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1 Mechanical Properties of Graphene

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19 The mechanical properties of graphene are reviewed with particular attention to what
20 is established and what is still uncertain. The thickness and the elastic constants are
21 clarified, and by considering also phonon frequencies it is argued that “best values”
22 come from graphite, when available. Properties not available from graphite include
23 the bending stiffness; this can be determined from studies of carbon nanotubes as
24 well as graphene. In many ways nanotubes provide access to fundamental properties
25 of graphene, not least as they are the only form of graphene that can be unsupported
26 (unstrained) in vacuum. Environmental effects are considered, both interactions with
27 substrates and with other solid and liquid media which may affect the geometrical
28 parameters defining graphene and associated elastic constant. Major uncertainties
29 persist whether slipping or sticking dominates experimental observation, both be-
30 tween graphene and solid media, and between the layers of bilayer and multilayer
31 graphene. The paper concludes with a short discussion of continuum and atomistic
32 models of graphene.

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81 I. INTRODUCTION

82 Graphene has attracted enormous attention (*e.g.* the 2010 Nobel Prize) and research
83 effort, because of its extraordinary properties, not the least of which is its two-dimensional
84 (2D) nature. While many layered materials such as graphite and MoS₂ were already known,
85 graphene was the first material in which all the atoms are in a single plane – so for theoreticians,
86 at least, who may ignore the electrons and consider the carbon nuclei as point masses,
87 it is a genuinely 2D material. Yet it is also a very familiar material. It is the material that
88 in stacks of millions or billions of layers, constitutes graphite, much as many sheets of paper
89 make a book.

90 There are many excellent review articles that cover the mechanical properties of graphene,
91 both experimental and theoretical, and which to be comprehensive have to have about 500
92 references.¹⁻³ Why another? Our purpose is different. We aim to clarify points that are often
93 confused in the literature, and where we deem appropriate to identify problems that are as
94 yet unsolved. Some properties of graphene are just what one might expect, given what we
95 know of graphite. Here we review primarily the accuracy to which this is known. Second,
96 some properties are expected to be different, for reasons that are understood. Third, and
97 perhaps most interesting, are the anomalies. By this, we mean the behaviours of graphene
98 that are well-established experimentally, yet which lack adequate explanation according to
99 our current understanding.

100 An interesting question is “to which extent is the continuum mechanics view applicable
101 to graphene?” Of course, this leads to the definition of a thickness for graphene, which is
102 comparable to trying to define the thickness of an atom. The quantum nature of matter,
103 predominant at this scale, will obviously only lead to an approximate or to a probabilistic
104 answer.

105 II. BASIC PROPERTIES

106 The most basic properties determining the mechanical behaviour of a piece of a mate-
107 rial are its physical dimensions and its elastic moduli. But how can we define or measure
108 these mechanical properties of graphene? This apparently naive question arises from the
109 fact that the one-atom-thick character of graphene challenges the science of the mechanics

110 of materials, a discipline based on continuum mechanics and which has developed its suc-
111 cessful non-atomistic view since the publication of the first book addressing the strength of
112 materials, “Two New Sciences”, written by Galileo Galilei in 1638.⁴ Of course, the discipline
113 has subsequently integrated the atomistic nature of materials into its thinking, as in the
114 concepts of theoretical strength and of dislocations, and in the use of molecular dynamics
115 (MD) and density-functional (DFT) modelling.

116 From the material mechanics point of view, a large part of the challenge of graphene
117 mechanics arises from the ambiguities of defining the thickness of a structure – here a one-
118 atom-thick surface. From this point of view, it is important to recognise that thickness is
119 not a material property, but a property of a structure, such as a plate. Moreover, even a
120 simple structure such as a corrugated-iron roofing sheet has more than one property equally
121 deserving of being called the thickness – maybe the 0.5 mm thickness of the sheet, maybe
122 the 30 mm depth of the corrugations. So the issue is not to define what the thickness of
123 graphene is, but to be clear what the context is and how the concept of thickness enters in, in
124 each context. If one wants to stack n corrugated-iron sheets at random angles, the height of
125 a stack of n sheets will be $30 \times n$ mm, while if they are aligned, the stack will be $0.5 \times n$ mm
126 high. So graphene in AB stacking as in graphite has a thickness of 3.35 \AA , unambiguously.
127 We should not be surprised if this value changes for other stackings⁵ (*e.g.* around 3.6 \AA for
128 random-angle stacking called turbostratic, and see Table I for some graphene thicknesses
129 as described by the distances from various substrates). Nevertheless, while the stacking of
130 corrugated-iron sheets may have no bearing on the physical characteristics of the individual
131 sheets, this can be a totally different issue for graphene. Depending on the geometrical
132 arrangements, modifications of the electronic structure of the individual graphene sheets
133 could be significant.

134 A. Thickness of graphene

135 The thickness of graphene is a vexed question. The common description of graphene as
136 a 2D material implies extension in two dimensions but not in the third (*i.e.* zero thickness).
137 Indeed, the very definition of the thickness of graphene is complicated from a quantum
138 mechanics point of view, as it pertains to defining the diameter of atoms. The Yakobson
139 paradox⁶ arose through the attribution of values as low as 0.6 \AA to the thickness – and hence

140 Young’s moduli as high as 5 TPa. It is interesting to see how the similar issue of the size of
141 atoms was addressed a century ago.

142 Sir William Bragg proposed the hypothesis that atoms of a given element could be con-
143 sidered to be spheres with a fixed radius.⁷ However, the crystal structures of some metals
144 and compounds led to proposals that some atoms have lower symmetry than spherical.⁸
145 Wyckoff re-examined the question and concluded that the hypothesis of constant radii (or
146 other shapes with fixed sizes) must be rejected but that the evidence supported atomic radii
147 that vary more or less according to their environment.⁹ This is the basis of the modern view.
148 Modern data compilations give for example the covalent radius of the carbon atom as 0.70
149 Å. More precisely, the covalent radius of carbon is largest for single-bonded carbon with the
150 C-C bonds in ethane and diamond both at 1.54 Å, smaller for sp²-bonded (graphite at 1.42
151 Å and ethene at 1.33 Å bond length) and smallest for triple-bonded carbon (acetylene, 1.20
152 Å). On the other hand, the van der Waals (vdW) radius of a carbon atom is given as 1.70
153 Å. An early measurement of the thickness of the benzene molecule gave 4.70 Å,¹⁰ while the
154 thickness of the larger pyrene molecule is 3.53 Å,¹¹ very close to the spacing of the graphene
155 sheets in graphite at 3.35 Å.

156 These considerations appear to give a clear meaning to the concept of the vdW thickness
157 of graphene, as much as of simpler molecules such as benzene and the higher polycyclic
158 aromatic compounds such as pyrene. It expresses the distance of closest approach of other
159 physisorbed atoms – whether carbon or anything else, because the repulsive interatomic
160 potential deriving from Pauli exclusion is largely independent of the nature of the interacting
161 atoms, and in the absence of a chemical bond, so is the vdW attractive potential. Following
162 Wyckoff,⁹ the thickness should be expected to vary with the environment, whether it is a
163 surrounding gas or a substrate, as can be seen in Table I – even quite considerably as the
164 vdW forces, while always weak, can vary by an order of magnitude.

165 B. Graphene elastic stiffness tensor

166 Only in graphite is graphene found in a symmetrical environment (sandwiched between
167 graphene sheets with only a vdW potential binding them) and with a known thickness. We
168 may then define the graphene elastic stiffness constants c_{ij} in this situation as a reference
169 system. To deal with possible variations in thickness in other environments, it makes sense

TABLE I. Experimental graphene-substrate distance for various substrates.

Substrate	Distance [Å]
Ir(111) ¹²	3.38
Graphite	3.35
Pt(111) ¹³	3.30
SiC(0001) ¹⁴	3.24
Ru(0001) ¹⁵	2.2
Co(0001) ¹⁶	2.2
Ni(111) ^{17,18}	2.1

170 to define the in-plane 2D elastic stiffness tensor $c_{ij}^{2D} = c_{ij}d_0$ with $i, j = 1, 2$, and d_0 is the
 171 graphite interlayer spacing at ambient pressure. This tensor comes simply from the sp^2 bond
 172 bending and stretching stiffnesses, and so is independent of the graphene thickness d *per se*.
 173 To see this, consider making the graphene layers in graphite thinner, spaced more closely
 174 and reducing a_{33} , as happens under pressure. Then there are more graphene layers per
 175 unit volume, a_{33}^{-1} , and the 3D constants c_{11} and c_{12} are increased proportionately – leaving
 176 c_{ij}^{2D} unchanged. The out-of-plane elastic constants, particularly c_{33} but also c_{13} , have to be
 177 considered separately. This is done in Sec. III C.

178 A crucial aspect is then to know if the graphene sp^2 bonds – which largely determine
 179 c_{ij}^{2D} – are significantly influenced either by the environment of the graphene (what it is in
 180 contact with) or by its geometry (for example, bending) leading to a modification of the
 181 reference elastic stiffness constants. The zeroth-order approximation would be that there
 182 are no such influences. But if there are, we will need also to consider what are the limits
 183 in such modifications which can be admitted before saying that we are dealing with a new
 184 system. These issues are addressed in Sec. VI.

185 III. GRAPHENE REFERENCE MECHANICAL PROPERTIES

186 A. Graphite 3D mechanical properties

187 Graphite consists of a macroscopic multilayer stack of graphene layers, held apart by the
 188 π -orbitals and Pauli exclusion, and held together by the vdW interaction, with a spacing of

189 3.35 Å.¹⁹ It is, consequently, highly anisotropic, while being isotropic in-plane. The elastic
 190 moduli reported in Table II were obtained from inelastic X-ray scattering,²⁰ but similar
 191 values were reported from ultrasonic and static mechanical testing.²¹

TABLE II. Elastic moduli values obtained from inelastic X-ray scattering²⁰ in Voigt notation

	c_{ij} [GPa]	c_{ij}^{2D} [Nm ⁻¹]
c_{11}	1109 ± 16	372 ± 5
c_{33}	38.7 ± 0.7	
c_{12}	139 ± 36	47 ± 12
c_{13}	0 ± 3	
c_{44}	5 ± 0.3	
c_{66}	485 ± 10	
Y	1092 ± 18	362 ± 27
ν	0.125 ± 0.033 (no units)	

192 Within experimental error, the c_{13} of graphite may be negative – and DFT calculations
 193 support this.²² While DFT may not evaluate the interlayer spacing accurately without vdW
 194 correction,²³ the related elastic constant c_{33} calculated at the experimental equilibrium spac-
 195 ing agrees well with the experimental c_{33} . That improves confidence in the reliability of the
 196 negative calculated value of c_{13} .²² If c_{13} is truly negative, it might be due to ripples in
 197 the measured graphite sample, like what happens with auxetics, prominent structures that
 198 have negative Poisson’s ratio.²⁴ This could be further linked to the negative thermal ex-
 199 pansion of graphene, which will be discussed later. In a general 3D anisotropic medium,
 200 the Young’s moduli for a stress σ_{11} are given by more complicated expressions than the
 201 usual $Y = c_{11} - 2c_{12}^2/(c_{11} + c_{12})$. The small or vanishing value of c_{13} simplifies this to
 202 $Y = c_{11} - c_{12}^2/c_{11}$ for the in-plane Young’s modulus of graphene. Similarly the in-plane
 203 Poisson’s ratio simplifies to $\nu = c_{12}/c_{11}$.

