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Statistics of jet drop production

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Key Points:

- We perform numerical simulations of the two-phase Navier-Stokes equations to resolve bubble bursting and jet drop production.
- We describe the jet drop distribution for a wide range of bubble conditions, with droplet production from 0.5 to 500 microns.
- The jet drop distribution from bubbles under breaking waves is compatible with laboratory measurements of drop distribution.

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Abstract

Bubbles bursting at the ocean surface are an important source of sea-spray aerosol. We describe jet drop production, from ensembles of high fidelity numerical simulations of bubble bursting, validated against experimental results. The number of jet drops, their size and velocity are controlled by the ratio of the bubble size, R_b , and the visco-capillary length, $l_\mu = \rho_w \gamma R_b / \mu_w^2$, where γ is the surface tension, ρ_w , μ_w the water density and viscosity. The mean drop size follows $\langle r_d \rangle \propto (R_b / l_\mu)^{5/4}$ and the ejected number of drops $n \propto (R_b / l_\mu)^{-1/3}$, accounting for temperature variations. We confirm that sub-microns jet drops are produced by bubbles in the 10-40 microns range. We compute the distribution of jet drops formed by a range of bubbles present under a breaking wave which compares well against laboratory experiments. We discuss the applicability of the proposed formulation in the context of sea spray generation function.

1 Introduction

Knowledge of the size production flux of primary ocean spray droplets and aerosol particles and its dependence on meteorological and environmental variables is necessary for modeling cloud microphysical properties and the influence of aerosol on radiative processes (de Leeuw et al., 2011; Quinn et al., 2015; Bertram et al., 2018). Biases and uncertainties in predicting sea spray aerosols are related to a lack of fundamental understanding in the production processes of aerosols, and the large range of scales involved going from wave statistics at O(1km to 1m), to the breaking dynamics at O(1-10m), air bubble entrainment and bursting at O(microns to mm); which directly impacts our ability to perform weather prediction and earth system modeling (de Leeuw et al., 2011; Deike et al., 2017; Deike & Melville, 2018). Ocean spray is composed of small liquid droplets formed through two main pathways (Veron, 2015): the spume drops, produced from the tearing of breaking wave crests by strong winds (Veron et al., 2012; Ortiz-Suslow et al., 2016; Troitskaya et al., 2018) and bubble bursting, itself decomposed into the film and jet drops production (Cipriano et al., 1983; Spiel, 1997; Lhuissier & Villermaux, 2012; Ghabache et al., 2014). These droplets transport water, heat dissolved gases, salts, surfactants, and biological materials between the ocean and atmosphere and these air sea-fluxes are dependent on the drop size distributions (Mueller & Veron, 2014; Peng & Richter, 2020), while their chemical composition is affected by the production mechanisms (Cochran et al., 2017; Wang et al., 2017).

The characterization of all drops requires a statistical approach, performing ensemble of experiments, or simulations of the same bubble bursting process. We present such statistical description of jet drops being ejected by bursting bubbles, including the mean number and size of jet drops being ejected, as well as their size distribution, controlled by the bubble size and water physical properties. This yields simple relationships for the resulting jet drops generated by a typical bubble size distribution under a breaking wave.

The production of droplets due to bubbles bursting at the surface of a liquid has long been considered as a fundamental mechanism controlling larger-scale fluxes between the liquid and the gas, and its importance on aerosol generation (Cipriano et al., 1983). Blanchard (1954); Blanchard and Woodcock (1957) measured the size and height of the ejected droplets, and Spiel (1994, 1997) reported on the statistics of number and sizes of the droplets produced by bubble bursting in both salt and tap water. Using high speed cameras (Ghabache et al., 2014, 2016; Ghabache & Séon, 2016; Brasz et al., 2018) have provided detailed experimental data on the size and velocity of the first droplet. These experimental developments on the dynamic of single bubble bursting have been accompanied by progresses in theoretical and computational description. Duchemin et al. (2002) presented the first numerical simulation of

the axisymmetric bursting of a single bubble solving for the two-phase Navier-Stokes equations. They demonstrated that the main controlling parameter of the speed and size of the first ejected droplet is given by the ratio of the bursting bubble size, R_b , and the visco-capillary length $l_\mu = \mu_w^2/(\rho_w\gamma)$, where μ_w is the water viscosity, ρ_w the water density and γ the surface tension. This ratio is called the Laplace number:

$$\text{La} = \frac{R_b}{l_\mu} = \frac{\rho_w\gamma R_b}{\mu_w^2}. \quad (1)$$

Duchemin et al. (2002) showed that there exists an optimal value $\frac{R_b}{l_\mu} \approx 1000$, (between $R_b = 10$ to 20 microns in salt water at 20°C) for which the capillary focusing is the most efficient and leads to the ejection of very fast and small droplets (several tens of m/s for air in water, and drops of $O(1\mu\text{m})$). Deike et al. (2018); Brasz et al. (2018); Lai et al. (2018) demonstrated the universal behavior of the first ejected droplet size and velocity, for a range of Laplace numbers from 900 to 100000, which correspond to the range of existence of jet drops in water (10 microns to 3mm at 20°C). The experimental and numerical results were used by Gañán-Calvo (2017) to propose scaling laws able to describe the size r_{d1} and velocity V of the first droplet, based on the inertio-capillary focusing of the capillaries along the cavity, and balance of forces during the cavity reversal and jet formation (with $V_\mu = \gamma/\mu$):

$$V/V_\mu = k_v \left(\text{La} \left(\text{La}_*^{-1/2} - \text{La}^{-1/2} \right) \right)^{-3/4} \quad (2)$$

$$r_{d1}/l_\mu = k_r \left(\sqrt{\text{La}} \left(\sqrt{\frac{\text{La}}{\text{La}_*}} - 1 \right) \right)^{5/4}. \quad (3)$$

