



HAL
open science

Millimeterwave and submillimeterwave spectra of sulfur dioxide $^{32}\text{S}^{16}\text{O}^{18}\text{O}$ and $^{32}\text{S}^{18}\text{O}_2$, centrifugal distortion analysis and equilibrium structure

Laurent Margulès, Roman Motiyenko, Jean Demaison

► **To cite this version:**

Laurent Margulès, Roman Motiyenko, Jean Demaison. Millimeterwave and submillimeterwave spectra of sulfur dioxide $^{32}\text{S}^{16}\text{O}^{18}\text{O}$ and $^{32}\text{S}^{18}\text{O}_2$, centrifugal distortion analysis and equilibrium structure. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 2020, 253, pp.107153. 10.1016/j.jqsrt.2020.107153 . hal-03206491

HAL Id: hal-03206491

<https://hal.science/hal-03206491v1>

Submitted on 15 Jul 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

A multi-modal competitive hub location pricing problem with customer loyalty and elastic demand

Mehdi Mahmoodjanloo^a, Reza Tavakkoli-Moghaddam^a, Armand Baboli^{b,*}, Atefeh Jamiri^c

^a School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

^b LIRIS laboratory, UMR 5205 CNRS, INSA of Lyon, 69621 Villeurbanne cedex, France

^c Department of Industrial Engineering, Nour Branch, Islamic Azad University, Nour, Iran

Abstract

This paper develops a multi-modal competitive hub location pricing problem whose target is the design of a transportation system for a company that plans to enter into a market with elastic demand, in which an existing transportation company operates its hub-and-spoke network. The entrant company aims to attract customers in the market by convenient locations for its hubs and proper pricing of its transportation services, while customer loyalty is different in the nodes. Hence, mixed-integer programming based on a multi-nominal logit model is proposed. Thereafter, to solve the single allocation hub-and-spoke model, it is decomposed into a bi-level model. In the new structure, the master problem is associated with hub location and assignment decisions, and the sub-problem is associated with pricing decisions. Moreover, upper and lower bounds are calculated to determine the price of transportation routes. Finally, based on a nested approach, a scatter search algorithm is used to search the solution space of the master problem, and a matheuristic method is designed to solve the pricing problem interactively. The proposed approach is employed to solve a case study in the postal service industry of Iran.

Keywords: Pricing; Hub location; Multi-modal transportation; Elastic demand; Customer loyalty.

1. Introduction

The number of companies that offer similar services has been increasing in recent years. As a result, competition among companies has increased to earn maximum profit and market share. Based on these conditions, companies must pay attention to the proper pricing of their services and design appropriate transportation routes to fulfill their customers' demands. For example, until recently, postal services in Iran were provided by a monopoly operator (Iran Post Company). To improve the quality of service and decrease prices, the government decided to open the market for other operators. The government believes that this policy is necessary to ensure the

* Corresponding author.

Email address: armand.baboli@insa-lyon.fr (A. Baboli).

development of effective competition. Based on this case, we assume in this study that a new transportation company intends to enter a monopoly market. The entrant tries to compete against the incumbent while having complete information about the location of the incumbent's hubs and spokes. The entrant applies optimal pricing and designs an appropriate network to obtain the maximum profit and market share. Therefore, to achieve this goal, the entrant has to consider the condition of a rival company (i.e., the incumbent) as well.

Hub location problems (HLPs) are an important research area of logistic problems. That is because a solution of these problems allows a decrease in the number of transportation links between origin and destination nodes. Moreover, it reduces overall transportation costs by consolidating traffic flows from various origins and transferring them to hubs with various destinations. HLPs are applied in many settings, such as airline industries (Aykin, 1994; Jaillet et al., 1996; Adler and Smilowitz, 2007), package delivery firms (Kuby and Gray, 1993), message delivery networks (Klincewicz, 1998), cargo industries (Taylor et al., 1995; Lumsden et al., 1999), telecom industries (Lee et al., 1996), emergency services and mobile post offices (Bashiri et al., 1996), and many other transportation systems.