204 B. In-plane graphene mechanical properties

205 Under the hypothesis that graphene can be treated as a continuous elastic medium, it
 206 is thus expected to have the 3D values given in Table II and hence the c_{ij}^{2D} values also

207 given there. Indeed, the in-plane Young’s modulus $Y^{2D} = 362 \text{ Nm}^{-1}$ is consistent with
208 the $Y^{2D} = 340 \pm 50 \text{ Nm}^{-1}$ of monolayer freestanding graphene measured by atomic force
209 microscopy (AFM)²⁵, which will be discussed in Sec. IV A.

210 For graphene in vacuum, the greatest difference from graphite is likely to be an increase
211 in the thickness, as the π -orbitals are no longer compressed by the vdW attractive force –
212 indeed, as discussed above, the thickness becomes defined by whatever convention is used
213 to specify where the π -orbitals end. The 3D elastic constants will vary inversely with the
214 thickness. The 2D elastic constants are unaffected. Neglecting any small effect on the sp^2
215 bond strength caused by the relaxation of the π -orbitals, and any small contribution of
216 the π -bonds to c_{11} and c_{12} , the 2D elastic constants will be unchanged; they are mostly
217 determined by the carbon sp^2 -network.²⁶

218 Direct experimental determinations of the in-plane elastic constants of graphene are much
219 less accurate than those of graphite, but are generally consistent with them within experi-
220 mental uncertainty (see Sec. IV). The results of *ab initio* theoretical calculations also show
221 very similar in-plane elastic constants of graphene to graphite, with the caveat that it is
222 much harder to assess the uncertainty of these calculations than to assess experimental
223 uncertainties.²²

224 Given the large uncertainties on experimental determinations of c_{ij}^{2D} in graphene, the
225 best information about possible perturbations of the in-plane sp^2 bonding comes from a
226 comparison of the phonon frequencies, and in particular the zone-centre E_{2g} optical phonon.
227 It is a Raman active mode, named the G-mode (G for graphite), at about 1600 cm^{-1} in all
228 sp^2 -bonded forms of carbon.²⁷ In graphite, the frequency of the G-mode is at $1575 - 1580$
229 cm^{-1} ,^{27–30} and it is reported at $1581 - 1592 \text{ cm}^{-1}$ in graphene.^{31–34} This difference of about
230 0.5% puts an upper limit of about 1% on any change in the stiffness of the sp^2 bonds or c_{ij}^{2D}
231 in going from graphite to graphene.

232 Chemical perturbations such as [substitutional doping and](#) doping by charge transfer from
233 a substrate or nitric acid, etc, mechanical perturbations such as bending, stretching or high
234 pressure, or structural perturbations such as stacking order, again scarcely perturb the GM
235 frequency [relative to graphite](#), a few tens of cm^{-1} at most. The sp^2 covalent bonding is
236 clearly robust. This is actually true of small molecules generally, which like graphene are
237 “all surface”, yet whose vibrational frequencies are little affected from the vapour phase,
238 through solvation or liquefaction, to crystalline solid forms. It is not surprising, then, that

239 this is so for graphene. Of course, the concept of a material that is all surface is not novel;
240 chemistry deals largely with such things. Some readers might be alerted here that shift of
241 tens of cm^{-1} in the G-mode could be considered to be huge in specific studies, but when
242 considering the corresponding change in the in-plane stiffness, such shifts are still very little
243 compared to the large experimental uncertainties from the direct measurements on graphene
244 by AFM (Sec. IV A).

245 We conclude that graphene and graphite are the same regarding in-plane stiffness within
246 experimental uncertainty, from the G-mode frequency. More details about the G-mode
247 frequency, such as the contributions from the deformed π -orbitals and up to the fifth nearest
248 neighbour C atoms, will be discussed in Sec. VIC and VIII, respectively.

249 C. Out-of-plane stiffness of graphene

250 On the assumption that graphene has the vdW thickness equal to the interlayer spacing
251 of graphite (Sec. II A), we may suppose that it has a stiffness in that direction, equal to that
252 of graphite, 38.7 ± 0.7 GPa.²⁰ This basic assumption is challenged by various objections.
253 For example, in graphite, c_{33} probes the interlayer potentials defined through the π -orbital
254 interaction in an AB stacking. What sense does this make when considering graphene in
255 vacuum or in a solid or liquid molecular environment? Deformation of solids is usually
256 described in terms of the changes in the distances between atoms, measured from nuclei to
257 nuclei, or in crystals, by the spacing of planes of atomic nuclei. It could be argued that with
258 only one plane of atoms, one plane of nuclei, out-of-plane deformation or strain of graphene
259 is meaningless, and with that, the c_{33} of graphene is a meaningless concept.

260 Against these challenges, thickness and stiffness certainly exist in monolayer graphene,
261 just as atomic radii exist (Sec. II A), and vary with pressure, which defines an atomic ra-
262 dial stiffness. Electronic orbitals extend out of the plane of graphene nuclei and they resist
263 compression. Such resistance (more accurately, compliance) consists of two parts. One is
264 the Pauli exclusion compliance from the overlap of the π -orbitals of graphene and the outer
265 orbitals of the medium that applies compression. It naturally depends on what that medium
266 is and how it is stacked onto graphene. Moreover, the compliance has to be apportioned
267 between the compliance of the graphene and the compliance of the medium. Surprisingly,
268 even if the compressing medium is another graphene monolayer AB-stacked to it, the out-

269 of-plane stiffness is calculated to be, not that of graphite, but only about half of it.³⁵ This
 270 softness is attributed to the squeezing of π -orbitals through the graphene plane in a bilayer
 271 system, whereas such squeezing-through is prohibited in graphite (infinite number of lay-
 272 ers) by symmetry.³⁵ In addition to the compliance of the Pauli exclusion for undeformed
 273 π -orbitals, the other contribution from the graphene to the total compliance is from the
 274 deformation of the π -orbitals of the graphene. This could be estimated by calculating the
 275 energy difference between relaxed and deformed π -electron distributions.

276 In the absence of a conventionally-defined elastic constant c_{33} based on internuclear dis-
 277 tances, one approach to define the out-of-plane stiffness of graphene is to use a related
 278 quantity that is itself unambiguously defined and measurable. The in-plane bonds stiffen
 279 under compressive in-plane strain, which can be expressed as a 2D strain and converted to
 280 a 2D stress by c_{ij}^{2D} . That has been measured by the increase in G-mode phonon frequency
 281 under pressure.^{31,34,36-38} In graphene as in graphite, the 2D in-plane stress can be applied by
 282 hydrostatic pressure, and the 2D stress is then directly proportional to the thickness. Since
 283 graphite and graphene are very soft out-of-plane, under hydrostatic pressure the thickness
 284 decreases significantly (the π -orbitals being considerably compressed). That gives a large
 285 reduction of in-plane force below the linear proportionality with pressure, and therefore a
 286 substantially sublinear shift of the G-mode frequency with pressure.³⁹ An experiment adopt-
 287 ing this approach had large experimental uncertainties, but within experimental uncertainty
 288 first confirmed from the shift-rate of the G-mode that the thickness of graphene is not sig-
 289 nificantly different from its thickness in graphite. Then the sublinearity of the shift-rate
 290 could not distinguish the graphene $c_{33} = 0 \pm 300$ GPa⁴⁰ from graphite (38.7 ± 0.7 GPa).²⁰
 291 Of course, there are also possible effects of the pressure medium on the graphene response;
 292 these are discussed further in Sec. VI below.

293 D. Properties not related to graphite

294 In contrast to the foregoing, there are some properties of graphene that are quite distinct
 295 from, or unrelated to, any properties of graphite. It is probably accurate to say that these
 296 are all properties related to the freedom graphene has to displace in the z -direction, out
 297 of plane, in ways that are unavailable to the layers in graphite. Briefly, these include the
 298 theoretical instability of a 2D sheet, the negative thermal expansion in-plane of graphene,

299 and the stiffness in bending of monolayer graphene and of multilayer graphene.

300 **1. Acoustic phonons in graphene; their effect on the thermal expansion and** 301 **stability of graphene samples**

302 In this section, and the following sections III D 2 to III D 4, we will review the nature
303 of the acoustic phonon modes in graphene. They are responsible for some key properties
304 of graphene: The observation of a negative thermal expansion coefficient under certain
305 conditions, lack of mechanical stability and – in consequence of this – static ripples in
306 the graphene monolayer. These properties reflect, in a fundamental way, the 2D nature
307 of graphene: The lack of restoring forces from adjacent atomic layers in the out-of-plane
308 direction, and the density of states for a 2D material varying in proportion to k rather than
309 k^2 .

310 To begin, we must briefly review the low-energy part of the phonon dispersion relation
311 of graphene. The dispersion relation has been studied for decades prior to the discovery of
312 graphene, as a simplified model for the phonons in graphite.^{41–43} Initially these calculations
313 were performed using traditional semi-classical “ball and spring” force constant models, and
314 the results obtained nowadays using DFT calculations²² are in reasonably good agreement
315 with those obtained previously using “pencil-and-paper” methods. Agreement is also good
316 with the experimental data obtained on graphite using electron energy loss spectroscopy,⁴⁴
317 inelastic neutron⁴² and X-ray scattering.²⁶ Fig. 1 shows the low-energy part of the dispersion
318 relation, calculated in Ref. 22 using DFT. In this section we shall regard low energy as up to
319 50 meV since we are interested in the phonons which can be excited in significant number
320 by thermal energy at $T \leq 300$ K. (At 300 K, $k_B T \approx 25$ meV).