where $\text{La}_* \approx 550$ is the drop ejection threshold, (about 5 to $10\mu\text{m}$ drops in salt water at 20°C), while k_r and k_v are empirical $O(1)$ constants fitted to the data. The asymptotic scaling at high La , for the first drop (or jet) velocity, is then $V/V_\mu \propto \text{La}^{-3/4}$, and for the first drop size, $r_{d1}/l_\mu \propto \text{La}^{5/4}$. It is important to realize that these scaling laws have certain limits. They do not capture the non-monotonic behavior of drop size and velocity below the optimal Laplace number, but are extremely valuable from a practical point of view. Gordillo and Rodríguez-Rodríguez (2019); Blanco-Rodríguez and Gordillo (2020) proposed an alternative set of scaling laws based on the ideas of capillary focusing and wavelength selection during the cavity collapse, combined with an inertio-capillary balance. The results are essentially compatible with eq. 4 and 5 at high La , but the treatment of $\text{La} \approx 1000 \sim \text{La}_c$ is different, and their formula reads

$$V/V_\mu \propto \text{La}^{-1} \left(1 - \left(\sqrt{\frac{\text{La}_c}{\text{La}}} \right)^{1/4} \right)^{-3/2}, \quad (4)$$

$$r_{d1}/l_\mu \propto \text{La} \left(1 - \left(\sqrt{\frac{\text{La}_c}{\text{La}}} \right)^{1/4} \right). \quad (5)$$

The effect of gravity on the bubble shape and selecting the capillaries has been discussed by Ghabache et al. (2014); Deike et al. (2018) and leads to corrections on the drop size and velocity once $R_b > 0.4l_c$, where $l_c = \sqrt{\gamma/(\Delta\rho g)}$ is the gravity-capillary length, which is 2.7mm in water at 20°C .

In Berny et al. (2020), we performed axi-symmetric numerical simulations of jet drop production by bubble bursting for a wide range of parameters, and extended the analysis to the subsequent droplets to describe the number and size of all jet droplets produced when a cavity collapses. These simulations showed that between 1 to 14 drops are ejected depending on La and $Bo = (R_b/l_c)^2$. The ejection speed of the droplets decreases as the production number increases, while the size of the droplets can be either smaller or larger than the first drop. The results are limited by the

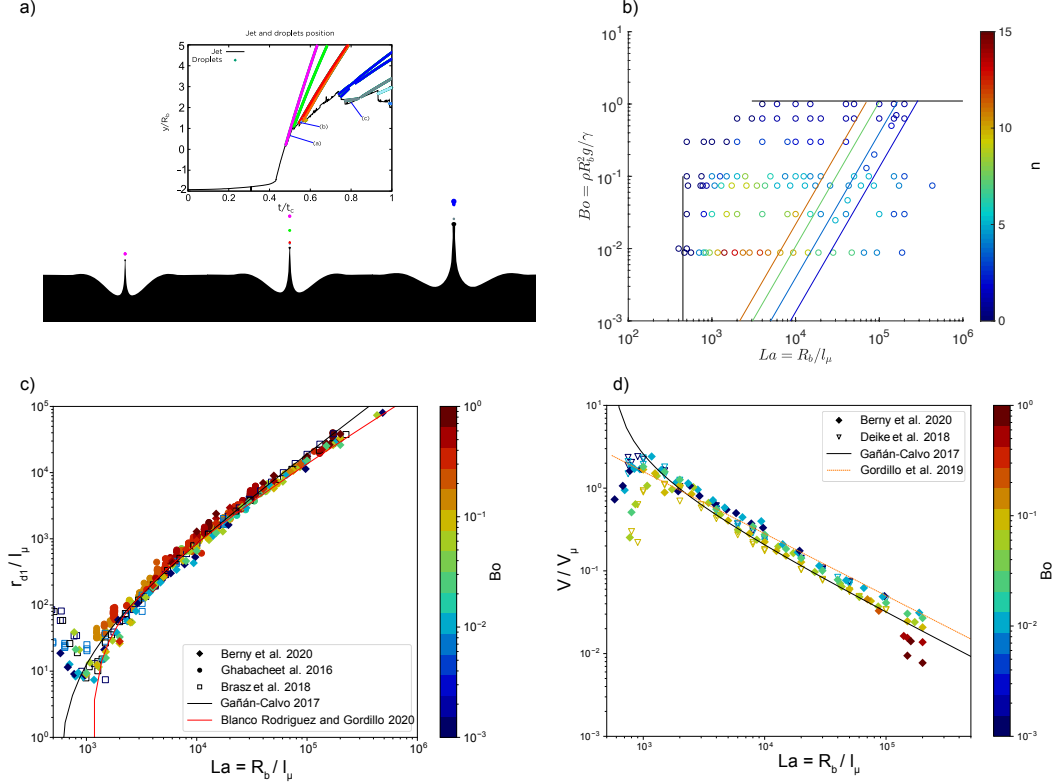


Figure 1. Jet drop dynamics and characteristics. (a) Numerical simulations of the two-phase flow process can be performed, and capture for a given initial cavity all jet drops being produced (Bery et al., 2020). Inset shows the position of the tip of the jet (black) and droplet position (the ejected drop being color-coded). (b) Phase diagram of existence of jet drops in terms of controlling parameters, R_b/l_μ and R_b^2/l_c^2 , from Bery et al. (2020). The number of ejected droplet is color-coded. The colored lines correspond to salt water conditions at 4, 10, 20 and 28°C. For $R_b^2/l_c^2 > 1$, no jet drops are being produced. (c) The first drop size, normalized by the visco-capillary length, r_{d1}/l_μ , as a function of the normalized bubble size R_b/l_μ . The first drop size can be described by a universal scaling (eq. 3) and all data collapse onto a single curve. Symbols are simulations and laboratory experiments from Ghabache et al. (2016); Brasz et al. (2018); Deike et al. (2018); Bery et al. (2020). (d) Same data for the jet drop velocity V/V_μ as a function of R_b/l_μ . Again, all data collapse onto a single curve and can be described by a universal scaling (eq. 2).

sensitivity of the subsequent drop properties to noise in the system, which was already discussed by Spiel (1994, 1997); Ghabache et al. (2014, 2016), with the existence of bi-modal distribution for subsequent drop size distribution.

