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# Rainflow-counting matrix interpolation over different operating conditions for hydroelectric turbine fatigue assessment

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## Abstract

The measurement of stress/strain on blade plays an essential role in the fatigue assessment of the hydroelectric turbine runner. However, a typical measurement campaign does not cover all the possible operating conditions due to the field measurement requirements and limitations, which leads to fatigue evaluation missing for unmeasured conditions. In this paper, kriging interpolation method is used to estimate the missing fatigue information based on the available measured operating conditions of the turbine. The rainflow counting matrix, which contains the fatigue cycles of strain signal, is considered as the input of the interpolation process. For validation, the fatigue cycles are randomly generated, using bivariate Gaussian distributions, from the interpolated matrix for comparison with the experimental values under similar operating conditions. The developed approach will help plan experimental campaign and might contribute to decreasing the measurement requirements for runner fatigue assessment.

**Keywords:** Hydroelectric turbine; Kriging interpolation; Runner fatigue assessment; Rainflow counting

## Nomenclature

$B$	Bin value of ARM histogram [probability]	ARM	Asymmetric Rainflow Matrix
$f$	From value of a strain cycle [ $\mu$ strain]	BEP	Best Efficiency Point
$t$	To value of a strain cycle [ $\mu$ strain]	BGD	Bivariate Gaussian Distribution
$o$	Operating condition [%OV]	CFD	Computational Fluid Dynamic
$p$	Bin position ( $p(f, v)$ )	DC	Decreasing Cycle
$n_o$	Number of experimental operating conditions	DPL	Deep Part-Load
$\vartheta_i$	Set of bin position $p_i$ and its neighbours	ES	Experimental Semivariogram
$\lambda_{kl}$	Regression weight	FEA	Finite Element Analysis
$\sigma_e^2$	Error variance	FPC	Four-Points Counting
$\gamma$	Semivariogram value	HCF	High Cycle Fatigue
$\hat{\gamma}$	Experimental Semivariogram	IC	Increasing Cycle
$h$	Histogram Space Increment	MAE	Mean Absolute Error
$u$	Operating Conditions Increment	RMSE	Root Mean Square Error
$v(h, u)$	Set of paired points	SNL	Speed No-Load
$N(h, u)$	Cardinal of $v(h, u)$	STK	Spatio-Temporal Kriging
$L(\cdot)$	Lagrange function	%OV	Guide Vane Opening Level
$\mu$	Lagrange multiplier	$\kappa$	Anisotropy correction
$\gamma_{Smetric}$	Sum-metric semivariogram model	$\gamma_o$	Semivariogram model for variable
$\gamma_{joint}$	Joint semivariogram	$\gamma_p$	$o$ Semivariogram model for variable
			$p$

## 1. Introduction

Fatigue is one of the damage mechanisms occurring during the operation of a hydroelectric turbine. Today, runner blades are experiencing increasing dynamic strains due to larger operating range of turbines caused by electrical network usage changes [1]. These changes mean that the turbine must

34 operate under conditions that deviate significantly from the Best Efficiency Point (BEP), which leads  
35 to a higher risk of fatigue crack. Monette et al. (2016) highlighted the reduced fatigue life of turbine  
36 runner and the high repair costs generated by this enlarged operating range [1]. A general overview of  
37 conditions and dynamic phenomena that significantly influence runner fatigue is presented by Liu et  
38 al. (2016), Seidel et al. (2014) and Gagnon et al. (2014) [2-4]. Steady state operating conditions  
39 containing a high stochastic component such as Speed No-Load (SNL) or Deep Part-Load (DPL) or  
40 Part load containing vortex rope phenomenon, and transient conditions such as turbine start-up can  
41 have major impact on runner fatigue assessment if not properly accounted for.

42 Fatigue is assessed based on several factors (runner material, stress level, initial crack), and  
43 requires a representative strain/stress history. Many approaches are available for improving runner  
44 fatigue life estimation. To limit fatigue failure, Gagnon et al. (2013) proposed a runner reliability  
45 model using the High Cycle Fatigue (HCF) onset as the limit [5]. Fatigue damage is linked to the  
46 propagation of cracks on the runner, and thus, an investigation of the metallurgical or mechanical  
47 behaviour of the material is also needed [6-9]. However, fatigue assessment cannot be carried out if  
48 the dynamic load information is missing. Computational Fluid Dynamic (CFD) combined with Finite  
49 Element Analysis (FEA) can be used to estimate the static and dynamic behaviour of the structure  
50 under such as SNL or DPL [10-12]. Nevertheless, numerical prediction is still challenging when  
51 turbulence models are not able to properly describe the behaviour of the turbine [13]. Moreover, the  
52 validation of numerical simulation depends on the availability of the experimental information.  
53 However, to recover this missing experimental information, other approaches, such as the  
54 interpolation and extrapolation of experimental data, can be used [14-16]. The use of indirect method  
55 can also help minimize the measurement requirement, Diagne et al. (2016) utilised the ARMAX  
56 (Autoregressive Moving Average with eXogenous input) model to predict the runner dynamic stress  
57 using shaft torsion information, which is more readily available [17]. For a detailed review of the  
58 fatigue life estimation methodologies used for Francis turbines, please refer to Presas et al. (2019)  
59 [13]. Our state-of-art summary shows that in situ field strains/stress during or after commissioning  
60 provide important and essential information for the study of the hydroelectric turbine runner fatigue.

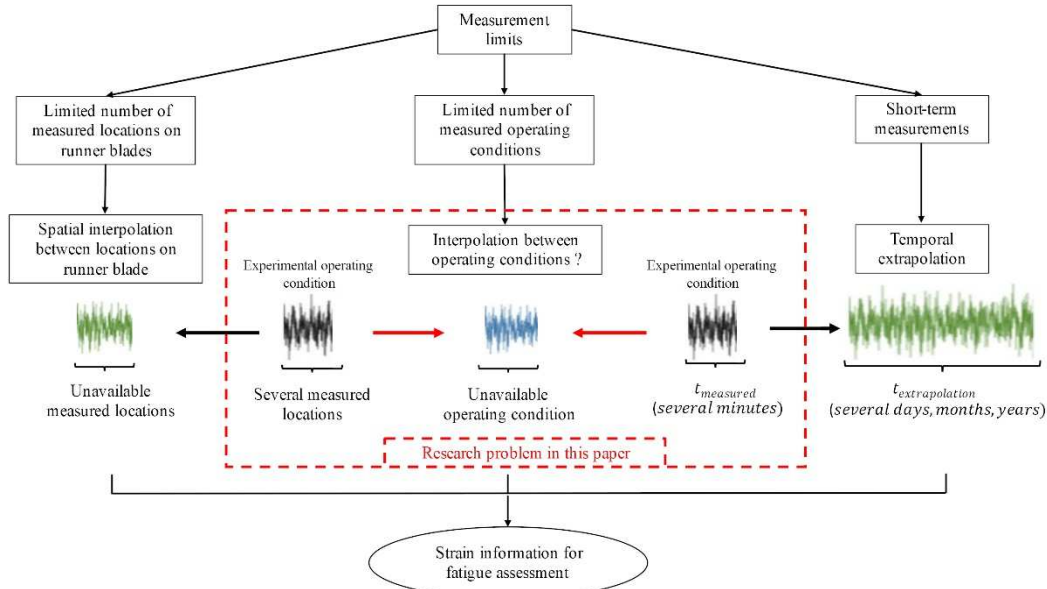
61 Finding a solution that can provide missing experimental information and improve knowledge on  
62 the dynamic behaviour remains a challenge. Obtaining experimental strains over all the possible states  
63 of the hydroelectric turbine operation would be overly costly and time-consuming; hence,  
64 experimental measurements are usually limited to a limited set of operating conditions, which leads to  
65 difficulties during runner fatigue assessment. The goal of this paper is to propose a methodology to  
66 minimize the requirements for experimental strain measurements. There are three difficulties  
67 associated with the measurement of strain for fatigue evaluation:

- 68 • Limited number of measured locations on runner blades
- 69 • Limited measurement length
- 70 • Limited number of measured operating conditions

71 The first was studied by Salah where the kriging method was used for the interpolation between  
72 different locations of the runner blade [14]. For the second, several approaches can be used for  
73 temporal extrapolation to obtain long-term fatigue assessments. Poirier et al. (2016) [15] extrapolated  
74 information on the cyclostationary decompositions of the short-term records of strain signals to  
75 evaluate runner fatigue cycles over a longer period of usage. Other researchers extrapolated the  
76 rainflow matrices directly over longer periods [19-22]. However, such strain interpolation between  
77 blade locations or a temporal extrapolation can be performed if and only if an estimate of the signal is  
78 available (generally in the form of measurements) at a given turbine operating condition. Therefore,  
79 the lack of measurements over the complete operating range should be studied as shown in *Figure 1*.