322 **2. Mechanical stability of graphene**

323 As shown in Fig. 1, there are three acoustic phonon modes which can be excited at 300
324 K. They are longitudinal (LA) and transverse (TA) polarization in-plane modes (both with
325 linear dispersion relations in the low k limit) and a transverse polarization out-of-plane mode
326 (ZA) with a quadratic dispersion relation in the low k limit ($E \propto k^2$).^{22,43} The softness of
327 this mode relative to the LA and TA modes is believed to be partially due to the lack of

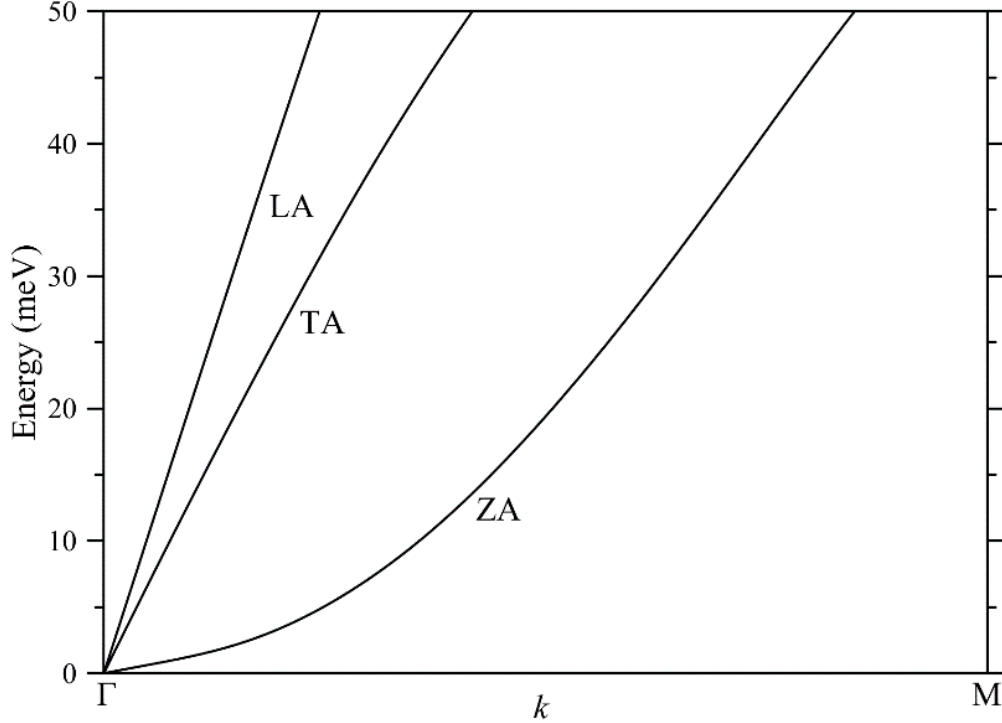


FIG. 1. Low-energy graphene phonon modes along the Γ -M direction in the first Brillouin zone, obtained in Ref. 22 using DFT. LA and TA modes are the longitudinal and transverse polarization in-plane modes and the ZA mode is the transverse polarization out-of-plane mode.

328 bending stiffness of graphene. However, the low bending stiffness is not necessary to explain
 329 the quadratic dispersion relation. A transverse wave on a string (or sheet) with zero tension
 330 but a bending stiffness D has $\omega(k) = k^2(D/\rho)^{1/2}$, so the quadratic dispersion relation is fully
 331 consistent with the recently measured 1.7 eV bending stiffness of graphene.^{39,45} The number
 332 of phonons present at temperature T from branch i of the dispersion relation (hence with
 333 energy $E_i(k)$) is obtained by multiplying the Bose-Einstein distribution by the density of
 334 states $D(k)$ and integrating over all available k (Eq. 1). The lower limit k_{min} corresponds to
 335 the longest wavelength mode that can exist on the graphene sheet. So $k_{min} \approx 2\pi/L$ where L
 336 is the diameter of the graphene sheet. For macroscopic graphene samples therefore $k_{min} \rightarrow 0$
 337 is a reasonable approximation. Referring to Fig. 1 and the known graphene reciprocal lattice,
 338 the path $\Gamma \rightarrow M$ in reciprocal space covers a range of $2.3 \times 10^9 \text{ m}^{-1}$, whilst $k_{min} \approx 10^5 \text{ m}^{-1}$
 339 for a $10 \mu\text{m}$ diameter graphene sheet.

340 The upper limit k_{max} should be set so as to integrate over all possible phonon modes, in
 341 reality the upper limit of the integral is set by the Bose-Einstein distribution term dropping

342 to zero upon increasing k , E .

$$N = \int_{k_{min}}^{k_{max}} \frac{1}{e^{\beta E(k)} - 1} D(k) dk \quad (1)$$

343 Here, we write $\beta = 1/k_B T$ for convenience. The densities of states in 1D, 2D and 3D
 344 systems, per unit area/volume, are given by:

$$\begin{aligned} D_{1D} &= \frac{1}{\pi} \\ D_{2D} &= \frac{k}{2\pi} \\ D_{3D} &= \frac{k^2}{2\pi^2} \end{aligned} \quad (2)$$

345 It has been shown that the value of the integral in Eq. 1 diverges to $+\infty$ when a 2D
 346 density of states function is utilized, combined with a quadratic dispersion relation.⁴⁶ This
 347 is due to the value of the integrand diverging in the low- k limit. The experimental obser-
 348 vations that genuinely free-standing graphene sheets do not exist, and that all graphene
 349 sheets are covered in static ripples (so, in at least one respect, not genuinely 2D) are at-
 350 tributed to this divergence.^{46,47} In addition, this divergence prevents the categorization of
 351 free-standing graphene sheets as a metastable form of carbon in an equivalent manner to
 352 diamond, nanotubes and fullerenes at ambient conditions.

353 In this section, we are going to explore the cause of this divergence in a little more
 354 detail – can it be observed in systems with other dimensionalities, and with other dispersion
 355 relations? In Table III we give the integrands that would be utilized in Eq. 1 for phonon
 356 modes with quadratic and linear dispersion relations for 1D, 2D and 3D systems. Since we
 357 seek only to evaluate which of these integrands diverge we have assumed a finite non-zero
 358 temperature and omitted all constant terms, where we assume β is a finite constant since
 359 $T > 0$ K. The integrands are evaluated in the low- k limit (equivalent to assuming that
 360 the sample is large enough to support phonons across essentially the entire Brillouin zone)
 361 using first order expansions of the exponential term ($e^x \approx 1 + x$), appropriate since we are
 362 considering exclusively acoustic phonons for which $E \rightarrow 0$ also in the low- k limit.

364 Summarising the findings from Table III, we observe that in a 3D system divergence is not
 365 observed for a linear or quadratic dispersion relation, for a 2D system divergence is observed
 366 only with a quadratic dispersion relation, and for a 1D system divergence is observed with
 367 either a linear or quadratic dispersion relation. These differences are due to the density of
 368 states function being different in each case.

TABLE III. Integrands $I_k^{(n)}$ to evaluate the number of phonons present at finite temperature for different dispersion relations, in systems with different dimensionality n . Constant terms and temperature dependence have been omitted.

	1D	2D	3D
$E \propto k$	$I_k^{(1)} = \frac{1}{e^k - 1}$ $\lim_{k \rightarrow 0} I_k^{(1)} = \frac{1}{k}$	$I_k^{(2)} = \frac{k}{e^k - 1}$ $\lim_{k \rightarrow 0} I_k^{(2)} = 1$	$I_k^{(3)} = \frac{k^2}{e^k - 1}$ $\lim_{k \rightarrow 0} I_k^{(3)} = k$
$E \propto k^2$	$I_{k^2}^{(1)} = \frac{1}{e^{k^2} - 1}$ $\lim_{k \rightarrow 0} I_{k^2}^{(1)} = \frac{1}{k^2}$	$I_{k^2}^{(2)} = \frac{k}{e^{k^2} - 1}$ $\lim_{k \rightarrow 0} I_{k^2}^{(2)} = \frac{1}{k}$	$I_{k^2}^{(3)} = \frac{k^2}{e^{k^2} - 1}$ $\lim_{k \rightarrow 0} I_{k^2}^{(3)} = 1$

369 To our knowledge, the consequences of this divergence have not been studied experimen-
370 tally in 1D systems. Certainly its observation would require the existence of an extremely
371 long system to ensure $k_{min} \rightarrow 0$, and for the system to be free-standing to allow these
372 phonons to propagate. The nearest humankind has got to a genuinely 1D system is carbyne
373 – in recent years carbyne chains up to 600 nm in length have been synthesized, satisfying
374 the first condition.⁴⁸ But the requirement that they are enclosed inside a carbon nanotube
375 probably prevents the second condition from being satisfied. Single-walled carbon nanotubes
376 (SWCNTs) themselves are not strictly one-dimensional as far as the density of states func-
377 tion is concerned. The quantization condition for the direction along the tube axis is the
378 requirement for a standing wave with allowed wavelengths determined by the tube length,
379 whilst the quantization condition for the direction tangential to the tube axis is for travel-
380 ling waves with allowed wavelengths determined by the tube circumference. Thus, whilst
381 the allowed quantum states are far more widely spaced in the tangential direction, the elec-
382 tron and phonon wavevectors do still have 2 degrees of freedom, and the density of states
383 function should follow a [2D form](#) rather than the 1D form.

384 As far as graphene is concerned, the divergence in the number of out-of-plane phonons at
385 finite temperature is believed to be responsible for the following experimental observations.
386 Firstly, real monolayer graphene samples can only exist when provided with some mechanical
387 support. Usually this is provided by a substrate. Samples may be described in the literature
388 as “freestanding” when there is an aperture in the substrate or some similar arrangement, but
389 there is always some mechanical support. Monolayer graphene samples may be suspended
390 in a liquid such as water, in which the viscosity of the liquid provides adequate support.^{47,49}

391 Secondly, real monolayer graphene samples are always rippled. These ripples are ex-
392 pected from the findings of atomistic simulations.⁵⁰ They have been observed using electron
393 diffraction⁵¹ and also scanning tunnelling microscopy,⁵² which has confirmed that they are
394 static ripples, with $\lambda \approx 5$ nm. The ripples become weaker for progressively thicker graphene
395 samples as the thickness of the graphene sample itself provides the required rigidity. These
396 ripples are a completely separate effect to the Brownian motion observed in graphene.^{53,54}

397 The role of ripples in ensuring stability can be understood in terms of the restoring forces.
398 The softness of the ZA mode shown in Fig. 1, compared to the in-plane modes, is because of
399 the lack of restoring forces due to bond-stretching in the low amplitude limit, **and of those**
400 **due to bending in the large-wavelength limit**. The curvature induced by the ripples ensures
401 that there is some restoring force due to bond-stretching even in the low amplitude limit,
402 making the mode — partially — analogous to the radial breathing mode in SWCNTs.⁵⁵

403 ***3. Thermal expansion coefficient of graphene***

404 The existence of thermal expansion is perhaps the most intuitive example of the interplay
405 between static and dynamic material properties. The lattice constant of a solid is considered
406 the archetypal static property, yet at $T > 0$ K it is altered (usually increased) by the presence
407 of phonons; the archetypal dynamic property. The observation that the vast majority of
408 materials expand upon heating is a consequence of the nature of the function $V(r)$ giving
409 the potential energy between two of the atoms comprising the solid as a function of their
410 separation r . The Lennard-Jones potential is frequently utilized as a good approximation for
411 covalently bonded solids even though it is only strictly correct for solids where the cohesion
412 is due solely to vdW forces. In any case, the potential will always have three key features in
413 common with the Lennard-Jones potential: (1) It will be attractive for moderate values of
414 r , with a minimum at $r = r_0$, the inter-atomic separation in the absence of phonon effects.
415 (2) In the limit $r \rightarrow \infty$, $V \rightarrow 0$. (3) In the limit $r \rightarrow 0$, $V \rightarrow +\infty$ to prevent atomic overlap.
416 As a result of these features, $V(r)$ is not symmetric about $r = r_0$ and this asymmetry will,
417 in the absence of other effects, favour thermal expansion rather than contraction.