We introduce variability in the initial conditions through a (axi-symmetric) random white noise in the liquid velocity, with a magnitude of 1% of the typical jet velocity. We show that the resulting droplet statistics compares well with the data from Spiel (1994, 1997) (§2). We discuss the drop size distributions for a range of non-dimensional control parameters, which accounts for bubble bursting at the ocean surface, considering typical ocean water temperature and their associated density, viscosity and surface tension (§3). Next, we discuss the resulting drop size distribution when considering the typical size distribution of bubbles under a breaking waves (§4). Finally, in §5, we compare the results to existing data set obtained in laboratory settings and discuss the assumptions made in this model, in particular the minimal and maximal bubble size and their implications for aerosol production.

2 Ensemble of numerical simulations of bubble bursting with noisy initial conditions.

2.1 Numerical methods

We perform numerical simulations of the two-dimensional axi-symmetric, incompressible two-phase Navier-Stokes equations (air and water) with surface tension. We use the free software Basilisk which is based on a spatial adaptive quad-octree grid allowing to save computational time while resolving the different length scales of the problem (Popinet, 2009; van Hooft et al., 2018). It is based on a momentum conserving scheme and a geometric Volume of Fluid (VOF) method to reconstruct the interfaces (Popinet, 2009, 2018) between the high density liquid (water) and the low density gas (air). These methods have been extensively described and validated in studies exploring complex multiphase flow, in particular wave breaking (Deike et al., 2016; Mostert & Deike, 2020) and bubble bursting (Deike et al., 2018; Lai et al., 2018; Berny et al., 2020).

The physical parameters of the problem are the bubble radius R_b , liquid and air density $\rho_{w,a}$ and dynamic viscosity $\mu_{w,a}$, surface tension γ and gravity g . The initial condition is given by the shape of a bubble at rest below an horizontal interface, obtained by the balance of forces described by the Young–Laplace equation (Toba, 1959; Lhuissier & Villiermaux, 2012; Ghabache et al., 2014) which we solve numerically (Berny et al., 2020). The problem is then controlled by the Laplace number, which compares the bubble size to the visco-capillary length $La = R_b/l_\mu$ and the Bond number, which compares the bubble size to the gravity-capillary length $Bo = (R_b/l_c)^2$. Figure 1a shows an example of bubble bursting simulation, displaying cavity collapse, jet formation and drop production. It is important to note that this approach solves the native two-phase flow equations without any sub-grid scale model. We use an adaptive mesh, critical to the success of these simulations as it allows an equivalent resolution of up to 4^{14} grid points (equivalent to 8192^2 and corresponding to 1638 grid points per bubble diameter, with the smallest grid size in each direction being $\Delta = R_b/3276$). More details of the numerical setup and validation are provided in Deike et al. (2018); Berny et al. (2020). These simulations resolve all relevant length scale without sub-grid scale models with the bursting process assumed axi-symmetric, as confirmed by laboratory experiments when the water and air are initially at rest.

The individual simulations are similar to Berny et al. (2020), which analyzed the number, size and velocity of ejected drops for various La and Bo . The first jet drop properties are summarized in figure 1(b,c,d), using numerical data from Berny et al. (2020); Deike et al. (2018); Brasz et al. (2018) and experimental data from Ghabache

et al. (2014, 2016); Brasz et al. (2018). The existence of jet drops as a function of La and Bo is shown in figure 1b together with the number of ejected drops in each conditions (Berny et al., 2020). The first drop size and velocity are shown in figure 1c and 1d, with excellent agreement between all data sets and with the scalings of eq. 2 and eq. 3. We note the slow-down of the drop velocity due to gravity for $Bo > 0.2$ ($R_b > 0.4l_c$).

In order to obtain a statistical ensemble of simulations for each configuration, i.e. for each La , Bo , we seed the water with random white noise of small amplitude. The amplitude of the noise is typically a few percent of the (expected) jet velocity, so that it presents a perturbation to the flow. We verified that the results are not sensitive to the particular form of the random noise imposed on the velocity field, as long as the perturbation is below a few percent of the characteristic ejection velocity.

We perform ensembles of 120 members for each conditions of interest. We consider 10 cases, with the Laplace number going from 1000 to 250000, and the associated Bond number in salt water at $28^\circ C$ ($10\mu m$ to $3mm$ corresponding to $Bo \rightarrow 0$ to $Bo=1$). For each ensemble, the data collection is as follows: each run is processed individually, and the number, size and velocity of each ejected drop is measured at ejection. All the data from a single ensemble are then gathered, leading to the definition of probability of existence and size distribution of each drop. The data set is also analyzed by gathering all runs together to define the ensemble averaged ejected number, drop size and size distribution.

2.2 Validation of the jet drop statistics against experimental data

We validate our numerical ensemble approach by comparison with the experimental data set from Spiel (1994, 1997), which are the only experimental studies that have considered the statistics of all ejected droplets. We perform ensembles of simulations corresponding to the data set presented by Spiel (1994, 1997) in both tap water and salt water.

The size distribution of each ejected droplet can then be analyzed and compared to the experiments from Spiel (1994, 1997), as shown in Fig. 2(a to f) with bubble radii going from $350\mu m$ to $1215\mu m$, with a water temperature of $28^\circ C$. We show the probability of existence of the n^{th} droplet (fig. 2a,c,e), as well as the distribution of drop size for the first 6 droplets (fig. 2b,d,f). The agreement in terms of probability of existence for all three cases (fig. 2a,c,f) is remarkable, showing that our setup, despite its strong simplification is able to reproduce the mean number of drops ejected by a bursting bubble in the laboratory for salt water conditions. For the smallest bubble (fig. 2a,b), the size distribution of the six drops are also very good, the numerical simulations capturing the size and statistics of all cases. The agreement is still reasonable for the two other cases with larger bubble size (fig. 2c,d), with a drop size difference between experiment and numerical simulation around 20%, while the simulations are capturing the bi-modal distribution of the droplet 2 and 3. Ensembles of simulations were carried out to reproduce the other cases from Spiel (1994, 1997) and similar agreement was observed, with good agreement in terms of mean number of ejected drop, and reasonable agreement in terms of drop size statistics, with variations of about 20% when considering individual drops reported by Spiel (1994, 1997).