Studies of various hub network models began in 1986. The first model of an HLP was introduced by O'Kelly (1986). Thereafter, O'Kelly (1987) introduced the first formulation of the HLP as an optimization problem. Generally, HLPs can be classified into four parts (Campbell, 1994): center, covering, fixed costs and median problems. Also, a hub median problem can be considered as a single allocation or multiple allocations (Ghaffarinasab et al., 2018). Alumur et al. (2012) presented a multi-modal hub location problem that jointly considered transportation costs and travel times. They studied decisions about how to design a hub network with various possible transportation modes. A complete literature review of HLPs can be seen in Alumur and Kara (2008), Campbell and O'Kelly (2012), Farahani et al. (2013), Contreras (2015).

Competition among firms that utilize hub networks is an interesting topic of research. The first model of a competitive hub location problem was introduced by Marianov, Serra and ReVelle (1999). Thereafter, related studies were carried out by Sasaki and Fukushima (2001), Adler and Smilowitz (2007), Eiselt and Marianov (2009), Gelareh, Nickel, and Pisinger (2010) and Sasaki et al. (2014). Lürer-Villagra and Marianov (2013) were the first to explicitly utilize a pricing policy in a competitive HLP. They studied a single-modal and multiple-allocation HLP in a competitive environment. They argued that the revenue of firms that are new entrants into competitive markets depend on network design and pricing strategy.

Competition in a market can be divided into three major types: static competition, dynamic competition, and competition with foresight (Farahani et al. 2014). The basic assumption in a static competition is that the existing rivals (i.e., incumbents) will not react to the entrance of a new competitor. However, the entrant should consider the effect of their rivals' activities. As in

the other types of competition, incumbents can react to the entrance of a new competitor by changes in their characteristics, such as pricing. In dynamic competition, competitors simultaneously determine their competitive factors. In competition with foresight, a competitor will react to an entrant's decisions sequentially. In the field of facility location, pricing has been studied for some time; however, to the best of our knowledge, there are a few studies in the case of hub location problems. [Table 1](#) represents a brief overview of the most closely related studies.

Table 1. Brief overview of competitive hub location and pricing problems

Publications	Competition type			Research questions and main contributions
	Dynamic	With foresight	Static	
Lüer-Villagra and Marianov, (2013)			*	<ul style="list-style-type: none"> - Entrant's hub location and pricing problem - Proving an equation to determine the optimal price of each entrant's route - Developing a genetic algorithm to solve the hub location problem
Čvokić et al. (2016)			*	<ul style="list-style-type: none"> - Both entrant's and incumbent's hub locations under fixed markups - Reformulating the lower level problem (LLP) to develop a matheuristic to solve the hub location problem
Čvokić et al. (2017)	*	*		<ul style="list-style-type: none"> - Hub location and pricing decisions for both competitors with and without relaxing the pre-commitment in terms of pricing - Providing that in the leader-follower hub location and pricing competition, where competitors are allowed to change their prices, there is a profit-maximizing solution for the leader
Esmaeili, M., & Sedehzade, S. (2018)			*	<ul style="list-style-type: none"> - Modeling a CHLP problem based on Stackelberg-game - Demand of each firm depends on its price under the Bernard's model - A solution approach is proposed using an imperialist competitive algorithm (ICA) and a closed expression
Čvokić et al. (2019)			*	<ul style="list-style-type: none"> - Modeling an r/p hub-centroid problem under the price war - Finding a unique finite Bertrand-Nash price equilibrium for the follower
This paper			*	<ul style="list-style-type: none"> - Developing a mathematical formulation for a multi-modal CHLP problem. - Considering the effect of customer loyalty and elastic demand. - Decomposing the model and proposing two nested evolutionary algorithms based on scatter search and differential evolution to solve a single allocation hub-and-spoke network.

