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Spatio-temporal dynamics of human behaviors during disasters : a mathematical and geographical approach

Valentina Lanza, Edwige Dubos-Paillard, Rodolphe Charrier, Nathalie Verdière, Damienne Provitolo, Oscar Navarro, Cyrille Bertelle, Guillaume Cantin, Alexandre Berred, M.A. Aziz-Alaoui

Abstract This chapter proposes to generalize to a geographical context an innovative SIR-based model describing human collective behaviors in situations of disasters. The novelty of this work is to adopt a complex network approach in order to model the influence of the space on the transitions among one behavior and the others. This network model will be applied on the specific case of a tsunami on the French Riviera caused by a seismic event.

1 Introduction

The current major challenge in the field of populations safety and risk management is to progress in the understanding and ability to anticipate human behavior in face of complex threats and disasters of all origins. Several mathematical models of crowd

V. Lanza, N. Verdière, G. Cantin, A. Berred and M.A. Aziz-Alaoui
Normandie Univ, UNILEHAVRE, LMAH, FR-CNRS-3335, Le Havre, France
e-mail: valentina.lanza@univ-lehavre.fr, nathalie.verdiere@univ-lehavre.fr, guillaume-cantin@mail.com, alexandre.berred@univ-lehavre.fr, aziz.alaoui@univ-lehavre.fr

R. Charrier and C. Bertelle
Normandie Univ, UNILEHAVRE, LITIS, Le Havre, France
e-mail: rodolphe.charrier@univ-lehavre.fr, cyrille.bertelle@univ-lehavre.fr

E. Dubos-Paillard
Géographie-Cités UMR 8504, Université Paris 1, Panthéon-Sorbonne
e-mail: Edwige.Dubos-Paillard@univ-paris1.fr

D. Provitolo
Université Côte d'Azur, CNRS, Observatoire de la Côte d'Azur, IRD, Géoazur
e-mail: Damienne.provitolo@geoazur.unice.fr

O. Navarro
Laboratoire de Psychologie des Pays de la Loire, Université de Nantes
e-mail: Oscar.Navarrocarrascal@univ-nantes.fr

dynamics, also in extreme scenarios as in a panic situation, have been proposed in literature (see the survey paper [3]). However, rare are the works that take into account several human behaviors at the same time and also transitions among these reactions [26]. To meet this challenge, a consortium grouping researchers of different disciplines (mathematicians, geographers, psychologists, computer scientists) has been formed in the framework of the ANR project Com2SiCa. The aim of this research program is to understand and simulate human behaviors in areas affected by disasters whatever their causes. The studied disasters are sudden, unexpected, without warning signs or with signs that are difficult to identify. A first work of this consortium proposed a fine analysis of human behaviors and their transitions according to the concerned brain areas [24, 25, 5].

The work presented in this chapter considers a psychologically-based model describing human collective behaviors and emotional contagion in situation of natural, societal or technological disasters. Indeed, according to geographers, neuro-scientists and psychologists [23, 7], during a catastrophe, people scarcely stay in the same category of behavior during the all event. Moreover, imitation or social comparison is an important factor in the processes of behaviors changes [11].

Furthermore, the role played by the spatial configuration and the constraints introduced by the hazard (cutting of roads, destruction of buildings etc.) is not negligible in the dynamics of human reactions during a catastrophe. Indeed, mathematical models in literature [17, 18] focus mainly on the influence of the spatial configuration (presence of obstacles or bottleneck formation, for instance) on the crowd flow dynamics, rather than on the crowd emotional states.

The aim of this chapter is to present a mathematical framework in order to have a spatial version of our model with a strong geographical meaning. In our approach, the space environment is modeled by a network that is adapted to the geographical area under study. The nodes represent a part of the geographical territory under study, with some properties and are filled with a group of individuals subject to the model behavior panel. The edges symbolize possible interactions between nodes. Each term in the network model is justified by geographical and/or psychological considerations. In particular, the coupling coefficients are here calculated using a physical and geographical approach and for each node of the network we introduce a capacity constraint [20], that is the maximal number of individuals that each node can host. We show how, depending on these parameters, the network dynamics can change and different effects, such as bottlenecks, can take place.

The chapter is organized as follows. Section 2 presents the a-spatial APC model. In Section 3, taking into account the geographical constraints, the network model is introduced. In Section 4 some numerical results for a network of three nodes are shown.

