



**HAL**  
open science

## Research problems

Jean-Claude Bermond

► **To cite this version:**

Jean-Claude Bermond. Research problems. *Discrete Mathematics*, 1988, 71 (3), pp.275-276.  
10.1016/0012-365X(88)90107-0 . hal-03203456

**HAL Id: hal-03203456**

**<https://hal.science/hal-03203456>**

Submitted on 20 Apr 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## RESEARCH PROBLEMS

**Problem 97.** Posed by J.-C. Bermond.

Correspondent: J.-C. Bermond,  
L.R.I., bât. 490,  
Université Paris Sud,  
91405 Orsay,  
France.

Let  $G$  be a regular graph with edge-set  $E(G)$ . The graph  $G$  is said to have a Hamilton decomposition if either its degree is even and  $E(G)$  can be partitioned into Hamilton cycles or its degree is odd and  $E(G)$  can be partitioned into Hamilton cycles and a single 1-factor.

**Conjecture.** If  $G$  has a Hamilton decomposition, then its line graph can be decomposed into Hamilton cycles.

There are a few results dealing with the conjecture. A. Kotzig has proved that if  $G$  is a cubic graph with a Hamilton decomposition (i.e. it has a Hamilton cycle), then its line graph can be decomposed into two Hamilton cycles. Jaeger [1] proved that if  $|E(G)|$  is even and  $G$  can be decomposed into Hamilton cycles, then the line graph of  $G$  has a 1-factorization. The key to the proof lies in the following result: If  $G$  can be decomposed into two Hamilton cycles, then its line graph can be decomposed into three Hamilton cycles.

The case when  $G$  is the complete graph seems to be unsolved. It is a particular case of a conjecture of A. Kotzig saying that the complete graph has a compatible eulerian decomposition.

### Reference

[1] F. Jaeger, The 1-factorization of some line graphs, *Discrete Math.* 46 (1983) 89–92.

**Problem 98.** Posed by J.-C. Bermond.

Correspondent: J.-C. Bermond,  
L.R.I., bât. 490,  
Université Paris Sud,  
91405 Orsay,  
France.

A digraph is said to be *strong* if for any ordered pair  $(u, v)$  of vertices, there is a directed path from  $u$  to  $v$ . The *independence number* of a digraph  $G$ , denoted  $\alpha(G)$ , is the maximum cardinality of a set of vertices no two of which are joined by an arc of  $G$ .

**Conjecture.** The minimum number of directed cycles needed to cover the vertices of a strong digraph  $G$  is less than or equal to the independence number of  $G$ .

The conjecture is true when  $\alpha(G) = 1$  because  $G$  contains a strong tournament. An old theorem of Camion states that a strong tournament has a directed Hamilton cycle. The conjecture is also true for  $\alpha(G) = 2$  by a result of Chen and Manalastas [1].

Let  $\alpha'(G)$  denote the maximum cardinality of an induced subdigraph without any directed cycles. The conjecture is true when  $\alpha(G)$  is replaced with  $\alpha'(G)$  [2]. See [3] for a recent survey dealing with the conjecture and generalizations.

## References

- [1] C.C. Chen and P. Manalastas, Every finite strongly connected digraph of stability 2 has a hamiltonian path, *Discrete Math.* 44 (1983) 243–250.
- [2] M.C. Heydemann, Minimum number of circuits covering the vertices of a strong digraph, *Annals Discrete Math.* 27 (1985) 287–296.
- [3] B. Jackson and O. Ordaz, Chvátal–Erdős conditions for paths and cycles in graphs and digraphs, a survey. Preprint.