80 In a previous work the authors developed an interpolation process for a subcomponent of the strain  
81 signal between different turbine operating conditions [16]. Only the periodic component of the signal  
82 was interpolated while the complete strain signal was overlooked as it is more complex and contains  
83 other hidden structures. The direct interpolation of complete strain signal (measurement form) is a  
84 complicated problem because of these hidden structures. However, in cases where only fatigue  
85 information is needed for further evaluation, the strain signal could be transformed into a cycle  
86 counting form (level crossing, peak counting, rainflow matrices, etc.) to simplify the interpolation.  
87 Among cycle counting methods, the rainflow algorithm is commonly used for fatigue assessment of

88 hydroelectric turbines [1, 10-11, 15-16]. Therefore, our goal is to interpolate the rainflow counting  
 89 matrices (instead of the signal itself), which contain information on the fatigue cycles, over the  
 90 complete range of operating conditions. Notice that there are two important variables that define the  
 91 operating conditions of turbine: opening vane level and head level. However, in this study, the aim is  
 92 to focus on the differences of stress level between different opening vane levels for the same head (the  
 93 data in the case study also comes from the measurements strain at relatively constant head). The head  
 94 level might be used in the future research if observed data become available for multiple head levels.  
 95



96  
 97 *Figure 1. Measurement limits and interpolation/extrapolation solutions for the fatigue assessment of hydroelectric turbine*  
 98 *runner*

99 The kriging method was chosen for this study to interpolate rainflow counting matrices (histogram  
 100 form), Then, the fatigue cycle reconstruction was generated via stochastic simulations using bivariate  
 101 Gaussian distributions. This reconstruction step helps recover the long-term information of fatigue,  
 102 more cycles are generated, more fatigue information is evaluated. The obtained results were compared  
 103 with the experimental fatigue cycles to evaluate the correctness of the proposed interpolation  
 104 methodology. In addition, the signal was reconstructed from the simulated cycles by using the inverse  
 105 method proposed in [19-20] to compare with the measured signal.

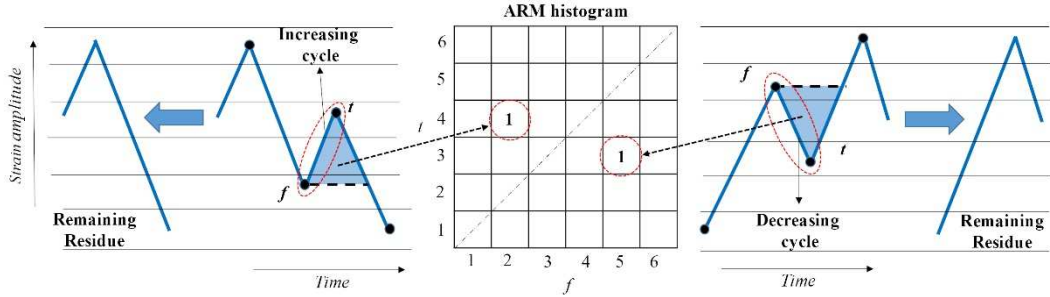
106 The paper is organized as follows: First, the methodology is presented with detailed explanations  
 107 of the principal methods (rainflow-counting, kriging interpolation, reconstruction simulation). Then, a  
 108 hydroelectric turbine runner case study is presented followed by interpolation results and a discussion.  
 109

## 110 2. Proposed Methodology

### 111 2.1 Asymmetric Rainflow Counting Matrices

112 The rainflow algorithm allows the analysis of signal amplitudes (or range) by assigning extreme (max,  
 113 min) values of closed loading cycles. To evaluate fatigue, only the range/amplitude information of  
 114 each cycle is often needed. In this study, the strain signal is transformed into an Asymmetric  
 115 Rainflow Matrix (ARM), which separates the cycles into two types: increasing and decreasing cycles  
 116 (or standing and hanging cycles in some references) [24]. These two types of cycles are defined based  
 117 on the order of occurrence of maximum and minimum values in the signal. To construct the ARM, a  
 118 rainflow-counting procedure called the Four-Point Counting (FPC) algorithm is used [25].  
 119 Considering four consecutive points (three consecutive cycles) in the loading signal, if the cycle range  
 120 created by the second and the third points is smaller than the range of the two other points, this cycle  
 121 is reduced and is added to the ARM. An ARM is constructed by two variables: from value (noted as  $f$ )  
 122 corresponding to the amplitude of the first point of the cycle (the first point of the cycle arrives on the  
 123 axis of time) and to value (noted as  $t$ ) corresponding to the amplitude of the final point of the cycle  
 124 (*Figure 2*). In our study, the created ARM is represented under histogram density form where each  
 125 bin contains the number of cycles or the probability of occurrence of cycles. The ARM is a

126 composition of max-min cycles (above the matrix diagonal) and min-max cycles (below the matrix diagonal).  
 127 Furthermore, the choice of the asymmetric rainflow matrix allows a distinction to be made  
 128 between increasing and decreasing cycles during the signal reconstruction. To properly apply the  
 129 FPC, the turning points, which represent the peak and the valley of the signal, are extracted before the  
 130 ARM is constructed. This extraction deletes variations that are neither peaks nor valleys (these small  
 131 variations in the rainflow approach are considered to have minor impact on the fatigue assessment).



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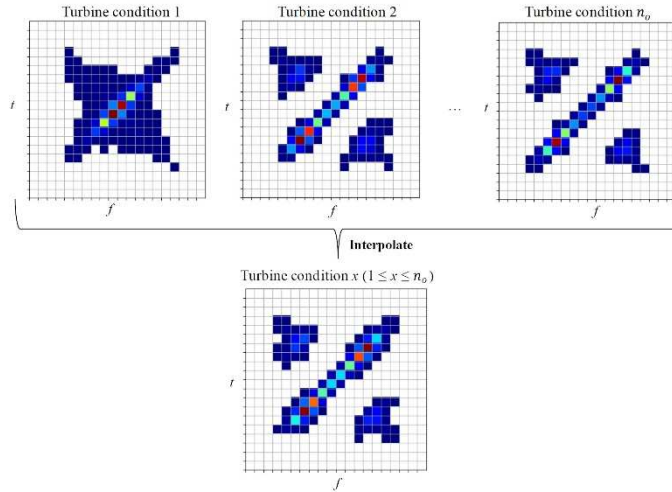
Figure 2. Four Point Counting algorithm applied for strain signal.

134 **2.2 Kriging Interpolation Method**

135 Before presenting the interpolation method, the context must be clarified. The interpolation input  
 136 is a set of ARM histograms, which are spread across the different turbine operating conditions. The  
 137 histogram bins are fixed for all the histograms. The goal is to estimate new ARM histograms at any  
 138 given operating conditions (see Figure 3).

139 The relation between the bins of each ARM matrix is an important bit of information that cannot  
 140 be ignored in the interpolation phase. Hence, the kriging interpolation method is a suitable method to  
 141 preserve the spatial covariance within the ARM histogram. However, there is also a covariance to  
 142 preserve between each operating condition. Thus, a multivariate kriging method, inspired by Spatio-  
 143 Temporal Kriging (STK), where the temporal and spatial coordinates are treated separately, is  
 144 applied, and developed for this study. Readers can consult the general definition of the kriging  
 145 principle in [27-30].

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Figure 3. Interpolation objective: new ARM histogram (at a given turbine operating condition) is interpolated from all experimental ARM histograms

150 In our case, the position of bins in the ARM histogram and the operating condition are considered  
 151 as coordinates. The estimator used in this study can be written as:

$$B^*(p_i, o_x) = \sum_{k \in \mathcal{D}_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) B(p_k, o_l) \quad (1)$$

152 where  $p_i$  is a given bin position in the ARM histogram ( $p$  depends on a 'from value'  $f_i$  and a 'to  
 153 value'  $t_i : p_i(f_i, t_i)$ ,  $o_x$  is a given operating condition of the turbine,  $B^*(p_i, o_x)$  is the interpolated

154 value of bin  $p_i$  at the condition  $o_x$ ,  $B(p_k, o_l)$  is the known bin value at location  $(p_k, o_l)$ ,  $\lambda_{kl}(p_i, o_x)$  is  
 155 the regression weight to optimize during the interpolation process (details below),  $n_o$  is the number of  
 156 experimental turbine conditions,  $\vartheta_i$  is a set of neighbourhood positions of  $p_i$  (including  $p_i$ ). It must  
 157 be noted that the spatial covariance map is obtained from the whole histograms of all the experimental  
 158 turbine conditions, while Eq. 1 searches for the input in the neighbourhood  $\vartheta_i$  of the target bin. This  
 159 reduces the calculation time and avoids the interpolation smoothing effect.