2 The aspatial behavioral model

Behaviors that can be observed during major disasters are non traditional. They are caused by hostile environmental pressure that imposes new ways of acting in a state of high or even extreme level of stress. They are conditioned by different parameters such as :

- The types of hazards and anticipation of the beginning of the disaster: some disasters can be anticipated and announced by different information channels (newspapers, radio, televisions...). It is often the case of hurricanes, floods, volcanic eruption. However, other disasters as earthquakes and nuclear explosions are sudden. In the first case, we mainly observe controlled behaviors [2, 10] since the authorities' actions allow the population to face the danger (organized evacuation, consideration of the potential effects of the disaster). In the second case, the reactions of the population are more instinctive [15] and individualistic (freeze response or panic flight for example), at least during the first moments of the disaster [24].
- The location of individuals in relation to the disaster [7]. The affected area is usually divided in four types of zones: the impact zone, where the material destructions, the number of casualties and the social and territorial disorganization are maximal; the destruction zone, where the material damages are very important, the social organization is very perturbed but the number of victims is less; and finally the marginal and external zones which are generally less impacted by the disaster.
- The temporality of the event which is at the origin of the disaster and the time of the day when it occurs. Many authors have proposed different stages to describe the course of a disaster. For example, W. Powell [21] distinguishes eight phases (pre-disaster, warning, threat, impact, inventory, rescue, remedy, recovery) as Henry W. Fischer [9] proposes five phases (pre-impact, impact, post-impact, recovery and reconstruction).
- The characteristics of the impacted zone: the human behaviors and associated displacements are generally constrained by the specificities of the territory. For example, the presence of open spaces or buildings permitting to ensure the security of populations, the number and the position of exits in buildings [12, 13] or the morphology of the network and its degree of deterioration [19] have an influence on people's reactions.
- The characteristics of individuals and the density of population: the behaviors vary with the physical characteristics of individuals (age, agility), their education and experiences (culture of risk). The local knowledge and the perception of the environment [28] are also important, as the familiarity with those present or the density of population. This density, which increases when the crowd is being formed, makes the situation more dangerous (i.e. reduction of the choices for the individual displacements, increasing of interactions between individuals and their neighbors) and can lead, for example, to extreme situations of trampling and suffocation.

These parameters highlight that for a same kind of disaster people's responses can be different. Moreover, during a disaster, individuals scarcely maintain the same behavior. We often observe a sequence of several behaviors (for example, freeze response followed by flight and containment). Nevertheless, we have few information

on the evolution of population response. Indeed, it is unthinkable to continuously monitor each individual expecting a catastrophic event, or in the hope of avoiding one [24]. It is also impossible, from an ethical point of view, to voluntarily trigger a catastrophe to study the real observed behavior of populations in the face of danger. That's why the first step of approach consisted in building a first model to analyze the question of behavioral change.

This model, called APC (Alert-Panic-Control), relies on the point of view of psychologists. It is an extension of the PCR (Panic-Controlled-Reflex) model proposed in [25, 24] and studied in [6]. At our knowledge PCR model is the first SIR-based model in literature that aims to describe the dynamics of the multiple human collective behaviors in the scenario of a catastrophe. In the new model we are presenting here, human behavior are governed by two elements: the emotional charge that can be more or less strong according to the level of stress, and the emotional regulation, i.e. the ability of people to control emotional charge.

Taking into account the respective influence of these two elements, three main classes of behaviors are considered in the APC model:

- i) The state of alert: people are on alert and in a situation of uncertainty. They are in search of information and try to identify the threat. They have often a low emotional charge and a strong emotional regulation. At the beginning of the catastrophe, all the people adopt an alert behavior that can last from few seconds to several minutes.
- ii) The state of panic: it corresponds to behaviors whose reasoning capacities are affected by a high level of stress. It gathers all automatic and uncontrolled behavior as freeze response, panic flight, automate behavior. In this state, the emotional charge is strong and is hardly regulated. This can lead to dangerous situations. That's why they must be quickly identified, not only they can disrupt the organization of relief, but also because they can endanger the lives of populations by pushing and running over people for example. These kinds of behaviors require a huge surge of nervous energy. So it often lasts at most one hour.
- iii) The state of control: the emotional charge is more or less intense but it can be regulated. As opposed to panic, people in a controlled behavior keep their self-control and their cognitive functions (sense of reality, emotional control, danger assessment, ability to choose a solution). It gathers for example search for help, mutual aid, evacuation, looting, etc.

It is worth noting that in the description and analysis of these behaviors we do not judge the behaviors with respect to the event. A control behavior can be inappropriate while a panic one can be efficient. Here we are interested in understanding the behaviors themselves and the transitions from a behavior to another.

Let us consider a population affected by a catastrophic event. We will note t_0 the time at which the catastrophe takes place and for $t \geq t_0$, $a(t)$, $p(t)$, and $c(t)$ the number of individuals in a state of alert, panic and control, respectively. In addition, we note $q(t)$ the number of individuals in the daily behaviors before the catastrophic event. In the same way we designate with $b(t)$ all the people that exit the impact zone. Furthermore, with respect to the previous model [24, 6], deaths are taken into

account in order to obtain a more realistic model. Thus, we note $d(t)$ the individuals who lost their lives during the disaster.

It is well known that human beings rarely get stuck in a single type of reaction during a catastrophic event [7]. First of all, each individual experiences a behavioral evolution of his own. Typically, the behavior of the state of alert a is adopted as soon as the event is triggered, then people evolve towards the state of panic p or the one of control c , depending on their life experiences and risk education. The course of events can induce sometimes a new analysis of the situation and a temporary return to the alert state. Furthermore, imitation processes and social comparison are important factors that can induce transitions from one behavior to another one [11].