418 This argument applies directly to any reasonably isotropic **and dense** 3D solid and, for
419 that matter, a 2D solid existing in a 2D world (in which case out-of-plane phonon modes
420 would not exist). However, graphene's position is as that of a 2D solid in a 3D world. In

421 this case, the excitation of an out-of-plane vibration does not cause any thermal expansion
422 in the out-of-plane direction. However it can cause contraction in the in-plane direction as
423 atoms are pulled inwards by the out-of-plane movement.

424 Thus, for graphene to exhibit a negative coefficient of thermal expansion coefficient
425 (CTE), all that is necessary is for the contribution from the out-of-plane phonons to dom-
426 inate over that from the in-plane phonons. We can see how this can be the case at low
427 temperature from Fig. 1. The quantum states available for all phonons are equally spaced
428 in k -space so, when phonons of all kinds (LA, TA, ZA) can be excited up to a certain energy,
429 the out-of-plane ZA phonons dominate as they cover a wider area of k -space. The qualitative
430 arguments proposed here are borne out by the findings of detailed theoretical calculations;²²
431 Graphene should indeed exhibit a negative CTE.

432 However, as shown earlier a graphene sheet which is genuinely freestanding, and therefore
433 free to expand and contract, cannot exist. Experimental measurements of the CTE of
434 graphene are therefore indirect and prone to large experimental, [theoretical and conceptual](#)
435 [uncertainties](#).⁵⁶ Most commonly, it has been measured by varying temperature whilst the
436 graphene is adhered to a substrate. The graphene is assumed to adhere perfectly to the
437 substrate due to its extremely high surface area to volume ratio so, upon temperature
438 increase the graphene would be forced to expand rather than contract and is therefore
439 under significant tensile strain. The extent of this strain is usually calculated using Raman
440 measurements on the G peak,⁵⁷ though grazing incidence X-ray diffraction has also been
441 utilized.⁵⁸ Potential sources of uncertainty in such experiments include, but are not limited
442 to:

- 443 • [The logic of such experiments is often not clearly presented. When a thin film fully](#)
444 [adheres to a substrate, the difference in thermal expansions causes a stress in the film,](#)
445 [not a strain. Stress cannot be measured by Raman. The CTE of the film should be](#)
446 [directly measured by Raman of the thermal strain in a *free-standing* specimen of the](#)
447 [film. Then to correct for the temperature effect on the phonon frequency, the phonon](#)
448 [shift at known strain and ambient temperature is compared with the data from the](#)
449 [film adhering to a substrate, at a known strain and elevated temperature. Given](#)
450 [the difficulty of studying free-standing graphene, data from theoretical simulations is](#)
451 [commonly used instead.](#)

- 452 • Commonly used substrates for graphene consist of layers of different materials with
453 different TEC. It is not always clear which layer dominates, and to what extent the
454 layers remain bonded.
- 455 • The out-of-plane phonons causing the negative TEC should be suppressed to some
456 extent by the presence of the substrate. It is thus not clear how applicable findings
457 regarding graphene on a substrate are to free-standing graphene.
- 458 • We assume a value for the Grüneisen parameter (for the phonon responsible for the G
459 peak) when the strain is calculated from the G peak Raman measurement.
- 460 • Graphene does not always adhere perfectly to the substrate.

461 Notwithstanding these problems, the experimental evidence is consistent with graphene
462 having a negative TEC at temperatures up to at least 500 K.⁵⁶ (and references therein) This is
463 also the case for the in-plane measurements on graphite. [Indeed, comparing reported values
464 and uncertainties of the TEC for graphene and for graphite in-plane,^{22,59–61} it is difficult to
465 find any justification for considering them to be different.](#) The small or negative CTE is
466 reflected in the contrasting characteristics of graphene grown by chemical vapour deposition
467 (CVD) and epitaxial growth (EG). Both growth processes take place at high temperature,
468 followed by the substrate contracting upon cooling. In the case of the CVD graphene, Raman
469 measurements at ambient conditions indicate that the graphene has “relaxed” *i.e.* slipped
470 over the substrate upon cooling to stay in equilibrium. However, Raman measurements
471 made on the EG graphene at ambient conditions indicate that it has remained adhered to the
472 substrate upon cooling after growth.⁶² As a consequence it is under significant compressive
473 strain; equivalent to several GPa pressure.⁵⁶

474 As briefly mentioned above, 3D materials with a layered structure can also exhibit a
475 negative thermal expansion coefficient along one axis or in one plane,^{63–65} although the
476 phonon modes that cause this will cause positive thermal expansion along some other axes.
477 In graphite at 300 K for example, the in-plane thermal expansion coefficient is *ca.* -1.4×10^{-6}
478 K^{-1} , but the out-of-plane thermal expansion coefficient is *ca.* $+25 \times 10^{-6} \text{K}^{-1}$.²²

479 **4. Grüneisen parameters and elastic bands**

480 The negative thermal expansion coefficient resulting from the ZA mode — by definition
481 — results in this mode having a negative Grüneisen parameter (Grüneisen parameters for
482 all phonon modes in graphene are calculated throughout the first Brillouin zone in Ref. 22).
483 In the Grüneisen approximation, the pressure and temperature dependence of the phonon
484 mode can both be incorporated into the volume or lattice parameter dependence on pressure
485 and temperature, linked by the Grüneisen parameter. For a 2D material such as graphene
486 we can write, following the approach in Ref. 34:

$$\frac{\omega(P, T)}{\omega_0} = \left(\frac{\alpha(P, T)}{\alpha_0} \right)^{-2\gamma_{ZA}} \quad (3)$$

487 where $\alpha(P, T)$ is the pressure and temperature-dependent lattice parameter, as projected
488 into the basal plane of the graphene lattice. In this case if $\gamma_{ZA} < 0$, $\omega(P, T)$ will decrease
489 under isothermal compressive strain and increase under isothermal tensile strain. Whilst
490 highly unusual on a microscopic level, it is the behaviour we are used to observing in common
491 macroscopic 1D systems in everyday life: When you stretch elastic bands, or guitar strings,
492 or the shrouds and stays of a yacht, they twang at a higher frequency!

493 **5. Bending stiffness**

494 In 3D systems, while the elastic constants discussed above are properties of a material, a
495 bending stiffness is a property, not of a material, but of a structure, i.e., related to geometry.
496 It is however defined in a similar way. The elastic constants are the second derivatives of the
497 potential energy with respect to deformation (strain). Graphene has the additional degree
498 of freedom, of bending to a curvature κ ($= 1/R$ where R is the radius of curvature). This
499 additional degree of freedom, bending, and hence a bending stiffness, has no analogue in
500 graphite. To account for the bending stiffness, D , defined by the energy of curvature κ ,

$$U = 1/2 D \kappa^2 \quad (4)$$

501 there is a large literature in which an effective Young's modulus Y_{eff} and an effective thick-
502 ness h_{eff} are introduced such that both the in-plane elastic moduli and the bending stiffness

503 can be expressed:

$$\begin{aligned} Yh &= Y_{eff}h_{eff} \\ D &= \frac{Y_{eff}h_{eff}^3}{12} \text{ or } \frac{Y_{eff}}{1-\nu^2} \frac{h_{eff}^3}{12} \end{aligned} \quad (5)$$

504 where in the second expression for D it is the plane-strain modulus that is used, as is correct
505 for a plate made of an isotropic material.

506 The model of Eq. 5 has had remarkable success in capturing the behaviour of graphene
507 and nanotubes, particularly in contexts where beam, plate and shell theory are used to un-
508 derstand buckling behaviour under load.⁶ Due to the uncertainties of a definition of graphene
509 thickness, on the other hand, it has led to claims that nanotubes have “an extremely large
510 Young’s modulus”,⁶⁶ for example the value of 5.5 TPa given by Yakobson *et al.* for Y_{eff} .⁶
511 Similarly, astonishment has been expressed at the small values of h_{eff} , as low as 0.066 nm,
512 “ultrathin compared with the C-C bond length 0.142 nm”.⁶⁶ The very wide range of values
513 reported for these parameters⁶⁷ has been described as a paradox (Yakobson’s paradox).^{68,69}
514 As some authors have recognised, there is no paradox,^{70–72} but much of the literature fails
515 to distinguish h and Y from h_{eff} and Y_{eff} . The wide ranges of values express only the im-
516 precision in the determination of D by different methods. The unphysical values found for
517 Y_{eff} and h_{eff} simply reflected the unphysical nature of these parameters, which correspond
518 to nothing in the real world but are simply convenient ways of representing D in structural
519 engineering computational packages that do not permit D to be entered independently of Y
520 and h (if any such packages exist, which is doubtful, as engineers routinely analyse structures
521 of this sort, such as honeycomb-filled or rib-reinforced plates).

522 Reported experimental methods of measuring D range from the collapse pressure P_C of
523 nanotubes under high pressure (1.7 ± 0.2 eV),⁴⁵ to the taper angle of a strip torn from an ad-
524 hesive substrate (2.1 ± 0.1 eV).⁷³ A value of 2 eV was estimated from the phonon dispersion
525 measured by high resolution electron energy loss spectroscopy (HREELS), on Pt (111) sup-
526 ported graphene.⁷⁴ Other measurements reported values as high as $10^3 - 10^4$ eV for rippled
527 monolayer⁷⁵ and $35.5_{-15.0}^{+20.0}$ eV for bilayer.⁷⁶ Torres-Dias *et al.* found that for nanotubes of
528 small diameters, the normalised collapse pressure $P_C R^3 = 3D$ dropped substantially below
529 the theoretical value, which could be due to the softened bending potentials at large bending
530 angles, or an effect of atomicity.⁴⁵ Carter *et al.*⁷⁷ study the Euler buckling load for a straight
531 pillar. When the compliance of a continuous pillar is concentrated at a few points (atoms,

532 or angular springs) between rigid portions, the buckling load is substantially reduced. The
 533 effect is hard to explain, but is readily derived from the TA phonon dispersion curve of the
 534 infinite linear chain, which predicts a decrease of the phase velocity of the TA mode as the
 535 wavelength decreases. In any case, this observation requires that the bending stiffness of
 536 flat graphene is obtained, as in Ref. 45, from the extrapolation of normalised experimental
 537 collapse pressures to $R^{-3} = 0$.^{45,77}