160 Kriging allows the unbiased estimation of values at given locations by determining the weights  
 161  $\lambda_{kl}$  which minimize the error variance  $\sigma_e^2(p_i, o_x)$  between the real value  $B$  and the interpolated value  
 162  $B^*$  at the same location,  $\sigma_e^2(p_i, o_x) = Var(B(p_i, o_x) - B^*(p_i, o_x))$ . The condition for non-bias  
 163 relating to the regression weights is  $\sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) = 1$ ; it ensures a constant local mean value  
 164 in every location. Therefore, to impose this non bias constraint, the minimization system is updated by  
 165 adding a Lagrange parameter  $\mu$ :

$$L(\lambda_{kl}, \mu) = \frac{Var(B(p_i, o_x) - B^*(p_i, o_x))}{\sigma_e^2(p_i, o_x)} + 2\mu \left( \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) - 1 \right) \quad (2)$$

166 where  $L(\lambda_i, \mu)$  is the Lagrange function. Making use of Eq.1, the minimization of Eq.2 leads to the  
 167 system that allows to obtain the  $\lambda_{kl}$  and  $\mu$ :

$$\begin{cases} \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) \gamma(h_{\|p_k - p_j\|}, u_{|o_l - o_m|}) - \mu = \gamma(h_{\|p_i - p_j\|}, u_{|o_x - o_m|}), & j \in \vartheta_i, m = 1, \dots, n_o \\ \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) = 1 \end{cases} \quad (3)$$

168 where

$$\gamma(h_{\|p_a - p_c\|}, u_{|o_b - o_d|}) = 0.5 \times Var[B(p_a, o_b) - B(p_c, o_d)] \quad (4)$$

169 is the semivariogram, which depends on the increments  $h$  and  $u$ ,  $p_a$  and  $p_c$  are two positions of the  
 170 bin in the histogram,  $o_b$  and  $o_d$  are two turbine operating conditions such that:  $h = \|p_a - p_c\|$  and  
 171  $u = |o_b - o_d|$ . Based on Eq. 4, the semivariogram is defined as the half variance between two values  
 172 separated by the increments  $h$  and  $u$ . This semivariogram can be considered as a simpler  
 173 representative of the covariance map that does not require information on the mean value of  $B$  (that is  
 174 often unknown in practice [29]). To obtain the semivariogram at any possible increment  $(h, u)$ , some  
 175 numerical models can be used to fit to the semivariogram using analysis estimations (e.g., Least  
 176 Squares method) [27][30]. However, to choose the best numerical model, one needs to create a  
 177 semivariogram from the experimental data. In practice, the quantity of data is not enough to provide  
 178 values exactly separated by  $h = \|p_a - p_c\|$  and  $u = |o_b - o_d|$ , as in Eq. 4. Consequently, to create  
 179 the Experimental Semivariogram (ES), Eq. 5 is used. This equation allows an estimation of the  
 180 semivariogram value associated with a set of  $(h, u)$  values, where  $h$  and  $u$  are midpoints of imposed  
 181 spatial increment intervals:

$$\hat{\gamma}(h, u) = \frac{1}{2N(h, u)} \sum_{(a,b,c,d) \in v(h,u)} [B(p_a, o_b) - B(p_c, o_d)]^2 \quad (5)$$

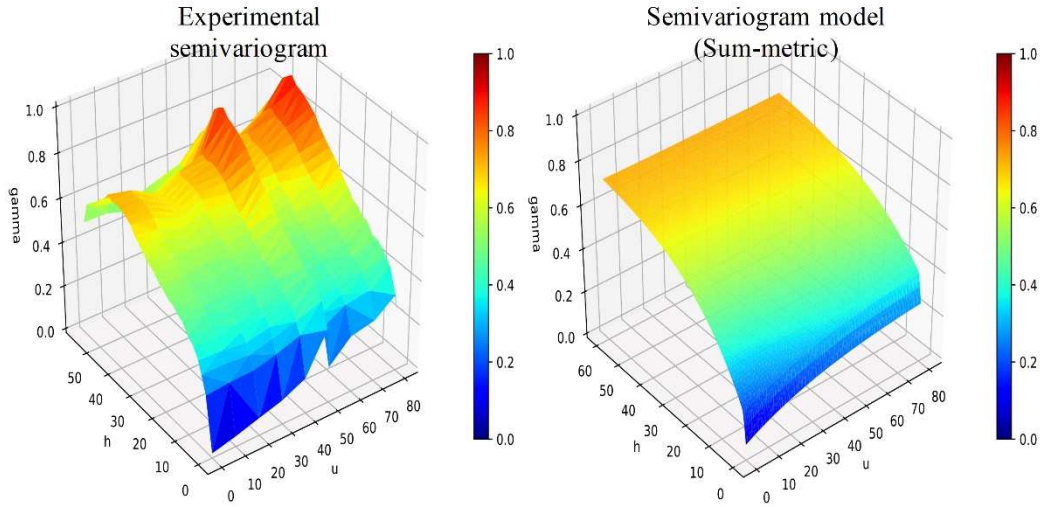
182 where  $\hat{\gamma}(h, u)$  is the experimental semivariogram,  $v(h, u)$  is a set of paired points separated such that  
 183  $\|p_a - p_c\| \in [h - dh, h + dh]$  and  $|o_b - o_d| \in [u - du, u + du]$  with  $dh > 0$ ,  $du > 0$ ,  $N(h, u)$  is  
 184 cardinal of  $v(h, u)$ . The next step is to fit the ES with a numerical model that will make a continuity  
 185 throughout the increments for the semivariogram. Classes of semivariogram models are presented in  
 186 [29-30]. For example, the sum-metric model (Figure 4), which turns out as one of the best fitting  
 187 models in our study, is expressed as:

$$\gamma_{Smetric}(h, u) = \gamma_p(h) + \gamma_o(u) + \gamma_{joint}(\sqrt{h^2 + (\kappa \cdot u)^2}) \quad (6)$$

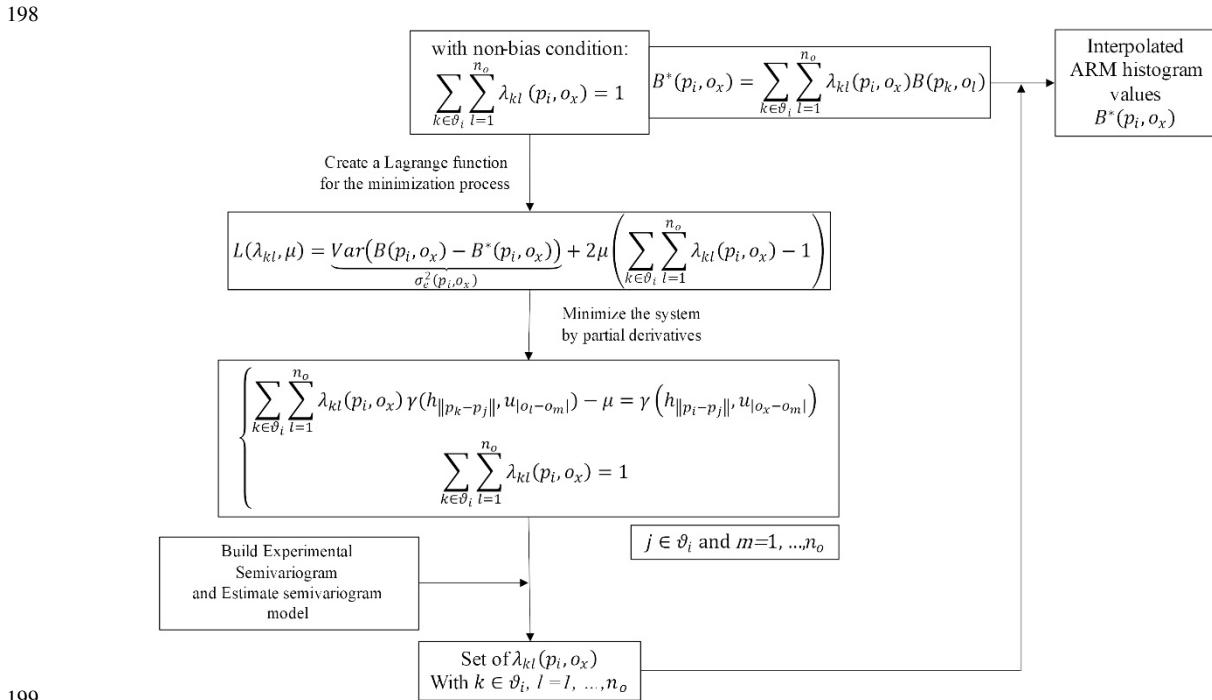
188 where  $\gamma_p(h)$  is the univariate semivariogram model for the variable  $p$ ,  $\gamma_o(u)$  is the univariate  
 189 semivariogram for the variable  $o$ , and  $\gamma_{joint}(\sqrt{h^2 + (\kappa \cdot u)^2})$  is a metric joint variogram model  
 190 including an anisotropy correction  $\kappa$  [30]. Many basic analytical models exist for the univariate  
 191 semivariograms such as spherical model, the exponential model, the Gaussian model, power model,