The scheme of the APC model can be found in Figure 1. Finally, the APC model equations are the following ($t \geq t_0$):

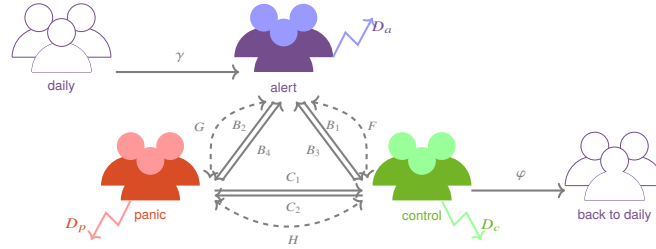


Fig. 1 APC model scheme

$$\begin{aligned}
 \dot{a}(t) &= \gamma(t)q(t) - (B_1 + B_2 + D_a)a(t) + B_3c(t) + B_4p(t) \\
 &\quad + F(a(t), c(t))a(t)c(t) + G(a(t), p(t))a(t)p(t), \\
 \dot{c}(t) &= B_1a(t) + C_1p(t) - (B_3 + C_2 + D_c)c(t) \\
 &\quad - F(a(t), c(t))a(t)c(t) + H(c(t), p(t))c(t)p(t) - \varphi(t)c(t), \\
 \dot{p}(t) &= B_2a(t) + C_2c(t) - (B_4 + C_1 + D_p)p(t) \\
 &\quad - G(a(t), p(t))a(t)p(t) - H(c(t), p(t))c(t)p(t), \\
 \dot{q}(t) &= -\gamma(t)q(t), \\
 \dot{b}(t) &= \varphi(t)c(t) \\
 \dot{d}(t) &= D_a a(t) + D_c c(t) + D_p p(t).
 \end{aligned} \tag{1}$$

We note B_1 and B_2 the proportions of individuals who, at time t , naturally evolve from the state of alert to a controlled behavior and to the state of panic, respectively. The proportions corresponding to the comeback from the control behavior to the alert, and from the panic to the alert are noted B_3 and B_4 , respectively. The rates of transition from panic to control, and control to panic, are noted C_1 and C_2 , respectively. Coefficients D_a , D_p and D_c are the death rates of the three populations.

Since imitation processes can be similar to contagion ones, the imitation functions have been built using a SIR model approach and taking into account that the behavior of the majority influences the behavior of the minority. For example, for the imitation between alerted and controls, we have:

$$F(a, c)ac = -\alpha_1 \xi \left(\frac{c}{a + \varepsilon} \right) \cdot \frac{c}{N} \cdot a + \alpha_2 \xi \left(\frac{a}{c + \varepsilon} \right) \frac{a}{N} \cdot c, \quad (2)$$

where

$$\xi(s) = \frac{s^2}{1 + s^2},$$

$N = N(t) = q(t) + r(t) + p(t) + c(t) + b(t)$ is the total population at time t , and $\varepsilon \ll 1$.

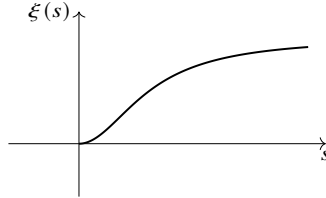


Fig. 2 The graph of function ξ , which is involved in the modeling of the imitation terms.

In particular,

$$-\alpha_1 \xi \left(\frac{c}{a + \varepsilon} \right) \cdot \frac{a}{N} \cdot c$$

represents the proportion of population in alert behavior that become controlled by mimicking the controlled people around them. The term $\frac{c}{N}$ is the classic "proportional incidence rate" found in many epidemic models [1, 4] which models the fact that each individual in the alert behavior does not meet all the people in the controlled behavior (this is impossible especially in the case of a dense crowd), but only a proportion of this sub-population. The term $\alpha_1 \xi \left(\frac{c}{a + \varepsilon} \right)$ is the probability that an individual in the alert behavior becomes controlled. The ξ function models the fact that only if there is a majority of controlled people, alerted individuals adopt a controlled behavior. Similarly

$$\alpha_2 \xi \left(\frac{a}{c + \varepsilon} \right) \frac{a}{N} \cdot c$$

represents the fraction of individuals in a controlled behavior that mimics those in an alert behavior.

Analogously, functions G and H are defined as

$$\begin{cases} G(a, p)ap = -\beta_1\xi\left(\frac{p}{a+\varepsilon}\right)\frac{p}{N}\cdot a + \beta_2\xi\left(\frac{a}{p+\varepsilon}\right)\frac{a}{N}\cdot p \\ H(c, p)cp = -\gamma_1\xi\left(\frac{p}{c+\varepsilon}\right)\frac{p}{N}\cdot c + \gamma_2\xi\left(\frac{c}{p+\varepsilon}\right)\frac{c}{N}\cdot p, \end{cases} \quad (3)$$

Finally, the function γ models the trigger of the catastrophe while the function φ describes exit from the impact zone, that we suppose can be achieved only being in a controlled behavior. They are defined as

$$\gamma(t) = \zeta(t, t_0, t_1) \text{ and } \varphi(t) = \zeta(t, t_2, t_3), \quad t \in \mathbb{R}$$

where ζ is defined as:

$$\zeta(t, \tau_0, \tau_1) = \begin{cases} 0 & \text{if } t < \tau_0 \\ 1 & \text{if } t > \tau_1 \\ \frac{1}{2} - \frac{1}{2} \cos\left(\frac{t - \tau_0}{\tau_1 - \tau_0}\pi\right) & \text{si } \tau_0 \leq t \leq \tau_1. \end{cases}$$

Table 1 sums up the notations of the APC model.