538 6. Folding

539 When a sheet of graphene is folded over onto itself, it adheres due to the vdW interaction.
 540 The radius of the fold is determined by the strength of the vdW attraction and by the value of
 541 D , and is of the order of the radius of C_{60} . An example is found in large-diameter SWCNTs,
 542 which are collapsed already at ambient pressure into the shape called “dogbone” or “peanut”.
 543 This presents a cavity of ≈ 0.6 nm in diameter at the edge.^{78,79} This small diameter reflects
 544 the very small value of the bending stiffness of monolayer graphene, in the range of 1 eV,
 545 but an accurate evaluation depends on knowing the strength of the vdW adhesion holding
 546 the fold folded. The atomic nature of the sheet plays a role, and folding is angle dependent
 547 as reported in the study of Zhang *et al.*⁸⁰ They reported that graphene sheet tends to fold
 548 along armchair (0° folding angle) and zigzag directions (30° folding angle). Consequently
 549 the spontaneous collapse of large nanotubes must depend on the chirality, which imposes
 550 the folding angle. The most unfavorable direction has the folding angle of 10° . Under very
 551 high temperature annealing (2000 °C) the unfavourable angle of folded graphene generates
 552 defects such as pentagons, relaxing the stress and leading to an irregular edge composed of
 553 zigzag and armchair directions.⁸¹

554 For bilayer and multilayer graphene with n layers ($n > 1$), the angle dependance disap-
 555 pears and this cavity size is governed by the number of layers, n . The length of the folding
 556 (shape of a pear) is well described by:⁸²

$$L(n) = \pi \sqrt{\frac{D}{\gamma}} = \left[\pi \sqrt{\frac{Y}{\gamma} \frac{d_{vdW}^3}{12}} \right] n^{\frac{3}{2}} \quad (6)$$

557 where D is the bending stiffness of the multilayer per unit length, d_{vdW} is the equilibrium
 558 distance between two graphene layers, Y is the Young’s modulus, γ surface adhesion energy
 559 per unit length around $260 \text{ mJ}\cdot\text{m}^{-2}$ (42.2 meV/atom).⁸² In Chen *et al.*,⁸³ the bending stiffness

560 $D_n(\text{eV})$ is well fitted by $6.7 \times (nh)^2$ with n between 2 to 6. These results are completely
 561 different even if for both relations, the bending stiffness of multilayer graphene is higher
 562 than a simple summation of the bending stiffness of each individual layer. The mechanical
 563 properties of bilayer and multilayer graphene depend critically on the issue of slipping or
 564 binding between layers. If layers slip freely, then the bending stiffness of n -layer graphene
 565 will be nD_1 which is not the case from simulation. On the other hand, if slipping does
 566 not occur, the bending stiffness of a bilayer, D_2 , will be largely unrelated to D_1 as it then
 567 derives directly from the in-plane stiffness (plus nD_1).⁸⁴ Neglecting D_1 for multilayers, we
 568 have $D_2 = 2(h/2)^2 c_{11}^{2D}$, $D_3 = 2h^2 c_{11}^{2D}$ and $D_4 = 2[(3h/2)^2 + (h/2)^2] c_{11}^{2D}$ and for large n , we
 569 recover the usual relation: $D_n = \frac{c_{33}}{1-\nu^2} \frac{(nh)^3}{12}$. The numerical values $D_1 = 0.7$, $D_2 = 131$ and
 570 $D_4 = 1308$ eV are to be compared to the value of $D_1 = 2.1$, $D_2 = 130$ and $D_4 = 1199$ eV
 571 reported by Sen *et al.*⁷³

572 7. *Shearing, sliding and friction between graphene layers*

573 In multilayer graphene, as mentioned in the previous Section, whether shearing or sliding
 574 occurs between layers determines the bending stiffness of the multilayer. In addition, a
 575 Raman shear mode is observed at low wavenumbers, between 31 cm^{-1} (bilayer) to 43 cm^{-1}
 576 (bulk).⁸⁵ Considering a linear chain model, it is possible to define an interlayer coupling
 577 stiffness, $\alpha = 12.8 \times 10^{18} \text{ N}\cdot\text{m}^{-3}$. The same value fits the Raman from bilayer through to
 578 graphite. It is a microscopic measure of the shear modulus, $\alpha d_0 = c_{44} = 4.3 \text{ GPa}$, close to
 579 the value of Table I. It corresponds to a spring constant $k = 0.419 \text{ N}\cdot\text{m}^{-1}$ for an unit cell.
 580 The existence of this mode shows the corrugation of the graphene surface at the atomic
 581 scale. It is also possible by friction to characterize atomically the surface. For a monolayer,
 582 the presence of ripples can increase the friction by 40% compared to bulk graphite where
 583 the layers are flat.⁸⁶ The friction results are well reproduced by calculation.⁸⁷ Finally, the
 584 sliding, corresponding to electrostatic interactions and dispersive forces, and its dependence
 585 on atomic direction has been calculated.⁸⁷ During the sliding, the interlayer distance changes
 586 by 0.04 \AA and the force is found to be in the range of 1.92 pN/atom considering no relaxation
 587 of the atoms, which is close to the experimental value of 2.11 pN/atom . These values are
 588 typical of flat 2D systems (h-BN).⁸⁷

589 **IV. MEASURING GRAPHENE MECHANICAL PROPERTIES**

590 The nanoscale thickness of monolayer graphene makes the accurate measurement of its
 591 mechanical properties a challenging task. A number of techniques have been proposed in
 592 the literature⁸⁸⁻⁹⁰ to measure the mechanical properties of graphene membranes. Some of
 593 the most important ones are described here.

594 **A. Atomic force microscopy**

595 The use of AFM in the study of the mechanical properties of graphene usually involves
 596 suspension of monolayer graphene over a substrate that has been previously patterned with
 597 holes and then applying a local force to the surface of graphene with high precision. In
 598 this way, the in-plane mechanical properties can be obtained. The work of Lee *et al.*²⁵
 599 was the first to measure the elastic modulus and fracture strength of graphene by using an
 600 AFM tip to indent graphene that was suspended over circular wells. Force-displacement
 601 (load-indentation) curves were obtained by indenting the membranes under constant speed.
 602 The applied force can then be calculated. However, the exact theory is far from simple⁹¹
 603 and many authors have used equations that appear to be over-simplified. The following
 604 equation, for example, has been frequently used,²⁵

$$F = \sigma_0^{2D}(\pi R)\left(\frac{\delta}{R}\right) + E^{2D}(q^3 R)\left(\frac{\delta}{R}\right)^3 \quad (7)$$

605 where R is the radius of the circular well, δ is the indentation depth, σ_0^{2D} is the pre-tension
 606 and E^{2D} is the 2D Young's modulus, and q can be expressed as $1/(1.05 - 0.15\nu - 0.16\nu^2)$,
 607 where ν is the Poisson's ratio of graphene.²⁵ Given the uncertainties in the other parameters,
 608 one may question the spurious precision implied by the inclusion of the factor q , which ranges
 609 only from 0.95 to 1.05 over the whole range of possible values, $0 < \nu < 0.5$, and has the value
 610 of 0.98 for the graphene value of $\nu = 0.16$. It is implausible that the contribution of the
 611 pre-tension to the force, for a given depth, is independent of the radius of the well. Fitting
 612 the data, values of the modulus and fracture strength of graphene are obtained. Similarly,
 613 the breaking strength can be calculated from:

$$\sigma_{max} = \left[\frac{F_{max} E^{2D}}{4\pi R_{tip}^2} \right]^2 \quad (8)$$

614 where R_{tip} is the radius of the AFM tip and F_{max} is the force at which the membrane breaks.

615 A variation of the AFM nanoindentation method where graphene suspended over a cir-
 616 cular hole, is the so-called beam bending method, where the 2D membrane is now in the
 617 form of a beam (or a stripe) and is suspended over a trough in the substrate. In this case
 618 the load-deformation relationship is:⁹²

$$F = \frac{Ew\pi^4}{6} \left(\frac{t}{L}\right)^3 \delta + \frac{\sigma_0 w \pi^2}{2} \left(\frac{t}{L}\right) \delta + \frac{Ew\pi^4}{8} \left(\frac{t}{L^3}\right) \delta^3 \quad (9)$$

619 where w , t , and L are the width, thickness and length of the beam, σ_0 is the intrinsic stress,
 620 δ is the beam deflection, F is the load applied in the beam centre and E is the Young's
 621 modulus.

622 Although the majority of the AFM-based results in the literature agree (within large
 623 experimental uncertainties) with values estimated from the bulk materials and with theo-
 624 retical calculations, it has been debated whether AFM nanoindentation can measure the
 625 mechanical properties of macroscopic 2D membranes in a meaningful way.⁹³ The AFM tip
 626 focuses on very small areas where the probability of defects is low. Intrinsic defects, rip-
 627 ples and crumples that are known to reduce the inherent properties of a “perfect” material
 628 are very common in 2D materials, yet are sometimes ignored or overlooked. Parameters
 629 that introduce uncertainties into the interpretation of data include the initial stress of the
 630 2D membranes, the position of the indenter (which needs to be in the exact centre of the
 631 membrane) and the indenter radius. Nevertheless, AFM nanoindentation is one of the most
 632 popular experimental methods to measure the mechanical properties of 2D membranes and
 633 has been used for a number of 2D materials.

634 Lee *et al.*²⁵ found the Young's modulus of monolayer graphene to be $E_{2D} = 340 \pm 50$ N
 635 m^{-1} , corresponding to $E = 1.0 \pm 0.1$ TPa for a thickness of 3.35 Å. However, not all graphene
 636 samples are flat. Nicholl *et al.*⁹⁴ and Ruiz-Vargas *et al.*⁹⁵ focused on crumpled and wrinkled
 637 graphene prepared by CVD and found that the stiffness obtained by AFM nanoindentation
 638 was reduced, compared to monocrystalline graphene.

639 A different, very useful application of AFM nanoindentation was reported by Cui *et al.*⁹⁶
 640 They performed a fatigue study using a combination of static and cyclic mechanical loading
 641 of a suspended 2D film (Fig. 2 (a)). Monolayer and few-layer graphene survived more than
 642 109 loading cycles at a mean stress of 71 GPa and a stress range of 5.6 GPa; that is higher
 643 than any material reported so far.