192 etc. [14][27]. The schema in *Figure 5* summarizes the kriging interpolation approach used in our case  
 193 study.  
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 196 *Figure 4. Experimental semivariogram and example of semivariogram model (Sum-metric). Semivariogram values in this*  
 197 *figure are normalized.*

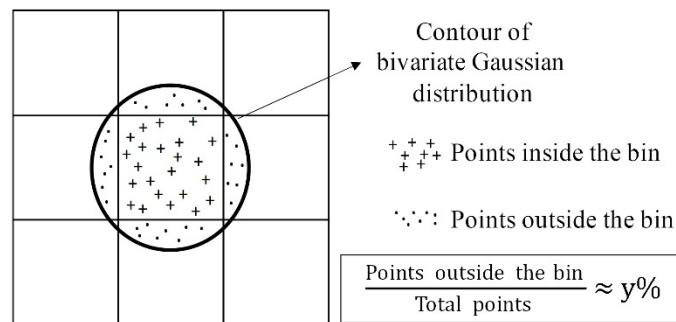


199  
 200 *Figure 5. Schema of the Kriging process in this paper applied for the studied variables*

### 201 2.3 Simulation Reconstruction

202 To generate strain cycles from an interpolated ARM histogram, it must be converted to a  
 203 continuous distribution. A smoothing process for the interpolated histogram is proposed by  
 204 considering that the cycles inside each bin follow a Bivariate Gaussian Distribution (BGD). The BGD  
 205 mean corresponds to the coordinates of the center point of the bin and the proposed covariance matrix  
 206 is  $\sigma^2 \mathbf{I}_2$ , where  $\sigma = \frac{0.5 \times \text{bin diagonal}}{\text{variability parameter}}$  and  $\mathbf{I}_2$  is the identity matrix. The variability parameter in the  
 207 denominator is a coefficient proposed to ensure that the simulated cycle distribution does not create  
 208 too much variability outside its bin. A variability test is proposed to select this variability parameter.  
 209 In this study, since the interpolation input comes from the experimental strains measured by gauges,  
 210 we impose the strain gauge uncertainty as a limited level of variability, which limits the variability of

211 generated point outside each bin. For example, if the relative error of the strain gauge is  $\pm y\%$ , the  
 212 imposed limited level for the variability will be  $y\%$ . It must be noted that the experimental  
 213 uncertainties are not the focused problem of this study. The test of variability is then launched for  
 214 several variability parameters. For each ARM, Monte Carlo simulation is generated from the bivariate  
 215 normal distribution with the mean  $[0,0]$  and the covariance  $\sigma^2 \mathbf{I}_2$ . If the percentage of the sample  
 216 values outside the bin dimension is approximately equals to the limited level  $y\%$  (Figure 6), this  
 217 variability parameter is chosen for the ARM histogram smoothing process. The reader can also  
 218 optimize the variability parameter by using or adding other uncertainty types (e.g., error of kriging  
 219 interpolation) if needed for other study. In addition, other approaches, such as the Kernel Density  
 220 Estimation (KDE), can also be useful for smoothing the histogram [19-20]. Finally, the cycles are  
 221 randomly generated from the set of BGDs by respecting the probability of occurrence of cycles in  
 222 each bin (the sum of all the probability values in the histogram is equal to 1). This generation process  
 223 allows an observation of a set of simulations, which is useful for the industry when evaluating the  
 224 fatigue. The risk of fatigue will be predicted with a set of possibilities instead of a unique prediction.  
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Figure 6. Percentage of the sample values outside the bin dimension is approximately equal to  $y\%$

228 In addition, an inverse method is used to reconstruct the strain signal from the simulated samples  
 229 [19-20]. The signal reconstruction consists of two steps: First, the largest cycles (the furthest points  
 230 from the ARM diagonal), either a Decreasing Cycle (DC) or an Increasing Cycle (IC), are used to  
 231 create an initial reconstructed signal (also called the *remaining residue* in Figure 2). Second, all the  
 232 other cycles are randomly inserted into the remaining residue descending order (from the furthest to  
 233 the closet point from the diagonal) by respecting the rule proposed by Socie et al. [19-20]. The smaller  
 234 IC is added to the position of the larger DC (peak and valley values of the smaller cycle fall within the  
 235 range of the larger cycle) and vice versa. Each cycle added to the remaining residue is removed in the  
 236 ARM until there are no cycles left. It is not expected that the reconstructed signal will be similar to  
 237 the experimental one because only the rainflow information is considered for the interpolation;  
 238 nevertheless, the signal should represent a time history that generates equivalent fatigue damage. The  
 239 flowchart in Figure 7 illustrates the interpolation scheme used in this study.  
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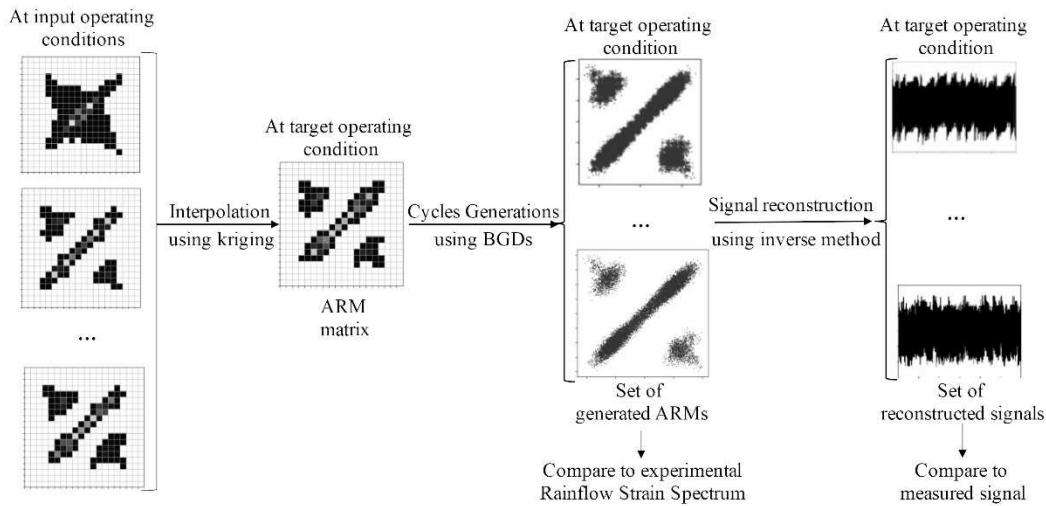


Figure 7. Flowchart displays the interpolation scheme

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### 3. Case Study

The data used in this case study were obtained from experimental measurements conducted using strain gauges, which were installed on the runner blade of a Francis hydroelectric turbine of a power plant in Quebec (Canada). The measured locations on the blade are close to critical locations where the highest stress levels are found. For the purpose of this paper, the experimental uncertainties (ambiance error, human error, error of installation, etc.) are ignored. The interpolation is performed between different

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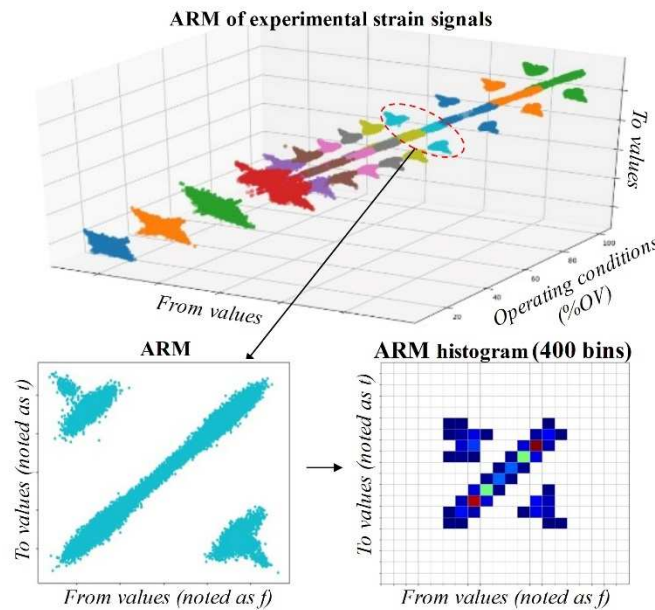


Figure 8. ARMs over different operating conditions for this case study and example of ARM histogram (the white bin corresponds to the null value)

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steady-operating conditions of the turbine. Transient operating conditions, such as start-ups or stops, are not considered in this research because their specific dynamic behaviours do not correlate with steady-operating conditions. A steady-operating condition is approximately defined as a regime of the turbine under which the guide vane opening, power and turbine head can be considered as quasi constant. For the interpolation, steady states are separated by different opening levels of the turbine guide vane (noted as %OV) (Figure 8). The guide vanes control the flow rate into the runner.

