25,25] models ($n = 2, \dots, 30$) for AR-part order selection. **b** BIC values of AT-FS-ARMA(16, n_c)_[25,25,25] $n_c = 1, \dots, 14$ for MA-part order selection

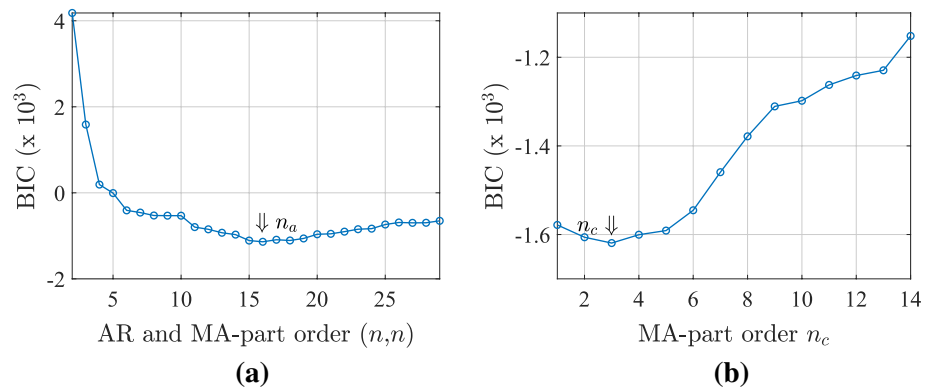
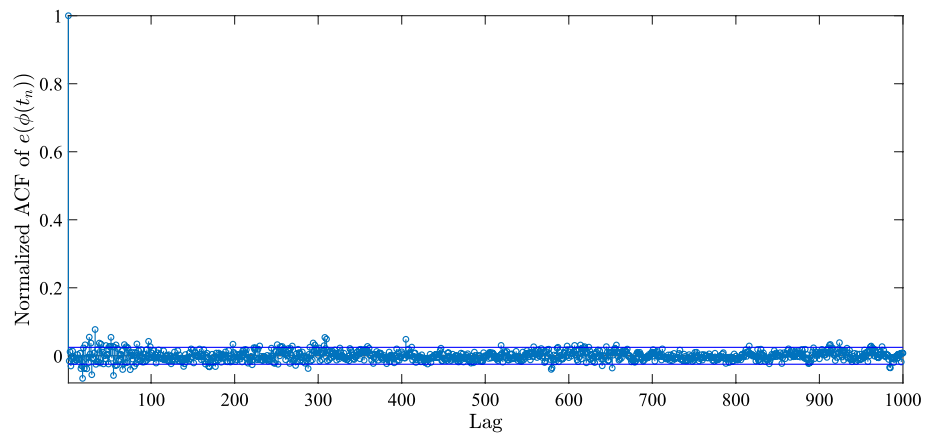


Fig. 7 AT-FS-ARMA model identification: normalized autocorrelation function (ACF) of the residual error of AT-FS-ARMA(16, 3)_[25,25,25] for 95% confidence level computed for a number of 1000 lags



obtained for the AT-FS-ARMA(n, n)_[25,25,25] candidate models from Test # 6 are presented in Fig. 6.

Once the model structure and orders are selected, the obtained model must be validated [27]. The acceptance of the model depends on its performance under procedures that check it for an example for the innovations sequence Gaussianity-whiteness. For stationary models, those tests usually check for the uncorrelatedness innovations sequence by estimating its autocorrelation function (ACF). However, for the nonstationary model due to nonstationarity variance, the usual tests are not applicable [43]. To overcome this

issue, the innovation variance may be normalized into a unit-variance sequence as [21, 43],

$$\epsilon(\phi_n) = \frac{e(t_n)}{\sigma_e(\phi_n)} \quad \epsilon(\phi_n) \sim \text{NID}(0, 1) \tag{8}$$

where $\epsilon(\phi_n)$ is the normalized variance.