Remark 2.1 It is easy to notice that

$$\forall t \geq t_0, \quad \dot{a}(t) + \dot{c}(t) + \dot{p}(t) + \dot{q}(t) + \dot{b}(t) + \dot{d}(t) = 0.$$

The population under study is thus constant over time

$$\forall t \geq t_0, \quad a(t) + c(t) + p(t) + q(t) + b(t) + d(t) = cst$$

and we have the following relation:

$$\forall t \geq t_0, \quad \dot{a}(t) + \dot{c}(t) + \dot{p}(t) + \dot{q}(t) + \dot{d}(t) = -\dot{b}(t).$$

This means that the 5th equation is a linear combination of the other equations of the model, so we can reduce our system to 5 equations in terms of the state variables $u = [a, c, p, q, v]^T$. \square

In the following, we suppose all the population in a daily behavior before the onset of the catastrophe, thus we consider as initial condition $u_0 = (0, 0, 0, N, 0)^T$.

3 A network of APC models

If the APC model makes it possible to reflect on the different behaviors that can occur depending on the characteristics of the disaster, risk culture and imitation phenomena, it remains a-spatial. However, the dynamics and movements observed during a disaster are also guided by territorial configurations (open/closed environment,

Table 1 Variables, functions and parameters of the APC model

Variables	Notation
Daily behavior before the catastrophe	$q(t)$
Alert behavior	$a(t)$
Control behavior	$c(t)$
Panic behavior	$p(t)$
Behavior outside the impact zone	$b(t)$
Functions	Notation
Beginning of the catastrophe	$\gamma(t)$
Leaving from the impact zone	$\varphi(t)$
Imitation functions	F, G, H
Parameters	Notation
Evolution from alert to control	B_1
Evolution from alert to panic	B_2
Evolution from control to alert	B_3
Evolution from panic to alert	B_4
Evolution from panic to control	C_1
Evolution from control to panic	C_2
Mortality rates	D_a, D_p, D_c
Imitation from alert to control	α_1
Imitation from control to alert	α_2
Imitation from alert to panic	β_1
Imitation from panic to alert	β_2
Imitation from panic to control	γ_1
Imitation from control to panic	γ_2

network connectivity, street widths, presence of squares, etc.) and the alternatives they offer, particularly in terms of evacuation, flight and accessibility to refuge areas.

3.1 The network structure

In our modeling process we work at a mesoscopic scale and we adopt complex networks and metapopulations frameworks. We suppose to split the environment under study in several patches, that will become the nodes of our network. The network edges symbolize interactions between nodes. They can be of two types. They can depict population displacements. This paper is mainly focusing on the methodological choices made to represent this kind of interaction. In the short term, it will be completed by a second kind of relationships permitting the dissemination of information between spatially non contiguous nodes to take into account the exchange of information by mobile phone or SMS alert systems for example [8].

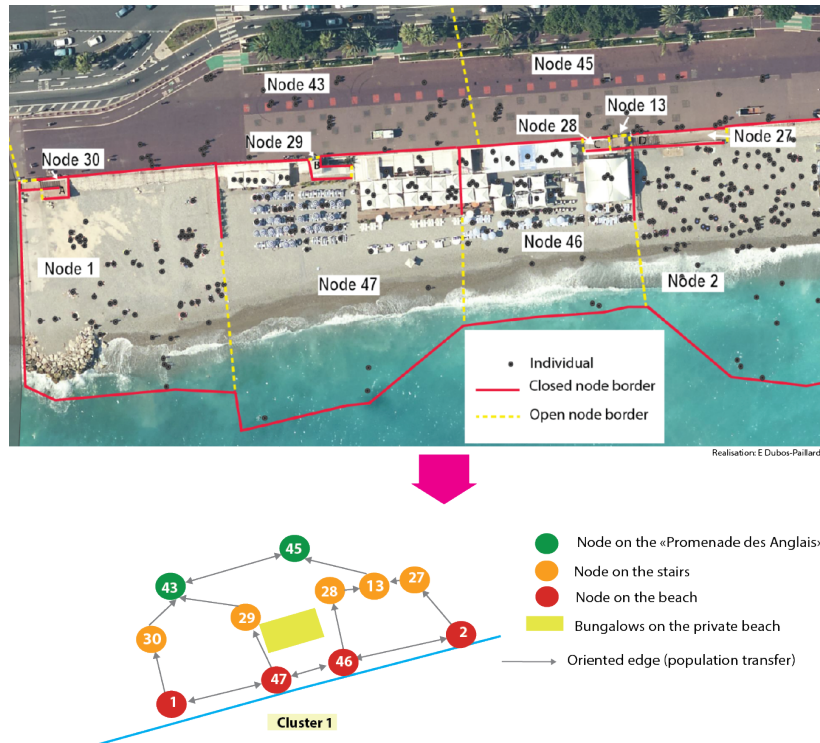


Fig. 3 From the spatial partition of the territory (Nice seafront in France) to the studied network