Due to the high quantity of cycles in each strain signal, the ARMs are presented under the histogram density form with a prior fixed bin number where each bin contains the probability of occurrence of cycles (Figure 8). The choice of input/output bin number is an important parameter. A higher number of bins significantly increases the calculation time, while a smaller one leads to

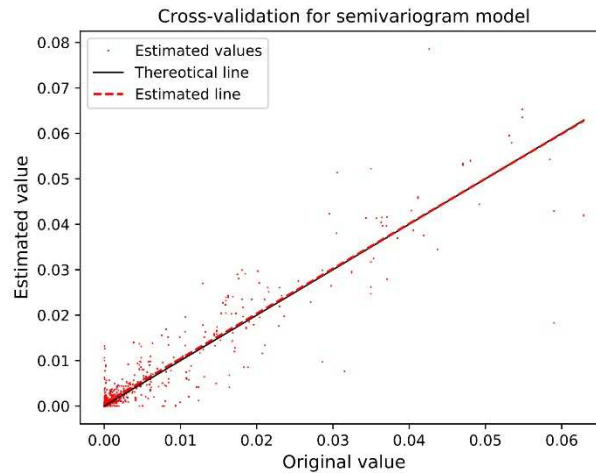
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263 imprecise results because of the large interval of values covered by each bin.

264 Fitting a model to the experimental semivariogram is an important step of the interpolation  
 265 process. We highly recommend using the cross-validation technique for this step [29]. This technique  
 266 removes some point values in the input set and then predicts these values from the remaining points  
 267 using fitted semivariogram models. This procedure is repeated several times for each semivariogram  
 268 model to observe the interpolation performance of this model. Moreover, other error estimations such  
 269 as Root Mean Square Error (RMSE) or Mean Absolute Error (MAE) are additionally used to obtain  
 270 the best semivariogram model. In our case study, based on a good cross-validation performance  
 271 (Figure 9) and on the small error estimation values (see MAE and RMSE in Table 1), the chosen  
 272 semivariogram model was sum-metric model ( see Eq. 6 and Table 1). For each univariate model,  
 273  $\gamma_p(h)$  was modeled by sum of an exponential model and a Gaussian model,  $\gamma_o(u)$  and  
 274  $\gamma_{joint}(\sqrt{h^2 + (\kappa \cdot u)^2})$  was respectively modeled by a Gaussian and an exponential model (Table 1).

$$\begin{cases} \gamma_{exponential}(h) = (Sill - nu) \cdot \left(1 - e^{-\frac{3 \cdot h}{range}}\right) + nu \\ \gamma_{Gaussian}(u) = (Sill - nu) \cdot \left(1 - e^{-\frac{(3 \cdot u)^2}{range^2}}\right) + nu \end{cases} \quad (7)$$

275 where *Sill* corresponds to the average variance where the experimental semivariogram stabilizes,  
 276 *range* presents the distance from which no more correlations exist between the data, and *nu* is a small  
 277 value added to the sill parameter to represent a very short-range variability in the dataset (Nugget  
 278 effect) [14][27].



279  
 280 Figure 9. Cross validation performance of sum-metric model (its parameters are presented in Table 1)

281 The neighbourhood set  $\vartheta_i$  (Eq. 1) is chosen using the cross-validation technique. Several sets with  
 282 different numbers of neighbourhood positions are tested and the optimized set is chosen based on the  
 283 smallest RMSE and based on the similarity between theoretical and estimated line (see APPENDIX).  
 284 Notice that the set of neighbourhoods is applied only for the histogram position “*p*”, but not for  
 285 operating conditions “*o*”. Thus, the points in a neighbourhood set correspond to the closest points  
 286 from the target points in ARMs. In this case study, the set of 9 nearest positions is chosen for the  
 287 neighbourhood set  $\vartheta_i$  in Eq. 1 (see APPENDIX).

288 Table 1 shows the optimized parameters used for this case study. To verify the quality of the  
 289 interpolated result, a validation is made using known experimental operating conditions (not included  
 290 in the set of interpolation inputs).

291

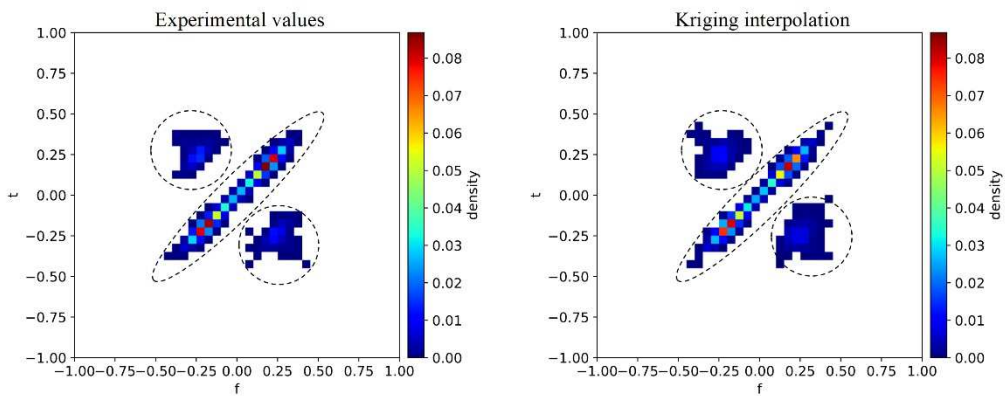
Parameters	Semivariogram model parameters (fitted to normalized ES- see Figure 4)					
Input/Output histogram bins number:	Optimized sum-metric model (RMSE = 0.0993, MAE= 0.0747)					
	Eq. 6: $\gamma_{Smetric}(h, u) = \gamma_p(h) + \gamma_o(u) + \gamma_{joint}(\sqrt{h^2 + (\kappa \cdot u)^2})$					
	Model type	<i>Sill</i>	<i>nu</i>	<i>range</i>	$\kappa$	

40 × 40 bins  Input operating conditions (%OV): 11, 20, 40, 50, 60, 70, 90, 100  Variability para. ≈ 3.87  Number of points in neighbourhood set: 9	$\gamma_p(h)$	Gaussian	0.116	3e-08	2.011	0.296
		Exponential	0.376	0.009	70.025	
	$\gamma_o(u)$	Gaussian	0.040	0.042	30.007	
		$\gamma_{joint}(\sqrt{h^2 + (\kappa \cdot u)^2})$	Exponential	0.230	7e-08	

292 *Table 1. Parameters of the proposed interpolation process adapted for the case study dataset*

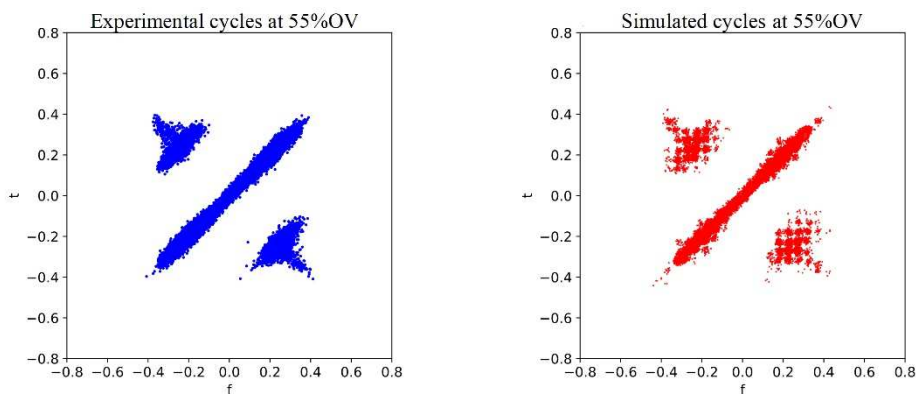
293 **4. Results and Discussions**

294 The interpolated results (corresponding to each step illustrated in *Figure 7*) and their comparison with  
 295 experimental ones at 55% OV are presented in the *Figures 10-12*. All the results are normalized to  
 296 respect the confidential information.  
 297



298 *Figure 10. Comparison between experimental and Interpolated ARM histogram at 55% OV*

300



301 *Figure 11. Experimental and generated cycles (one simulation) at 55 %OV*  
 302