A specific amount of price change does not always have the same effect on differences in demand. That is because the demand for a product/service can be elastic or inelastic, depending on the rate of change in demand relative to the change in the price of a product/service. Demand is elastic in the market when the response of demand is greater than a small proportional change in the price. Inelastic demand occurs when there is relatively less change in demand with greater change in price. There are a few studies that consider the effect of elastic demand. [Redondo et al. \(2012\)](#) studied the effect of elastic demand on competitive location problems. They showed that the assumption of fixed demand significantly affects location decisions; therefore, the correct type of demand (elastic or inelastic) must be considered for modeling location problems. [Kaveh et al. \(2016\)](#) designed a hub network with elastic demand, in which the demand is dependent on a utility that is related to the location of the hubs. Later, [Rahimi et al. \(2019\)](#) and [Kaveh et al. \(2019\)](#) utilized this concept in their hub location models; however, they did not separately consider the

effect of the price on demand. Two different hub location models considering the price-sensitive demand were studied by O'Kelly et al. (2015) and Čvokić and Stanimirović (2020); however, they developed their models for a non-competitive market.

Another concept that can be useful for studying demand behavior in a market is customer loyalty. This occurs when customers prefer buying a specific product or using a specific shop, rather than buying products that are related to other brands or using other shops. Customers display such behavior if they purchase a specific brand or product consistently over an extended period. For example, many customers prefer to use a specific travel agent because of good experiences with their services (Tasci, 2017). To the best of our knowledge, customer loyalty has not yet been considered in location and pricing models.

In the present paper, we develop a multi-modal competitive hub location pricing problem (CHLP) considering customer loyalty and elastic demand. The main contributions of the paper are as follows:

- Developing a mathematical formulation for a multi-modal CHLP problem.
- Modeling elastic demand in a competitive market.
- Considering the effect of customer loyalty on the decision-making process.
- Deriving a closed-form expression for the pricing problem when demand is inelastic.
- Deriving an upper bound and a lower bound for the price of each route in the hub network when demand is elastic.
- Decomposing the proposed model to a hub location problem (i.e., master problem) and a pricing problem (i.e., sub-problem). Then, proposing two nested evolutionary algorithms to tackle the complexity of solving this structure.
- Developing a scatter search algorithm to solve the single allocation multi-modal HLP.
- Developing a metaheuristic based on differential evolution algorithms to solve the pricing sub-problem.
- Investigating the characteristics of the model in a real-world example by exploring the sensitivity analyses.

The rest of this paper is organized as follows. Section 2 describes a generic mathematical model of the problem and two extended versions considering the effect of elastic demand and customer loyalty. In Section 3, the problem has been decomposed to a master and a sub-problem based on location and pricing decisions. Thereafter, two solution approaches have been proposed to tackle the problem with/without elastic demand and customer loyalty considerations. Computational experiments have been explained in Section 4. Finally, in Section 5 conclusions of the research have been provided.

2. Mathematical formulation

2.1. Problem definition

It is assumed that there is a situation where a new distribution company (entrant) intends to enter into a monopoly market. In this market, an existing company (incumbent) serves customers by a distribution network utilizing a hub-and-spoke topology. The entrant wants to design its service network and determine the related prices to compete with the incumbent's servicing system. Both the entrant and incumbent offer their own servicing time and price for each pair of origin and destination (OD).

The problem can be defined on a directed graph $G = (N, A)$, where A is the set of admissible arcs and N is the set of nodes. Each node represents a customer zone (e.g., demand for a city or a region). To give a more clear representation of the model, we explain the developing process in three phases. First, we develop the proposed model by [Lüer-Villagra and Marianov, \(2013\)](#) adding the possibility of considering transportation modes. Moreover, to accommodate it with our case, a constraint is added to convert the model to a single-allocation HLP. This can be considered as a generic model of our case. Thereafter, we extend the generic model considering the effect of elastic demand and the effect of customer loyalty in subsections [2.3](#) and [2.4](#) respectively. To obtain insightful results, the following assumptions are made:

- 1) In this research, we focus on designing an uncapacitated single-allocation hub network to have more adapt with the conditions of our case study in postal services.
- 2) The hub nodes are not interconnected ([Luer-Villagra and Marianov, 2013](#)).
- 3) On each OD, the fraction of demands absorbed by each company can be predicted based on a utility function using a logit model. In the related literature, logit models are extensively used to accommodate several attributes to select some alternatives ([Zambrano-Rey et al., 2019](#); [Čvokić et al., 2019](#)).
- 4) The customers have an overview of all service times and prices when they want to select their services. For example, the required information is presented on the website's online store of each company.

Notations for the model are defined as follows:

Sets:

- N Set of nodes ($i, j, k, l \in N$)
- P Set of the incumbent's hub nodes
- M Set of transportation modes ($m \in M$)

Parameters:

- K_{ij}^m Fixed cost of arc ij for transportation mode m .

d_{ij}	Distance of arc ij
φ_{ij}^m	Travel cost per unit of transportation mode m along arc ij
τ_{ij}^m	Travel time per unit of transportation mode m along arc ij
σ_{kl}	Discount factor associated with the hub-to-hub travel cost along arc kl
δ_{kl}	discount factor associated with the hub-to-hub travel time along arc kl
F_k	Cost of locating a hub at node k
D_{ij}	Inelastic demand, the flow that should be sent from node i to node j
D'_{ij}	Elastic demand, the flow that should be sent from node i to node j
D_{ij}^{max}	Maximum possible demand, the maximum flow that can be sent from node i to node j
β_1	Sensitivity parameter of customers to service time
β_2	Sensitivity parameter of customers to service price
q_{ij}	Incumbent's price to flow from node i to node j
t'_{ij}	incumbent's time to flow from node i to node j
η_{ij}	Total utility of the incumbent's servicing system from node i to node j
CLL_j	Customer loyalty index for incumbent's services in node j ($0 \leq CLL_j \leq 1$)

Decision variables:

Y_k	1 if a hub at node k is located by the entrant; 0, otherwise
H_{ij}^m	1 if a direct connection by transportation mode m on arc ij ($i, j \in N$) is established by the entrant; 0, otherwise
$P_{ij/kl}$	Entrant's price to service the demand along arc ij , using intermediate hubs k and l
$x_{ij/kl}$	Fraction of the demand flow along arc ij through the entrant's hubs that are located at $k, l \in N$
z_{ij}	Fraction of the demand flow along arc ij through the incumbent's hubs
$c_{ij/kl}$	Variable cost of the flow between nodes i and j , using hubs $k, l \in N$
$T_{ij/kl}$	Response time for demand in node j from node i , using hubs $k, l \in N$

2.2. Generic model: Competitive Hub Location Pricing Problem with Transportation Modes

The competitive hub location pricing problem with transportation modes (CHLP-TM) can be formulated by:

$$Z = \text{Max} \sum_{i,j,k,l \in N} (P_{ij/kl} - c_{ij/kl}) D_{ij} x_{ij/kl} - \sum_{(i,j) \in A, m \in M} K_{ij}^m H_{ij}^m - \sum_{k \in N} F_k Y_k \quad (1)$$

s.t.

$$x_{ij/kl} = \frac{Y_k \cdot Y_l \cdot \sum_{m_1 \in M} H_{ik}^{m_1} \cdot \sum_{m_2 \in M} H_{kl}^{m_2} \cdot \sum_{m_3 \in M} H_{lj}^{m_3} \cdot \exp(-\beta_1 T_{ij/kl} - \beta_2 P_{ij/kl})}{\sum_{s,t \in N} Y_s \cdot Y_t \cdot \sum_{m_1 \in M} H_{ik}^{m_1} \cdot \sum_{m_2 \in M} H_{kl}^{m_2} \cdot \sum_{m_3 \in M} H_{lj}^{m_3} \cdot \exp(-\beta_1 T_{ij/st} - \beta_2 P_{ij/st}) + \eta_{ij}} \quad \forall i, j, k, l \in N \quad (2)$$