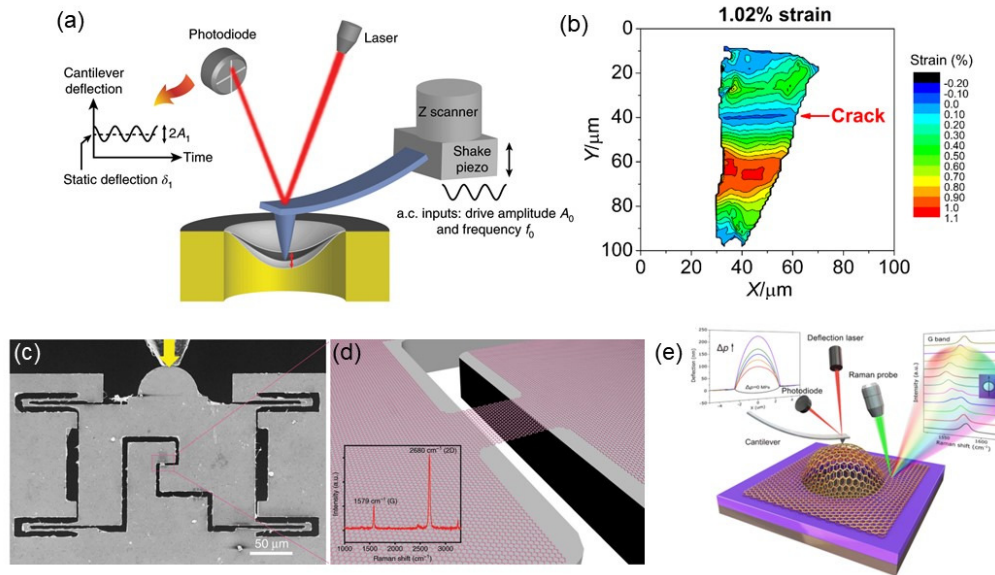


FIG. 2. (a) Schematic of the AFM fatigue testing set-up. Reproduced with permission from Nature Materials **19**, 405 (2020).⁹⁶ Copyright (2020) Springer Nature. (b) Strain contour map of a monolayer graphene flaked under 1.02% strain, where the strain distribution can be identified. The presence of a crack running through the upper part of the flake can be realised by the blue 0% strain line. Reproduced from Ref. ⁹⁷ with permission from The Royal Society of Chemistry. (c) A deposited graphene sample in the centre of a push-to-pull micromechanical device, actuated by an external pico-indenter. The yellow arrow indicates the indentation direction during a tensile testing process. (d) Illustration of the graphene sample suspended between the device gap. (c) and (d) are reproduced from K. Cao, S. Feng, Y. Han, L. Gao, T. Hue Ly, Z. Xu, and Y. Lu, Nature Communications **11**, 284, 2020,⁹⁸ licensed under a Creative Commons Attribution (CC BY) license. (e) Schematic diagram of the bilayer graphene balloon. The left inset shows the increasing pressure on a graphene bubble, while the right inset shows the Raman G band of graphene across the balloon (line scan). Reproduced with permission from Phys. Rev. Lett. **119**, 036101 (2017).⁹⁹ Copyright (2017) The American Physical Society.

644 B. Raman spectroscopy

645 Raman spectroscopy has been used extensively for the study of the mechanical properties
646 of carbon-based materials as a result of their strong resonant Raman scattering. The well-
647 defined characteristic Raman peaks enable the observation of a number of very interesting
648 phenomena and properties under uniaxial or biaxial strain.

649 For uniaxial strain, these experiments involve the deposition of monolayer or multilayer
650 graphene flexible onto a polymeric substrate, which is subsequently strained by 2-, 3- or 4-
651 point bending under a Raman spectrometer. The application of strain leads to an elongation
652 of the C-C bonds and the lattice deformation is clearly and accurately evidenced by down-
653 shifts of the Raman peaks. Mohiuddin *et al.*³³ first studied the deformation of monolayer
654 graphene deposited onto a flexible substrate under uniaxial tensile strain and found that the
655 shift of the 2D-mode is about $-60 \pm 5 \text{ cm}^{-1}/\%$ strain. This downshift corresponds to the
656 material having an elastic modulus in the order of 1 TPa and was subsequently confirmed
657 by a number of research groups.^{1,97,100,101} The G band is split by the uniaxial strain into
658 G^+ and G^- peaks. The shift of the G^+ band was $-10.8 \text{ cm}^{-1}/\%$ strain while the shift of
659 the G^- band was $-31.7 \text{ cm}^{-1}/\%$ strain. The frequency of the G-mode is related to the C-C
660 bond stiffness, though it contains non-negligible contribution from up to the fifth nearest
661 neighbour (more details in Sec. VIII).²⁶ It is nevertheless reasonable to consider that its shift
662 with strain has contribution only from the nearest neighbour, and is therefore determined
663 by the anharmonicity of the C-C bond. The physical meaning of the shift of the 2D-mode is
664 less clear as it is related to the evolution of the LO/TO phonon dispersion under strain.¹⁰²

665 The strength of monolayer graphene can be also studied by *in situ* Raman mapping. Zhao
666 *et al.*⁹⁷ prepared monolayer graphene by mechanical exfoliation, deposited the samples onto
667 polymer substrates and performed *in situ* Raman mapping at different strain levels to obtain
668 the strain distributions over the graphene flakes. Strain contour maps showed significant
669 events such as strain build up, edge effects and cracks which developed with increasing strain
670 (Fig. 2 (b)). Two main mechanisms of failure were observed: flake fracture, and failure
671 of the graphene/polymer interface. Low strengths were observed for these macroscopic
672 monolayer samples, only 10 – 15 GPa, an order of magnitude lower than the value of 130
673 GPa that was reported by Lee *et al.*²⁵ Simulations suggest that this was due to the presence
674 of defects.⁹⁷ Under large deformation, Raman observations also suggest a manifestation of

675 large nanometer-scale strain inhomogeneity within the laser spot size.^{103,104}

676 In multilayer graphene, Gong *et al.*¹⁰⁵ studied the effect of the layer number on the
677 downshift of the 2D band and found that monolayer and bilayer graphene displayed almost
678 the same redshift rate. On the other hand, with further increasing layer number, the shift
679 rate decreases significantly. The 2D Raman profile was fitted by a single Lorentzian, although
680 a broadened 2D profile is expected with increasing number of layers as it contains more
681 components,¹⁰² and the decrease of the ‘average’ shift rate was interpreted as showing some
682 slippage between the layers, *i.e.* reduced internal stress transfer. However, these spectra
683 were recorded at the centres of the flakes. Similarly to shear lag, slippage between layers or
684 between the lowest layer and the substrate should reduce the strain at the edge of the flake
685 first and propagate inwards as the strain is increased (Fig. 2 (b)). More detailed studies
686 would be desirable.

687 For biaxial strain, blister testing as described in Sec. IV D below is appropriate. G-mode
688 shifts as large as -80 cm^{-1} are obtained and the deduced Gruneisen parameter of $1.8 \pm 10\%$
689 is compatible with biaxial strain.¹⁰⁶ The calculated slope (with some approximation on Y , D
690 and d_{vdW}) $\Delta\omega_G/\epsilon = -57 \text{ cm}^{-1}/\%$ can be compared with later similar studies. Shin *et al.*¹⁰³
691 achieved biaxial reversible strain up to $\approx 2\%$. Using the same approximations, they found
692 $\Delta\omega_G/\epsilon = -62 \text{ cm}^{-1}/\%$. With large biaxial strain, the linewidth of the G-mode increases.

693 Interlayer modes, the layer breathing mode and shear mode, are convenient measures of
694 adhesion and shear strength between graphene layers. A linear chain model describes the
695 frequency of the shear mode, and its change with number of graphene layers very well, as
696 discussed in Sec. III D 7. The model applies to the layer breathing mode too.¹⁰⁷ The layer
697 breathing mode becomes Raman-active, when graphene layers are twisted from AB stack-
698 ing. Its intensity is usually very weak, and requires resonance condition to be observable.
699 Resonance Raman spectroscopy is particularly useful to study graphene and CNTs samples,
700 where the resonance condition is that the energy of the in-coming or out-going laser matches
701 the gap between van Hove singularities in these low-dimensional samples.^{108,109}

702 C. *In situ* tensile tests

703 For *in-situ* tensile tests, nanomechanical testing devices are usually introduced within a
704 scanning electron microscope (SEM) or transmission electron microscope (TEM) and the

705 deformation of the graphene is followed in the images. To study its fracture toughness, Zhang
706 *et al.*¹¹⁰ suspended nanocrystalline graphene over the jaws (gap) of a micromechanical device
707 driven by a nanoindenter, within an SEM chamber. This imposed uniaxial tension on the
708 graphene. Brittle fracture was observed when a central crack had been machined by FIB
709 in the graphene samples prior to testing. The fracture toughness of graphene (important
710 for engineering applications) was found to be $K_c = 4.0 \pm 0.6$ MPa, while the critical strain
711 energy release rate was $G_c = 16 \pm 5$ J·m⁻² (where we estimate the uncertainty on G_c from
712 their data). Cao *et al.*⁹⁸ reported *in situ* tensile tests within an SEM chamber and measured
713 the elastic properties and stretchability of monolayer CVD graphene (Fig. 2 (c,d)). The
714 Young's modulus was ~ 1 TPa, while the tensile strength was around 50 – 60 GPa, when
715 the elongation of the sample was $\sim 6\%$. Once again the actual strength of macroscopic
716 graphene samples is significantly lower than the value of 130 GPa reported by Lee *et al.*²⁵ in
717 AFM nanoindentation. Brittle fracture initiated from the edges of the samples, suggesting
718 that control of the edge states and edge effects could lead to greater strength.

719 *In situ* TEM can offer useful information on cracks and defects in graphene. For example,
720 Fujihara *et al.*¹¹¹ observed that crack propagation takes place along a specific crystallographic
721 direction, in order to create zigzag edges. Kim *et al.*¹¹² showed that the presence of grain
722 boundaries influences crack growth. When the stress is normal to the grain boundaries,
723 the crack can follow the boundary; however, if the crack is initiated away from the grain
724 boundaries and at some random orientation with respect to it (with the strain direction not
725 normal to the grain boundary), the stiffness the tear experiences is more or less unaffected
726 by the grain boundary and the crack will pass through the grain boundary, switching to the
727 most favourable direction in the next grain.

728 D. Pressurized blister method

729 The use of the pressurized blister (bulge, bubble, or balloon) method can provide infor-
730 mation on the mechanical properties of 2D membranes and the interfacial adhesion between
731 the substrate and the membrane. For these experiments, graphene is again suspended on
732 top of a hole, or microcavity, in the substrate. The vdW forces between the substrate and
733 the membrane hold the sample in place. Gas is fed in the hole thus pressurising the mem-
734 brane. This leads to a spherical blister with a radius R . AFM is used to measure the blister

735 (compare Raman methods in Sec. IV B). The relation between its height, δ , and the pressure
736 difference inside and out, Δp , is,¹¹³

$$\Delta p = K(\nu)(Ed\delta^3)/R^4 \quad (10)$$

737 where E is the Young's modulus, and d is the graphene thickness. $K(\nu)$ is a coefficient
738 depending on Poisson's ratio only and is very close to 3. Thus the elastic modulus can be
739 calculated from the measured AFM deflections.

740 The group of Bunch¹¹⁴ first measured the adhesion of graphene on a silicon oxide substrate
741 in this way. The adhesion strength is revealed by the pressure at which the blister diameter
742 begins to exceed the hole diameter, i.e. when the graphene begins to peel away from the
743 substrate. The relationship is given by Bodetti *et al* as Eq. 14 of their paper.¹¹⁵ They obtain
744 adhesion energies of 0.45 ± 0.02 J/m² for monolayer graphene and 0.31 ± 0.03 J/m² for
745 few-layer graphene (2 – 5 sheets). With its high flexibility, graphene, especially monolayer
746 graphene, is able to conform to the topography of very smooth surfaces, thus leading to high
747 values of adhesion energy. However, much lower values may be observed, as adhesion between
748 a graphene sheet and a substrate (or surface) is highly dependent on the surface conditions
749 such as moisture, roughness, chemical reactivity and others and a considerable spread on
750 the adhesion energies has been reported in the literature (see Ref. 2 and references therein).
751 Similarly, weakening of the adhesion in turbostratic graphite compared with Bernal graphite
752 is expected.^{5,116} In a more recent report, Wang *et al.*⁹⁹ measured the interlayer shear stress
753 of bilayer graphene by monitoring the strains in the graphene next to the blister but not
754 lifted off the substrate (Figure 2 (e)). Here strain develops as a consequence of sliding. Their
755 data provided evidence of both the lower monolayer sliding on the substrate, and the upper
756 layer sliding on the lower layer. Analysis of the data gave the interfacial shear stress of
757 monolayer graphene on SiO₂ as 1.64 MPa. This was much higher than the interlayer shear
758 stress of bilayer graphene, 40 kPa (1.04 fN/atom). The implication is that the graphene-
759 SiO₂ vdW bonding is much stronger than the weak dispersion vdW bonding that holds the
760 two graphene sheets together.