3.2 The model equations

Here are our main assumptions:

- i) we consider a network of n non-identical nodes;
- ii) the population of the network is constant (each node having its own victims and assuming that there are no individuals coming in or out of the network). On the other hand, the population will not be constant on each node;
- iii) each node k has a maximum carrying capacity, that is a maximum number of individuals N_k^{max} that can be received;
- iv) we consider only linear couplings, that is to say physical displacements. In addition, we suppose there is no behavioral transition during the transfer;
- v) the dead people and people in a daily behavior do not move. People in a state of alert hardly move.

In the following we note

- $N_i = N_i(t) = q_i(t) + a_i(t) + p_i(t) + c_i(t) + b_i(t)$ the total population of the i -th node at time t ;
- $\mathcal{N}^+(i)$ the out-neighbors set of node i (that is the nodes that are adjacent to node i and whose edge starts from i) [27];
- $\mathcal{N}^-(i)$ the in-neighbors set of node i (that is the nodes that are adjacent to node i and whose edge comes into i).

Then, the equations for the APC network model are the following ($i = 1, \dots, n$):

$$\left\{ \begin{array}{l}
 \frac{dq_i}{dt} = \gamma_i q_i - (B_1^i + B_2^i + D_a^i) a_i + B_3^i c_i + B_4^i p_i + F^i(a_i, c_i) a_i c_i + G^i(a_i, p_i) a_i p_i \\
 \quad + \sum_{k \in \mathcal{N}^-(i)} \left(1 - \frac{N_i}{N_i^{max}}\right) \eta_{ki}^a a_k - \sum_{k \in \mathcal{N}^+(i)} \left(1 - \frac{N_k}{N_k^{max}}\right) \eta_{ik}^a a_i, \\
 \\
 \frac{dc_i}{dt} = B_1^i a_i + C_1^i p_i - (B_3^i + C_2^i + D_c^i) c_i - F^i(a_i, c_i) a_i c_i + H^i(c_i, p_i) c_i p_i \\
 \quad - \varphi_i c_i + \sum_{k \in \mathcal{N}^-(i)} \left(1 - \frac{N_i}{N_i^{max}}\right) \eta_{ki}^c c_k - \sum_{k \in \mathcal{N}^+(i)} \left(1 - \frac{N_k}{N_k^{max}}\right) \eta_{ik}^c c_i, \\
 \\
 \frac{dp_i}{dt} = B_2^i a_i + C_2^i c_i - (B_4^i + C_1^i + D_p^i) p_i - G^i(a_i, p_i) a_i p_i - H^i(c_i, p_i) c_i p_i \\
 \quad + \sum_{k \in \mathcal{N}^-(i)} \left(1 - \frac{N_i}{N_i^{max}}\right) \eta_{ki}^p p_k - \sum_{k \in \mathcal{N}^+(i)} \left(1 - \frac{N_k}{N_k^{max}}\right) \eta_{ik}^p p_i, \\
 \\
 \frac{dq_i}{dt} = -\gamma_i q_i, \\
 \\
 \frac{db_i}{dt} = \varphi_i c_i, \\
 \\
 \frac{dv_i}{dt} = D_a^i a_i + D_c^i c_i + D_p^i p_i,
 \end{array} \right. \quad (4)$$

with the dependencies on t being omitted to lighten notation and where

$$\left\{ \begin{array}{l}
 F^i(a_i, c_i) = \frac{1}{N_i} \left(-\alpha_1^i \xi \left(\frac{c_i}{a_i + \varepsilon} \right) + \alpha_2^i \xi \left(\frac{a_i}{c_i + \varepsilon} \right) \right) \\
 \\
 G^i(a_i, p_i) = \frac{1}{N_i} \left(-\beta_1^i \xi \left(\frac{p_i}{a_i + \varepsilon} \right) + \beta_2^i \xi \left(\frac{a_i}{p_i + \varepsilon} \right) \right) \\
 \\
 H^i(c_i, p_i) = \frac{1}{N_i} \left(-\gamma_1^i \xi \left(\frac{p_i}{c_i + \varepsilon} \right) + \gamma_2^i \xi \left(\frac{c_i}{p_i + \varepsilon} \right) \right).
 \end{array} \right. \quad (5)$$

Remark 3.1 According to our assumptions, we have capacity constraints on each node. This means that when the maximum capacity of one node is reached at time

t , then this node cannot receive other population from the others nodes, so at time t the incoming flux for this node must be zero.