$$\eta_{ij} = \exp(-\beta_1 t'_{ij} - \beta_2 q_{ij}) \quad \forall i, j \in N \quad (3)$$

$$c_{ij/kl} = \sum_{m \in M} \varphi_{ik}^m d_{ik} H_{ik}^m + \sum_{m \in M} \varphi_{kl}^m \sigma_{kl} d_{kl} H_{kl}^m + \sum_{m \in M} \varphi_{lj}^m d_{lj} H_{lj}^m \quad \forall i, j, k, l \in N \quad (4)$$

$$T_{ij/kl} = \sum_{m \in M} \tau_{ik}^m d_{ik} H_{ik}^m + \sum_{m \in M} \tau_{kl}^m \delta_{kl} d_{kl} H_{kl}^m + \sum_{m \in M} \tau_{lj}^m d_{lj} H_{lj}^m \quad \forall i, j, k, l \in N \quad (5)$$

$$\sum_{m \in M} H_{ij}^m \leq 1 \quad \forall i, j \in N \quad (6)$$

$$Y_k \in \{0, 1\} \quad \forall k \in N \quad (7)$$

$$H_{ij}^m \in \{0, 1\} \quad \forall (i, j) \in A, m \in M \quad (8)$$

$$P_{ij/kl} \geq 0 \quad \forall i, j, k, l \in N \quad (9)$$

Objective function (1) maximizes the profit of the entrant's decisions. The first term of the objective function indicates the net profit of transportation services. The second and third terms indicate the fixed cost of transportation routes and the fixed cost of locating hubs, respectively. According to a logit model, Constraint (2) assigns the entrant's flows. Eq. (3) defines the utility of an incumbent's network for the demand that should be sent from origin i to destination j . Constraints (4) and (5) state the transportation costs and times over a route $i \rightarrow k \rightarrow l \rightarrow j$, respectively. Constraint (6) ensures that one transportation mode can be maximally assigned between every two nodes. Constraints (7) to (9) represent the domain of the decision variables.

Also, if we want to have a single-allocation hub-and-spoke network, Constraint (10) should be considered as well:

$$1 - Y_i \leq \sum_{\substack{j \in N, j \neq i \\ m \in M}} H_{ij}^m \leq 1 - Y_i + (|N| - 1) \cdot Y_i \quad \forall i \in N \quad (10)$$

2.3. Considering the effect of elastic demand

In the previous model (CHLP-TM), demand D_{ij} is assumed to be fixed at all demand points. Now, let us make a more realistic assumption that the demand at each node is affected by the price. There are different possible expressions for the expenditure function in the literature. As [Berman and Krass \(2002\)](#) proposed, elastic demand based on an exponential expenditure function is defined by:

$$W_i(U_i) = W_i^{min} + (W_i^{max} - W_i^{min}) \times (1 - \exp(-\rho_i \cdot U_i)) \quad (11)$$

where U_i is the utility of service i , and expenditure function $W_i(U_i)$ is a non-negative function of the utility vector, which is non-decreasing for all components of U_i . Also, W_i^{max} and W_i^{min} are the maximum and minimum demands of service i , respectively. $\rho_i > 0$ is a positive constant.

In Eq. (11), it is assumed that utility has an additive effect on demand. However, we assume that demand is dependent on price. Despite utility, the price has a decreasing effect on demand. Considering this point, we can customize Eq. (11) as follows:

$$D'_{ij} = D_{ij}^{min} + (D_{ij}^{max} - D_{ij}^{min}) \times (\exp(-\beta_2 \cdot \bar{P}_{ij})) \quad (12)$$

where \bar{P}_{ij} is the mean of the prices that can be proposed for the demand in node j from node i .