Given the large uncertainties remaining in the sea spray production processes, and the relatively small size of the experimental samples, this is deemed reasonable. We conclude that our simulations are capturing the main properties of the ejected jet drops: the first drop size and velocity is very well captured, together with the mean total number of drops for particular conditions, as well as the size of the subsequent droplets. Moreover, our methodology has the advantage of being easy in terms of computations (compared to performing the corresponding more than 1000 bubble bursting

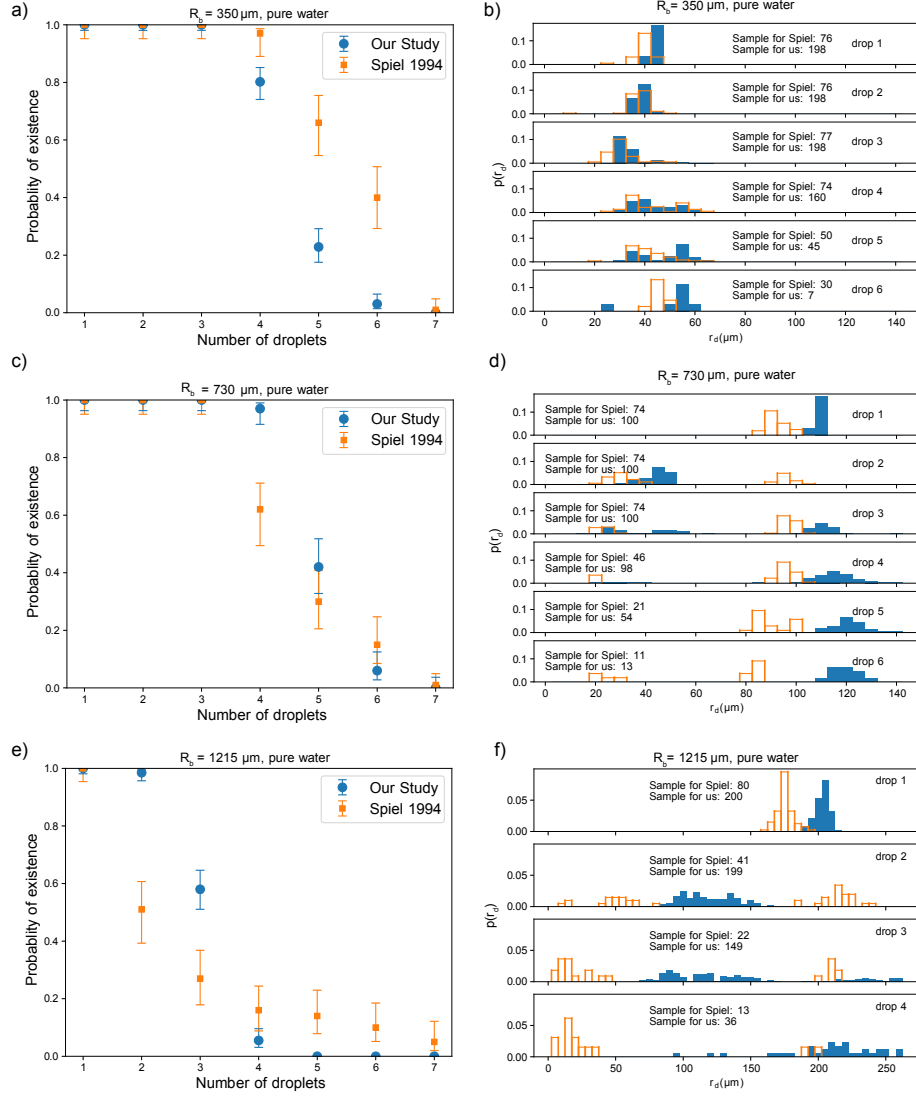


Figure 2. Ensemble of simulations and comparison with experimental data. (a to f) Probability of existence of the n -th drop from the simulations, compared to Spiel’s data, for the $R_b = 354$ (a), 728 (c) and $1213 \mu\text{m}$ (e) (at 28°C); and probability distribution of the first six drops (b,d,f). Good agreement is observed in terms of probability of existence together with reasonable agreement for the ejected droplet distribution between the simulations and experimental data. This validates our ensembles of simulations to study the statistics of jet drops.

experiments). We should also keep in mind that both the simulation setup and the laboratory experiment in quiescent condition represent an idealization of the bursting bubble process occurring at the surface of the ocean. The cavity collapse occurs over a time scale of ms to s, and the possible influence of the atmospheric or oceanic turbulent boundary layers on the bursting process and jet destabilization have not yet been characterized. The role of the surrounding bubbles, both just below and at the surface also remain to be fully understood.

3 Number, mean size and distribution of jet drops for ocean water

From the practical point of view of ocean sea spray production, the universal control of the Laplace number, $La = R_b/l_\mu$ on the drop properties (size and velocity) allows to naturally consider variations in temperature and salinity in the ocean, through their influence on the density, viscosity and surface tension.

We analyze the jet drop statistics obtained from the ten ensembles described above, corresponding to non-dimensional bubble size R_b/l_μ from 1000 to $2.5 \cdot 10^5$. As shown in fig. 1b, this range corresponds to bubbles in salt-water, from $10\mu\text{m}$ to close to the capillary size, 2.7mm , and cover the full range of jet drop production in ocean water. We consider the ratio R_b/l_μ as the main controlling parameter as the Bo number only plays a role for bubbles close to the capillary size and will mainly serves as upper cut-off of our scaling relationships, with $R_b/l_\mu > \approx 10^5$ corresponding to $Bo \approx 1$ and the limit of jet drop production.

Figure 3a shows the number of droplets ejected as a function of the bubble Laplace number, R_b/l_μ , aggregating numerical data from (Berny et al., 2020), the ensemble performed here, as well as experimental data from Spiel (1994); Ghabache (2015). The agreement between simulations and experiments is excellent. The total number of drops is about 15 for $R_b/l_\mu = 1500$ and decreases through an algebraic relationship until $R_b/l_\mu = 10^5$,

$$n(R_b) = \chi \left(\frac{R_b}{l_\mu} \right)^{-1/3}, \quad (6)$$

with $\chi = 145$ a non-dimensional parameter adjusted to the data. For $R_b/l_\mu > 10^5$, a sharp decrease is observed, with one drop produced at $R_b/l_\mu = 2 \cdot 10^5$ and no drop above. This is related to the effect of gravity for large bubbles and corresponds to $R_b/l_c = 1$, or $R_b \approx 2.7\text{mm}$, above which gravity prevents drop formation, as illustrated in fig. 1b.