This property is taken into account by the terms

$$\left(1 - \frac{N_i}{N_i^{max}}\right) \quad i = 1, \dots, n.$$

It is worth notice that if we suppose, for instance, that at time t the node i is full (i.e. $N_i(t) = N_i^{max}$), we will have

$$\sum_{k \in \mathcal{N}^-(i)} \left(1 - \frac{N_i}{N_i^{max}}\right) \eta_{ki}^a = \sum_{k \in \mathcal{N}^-(i)} \left(1 - \frac{N_i}{N_i^{max}}\right) \eta_{ki}^p = \sum_{k \in \mathcal{N}^-(i)} \left(1 - \frac{N_i}{N_i^{max}}\right) \eta_{ki}^c = 0,$$

thus there is no incoming flux for node i . \square

3.3 Coupling coefficients

As we have said before, a link between two nodes of the network represents the fact that the population in one node can move to the other. The proportion of the population in a behavior state which moves from node i to node j is modeled by a linear coupling coefficient, η_{ij}^c or η_{ij}^p depending on the behavior under study, in the network mathematical model (4). We recall that, according to our hypotheses, panic and control behaviors are the only ones that can move.

One of the main issue is therefore to quantify this linear coupling between two given nodes, that is between two spatial zones, according to the geographical configuration of these zones. Our study has led to a specific formulation for this coupling, which depends on some quantities when one considers some population going from one given node to another:

- the width L of the exit from one node to another;
- the average speed $\langle v \rangle$ of the population leaving the node under study (it is noteworthy that this quantity is computed here only with the horizontal components of speed vectors);
- the global surface S of the node under study from whom population is leaving.

Thus, coupling coefficients can be viewed as the analogous of mass flow rates.

Figure 4 represents an example of two nodes. In this configuration we have

$$\eta_{12} = \frac{\langle v \rangle L}{S_1}.$$

To get this formula, we have to consider the following definition of this coupling coefficient, that is the proportion of people who leave the space per minute:

$$\eta_{12} = \frac{1}{N_1} \frac{dN_{12}}{dt}.$$

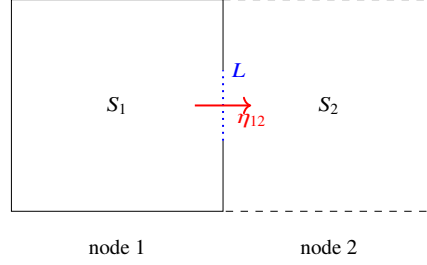


Fig. 4 The coupling coefficient value η_{12} depends on the global surface of node 1, on the width of the exit from node 1 to node 2, and on the average speed $\langle v \rangle$ of the population leaving node 1.

The quantity dN_{12} corresponds to the population quantity that goes through the exit per time unit dt , that is $dN_{12} = n_1 \langle v \rangle L dt$, where n_1 is the population density on node 1. To finally get the result in terms of densities, we have to divide this number of people by the global number of people on node 1 (having a surface S_1), that is $N_1 = n_1 S_1$. Finally this gives the expression of the coupling.

Remark 3.2

- $\langle v \rangle$ is the average speed but each behavior has its own average speed, that is the way why we have to consider $\eta_{12}^a, \eta_{12}^c, \eta_{12}^p$,
- in case of multiple exits from node i to node j , we have multiple L_i , so we have multiple contribution for the calculation of the coupling coefficient. \square

4 Case study and numerical results

Here we are interested in the scenario of a tsunami on the French Riviera caused by a seismic event. The arrival time of the first wave would vary from 15 to 30 minutes [14]. This short time would make it difficult to alert and organize the evacuation of the population by public authorities. The impact zone under study is the seafront of Nice (France) [22]. In the following, let us consider a network of three nodes such as in Figure 5. We have selected three nodes that correspond to a small part of the beach (Node 1), a staircase that allows someone in node 1 to leave the beach (Node 2) and a part of the *Promenade des Anglais* (Node 3). The goal for people who are on the beach is to reach the *Promenade des Anglais* where they will be in security. We consider this disaster to be unexpected due to a Ligurian fault rupture [16], thus we use the following functions γ_i and φ_i :

$$\gamma_i(t) = \gamma(t) = \zeta(t, 1, 3) \quad \text{and} \quad \varphi_i(t) = \varphi(t) = \zeta(t, 70, 100), \quad t \in \mathbb{R},$$

with ζ defined as: for $\tau_0 \leq t \leq \tau_1$, $\tau_0, t, \tau_1 \in \mathbb{R}$,

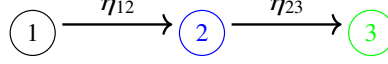


Fig. 5 The network under study.

$$\zeta(t, \tau_0, \tau_1) = \begin{cases} 0 & \text{if } t < \tau_0 \\ 1 & \text{if } t > \tau_1 \\ \frac{1}{2} - \frac{1}{2} \cos\left(\frac{t - \tau_0}{\tau_1 - \tau_0} \pi\right) & \text{if } \tau_0 \leq t \leq \tau_1 \end{cases}$$

In the selected scenario, we have chosen the function γ very sharp such that in three minutes the majority of the population in the daily behavior becomes alert. Furthermore, we suppose that people can exit from the impact zone only after 70 minutes from the beginning of the catastrophe.