$$\bar{P}_{ij} = \frac{q_{ij} + \sum_{\substack{m_1, m_2, m_3 \in M \\ k, l \in N}} P_{ij/kl} Y_k Y_l H_{ik}^{m_1} H_{kl}^{m_2} H_{lj}^{m_3}}{1 + \sum_{\substack{m_1, m_2, m_3 \in M \\ k, l \in N}} Y_k Y_l H_{ik}^{m_1} H_{kl}^{m_2} H_{lj}^{m_3}} \quad \forall i, j \in N \quad (13)$$

Based on the assumption of single allocation of the hub network, there is just one unique route between each arbitrary pair of nodes i and j . Also, based on Eq. (10), the transportation mode for the mentioned route will be unique. So, the price for the paired nodes i and j will be unique.

$$\bar{P}_{ij} = \frac{q_{ij} + p_{ij}}{2} \quad \forall i, j \in N \quad (14)$$

Since the market was a monopoly before the presence of an entrant, we can assume that the inelastic demand (D_{ij}) in the previous model is related to the incumbent's price (q_{ij}). Therefore, the elastic demand (D'_{ij}) should consider the following conditions:

$$D'_{ij} = \begin{cases} D_{ij}^{max} & \text{if } \bar{P}_{ij} \rightarrow 0 \\ D_{ij} & \text{if } \bar{P}_{ij} = q_{ij} \\ D_{ij}^{min} & \text{if } \bar{P}_{ij} \rightarrow +\infty \end{cases} \quad (15)$$

It is also assumed that $D_{ij}^{min} = 0$. Considering $\bar{P}_{ij} = q_{ij}$ and replacing the second condition of (15) in (12), we will have:

$$D_{ij} = D_{ij}^{max} \times \exp(-\beta_2 \cdot q_{ij}) \Rightarrow D_{ij}^{max} = D_{ij} \times \exp(\beta_2 \cdot q_{ij}) \quad (16)$$

Finally, we can calculate the elastic demand by:

$$D'_{ij} = D_{ij} \cdot \exp(\beta_2(q_{ij} - \bar{P}_{ij})) \quad (17)$$

The competitive hub location and pricing problem with elastic demand and transportation modes (CHLP-ED/TM) problem can be formulated by:

$$Z = \max \sum_{i,j,k,l \in N} (P_{ij/kl} - c_{ij/kl}) D'_{ij} x_{ij/kl} - \sum_{(i,j) \in A, m \in M} K_{ij}^m H_{ij}^m - \sum_{k \in N} F_k Y_k \quad (18)$$

s.t.

Constraints (2) – (9), (12) and (13).

Moreover, if we want to have a single-allocation network in the CHLP-ED/TM, Constraints (12) and (13) should respectively be replaced by Constraints (17) and (14), and Constraint (10) should be added to the recent model.

2.4. Considering the effect of customer loyalty

Customer loyalty is the result of an affirmative perceived value of an experience, physical attribute-based satisfaction, or a consistently positive emotional experience that contains products or services. In this model, customer loyalty has been considered in determining the

market share of the entrant and incumbent. Hence, we characterize CLI_i as the loyalty index of the customer situated in node i to the incumbent's services. Therefore, we can modify Eq. (2) to Eq. (19).

$$x_{ij/kl} = \frac{(1 - CLI_i) \cdot (\sum_{m_1, m_2, m_3 \in M} Y_k Y_l H_{ik}^{m_1} H_{kl}^{m_2} H_{lj}^{m_3}) \cdot \exp(-\beta_1 T_{ij/kl} - \beta_2 P_{ij/kl})}{(1 - CLI_i) \cdot \sum_{\substack{m_1, m_2, m_3 \in M \\ s, t \in N}} Y_s Y_t H_{is}^{m_1} H_{st}^{m_2} H_{tj}^{m_3} \cdot \exp(-\beta_1 T_{ij/st} - \beta_2 P_{ij/st}) + CLI_i \cdot \eta_{ij}} \quad (19)$$

Note that if $CLI_i = 0.5$, Eq. (19) will be returned to Eq. (2) again. That is while for $CLI_i < 0.5$, where the customer has low satisfaction to the incumbent services, more flows of demand can be absorbed by the entrant. On the other hand for $CLI_i > 0.5$, the entrant hardly can increase its market share because of the customer's loyalty to the incumbent services.