Figure 7 shows the ACF of the normalized innovations sequence for 95% confidence level. The impulse-like signature for the ACF indicates the uncorrelatedness of the normalized innovations sequence and, therefore, an overall adequate model fitting. Another procedure is the residual

Fig. 8 AT-FS-ARMA model identification: the residual error sequence $e(\phi(t_n))$

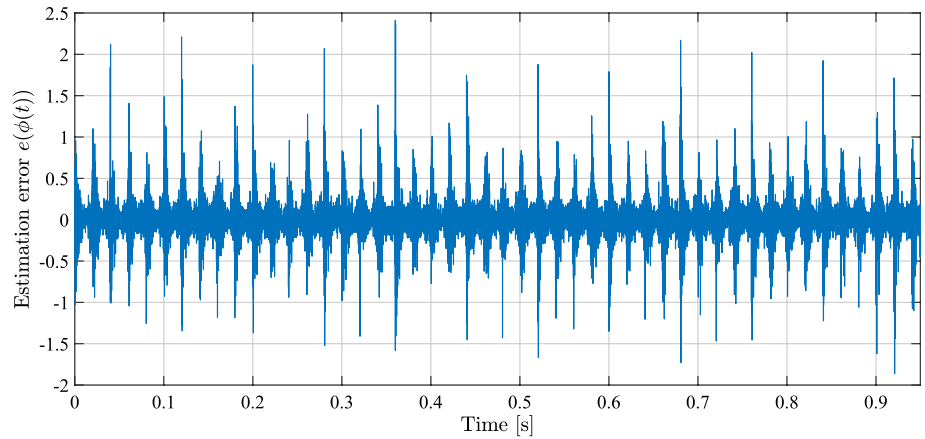


Fig. 9 AT-FS-ARMA model identification: the angle-/time-dependent innovations variance $\sigma_e^2(\phi(t_n))$

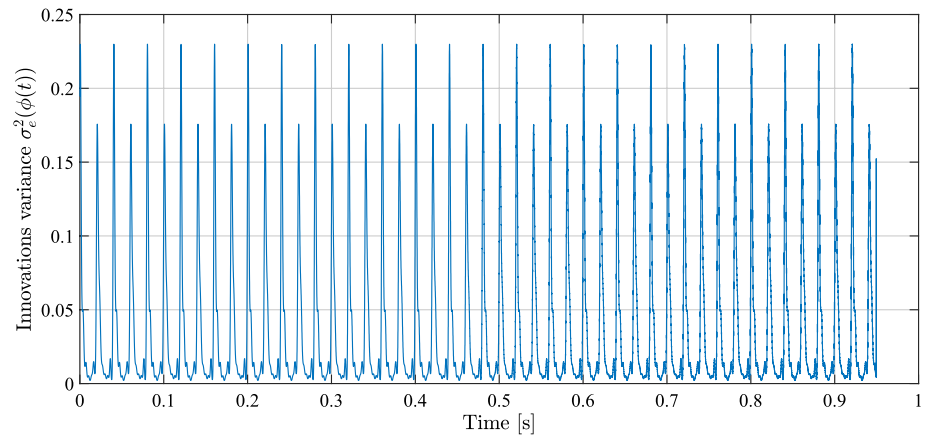
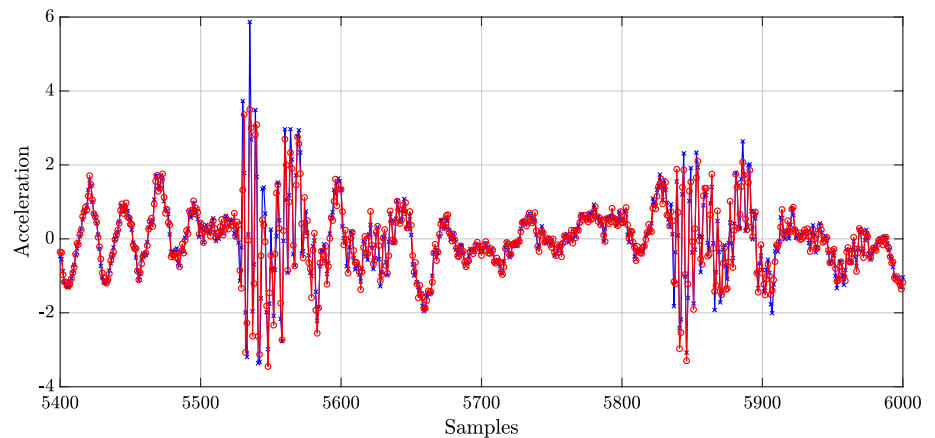


Fig. 10 AT-FS-ARMA identification: segment of the vibration response $\rightarrow x$ and AT-FS-ARMA(16, 3)_[25,25,25] prediction $\rightarrow \circ$ (color figure online)



sign test (run test for randomness) [34, 47] in which the AT-FS-ARMA(16, 3)_[25,25,25] innovations sequence is tested for 95% confidence level and attests the hypothesis that the uncorrelated residual series is valid.