Table 2 shows the parameters values for the numerical simulations we will show in this paper. We remark that here the superscript denotes the node number, e.g. B_1^i is the coefficient B_1 of the node i .

Table 2 Parameters values for the numerical simulations

Parameters	Values	Parameters	Values	Parameters	Values
$B_1^1 = B_1^2 = B_1^3$	0.1	$C_1^1 = C_1^2 = C_1^3$	0.1	$D_a^1 = D_a^2 = D_a^3$	0.001
$B_2^1 = B_2^2 = B_2^3$	0.1	$C_2^1 = C_2^2 = C_2^3$	0.1	$D_c^1 = D_c^2 = D_c^3$	0.001
$B_3^1 = B_3^2 = B_3^3$	0.05	$B_4^1 = B_4^2 = B_4^3$	0.05	$D_p^1 = D_p^2 = D_p^3$	0.001
$\alpha_1^1 = \alpha_1^2 = \alpha_1^3$	0.3	$\alpha_2^1 = \alpha_2^2 = \alpha_2^3$	0.2	$\beta_1^1 = \beta_1^2 = \beta_1^3$	0.6
$\beta_2^1 = \beta_2^2$	0.1	β_2^3	0.2	$\gamma_1^1 = \gamma_1^2$	0.6
γ_1^3	0.3	$\gamma_2^1 = \gamma_2^2$	0.1	γ_2^3	0.3
$q_1(0)$	300	$q_2(0)$	5	$q_3(0)$	195
N	500	$\eta_{12}^a = \eta_{12}^p = \eta_{12}^c$	0.5	$\eta_{23}^a = \eta_{23}^p = \eta_{23}^c$	0.5

The parameters have been chosen in order to model a tendency to adopt a panic behavior in nodes 1 and 2, since the first node is the one directly impacted by the catastrophe while in the second one the narrowness of the staircase could play an important role. As initial condition, we suppose all the population in the daily behavior before the catastrophe, distributed in the three nodes as following: $q_1(0) = 300$, $q_2(0) = 5$ and $q_3(0) = 195$, thus in node 1 we have 60% of the total population, in node 2 we have 1% and 39% in node 3, respectively. Finally, we have considered the same coupling coefficients for the different populations. Of course this is just a possible scenario and different values of the parameters can be considered.

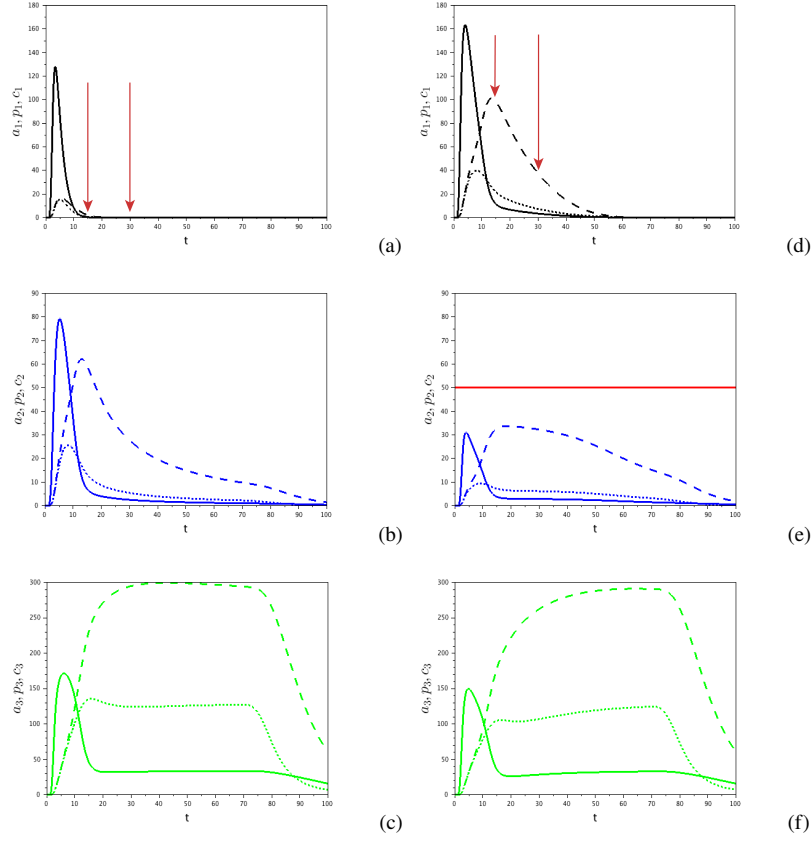


Fig. 6 Time course of the three most important populations a (solid line), p (dashed line), c (dotted line), in node 1 (a)-(d), node 2 (b)-(e) and node 3 (c)-(f), for two different values of N_2^{max} : $N_2^{max} = N$ in (a)-(b)-(c) and $N_2^{max} = 0.1N = 50$ in (d)-(e)-(f), respectively. We have chosen the following color code: node 1 is in black, node 2 is in blue and node 3 is in green.