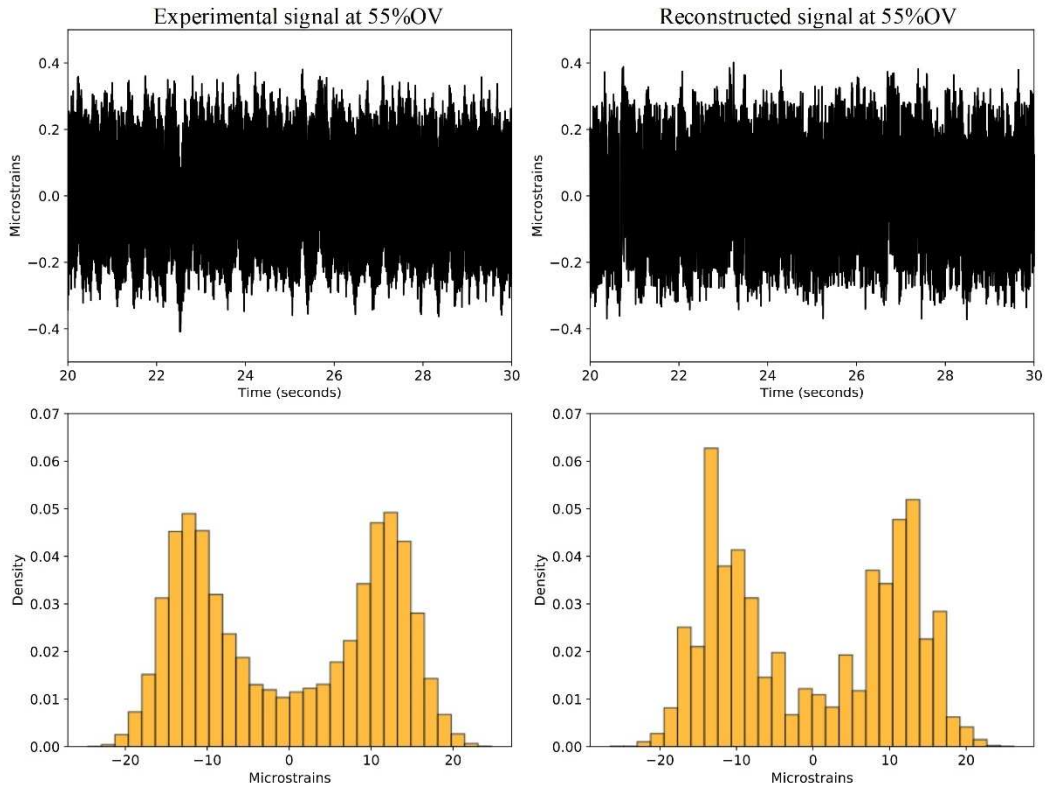


Figure 12. Comparison between experimental signal and one reconstructed signal at 55 %OV.  
Above: 10 seconds of signals. Below: Histogram of whole signal

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304  
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306 The interpolated ARM shows the same number of distribution modes (dashed line in *Figure 10*) as  
307 in the experimental ones; this means that this interpolation did not generate large overestimation in  
308 positions the null bins. Nonetheless, some differences are observed at the frontier positions of each  
309 mode in the interpolation result (*Figure 10, 11*). Graphically, the reconstructed signal (*Figure 12*) is  
310 not similar to the experimental one like we have foreseen, however its amplitudes have the same level  
311 as the experimental one. To verify the quality of interpolation process, we have compared the rainflow  
312 strain spectrums (strain range as a function of cumulative number of fatigue cycles) between 500  
313 simulations (sampled from the set of BGDs) and the experimental values at some operating conditions  
314 (*Figure 13*).

315 For fatigue assessment, the expected goal in this study is to capture the amplitude/range level of  
316 fatigue cycle (especially high range cycles which cause more damage). At 55 %OV and 65 %OV, the  
317 experimental rainflow (the blue line in *Figure 13*) is mostly within of the set of simulations while  
318 there seems to be some overestimations at the high ranges at 80 %OV. This difference might be  
319 explained by that there are more operating conditions close to the 55 %OV, 65 %OV in the  
320 interpolation input set (see *Table 1*).

321 An unusual variability is observed at 30 %OV. The interpolation result depends on the nature of  
322 known data. The more complicated the combination of phenomena hidden in the signal, the less  
323 confident we might be in the interpolation result. Therefore, at some partial load conditions of turbine,  
324 this unusual variability of the interpolated result occurs when an operating condition with a  
325 complicated dynamic behavior is presented next to the interpolation target condition. Some of those  
326 conditions contain specific fluctuation coming from the vortex in the draft tube [3], which appears and  
327 disappears over limited set of measured operating conditions rendering the interpolation difficult. This  
328 problem provides new challenge for further research, in which we would like to recover such  
329 fluctuation information.

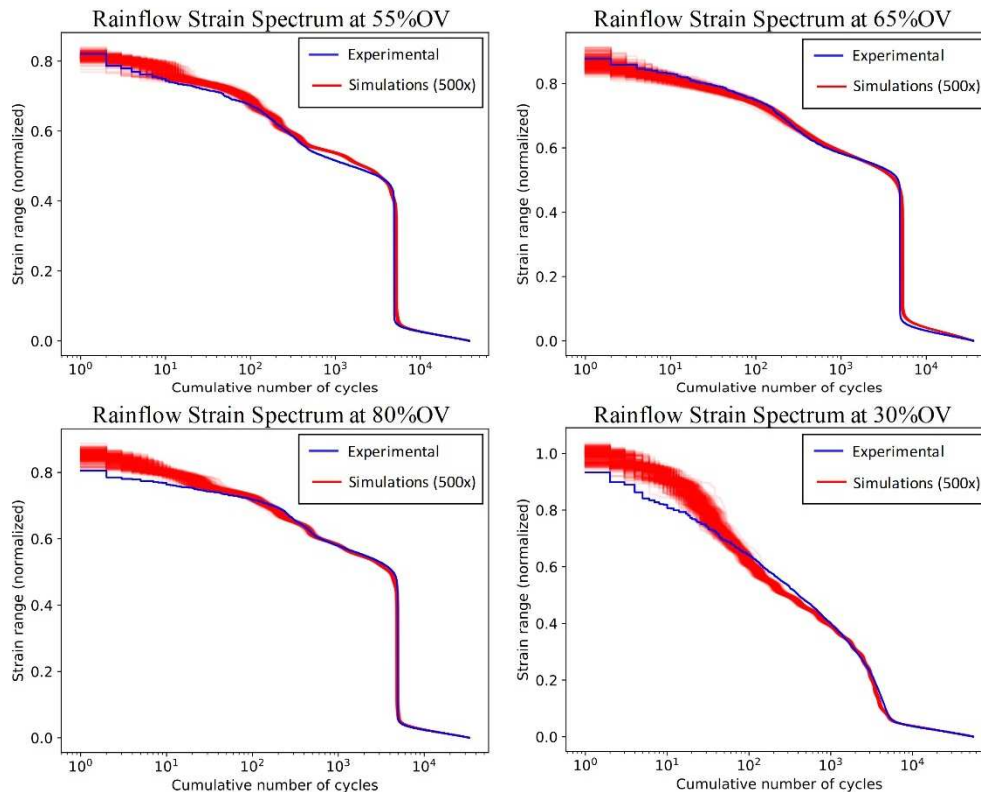


Figure 13. Rainflow strain spectrum between experimental values and simulations

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332 For the runner maintenance strategy, even if the reconstructed signal is not similar to the  
333 experimental one (Figure 12), the interpolated strain ranges might be more representative of data  
334 variability and uncertainty than reusing experimental measurements from nearby operating conditions.  
335 Also, the proposed interpolation can be a solution to reduce the measurement requirements for runner  
336 fatigue assessment.

337

### 338 5. Conclusions

339 In this article, an interpolation process using kriging has been presented to estimate the strain range at  
340 any steady state operating condition based on rainflow cycle counting for hydroelectric turbine runner  
341 fatigue assessment. A set of bivariate Gaussian distributions were used to smooth the histogram,  
342 allowing quicker simulations of the loading cycles. It must be noted that the proposed interpolation  
343 process is sensitive to the parameters used (i.e., the semivariogram model, etc.), using the error  
344 optimizing methods (such as cross-validation, Mean Absolute Error, etc.) are therefore highly  
345 recommended. The results of the case study show that even when the reconstructed signal is not  
346 similar to the experimental one, our proposed interpolation might be useful to estimate the dynamic  
347 strain level using only the load cycles information of strain signal.

348 The advantage of the proposed method is that it creates a stochastic process accounting for  
349 uncertainty hence contributing to fatigue risk assessment by generating loading cycles representative  
350 of observed behaviours. Moreover, it can help to optimize the requirements for experimental  
351 measurements.

352 In the result of this case study, the complicated phenomena hidden in strain signal caused the  
353 unusual variability at some partial load conditions. For further research, preserving other information  
354 of strain signal (not only the strain range information) during the interpolation phase will be interested  
355 to improve the quality of missing data recovery at unmeasured operating conditions.

356

### 357 Acknowledgments

358 The author would like to thank the Hydro-Québec Research Institute (IREQ), Andritz Hydro  
359 Canada LTD, INSA de Lyon (France), École de Technologie Supérieure de Montréal (Quebec,  
360 Canada) and the Mitacs Acceleration program for their support and financial contribution.