The relationship, eq. 6, can be used to compute the number of drops ejected as a function of the bubble size for given water properties, mainly controlled by salinity and temperature in the ocean. An example of the resulting curves is shown in figure 3b, where we have considered l_μ for salt water at temperatures of 4°C to 28°C . The water temperature shifts the maximum number of drops ejected from about $20\mu\text{m}$ at higher temperature to about $100\mu\text{m}$ at lower temperature, due to the increase in water viscosity when the temperature is decreased. The cut-off visible in figure 3b corresponds to the effect of gravity preventing drop ejection for $R_b/l_c > 1$.

Figure 3c shows the first moment of the drop size distribution, the mean droplet size $\langle r_d \rangle = \int r_d n_d(r_d) dr_d$ as a function of the bubble radius R_b/l_μ , together with the scaling relationship eq. 3 (and 5) of the first drop (shown in figure 1). The mean droplet size increases with the bubble size, and follows closely the scaling observed for the first drop. The scaling can be simplified and for the full range of non-dimensional bubble sizes, $900 < R_b/l_\mu < 3 \cdot 10^5$, can be written as,

$$\frac{\langle r_d \rangle}{l_\mu} = \eta \left(\frac{R_b}{l_\mu} \right)^{5/4}, \quad (7)$$

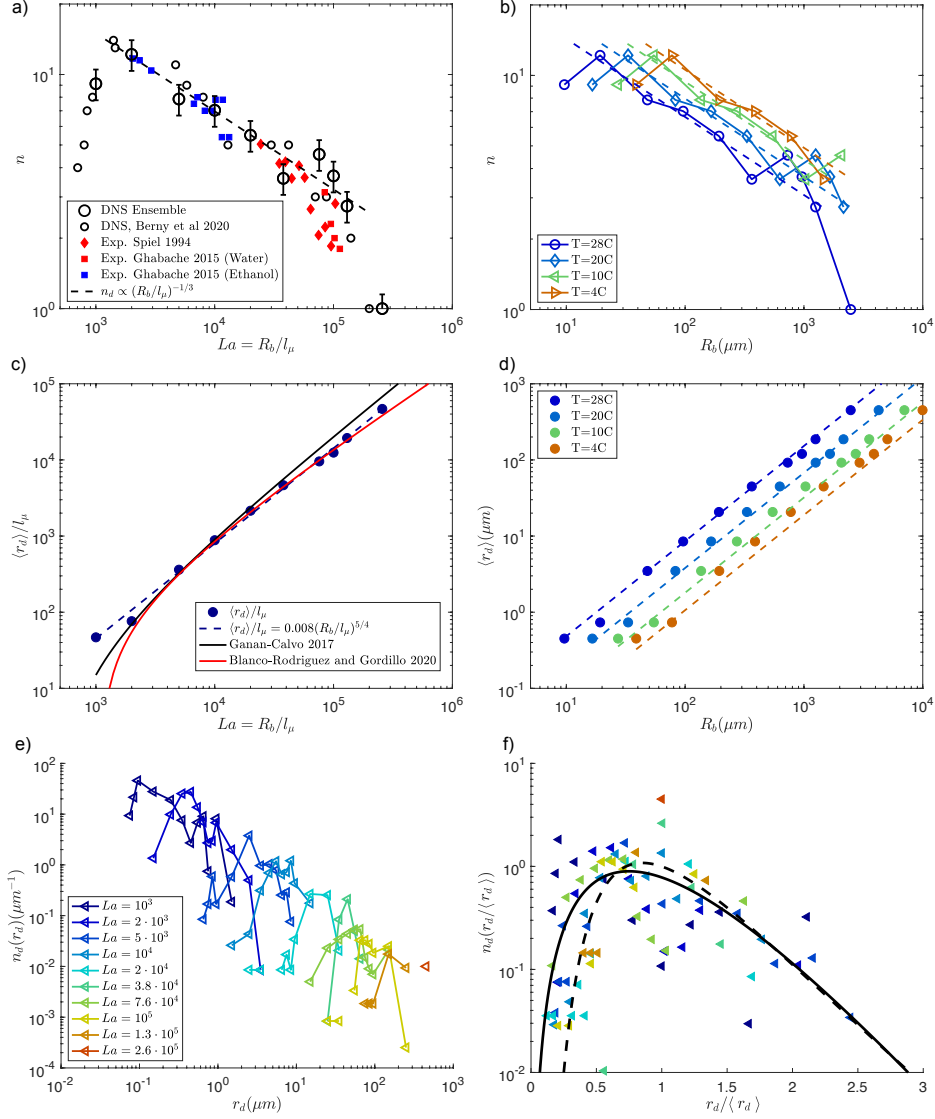


Figure 3. (a) Number of jet drops ejected, as a function of the non-dimensional bubble size R_b/l_μ . In a) we superpose the experimental data from Spiel (1994, 1997); Ghabache (2015) for water as well as more viscous fluids allowing to explore smaller values of R_b/l_μ (only data with $Bo < 0.05$ are considered to be in the regime where only R_b/l_μ controls the drop production). Data can be described as $n(R_b) = \chi \left(\frac{R_b}{l_\mu} \right)^{-1/3}$. (b) Numerical data presented as a function of the dimensional R_b for 4 different water conditions, calculated from (a) using the water properties at these temperatures to compute l_μ , showing that changes in temperature shift the drop production efficiency to larger sizes. (c) Mean ejected drop size $\langle r_d \rangle / l_\mu$ as a function of R_b / l_μ . The data can be described by $\frac{\langle r_d \rangle}{l_\mu} = \eta \left(\frac{R_b}{l_\mu} \right)^{5/4}$, close to the observed scaling for the first drop size. (d) Same data expressed in dimensional units using the water properties at these temperatures to compute l_μ . (e) Size distribution of ejected droplets obtained for the ten ensembles considered, using l_μ at $T=28^\circ C$, so that bubble radii vary from 10 to $2400 \mu m$. The distribution of ejected droplet size varies from 0.1 to $100 \mu m$. (f) Normalized distributions, with a Gamma distribution of order 4 superimposed (solid black line) and a log-normal distribution (dashed black line).

with $\eta = 0.008$ a non-dimensional coefficient adjusted to the data.