The comparison between the vibration response and its modeling using AT-FS-ARMA(16, 3)_[25,25,25] is presented in terms of the residual error sequence in Fig. 8. The estimated angle-/time-dependent innovation variance is in Fig. 9 and

is periodic, as expected. The vibration response and model prediction in Fig. 10 were in a good accordance.

The obtained value for the residual sum of squares normalized by the series sum of squares (RSS/SSS) is approximately 5.96%, which indicates a high modeling accuracy. For the effect of comparison, the same signal was modeled using a traditional FS-TARMA (with a time-based basis) with precisely the same orders and basic dimensions, and

Fig. 11 Averaged smoothed pseudo-WVD $|W_{xx}(\phi, f)|$ computed over the signal resampled in the angular domain synchronously averaged on 189 cycles

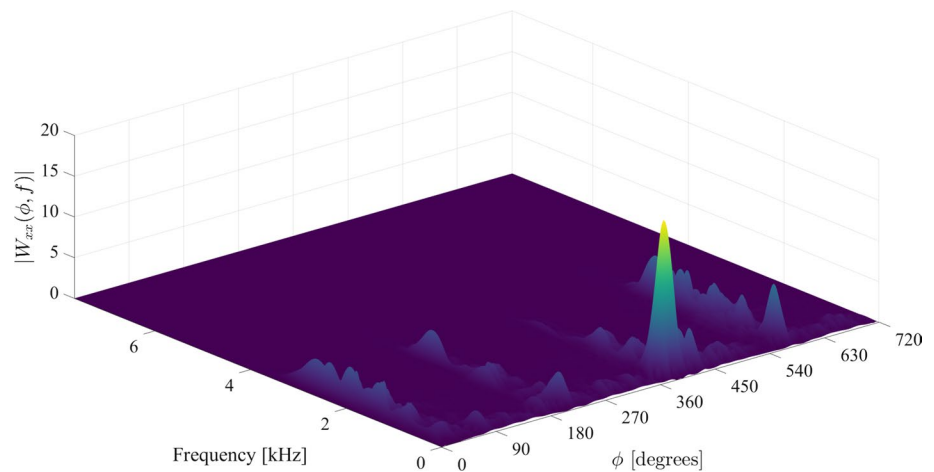
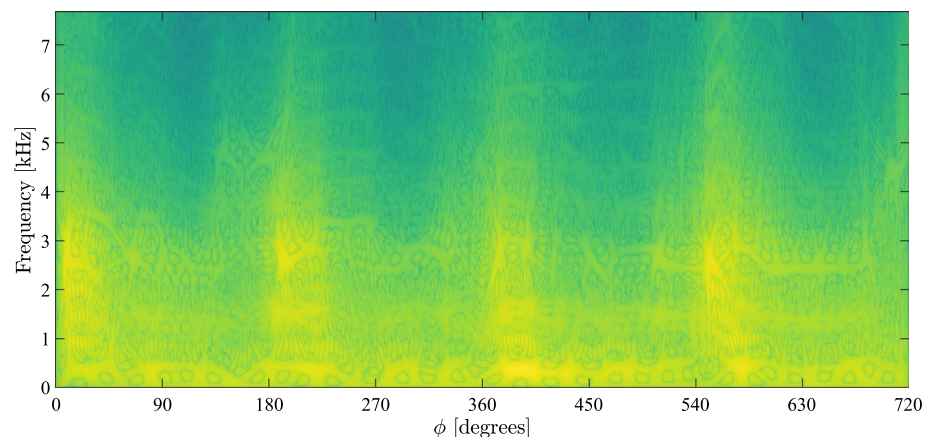


Fig. 12 Averaged smoothed pseudo-WVS $\log_{10}(W_{xx}(\phi, f))$ computed over the signal resampled in the angular domain synchronously averaged on 189 cycles



the RSS/SSS obtained was equal to 11.30%. This does not exclude, however, the possibility that an even more parsimonious FS-TARMA may be identified with RSS/SSS lower than that obtained here. Also, as expected, the success of the estimated model depends on the tachometer signal quality or, more generally, on the angular displacement estimation. Thus, poor tachometer signal features may result in problems in selecting the orders or may compromise the model's statistical characteristics. Some preliminary tests, not included in this work, were affected by a weak tachometer signal. To avoid that, the first 1000 samples of the signals were dropped due to missing tachometer pulses at the beginning of the acquisition.