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**About 4410 words (excluding abstract, figure captions and references). Figure 10, 13 must be printed in color**

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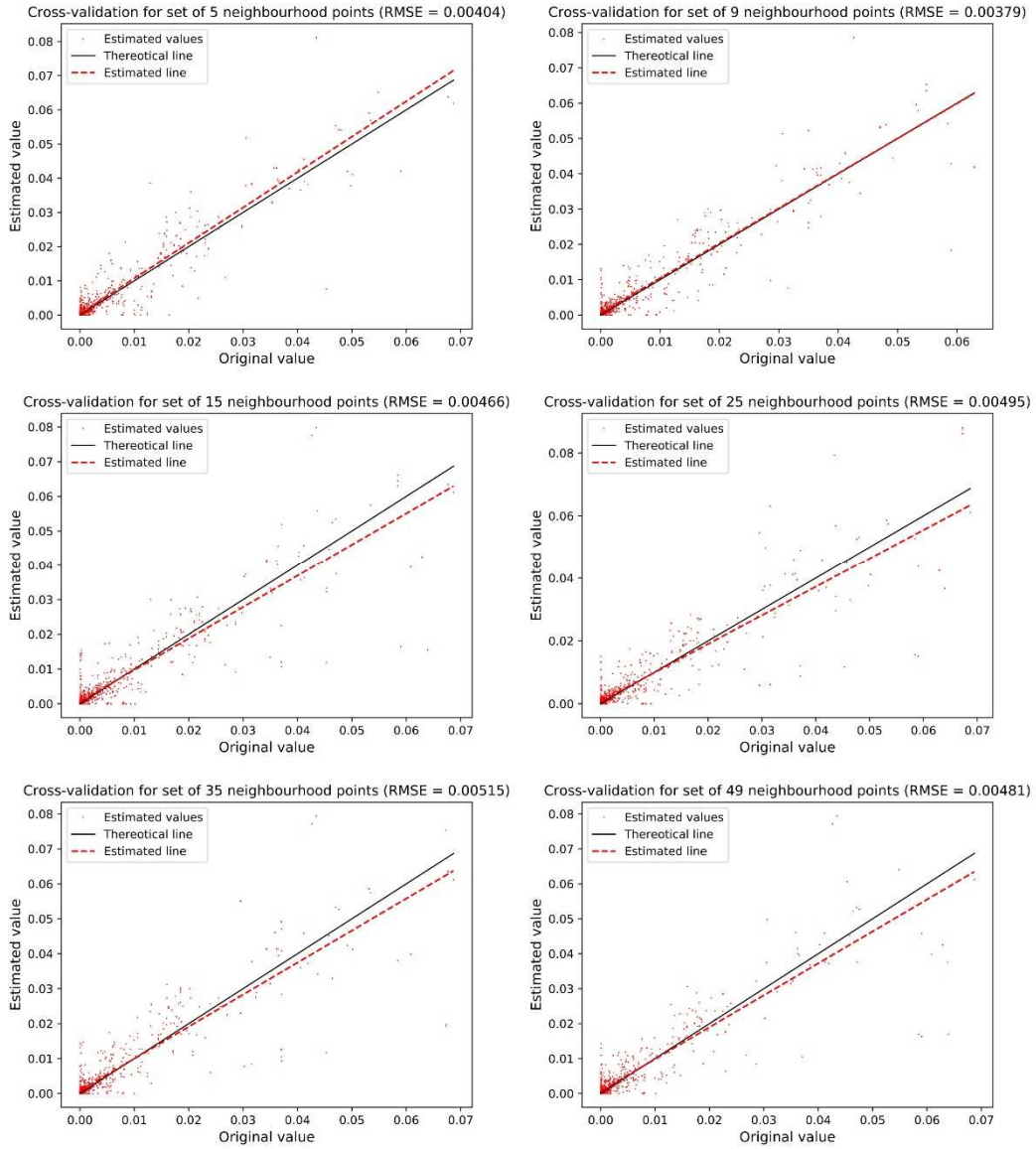


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- 446

#### 447 **Appendix: The cross-validation performances for several neighbourhood sets**

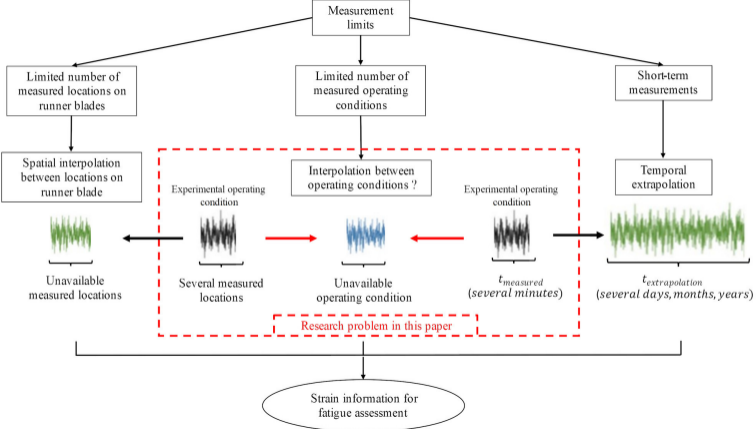
448 The cross-validation technique was performed for several neighbourhood sets (with the same  
449 semivariogram model). The optimized neighbourhood set for the interpolation is chosen based on the  
450 smallest RSME and based on the similarity between theoretical (black) and estimated line (red) in the  
451 cross-validation. In observing *Figure 14*, if one chooses a small set (the case of 5 points) or a big set  
452 (the cases of 35 or 49 points), the variation and the RMSE values are larger. In this case study, the set  
453 of 9 neighbourhood points is chosen (row 1 and column 2 in *Figure 14*).

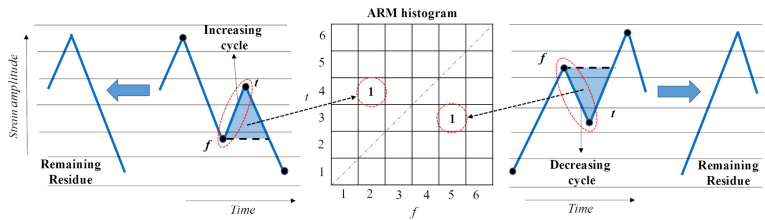
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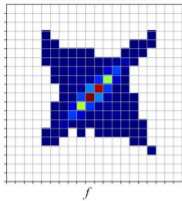
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Figure 14. Cross-validation performances for several neighbourhood sets

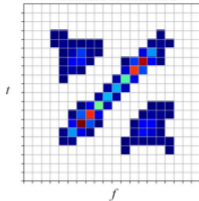
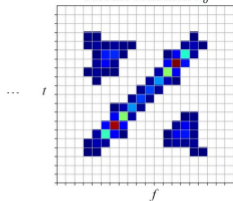




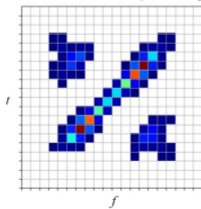
Turbine condition 1



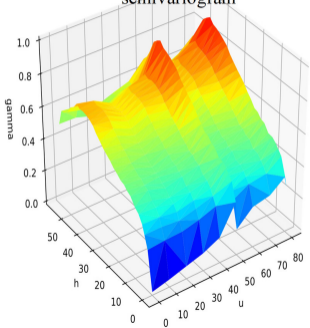
Turbine condition 2

Turbine condition  $n_o$ 

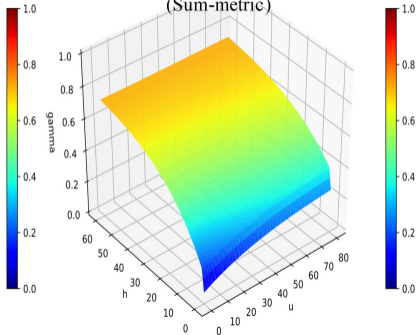
Interpolate

Turbine condition  $x$  ( $1 \leq x \leq n_o$ )

Experimental  
semivariogram



Semivariogram model  
(Sum-metric)





with non-bias condition:

$$\sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) = 1$$

$$B^*(p_i, o_x) = \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) B(p_k, o_l)$$

Interpolated  
ARM histogram  
values  
 $B^*(p_i, o_x)$

Create a Lagrange function  
for the minimization process

$$L(\lambda_{kl}, \mu) = \underbrace{\text{Var}(B(p_i, o_x) - B^*(p_i, o_x))}_{\sigma_e^2(p_i, o_x)} + 2\mu \left( \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) - 1 \right)$$

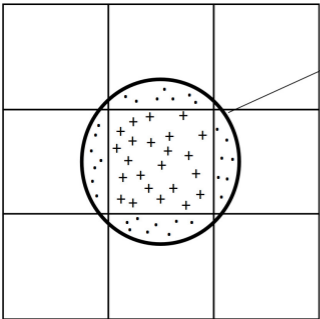
Minimize the system  
by partial derivatives

$$\begin{cases} \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) \gamma(h_{\|p_k - p_j\|}, u_{|o_l - o_m|}) - \mu = \gamma(h_{\|p_i - p_j\|}, u_{|o_x - o_m|}) \\ \sum_{k \in \vartheta_i} \sum_{l=1}^{n_o} \lambda_{kl}(p_i, o_x) = 1 \end{cases}$$

Build Experimental  
Semivariogram  
and Estimate semivariogram  
model

$j \in \vartheta_i$  and  $m=1, \dots, n_o$

Set of  $\lambda_{kl}(p_i, o_x)$   
With  $k \in \vartheta_i, l=1, \dots, n_o$



Contour of  
bivariate Gaussian  
distribution

+ + +  
+ + + Points inside the bin

. . . . Points outside the bin

$$\frac{\text{Points outside the bin}}{\text{Total points}} \approx y\%$$

At input operating conditions



...