761 E. Inelastic X-ray scattering

762 As discussed in Section III, some key elastic moduli of graphene come directly from the
763 experimental values for graphite, measured by inelastic x-ray scattering (IXS). This measures

764 the acoustic phonon branches of graphite.²⁰ The initial slope of these branches along high-
765 symmetry directions gives the sound velocity, and therefore the moduli by Christoffel's
766 equation.¹¹⁷ IXS is less sensitive to structural defects than ultrasonic methods and it does
767 not have the difficulties in sample size and energy transfer limitations that inelastic neutron
768 scattering suffers.²⁰ Experimentally IXS employed to measure graphite is not suitable for
769 measurements on graphene due to the small sample volume. The alternative approaches
770 to measure the in-plane elastic moduli of graphene are introduced above, and the results
771 largely depend on interactions of graphene with its surroundings (substrates and/or [pressure](#)
772 [transmitting media](#), PTM). Details will be discussed in Section V.

773 F. Density functional theory

774 This section particularly addresses properties that are anisotropic in-plane, which have
775 been calculated but not confirmed by experiments. Whereas it is extremely difficult to
776 do any mechanical testing on free-standing graphene, the in-plane graphene mechanical
777 properties calculated by DFT are obtained naturally freestanding, and in vacuum. It is
778 commonly considered that the calculated in-plane elastic constants are accurate (and in-
779 deed they are close to the well-established experimental results on graphite)²² and they
780 are often used to parameterise various empirical models and evaluate the validity of classic
781 simulations.^{22,118–121} Strain is the input in a computational model and the resulting energy
782 and stress are calculated. The ease with which the positions of the atoms can be specified
783 enables the investigation of a number of anisotropic in-plane properties that result from
784 strain along specific directions (armchair or zigzag).

785 The undeformed hexagonal lattice is isotropic. While in-plane uniaxial strain breaks
786 the symmetry, the isotropy remains in the sense that most properties vary with the
787 strain along most directions very similarly, except two specific directions.^{122,123} We present
788 clear anisotropic response of four properties to uniaxial strain along zigzag or armchair
789 directions.^{121,124,125} Fig. 3 (a) presents that the in-plane Poisson's ratio (corresponding to
790 the elastic modulus c_{12}) shifts differently with uniaxial strain along armchair or zigzag
791 direction, and the stiffness (corresponding to c_{11}) becomes different after 15% strain along
792 armchair or zigzag direction.¹²⁴ Fig. 3 (b) shows that stress-strain curves of the shear de-
793 formation (relevant to c_{66}) along these two directions are different.¹²¹ The ultimate stress of

794 graphene along these two directions is also calculated to be different by about 20%.¹²⁰ Zhou
795 *et al.* plotted the electron density of graphene at zero, biaxial and uniaxial stress, as shown
796 in Fig. 4.¹²⁵ In Fig. 4 (c), the high electron density area (red area) at uniaxial stress along
797 zigzag direction attracts the nuclei at corners more than the central areas of those stretch
798 bonds attracting those corresponding nuclei. That extra attraction stiffens the three-atom
799 bending force constant labeled by k_{angle} , which supports the uniaxial stress. Differently,
800 along uniaxial stress along the armchair direction, the supporting bending force constant
801 (labeled as k_{angle} in Fig. 4 (d)) is stiffened by two high electron density areas (two red areas
802 on the right). This was proposed as a possible interpretation for the anisotropy in ultimate
803 strength¹²⁵ and could be relevant to other anisotropic properties.

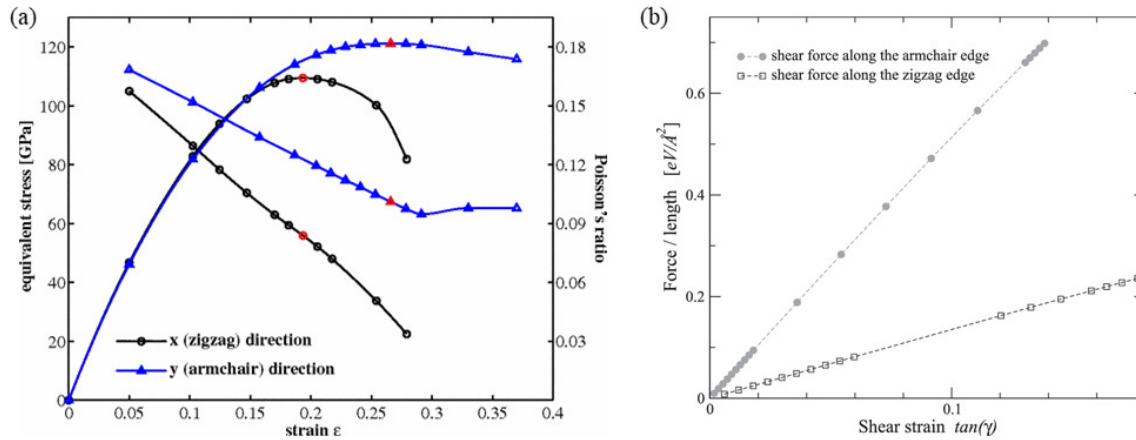


FIG. 3. (a) “The curves connected to the origin are the equivalent tensile stress ($d_0=3.34 \text{ \AA}$) versus uniaxial strain in the x and y directions, respectively. The lines with initially negative slopes (scale labels to the right) are the finite-deformation Poisson’s ratios as functions of the uniaxial strain in the x and y directions, respectively. The red circles and triangles indicate the condition where peak stress could be attained for zigzag and armchair nanotubes, respectively.” Reproduced with permission from Phys. Rev. B **76**, 064120 (2007).¹²⁴ Copyright (2007) The American Physical Society. (b) “Stress-strain curves for shear deformations of graphene monolayers, obtained through MD simulations. Filled circles (open squares) show results corresponding to shear forces acting on the armchair edge (zigzag edge) of graphene. Dashed lines are guides to the eye.” Reproduced from G. Kalosakas, N. N. Lathiotakis, C. Galiotis, and K. Papagelis, Journal of Applied Physics **113**, 134307 (2013),¹²¹ with the permission of AIP Publishing.

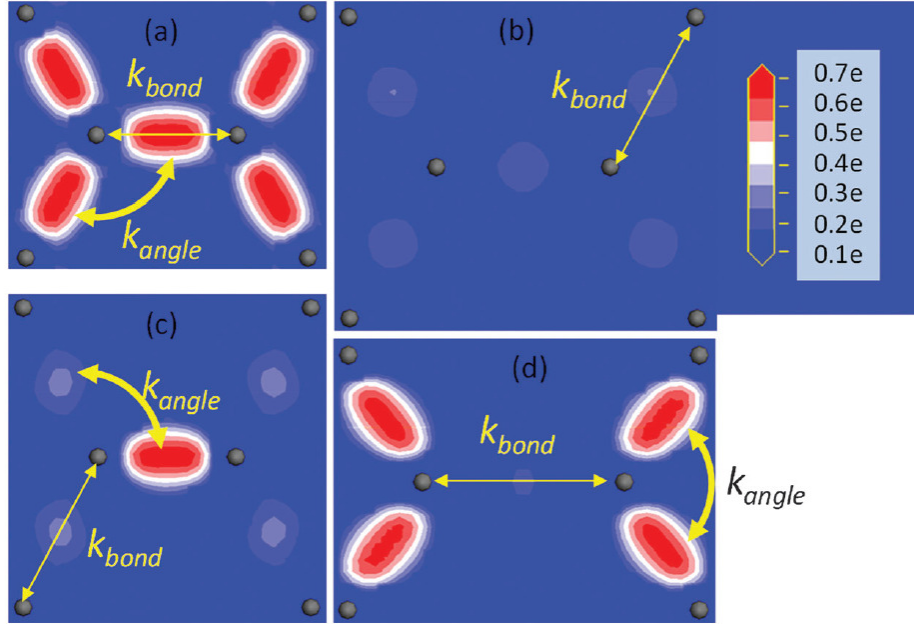


FIG. 4. “The electron density distribution contour of graphene under different tensile loads, (a) $\epsilon=0$; (b) biaxial tension at $\epsilon=0.28$ (the bond stretching ratio $\xi=0.25$); (c) uniaxial tension along the zigzag direction at $\epsilon=0.28$ (the bond stretching ratio $\xi(\text{Bond I})=0.2$, $\xi(\text{Bond II})=-0.03$); (d) uniaxial tension along the armchair direction at $\epsilon=0.28$ (the bond stretching ratio $\xi(\text{Bond I})=0.36$, $\xi(\text{Bond II})=0.003$). Reproduced with permission from Phys. Chem. Chem. Phys. **18**, 1657 (2016).¹²⁵ Copyright 2016 The PCCP Owner Societies.

804 **V. GRAPHENE IN INTERACTION WITH ITS ENVIRONMENT AT**
 805 **HIGH PRESSURE**

806 The mechanical properties of graphene can be probed by high pressure. In bulk materials,
 807 high-pressure experiments can characterize the bulk modulus and its pressure derivative
 808 through the equation of state, which links the volume variation to pressure. Many bulk
 809 systems are anisotropic, and have an anisotropic elastic stiffness tensor c_{ij} . With more than
 810 one atom per primitive cell, as in graphene, internal strains may occur. Particularly for 2D
 811 systems, bond or directional compressibilities can be defined to fully describe the changes in
 812 atomic positions due to the deformation under hydrostatic pressure.¹²⁶ Graphene is however
 813 not only highly anisotropic but also is only one atom thick. While its in-plane deformation
 814 modifies the C-C separation or sp^2 bond length, its out-of-plane deformation can only be

815 described as deformation of the π -orbitals.