To discuss the potential of the AT-FS-ARMA for vibration analysis, a comparison between parametric and non-parametric angle–frequency distribution is performed. Since STFT-based methods would present time–frequency resolution limitations due to the uncertainty principle, the smoothed pseudo-WVD is used for the time/angle–frequency analysis.

When compared to the traditional Wigner–Ville distribution, the smoothed pseudo-WVD has reduced occurrence

cross-terms when analyzing multicomponent signals, at the cost of loss on its time–frequency resolution and on its marginal properties [15, 16].

Angular resampling is performed using the raw vibration signal on its original sampling frequency and length. Then, its angular–synchronous average is calculated and the smoothed pseudo-WVD is computed from it.

The smoothed pseudo-WVD of a signal $x(\phi)$ in its continuous form is defined as [6],

$$W_{xx}(\phi, f) = \int_{-\infty}^{+\infty} g(\phi)H(f)x\left(\phi + \frac{\tau}{2}\right)x^*\left(\phi - \frac{\tau}{2}\right) \exp^{-i2\pi f\tau} d\tau \quad (9)$$

where $g(\phi)$ and $H(f)$ are independent smoothing windows in angle and frequency, respectively.

Figure 11 shows the obtained smoothed pseudo-WVD computed over the angular–synchronous averaged signal over 189 cycles. The frequency axis is limited to 7680 Hz for a direct comparison with the parametric spectra that will be presented below. The highest peak in amplitude is related to the combustion phenomena in the cylinder, which is closer to the accelerometer's position. The

Fig. 13 “Frozen” angle–frequency distribution for the fifth engine cycle. Angular scale is mapped between 0° and 720°

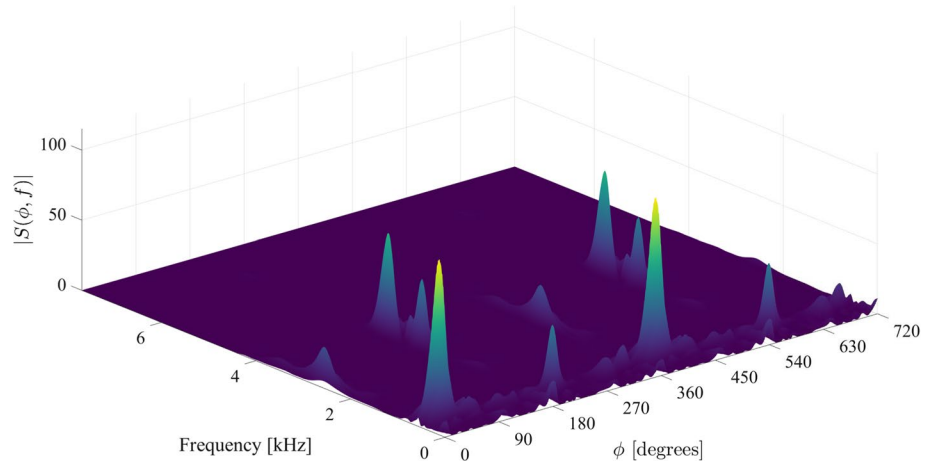
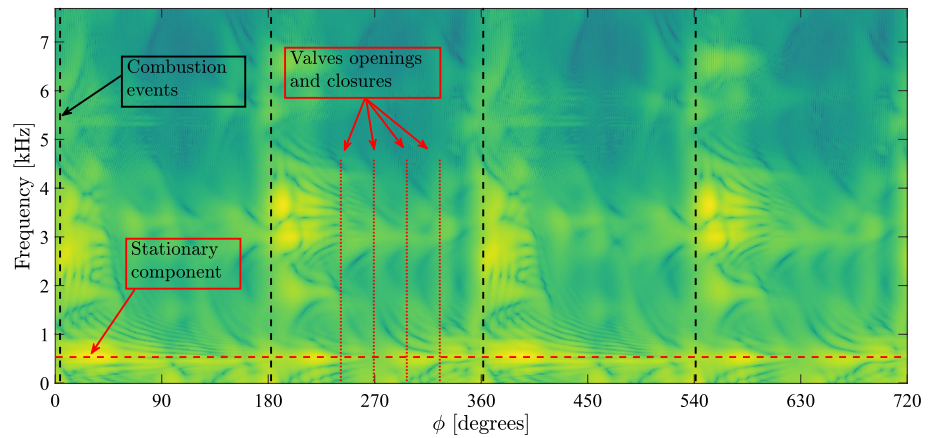