Interpolation  
using kriging

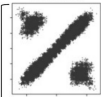
At target operating condition



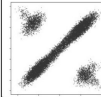
ARM  
matrix

Cycles Generations  
using BGDs

At target operating condition



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Set of  
generated ARMs

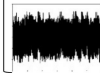
Compare to experimental  
Rainflow Strain Spectrum

Signal reconstruction  
using inverse method

At target operating condition



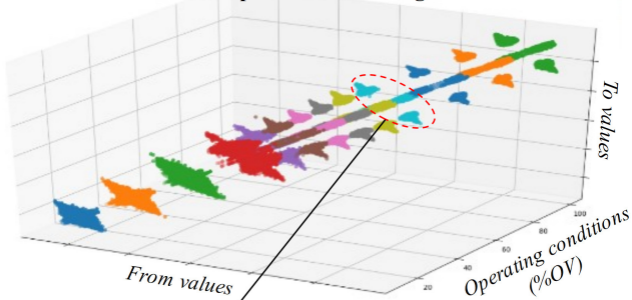
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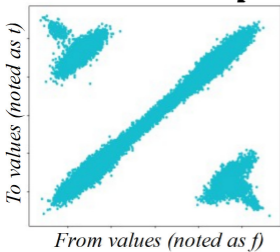
Set of  
reconstructed signals

Compare to  
measured signal

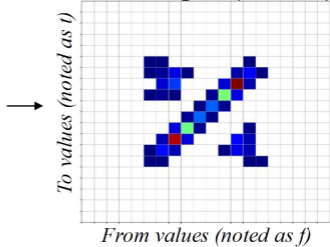
# ARM of experimental strain signals



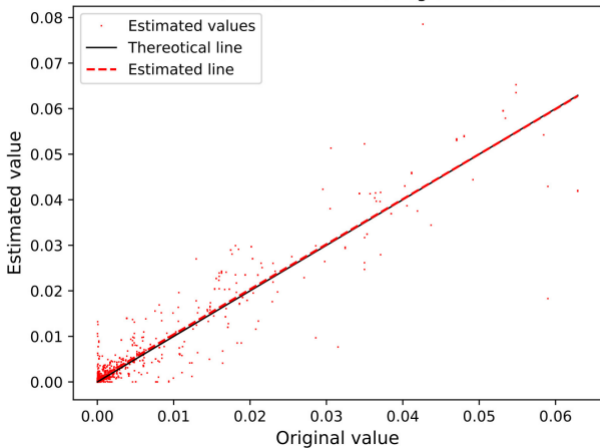
## ARM



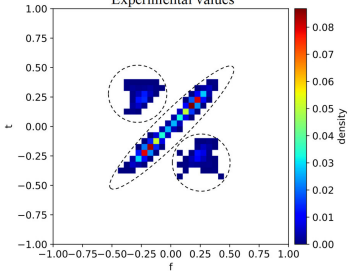
## ARM histogram (400 bins)



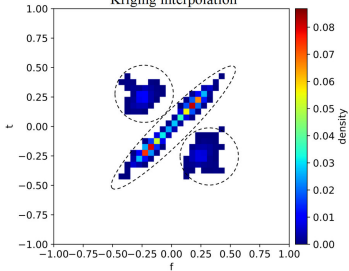
Cross-validation for semivariogram model



Experimental values

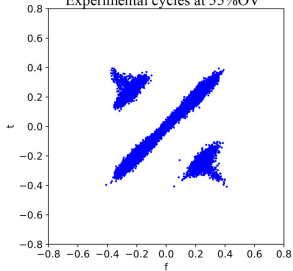


Kriging interpolation

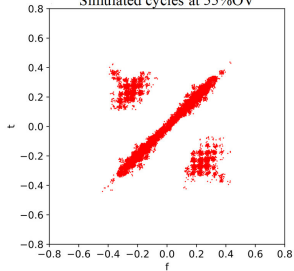




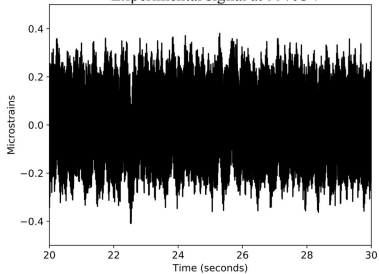
Experimental cycles at 55%OV



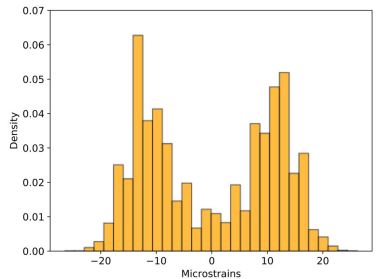
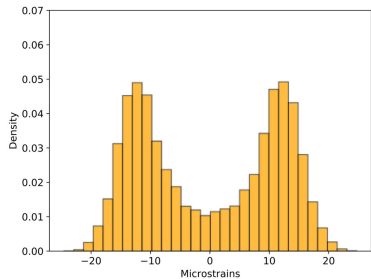
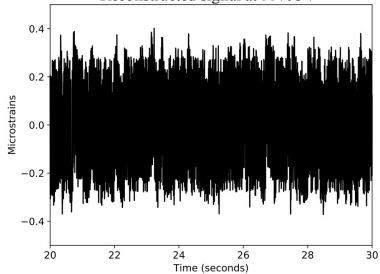
Simulated cycles at 55%OV



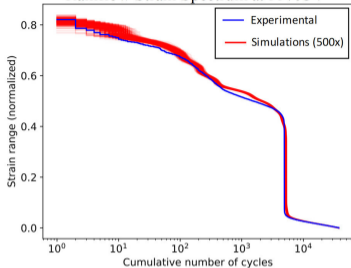
Experimental signal at 55%OV



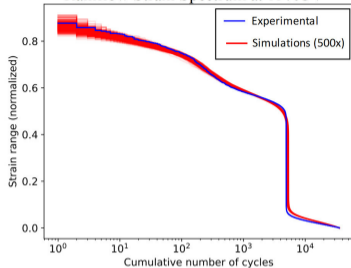
Reconstructed signal at 55%OV



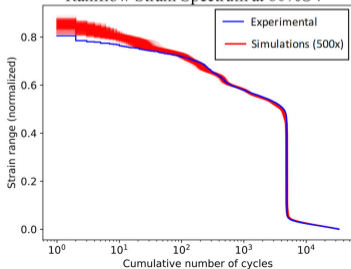
Rainflow Strain Spectrum at 55%OV



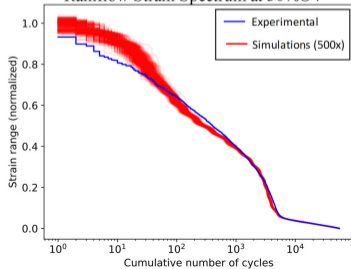
Rainflow Strain Spectrum at 65%OV



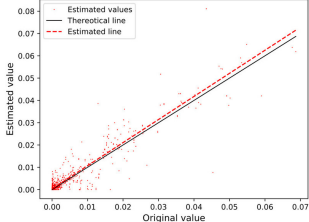
Rainflow Strain Spectrum at 80%OV



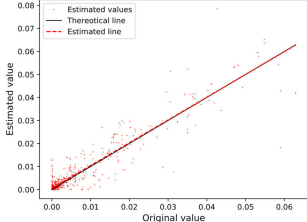
Rainflow Strain Spectrum at 30%OV



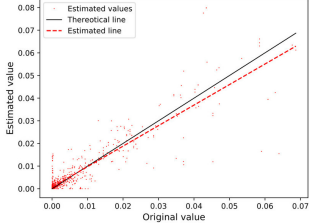
Cross-validation for set of 5 neighbourhood points (RMSE = 0.00404)



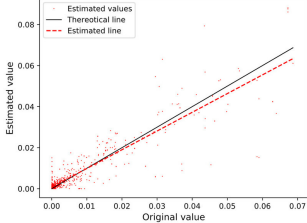
Cross-validation for set of 9 neighbourhood points (RMSE = 0.00379)



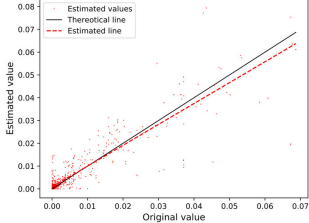
Cross-validation for set of 15 neighbourhood points (RMSE = 0.00466)



Cross-validation for set of 25 neighbourhood points (RMSE = 0.00495)



Cross-validation for set of 35 neighbourhood points (RMSE = 0.00515)



Cross-validation for set of 49 neighbourhood points (RMSE = 0.00481)