Throughout these simulations we have chosen the following color coding: the variables of node 1 are represented in black, those of node 2 in blue and the ones of node 3 are in green, respectively.

Furthermore, in the following we have considered two different values of N_2^{max} , the maximum capacity of node 2, in order to investigate its effect on the global dynamics of the network. In the first scenario we have taken $N_2^{max} = N = 500$, the extreme case where node 2 can hold all the population. In the second one, we have supposed a capacity constraint on node 2 and put $N_2^{max} = 10\%N = 50$.

In Figure 6 the time dynamics for the three populations a (solid line), p (dashed line) and c (dotted line) of the three nodes are shown, for the two different values of N_2^{max} . In (a)-(b)-(c) we have the time courses for $N_2^{max} = N$, while in (d)-(e)-(f) we have the results when we consider the capacity constraint on node 2 equal to $N_2^{max} = 0.1N = 50$.

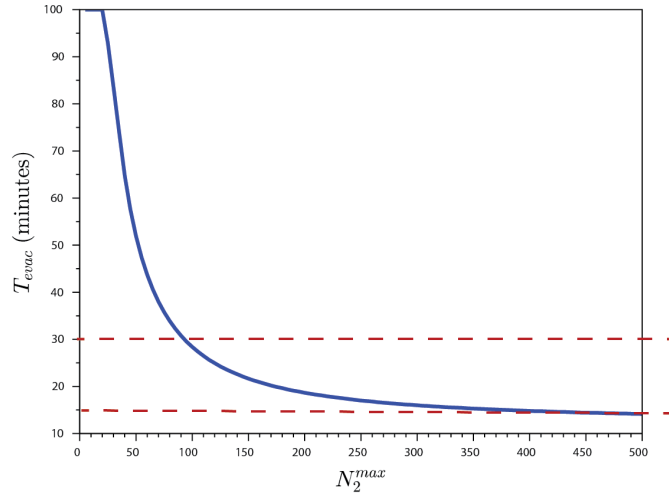


Fig. 7 Evacuation time for node 1 as function of N_2^{max} .

We can see that the stair capacity constraint has a great influence on the course of the event. While all the population has left the beach after 15 minutes in the first scenario, more than 130 people remain in danger after this period in the second one due to the inability to use the stairs that are overcrowded ($N_2^{max} = 50$). After 30 minutes about 50 people are still trapped on the beach. During these 15 minutes, the percentage of population in panic increases from 75% to 77%. These data are important because people who are still on the beach at these selected times will be affected by the tsunami wave and could possibly lose their lives. The effect of the stair capacity on the evacuation dynamics of node 1 can be better observed if we look at the evacuation time of node 1, defined as

$$T_{evac} = \inf \{t \in]0; T] \mid a_1(t) + p_1(t) + c_1(t) < 10\% q_1(0)\}.$$

In Figure 7, the evacuation time of node 1 as function of the maximum capacity of the middle node N_2^{max} is represented. According to the spatial configuration and the scenarios under study, with a stair maximal capacity of 50 individuals, it appears

that to evacuate about 300 people from the beach by one stair we should multiply the stair capacity by two to evacuate in 30 minutes and by eight to evacuate in 15 minutes.

5 Conclusion

In this paper, we have adopted a complex network approach in order to take into account the spatial variable in the modeling and analysis of the dynamics of human behaviors during catastrophic events. As case study, we considered a scenario consisting in a tsunami on the French Riviera caused by a seismic event. In this type of events the population must quickly evacuate the beach because the estimated time before the arrival of the first wave vary from 15 to 30 minutes. Nevertheless, this evacuation depends on the capacity of the stairs that allow to leave the beach. To analyze this aspect, we have considered an elementary network made up of three nodes that correspond to a small part of the beach where 300 people are located at the beginning of the event, a staircase that allows to leave the beach and a part of the Promenade des Anglais, considered as safe. Two scenarios about the arrival timeline of the first tsunami wave have been studied (15 and 30 minutes). We have also distinguished a model without capacity constraint and another where the number of individuals in the staircase can not be larger than 50. The numerical simulations show that the capacity constraints on the nodes of the network influence the global dynamics of the model. Without any constraint, people can evacuate in 15 minutes whereas there is still more than one third of the population on the beach when we add a capacity constraint in the stairs (50 persons). 74% of them are in panic. After 30 minutes one in six people is still trapped on the beach and the panic reaches 77% of them. To allow a rapid evacuation (about 15 minutes) of about 300 people the stair capacity should be multiplied by eight.

The next steps of this research is to take more into account the field reality, to analyze the necessary time to evacuate different number of people according to the number of stairs and the possibility to go toward another stair when one of them is overcrowded. We also want to introduce in the model a second kind of relationships between non adjacent nodes to build scenarios permitting the dissemination of information by mobile phone or SMS alert systems, since these kinds of exchange also have an influence on people behaviors during a disaster.

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