Fig. 14 “Frozen” log-scaled angle–frequency spectrum for the fifth engine cycle. Angular scale is mapped 0° and 720°. Events related to the engine’s kinematics are indicated in the figure (color figure online)



obtained spectrum is represented on the logarithm scale in Fig. 12, where it is possible to verify the success of the four combustion events that excite the system over a wide frequency range. Also, a stationary component around 570 Hz is noticeable which may be related to the gearbox of the vehicle. Finally, transients are detected around 3 kHz after each of the combustion events. Those transients may be related to the openings and closures of the inlet and exhaust valves.

The “frozen” configuration angle/time–frequency distribution is presented in Fig. 13, and is obtained running realizations of the AT-FS-ARMA(16, 3)_[25,25,25] model for angle-periodic inputs. Hence, the previously estimated trajectories $a_i(\phi_n)$, $c_i(\phi_n)$ and innovations variance $\sigma_e^2(\phi_n)$ are used to estimate the new output assuming a sinusoidal innovations sequence, modulated by an angle-periodic amplitude $\sigma_e(\phi_n) = \sigma_e(\phi_n + \Phi)$, as

$$e(t_n, f_k) = \sigma_e(\phi_n) \exp^{i2\pi f_k t_n} \tag{10}$$

for given frequencies $f_k \geq 0$. Then, an output $x(t_n, f_k)$ is computed for each frequency f_k and the “frozen” angle-/time-varying map is obtained by

$$S(\phi_n, f_k) = |x(t_n(\phi_n, f))|^2. \tag{11}$$

Figure 14 presents the “frozen” angle–frequency map. In comparison with Fig. 12, the parametric spectrum in Fig. 14 seems smoother along the angle axis. In Fig. 14, the events related to the engine cycle are evidenced, where the combustion events and the transients due to the openings/closures of the valves are evidenced by vertical lines, while the stationary component around 570 Hz is signaled by the horizontal red dashed line (Fig. 13).

One advantage of the parametric spectra is that they are free from cross-terms. Hence, they may be used as a helpful complementary tool for identification and characterization of vibrations of rotating and reciprocating machines (Fig. 14).

4 Summary and conclusions

Time-varying parametric models have been a topic of research in recent decades. Some of those methods are DPE-based, where the parameters of the models are expanded in terms of projection coefficients that vary, respectively, to a deterministic FS, which is chosen a priori according to the nature of the signal. The FS-TARMA models have been successfully applied to vibration analysis, and the Fourier-based (or trigonometric) basis is often used to model vibration signals. In the vibration analysis field, the evolution of this FS is often in terms of the time variable. However, their applications in rotating and reciprocating machinery are still not typical. Those machines frequently perform angle-/time-dependent dynamics, and their signals have angle-periodic data, i.e., angle-dependent cyclostationarity. To overcome these drawbacks, an angle-/time-varying FS-ARMA model was proposed in this paper.

The method attempts to couple the advantages of parametric methods of identification with concepts from the cyclostationary framework. The formulation was based on the well-known FS-TARMA models, and an expansion basis based on the angle-dependent Fourier series. The method was illustrated by an application using experimental data from an diesel ICE operating at a constant speed. The method demands both vibration and angular position information, which may be acquired by a tachometer. The analysis was performed through the estimation of parametric angle–frequency spectra and distributions and comparing those to the obtained ones through smoothed-pseudo-WVS/WVD. The AT-FS-ARMA models may be used as complementary tools for analysis of reciprocating and/or rotating machines. However, further investigations are required to assure the capacity of the proposed model when modeling signals with time-varying speed but where the cyclostationarity is maintained.

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Compliance with ethical standards

Conflict of interest The authors declares that there is no conflict of interests regarding the publication of this article.

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