Finally, to model the competitive hub location and pricing problem considering elastic demand and customer loyalty (CHLP-ED/CL/TM), we can reformulate (18) subject to Eqs. (3) – (9), (13), (17) and (19).

3. Solution approach

The mathematical models proposed in the previous section are NP-hard. Even without pricing decisions, solution of a hub location problem is very complex. Therefore, considering pricing decisions and related nonlinear terms in the model increases problem complexity. The literature has proposed some hybrid approaches to solve such models. L uer-Villagra and Marianov (2013) decomposed their proposed competitive hub location pricing model to a master problem and a sub-problem with a bi-level structure. The master problem was related to hub-and-spoke location decisions, and the sub-problem was related to pricing decisions based on a multinomial logit (MNL) function. Consequently, they proposed a hybrid solution approach based on a genetic algorithm (GA) to solve a single-level reduced model. Zhang (2015) presented a competitive location and pricing model for a retailer based on an MNL function, and proposed a two-phase solution framework based on decomposition that contains two major components, location and pricing problems. They examined the performance of three pricing heuristics, including a path-following approach (PF), a gradient descent method (GD), and a gradient descent method with multiple random starting points (GDM), and three location heuristics, including a tabu search procedure (TS), greedy search (GS), and GA. Based on the computational experiments, the two hybrid approaches (i.e., GA+PF and TS+PF) outperformed the other approaches.

Bi-level decomposition algorithms are extensively used to solve certain nonconvex large-scale optimization problems. Based on the literature, decomposing the location and pricing model to a bi-level structure and interactively solving these sub-problems are the main alternative for tackling such problems. As a result, we can transform the proposed hub location and pricing

models into a bi-level structure in which a pricing problem serves as a sub-routine for the location problem. This structure is represented in Fig. 1.

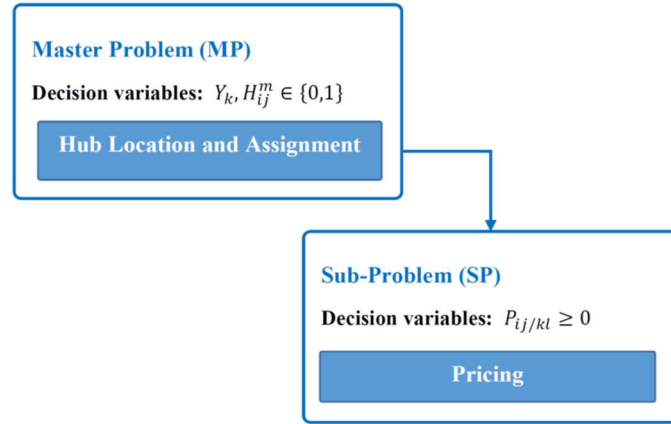


Fig. 1. Decomposition of the CHLP problem

Accordingly, we are faced with a bi-level model. There are various approaches to solving bi-level problems. For a complete literature review on bi-level optimization problems (BOPs), see [Sinha et al. \(2017\)](#). These authors reviewed the classical and evolutionary approaches to solving BOPs. They emphasized evolutionary methods because of a high degree of difficulty in real-world applications that usually lead to failure of the classical methods for solving BOPs. Hence, using heuristic/meta-heuristic and classical methods simultaneously (in a nested approach) is the main alternative for tackling BOPs ([Talbi, 2013](#)). Descriptions of some successful applications of the mentioned approach for solving multi-level problems can be found in ([Parvasi et al., 2017](#); [Akbari-Jafarabadi et al., 2017](#); [Fard & Hajaghaei-Keshteli, 2018a](#); [Fard & Hajaghaei-Keshteli, 2018b](#)).