Figure 3d shows the same data for temperatures between 4 to 28°C, which illustrates that in the range of production, larger drops will be produced at lower temperatures, as a consequence of the increased viscosity.

Figure 3e shows the corresponding size distribution of ejected drops, for the ten ensembles. For simplicity, we consider here the dimensional variables for the 28°C configuration, so that bubble sizes range from 10µm to 2.5mm. We write the size distribution $n_d(r_d) = n(R_b)p(r_d, R_b)$, where $n(R_b)$ is the total number of drops ejected for a bubble of size R_b (shown in figure 3b, T=28°C) and $p(r_d, R_b)$ is the (normalized) probability distribution function of drop size ejected by the bubble of size R_b . As already discussed, we observe that smaller drops are generated by smaller bubbles, while the distribution appear as skewed exponentials, as a consequence of the jet fragmentation process.

Following the work of Villermaux (2020) on drop formation by fragmentation and Lhuissier and Villermaux (2012) on film drop production by bubble bursting, we seek to represent the drop size distribution of each ensemble by a Gamma distribution, which is a two-parameter distribution controlled by the mean drop size $\langle r_d \rangle$ and its order m , which in the fragmentation context correspond to the rugosity of the jet before fragmentation:

$$\mathcal{P}(r_d/\langle r_d \rangle) = \frac{m^m}{\Gamma(m)} \left(\frac{r_d}{\langle r_d \rangle} \right)^{m-1} e^{-m \frac{r_d}{\langle r_d \rangle}} \quad (8)$$

Figure 3f shows the drop size distribution $p(r_d/\langle r_d \rangle, R_b)$ as a function of $r_d/\langle r_d \rangle$ and that our data can be relatively well described by a Gamma distribution of low order (low rugosity), here $m = 4$ (solid line). This statement is more accurate for smaller bubbles with production of O(10) drops, where the fragmentation scenario is more accurate, while for larger bubbles with O(1) drop being produced, the process is more deterministic, with the drop size selected by the cavity collapse and jet dynamics, and the distribution resulting from the large ensemble is narrower. Note that our data set remains relatively limited in terms of statistical convergence and we do not try to demonstrate that the Gamma distribution is a better statistical model than other options that could have been considered to represent the jet drop distribution, such as a log-normal distribution (dashed line) which is indicated in 3f and also describes the data. The choice of the Gamma distribution is justified by analytical integrability with bubble size distribution described as power laws in the next section.

4 Application to the distribution of jet drops generated by a distribution of bursting bubbles characteristic of a breaking wave

Having characterized the drop size distributions for various bubble sizes, we aim to apply the proposed scalings to a canonical bubble size distribution generated by a breaking wave. The bubble size distribution $q(R_b)$ under a breaking wave has been studied by generating breaking waves in laboratory settings (Deane & Stokes, 2002; Rojas & Loewen, 2007; Blenkinsopp & Chaplin, 2010; Prather et al., 2013) as well as through numerical studies (Deike et al., 2016; Mostert et al., 2021). It is usually described by two scalings, separated by the Hinze scale (Hinze, 1955; Rivière et al., 2021), which compares the action of surface tension and turbulence and corresponds to the critical size below which bubbles do not break under the water turbulence, $r_H = \mathcal{C}(\gamma/\rho)^{3/5}\varepsilon^{-2/5}$, where ε is the water side turbulence dissipation rate, \mathcal{C} an order 1 constant, leading to $r_H \approx 1 - 2\text{mm}$ in typical conditions. From the discussion above, jet drop production is mainly controlled by bubbles below the Hinze scale. For bubbles larger than the Hinze scale, experiments and simulations support $q(R_b) \propto R_b^{-10/3}$ which can be explained by a turbulent breakup cascade argument (Garrett et al.,

2000). For bubbles below the Hinze scale, the bubble size distribution can be described by (Deane & Stokes, 2002; Prather et al., 2013; Mostert et al., 2021; Rivière et al., 2021)

$$q(R_b) \propto R_b^{-3/2}. \quad (9)$$

Experimental measurements of drops produced by a bubble plume characteristic of the bubble population under a breaking wave have been performed by various authors. We consider the recent experiments from Prather et al. (2013) and Erinin et al. (2019) which have measured the drop size distribution resulting from the bursting of bubbles with a bubble bulk size distribution comparable to $q(R_b) \propto R_b^{-3/2}$.

Prather et al. (2013) presents measurements of dry aerosol particles between 0.01 and $10\mu\text{m}$, resulting from bubble bursting and are shown in figure 4 (main). Data are obtained from two sets of experiments where the bubble size distribution is also measured, a breaking wave obtained by linear focusing as well as from a waterfall experiment, with bubble distribution closely matching the $R_b^{-3/2}$ scaling (Prather et al., 2013). The similarity of the drop size distribution demonstrated the importance of the bubble size distribution to understand the resulting drop statistics (Prather et al., 2013). We convert the dry-aerosol diameter into liquid droplet radius by considering the conversion factor $2D_d^{\text{dry}} = 2r_d^{80\%} = r_d$ used in the literature (Tang et al., 1997; Lewis & Schwartz, 2004; Veron, 2015), and keep the absolute count per unit bin size per unit volume from Prather et al. (2013); Quinn et al. (2015).

Erinin et al. (2019) measured the size distribution of liquid drops from $50\mu\text{m}$ to 1mm resulting from a breaking wave, and separated their data into stage I at early time, which corresponds to spume drops generated at impact, and stage II at later times which coincides with the bubble plume rising and bursting at the surface. While Erinin et al. (2019) do not report bubble size distribution data, the breaker is obtained by linear focusing in a similar way as Deane and Stokes (2002) and it is reasonable to assume that the bubble size distribution should be comparable. The data from Erinin et al. (2019) are shown in fig. 4 (main).

We aim to compare the predicted drop size distribution with these experimental data. We can write the jet drop size distribution by integrating over all bubble sizes,

$$N_d(r_d) = \int \frac{q(R_b)n(R_b)}{\langle r_d \rangle} p(r_d/\langle r_d \rangle, R_b) dR_b. \quad (10)$$

This assumes that all bubbles will burst and that the bursting process is well captured by the single bubble bursting studies, which are rather strong assumptions that could be modified by considering the bubble plume evolution and measurements of the surface bubble size distribution actually bursting (see the discussion in Neel and Deike (2021) on collective effects).

The bounds of integration are particularly important, as they will set the smallest and largest drop being produced. From the consideration above, we have the upper bound, say R_b^{up} which will be around 2.7mm , which is slightly larger than the typical Hinze scale, and close to the end of the $q(R_b) \propto R_b^{-3/2}$ regime. The lower bound can be discussed in terms of the absolute smallest bubble able to produce a jet drop, $R_b^{\text{low}}/l_\mu \approx 900$, which at 28°C will be around $10\mu\text{m}$. However, two comments are necessary. First, measurements of the bubble size distribution under a breaking wave, or plunging waterfall used to study drop production (Deane & Stokes, 2002; Prather et al., 2013) rarely extend below $20\mu\text{m}$. Second, under a breaking wave in the field, a bubble below a certain size will have a rise velocity smaller than the water turbulent fluctuations, hence will not be able to rise back to the surface. This threshold is uncertain but can be estimated to be for a rise velocity of a few cm/s , i.e. bubble sizes close to $100\mu\text{m}$). We will consider both the absolute lower threshold of $10\mu\text{m}$ and the probably more realistic one of $100\mu\text{m}$ in the following for discussion.

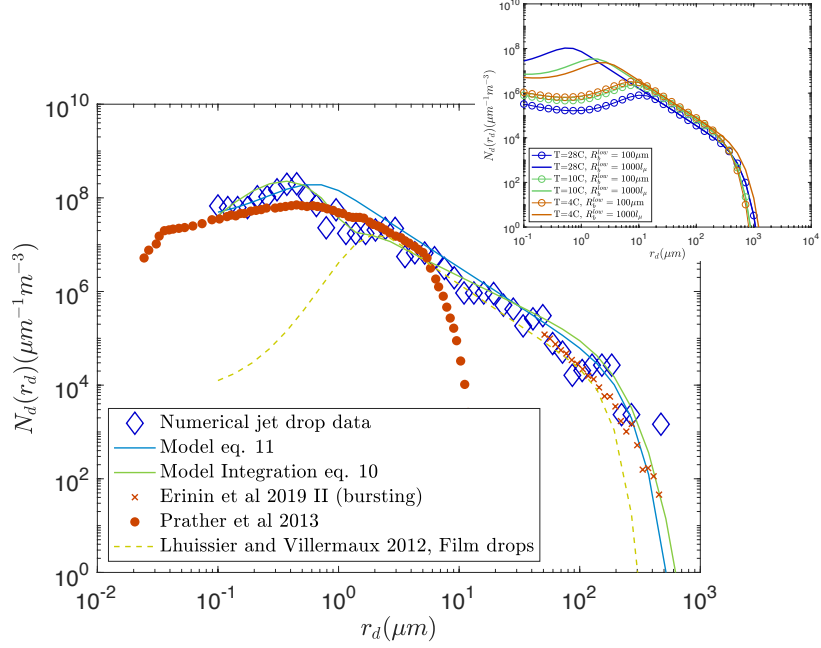


Figure 4. (a) Jet droplet size distribution due to a distribution of bubbles entrained by the breaking wave, assumed to follow the Deane and Stokes (2002) and Prather et al. (2013) bubble size distribution. We assume that all bubbles burst and produce jet drops. Blue open diamond is the predicted distribution using our modeled data. We compare the shape of the distribution with two data sets from the literature obtained for similar breaking wave conditions and bubble size distribution. Note that the vertical normalization is arbitrary as it was not provided in the experimental data sets, and so only the shape can be commented. Red dots are measurements from Prather et al. (2013) of dry aerosol. Conversion to liquid droplet size is a factor 2. Solid and dotted red are large drops measurements from Erinin et al. (2019) using holographic measurements for larger drops. (b) Sensitivity of the model jet drop distribution, given by eq. 11, for temperature from 4°C to 28°C . For each temperature, two low bubble size cut-off are considered, either $100\mu\text{m}$ (circle lines) or $R_b^{\text{low}} = 1000\mu\text{m}$.

Figure 4 (main) shows the result of integration of Eq. 10, using the numerical data presented in fig. 3, the bubble size distribution eq. (9) and considering $R_b^{up} = 2\text{mm}$ and $R_b^{low} = 10\mu\text{m}$. Again, for simplicity we consider the $T=28^\circ\text{C}$ conditions for now, but other conditions can be obtained by using the appropriate visco-capillary length-scale l_μ . The resulting jet drop size distribution extend from sub-micron drops, with a peak at $0.5\mu\text{m}$ before falling off for smaller drops, and extend up to about $500\mu\text{m}$. This demonstrates the ability of the jet drop mechanism to produce a wide spectrum of aerosol sizes, reaching the sub-micron range.

Since we consider the absolute prefactor from (Prather et al., 2013), we can compare quantitatively the droplet size distribution. Figure 4a shows the data set from Prather et al. (2013) and they agree quantitatively with our numerical results in the 0.5 to $10\mu\text{m}$ range. We comment that film drops are also important in this range and in the data set from Prather et al. (2013) but that precise separation between the two processes will depend on the efficiency of the ejection, which might require direct measurements of the surface dynamics. Our distribution is also in good agreement with the data from Erinin et al. (2019), but we note that due to different normalizations, only the shape and range of droplets can be discussed.