In the next section, a scatter search (SS) algorithm for the hub location problem is designed. Because of different concavity conditions in the pricing problems of the CHLP-TM and CHLP-ED/CL/TM, we discuss them separately, while the proposed approach to solve the hub location problem is same. Hence, an exact method for the pricing problem of the CHLP-TM and a matheuristic approach (named EGPSDE) for the pricing problem of CHLP-ED/TM and CHLP-ED/CL/TM are suggested. [Table 2](#) presents a general view of the combination of the proposed approaches to solve the problem.

Table 2. Combination of the proposed solving approaches

Model types	Master problem (Hub location and assignment)	Sub-problem (Pricing)
CHLP-TM	Scatter search	Extracted closed-form expression
CHLP-ED/TM	Scatter search	Metaheuristic (EGPSDE)
CHLP-ED/CL/TM	Scatter search	Metaheuristic (EGPSDE)

3.1. Solution approach for the CHLP-TM model

3.1.1. Theoretical discussion

For each solution generated in the master problem (i.e., the values $\{\hat{Y}_k\}_{k \in N}$ and $\{\hat{H}_{ij}^m\}_{(i,j) \in A}$), we can define S_{ij} as the set of feasible pairs of hubs (k, l) that can connect the origin-destination (O-D) pair (i, j) , that is:

$$S_{ij} = \left\{ (k, l) \in N^2 \mid \exists m_1, m_2, m_3 \in M, \hat{Y}_k = \hat{Y}_l = \hat{H}_{ik}^{m_1} = \hat{H}_{kl}^{m_2} = \hat{H}_{lj}^{m_3} = 1 \right\} \quad (20)$$

Substituting Eq. (2) in Eq. (1), and using Eq. (20), the objective function of the entrant's pricing problem in the sub-problem is:

$$\text{Max}_P f(\hat{Y}, \hat{H}, P) = \sum_{i,j \in N} \frac{D_{ij} \sum_{(k,l) \in S_{ij}} (P_{ij/kl} - c_{ij/kl}) \exp(-\beta_1 T_{ij/kl} - \beta_2 P_{ij/kl})}{\sum_{(s,t) \in S_{ij}} \exp(-\beta_1 T_{ij/st} - \beta_2 P_{ij/st}) + \eta_{ij}} - \omega \quad (21)$$

where ω is a constant value that can be obtained from Eq. (22).

$$\omega = \sum_{(i,j) \in A, m \in M} K_{ij}^m \hat{H}_{ij}^m + \sum_{k \in N} F_k \hat{Y}_k \quad (22)$$

The entrant's optimal prices can be derived based on the first-order conditions that are represented in the following proposition.

Proposition 1. The optimal price of the entrant's service for each route $i \rightarrow k \rightarrow l \rightarrow j$ can be obtained from Eq. (23).

$$P_{ij/kl}^* = c_{ij/kl} + \frac{1}{\beta_2} \left\{ 1 + W_0 \left[\frac{1}{\eta_{ij}} \sum_{(s,t) \in S_{ij}} \exp(-\beta_1 T_{ij/st} - \beta_2 c_{ij/st} - 1) \right] \right\} \quad (23)$$

where W_0 is the principal branch of the Lambert W function, which is defined as the inverse function of $f(\zeta) = \zeta e^\zeta$.

Proof. Lüer-Villagra and Marianov (2013) derived a closed-form expression for optimal pricing in the case of multiple bundles (i.e., hub pairs) with a single attribute (i.e., price) in the MNL utility function. Our proof and formula are an extension of the case of a multi-attribute utility function considering an extra attribute (i.e., non-price option). The objective function (21) can be separated in some independent expressions for each origin-destination pair (i, j) . Hence, the expression (24) for a special route $i \rightarrow s \rightarrow t \rightarrow j$ can be obtained based on using the first-order conditions $\frac{\partial Z}{\partial P_{ij/kl}} = 0; \forall i, j, k, l \in N$.