The final distribution can be discussed theoretically, by considering the various scaling laws described above, which we have expressed as algebraic relationships. The mathematical construction is similar to the one proposed by Lhuissier and Villermaux (2012) for film drops, except all scalings are different and the nature of jet drop production is coming from the jet fragmentation and not the rim dynamics as it is the case for film drops. We use the scalings described above, $q(R_b) = A_b R_b^{-3/2}$, with A_b adjusted to match the distribution from Prather et al. (2013), $n(R_b) \propto R_b^{-1/3}$ (eq. 6), and $\langle r_d \rangle \propto R_b^{5/4}$ (eq. 7); and defining α and β such that $q(R_b)n(R_b) \propto R_b^{-\alpha}$ and $\langle r_d \rangle \propto R_b^\beta$, and introducing $\zeta = (\alpha - 1)/\beta = 2/3$, Eq. 10 can be integrated yielding:

$$N_d(r_d) = \mathcal{A} r_d^{-1-\zeta} [\Gamma_{inc}(m + \zeta, m r_d / r_d^{up}) - \Gamma_{inc}(m + \zeta, m r_d / r_d^{low})] \quad (11)$$

where $\mathcal{A} = A_b(\chi/\eta)l_\mu^{7/12}$ is non-dimensional and aggregates the pre-factors from the bubble size distribution, the mean drop size and average number of drops (with $(\chi/\eta)l_\mu^{7/12} \approx 0.38$ at 28°C), and Γ_{inc} is the incomplete Gamma function, $m = 4$ the order of the Gamma function for the individual bubble bursting jet drop statistics. This leads to a jet drop size distribution following $N_d(r_d) \propto r_d^{-1-\zeta} \propto r_d^{-5/3}$ between the smallest and highest drop size ejected, determined as $r_d^{low} = \langle r_d \rangle (R_b^{low}) = \eta R_b^{low 5/4} l_\mu^{-1/4}$, and similarly for the largest drop size.

The integration of eq. 10 and the final model, eq. 11, are displayed in figure 4a and describe very well the numerical data. Note that the integration of Eq. 10 uses the same bubble size resolution as the one from the numerical data (with bubble size resolved every $10\mu\text{m}$ at small size) and therefore matches small drop behavior more closely than the final eq. 11, which would correspond to effectively a higher bubble and droplet sampling.

The same mathematical construction performed by Lhuissier and Villermaux (2012) for film drops is also shown in figure 4a (with y-axis arbitrarily scaled). This led to $\zeta^{film} = 3/4$, so that $N_d(r_d)^{film} \propto r_d^{-7/4}$ which is slightly steeper than the jet drop distribution, between the upper and lower bound of film drop production. Film drops are produced by the retraction of the cap of the bubble, and hence need to be for bubbles $R_b \sim l_c$, the capillary length. Lhuissier and Villermaux (2012) discuss film drop production from $R_b \approx 1\text{mm}$ to $R_b \approx 10\text{mm}$, and the corresponding mean drop size is $\langle r_d \rangle \propto R_b^{3/8} h_b^{5/8}$, where h_b is the film thickness at rupture. The film thickness at rupture is itself a function of the bubble size and lifetime, and in non-saline water, the minimal measured thickness is about 0.1 to $1\mu\text{m}$ (Lhuissier & Villermaux, 2012; Poulain et al., 2018; Néel & Villermaux, 2018), leading to an estimation of the

smallest liquid drop size around 1 to $4\mu\text{m}$. Note that smaller films in salt water could lead to smaller drops and evaluations of the smallest film size remains a fundamental open question (Lhuissier & Villermaux, 2012; Néel & Villermaux, 2018). Separately numerous measurements have reported solid dry particles attributed to film drops down to $0.01\mu\text{m}$ (Cipriano & Blanchard, 1981; Resch & Afeti, 1991; Cipriano et al., 1983; Sellegri et al., 2006; Mårtensson et al., 2003; Wang et al., 2017). Below $0.1\mu\text{m}$, the sea spray aerosols are mainly of organic composition (Quinn et al., 2015; Bertram et al., 2018), their size not directly proportional to the liquid drop size contrary to the sea salt aerosols.

The inset of Figure 4 shows the modeled distribution eq. 11 for temperature conditions from 4°C to 28°C (i.e. various l_μ). For drops in the 10 to $500\mu\text{m}$ range, the lower temperature leads to an increase in production, while the smallest drop produced depends on the choice of lower bound and temperature (through the viscosity variations). Indeed, the smallest bubble able to produce jet drop is likely to be given by the smallest bubble size able to reach the surface after a breaking wave event, which might be larger than $10\mu\text{m}$. Considering the rise velocity of bubbles in quiescent flow, compared to typical turbulence background velocity, (Woolf & Thorpe, 1991) have argued that bubbles below 50 to $200\mu\text{m}$ will fully dissolve, which would lead to a cut-off in the ocean of jet drops between 0.5 to $20\mu\text{m}$ depending on the water temperature. Assuming a cut-off at $100\mu\text{m}$ for the bubbles bursting at the ocean surface, lower temperatures lead to slightly smaller bubbles, while the opposite effect would take place if the lower bound considered is the peak of production at $R_b/l_\mu \approx 1000$. This highlights the importance of understanding the role of temperature on the bubble size distribution, and which bubbles actually rise and burst at the ocean surface.

5 Conclusion

In this paper, we quantify the size distribution of jet drops generated by bursting bubbles, initially entrained by breaking waves. We demonstrate that the mechanism of jet drop can produce a wide range of liquid droplet sizes, ranging from $0.5\mu\text{m}$ drops produced by $10\text{-}20\mu\text{m}$ bubbles (at 20°C), up to $500\mu\text{m}$ drops produced by millimetric bubbles. This confirms the recent experimental discussion from Wang et al. (2017), which demonstrated that both film and jet drops can produce sub-microns aerosols, with different chemical compositions. The relative importance of film and jet drops remains to be better quantified, and will likely depend on the physico-chemical conditions of the water, including water temperature, humidity conditions, and contamination, which will affect the film drainage process and the properties of the resulting film drops. The lower bound of bubble size for jet drop production is shown to depend on water viscosity, through the visco-capillary length. However, we note that sub-microns drops result from bubbles in the 10 to $50\mu\text{m}$ range which might not rise to the ocean surface in the field. The characterization of the surface bubble size distribution in the field would bring invaluable data to clarify this question. The proposed formulations present a natural dependency on temperature, through their dependency on water viscosity, but do not account for other physico-chemical effects, such as surfactants, and neglect any collective behavior at the water surface.

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