816 In high-pressure experiments, both the C-C sp^2 bonds and the spatial extension of the
817 π -orbitals will be modified by interactions with the surrounding PTM (anything from he-
818 lium, argon, through to water, ethanol or various oils), which itself increases in density
819 with pressure. Interactions between graphene and the surrounding medium can range from
820 weak vdW to strong covalent interactions. In particular, the vdW interactions will become
821 stronger under pressure. The π -orbitals may become highly modified under pressure, per-
822 haps creating strong dipolar interactions with neighboring molecules. At what point could
823 we consider the system to be something other than graphene (as, *e.g.* graphene oxide)? The
824 effects of pressure may be simply to change the graphene thickness, or may include doping
825 or hybridization. As in any material, the c_{ij} are expected to increase with pressure, but
826 since all the carbon atoms in graphene are in contact with the environment, we may expect
827 that c_{ij} will depend on pressure and temperature, and also on the environment. It is likely
828 that c_{33} will be most influenced by the pressure and the environment. But are the pressure
829 dependencies of other parameters dominated by the pressure or by the environment? How
830 best to compare the pressure responses of graphene and graphite? These issues are addressed
831 here.

832 Graphene can be studied in suspension, with PTM on both sides, and when the PTM
833 is liquid, the graphene is free of non-hydrostatic strains. When graphene is supported by a
834 substrate, the situation is quite different. We must consider what differences are significant,
835 and what can be learned from these different environments.

836 This section has three parts treating the pressure response of graphene in the different
837 environments depicted in Fig. 5. First, we will discuss the case of graphene in suspension
838 in a fluid as shown in Fig. 5 (a). Then, the case of supported graphene immersed in a fluid
839 PTM (b). Finally, the case of graphene sandwiched between two different (Fig. 5 (c.1)) or
840 identical (Fig. 5 (c.2)) solids. The Fig. 5 (c) cases are also relevant to help understand the
841 mechanical response of graphene in composite materials.

842 A. Suspended graphene in a fluid PTM

843 This may be considered as the paradigmatic pressure experiment, with the two variants
844 of Fig. 5(a). A few Raman scattering experiments have been reported corresponding to

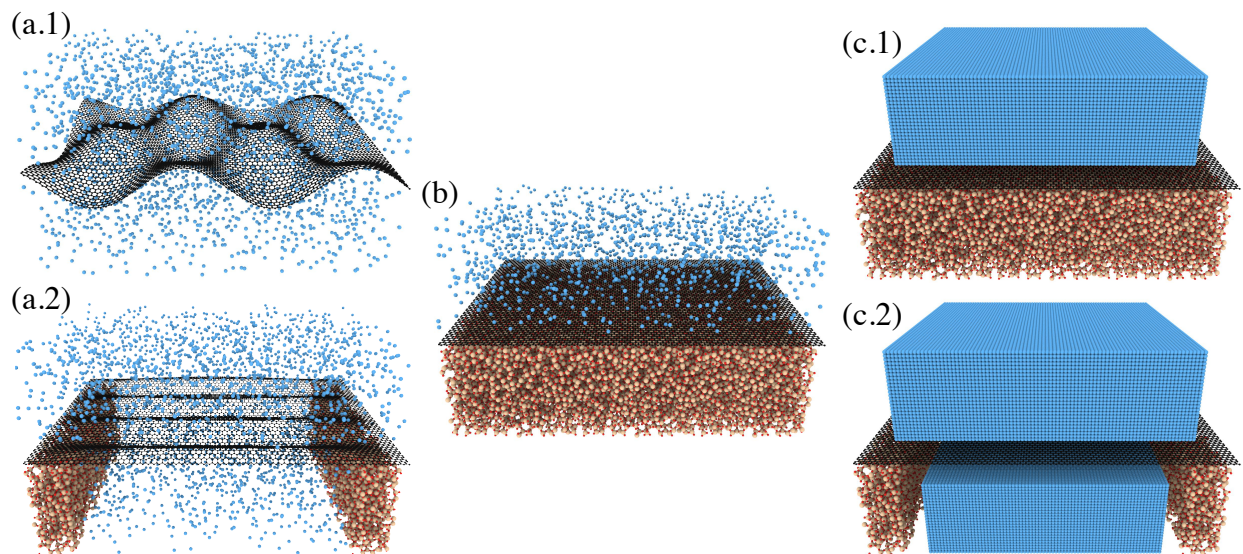


FIG. 5. The various cases encountered in high-pressure experiments and discussed in this section are: (a) suspended graphene in a fluid PTM, (b) supported graphene in a fluid PTM, and (c) graphene sandwiched between two solids. In cases (a) and (c), one needs to distinguish between (a.1) free-standing graphene and (a.2) graphene across a hole ; and (c.1) graphene sandwiched between two different solids and (c.2) graphene in a single solid. The presence of ripples is shown in (a.1) and wrinkles in (a.2).

845 these configurations: an ensemble of graphene layers in suspension and considered separated
 846 each from the others (Fig. 5(a.1)) or supporting an individual graphene layer
 847 substrate (Fig. 5(a.2)). In both cases, we may note that the graphene may not be flat due to
 848 the spontaneous formation of ripples (Fig. 5(a.1)) and wrinkles (Fig. 5(a.2)).^{127,128} Wrinkles
 849 and ripples differ by their aspect ratio.¹²⁷ Ripples are isotropic, with an amplitude \sim nm,
 850 and an aspect ratio \sim 1. Wrinkles are more aligned and larger, having an aspect ratio $>$ 10,
 851 due to the partial decoupling of bending and stretching modes.^{50,127,128}

852 In Fig. 5 (a.2), while the graphene sheet may be under tension at ambient pressure
 853 (resulting in wrinkles with an axis perpendicular to the trench),¹²⁷ we might expect that
 854 differential contraction under pressure of the support and the graphene (unless the support
 855 is made of diamond) would result in a loss of tension and eventually buckling in a different
 856 direction or even crumpling.¹²⁷ However, this may be prevented by the evolution of the
 857 adhesion of graphene to the internal walls of the trench. [Bunch *et al.* observe a minimum](#)

TABLE IV. Summary of high-pressure Raman experiments on suspended graphene.

Type	PTM ¹	$P_{hydrostatic}$ (P_{Max}) [GPa]	$\frac{\partial\omega_G}{\partial P}$ hydro (P_{Max}) [cm ⁻¹ /GPa]	Ref
suspension ⁽²⁾	N ₂	2 (8)	4.7	34
supported ⁽³⁾	Fluorinert	1.0 (4.2)	- (5.6)	37
suspension	DMF	1.6 (7)	5.4	40
supported ⁽⁴⁾	H ₂ O	1.0 (35)	- (3.4)	131

(1) see text

(2) The sample were films with a mixture of monolayer, bilayer and few-layer graphene and having 2D characteristic signature of few-layer graphene rather than single-layer graphene.

(3) From the detachment from graphene supported on Si/SiO₂ using as PTM a 1:1 mixture of Fluorinert FC70 and FC77 (nonpolar).

(4) A gold microscopy grid was used to suspend graphene.

858 non-zero tension,¹²⁹ due to this, and analysed by Lu and Dunn.¹³⁰

859 All high-pressure experiments on suspended graphene have used Raman spectroscopy as a
860 probe. Raman spectroscopy does not give direct access to the pressure evolution of the unit
861 cell parameters which would determine the elastic constants. Raman spectroscopy provides
862 nevertheless a signature of the response of the C-C interatomic potential through the G-
863 mode pressure dependence. Table IV summarizes the most relevant results from suspended
864 graphene studies.

865 All the studies in Table IV were done using different PTM (FC-70 is perfluorotripropy-
866 lamine and FC-77 is a perfluorocycloether; DMF is N,N-dimethylformamide). Only two
867 studies (using nitrogen and DMF as the PTM) were performed in hydrostatic conditions
868 throughout the pressure range (*i.e.* with a liquid PTM). On the other hand, the nitrogen
869 PTM study was of a mixture of monolayer, bilayer and few-layer graphene, which makes its
870 results difficult to interpret. The DMF PTM study is the only one in which a G-band pres-
871 sure dependence has been obtained for a monolayer graphene sample suspended in liquid.
872 This study was able to obtain a $\frac{\partial\omega_G}{\partial P}$ in hydrostatic conditions from a careful analysis from
873 a total of 4 pressure points in the DMF hydrostatic domain up to 1.6 GPa. The value of
874 5.4 cm⁻¹/GPa was obtained with a linear fit and 5.6 cm⁻¹/GPa with a quadratic fit using

875 the quadratic coefficient of the graphite fit.²⁸ These are 15 to 25 % higher than the graphite
876 G-mode slope which is $4.3 - 4.7 \text{ cm}^{-1}/\text{GPa}$ using the same quadratic coefficient.²⁸ This
877 difference is partly due to the non-negligible impact of interlayer-coupling on the in-plane
878 vibrations in graphite (see Sec. VI C). There are reports pointing to PTM-induced charge
879 transfer to graphene in polar media^{132,133} even from the first stages of compression,¹³³ and of
880 pressure-induced reactivity with water.¹³⁴ Doping, p or n , leads to an enhanced value of the
881 G-mode frequency¹³⁵ which may explain an increase of the G-mode pressure coefficient in
882 graphene. Experiments using an inert PTM would clarify whether there is a difference be-
883 tween graphene in-plane elastic constants and those of graphite, or if the observed differences
884 are related to polarization-induced modifications of the electronic structure of graphene.

885 Another interesting aspect is that, as pressure is increased, the PTM fluid viscosity
886 increases. The vdW graphene-fluid interaction may overcome the thermal energy of the
887 fluid molecules. An organized fluid layer may then appear in contact with graphene. This
888 constitutes a loss of 2D character which may also affect the mechanical properties of graphene
889 and may explain differences in the Raman response with different PTM. DFT and MD
890 modelling - DFT is performed at zero temperature - have in fact shown in carbon nanotubes
891 under high pressure the formation of a structurally coherent contact layer of CO_2 or water
892 around carbon nanotubes.^{136,137} In fact, DFT modelling shows that in carbon aromatic
893 systems such as benzene, vdW interaction plays an important role in phase stability at high
894 pressures.¹³⁸

895 We may conclude this part by underlining that dipole or other enhanced VdW interactions
896 with the PTM molecules and the possible loss of 2D character through the formation of a
897 PTM contact layer need to be explored as possible mechanisms modifying the graphene
898 mechanical properties at high pressure.

899 B. Supported Graphene in a fluid PTM

900 This is the case depicted in Fig. 5 (b), *i.e.* graphene on a substrate and immersed in a
901 fluid PTM. In this case, the PTM applies hydrostatic pressure on the graphene-substrate
902 system. With increasing pressure, the substrate (unless it is diamond) contracts much faster
903 than the graphene, which puts the graphene under a large biaxial compression. How much
904 graphene contracts and is strained in response to the substrate shrinking is governed by the

905 graphene-substrate adhesion and friction, graphene/PTM adhesion, graphene stiffness, and
906 graphene bending modulus. We will now discuss the role of each parameter.

907 **1. Role of the substrate**

908 Graphene stiffness is one of the largest-ever measured, with a Young’s modulus of ~ 1 TPa
909 (Table II),^{25,139} while its bending stiffness is often considered as negligible,^{140,141} indeed so
910 small that it is hard to measure in a direct manner. Many reported values (depending on
911 the temperature and flake size) are often about 1 eV,¹⁴² and it was measured in carbon nan-
912 otubes as 1.7 eV (see Sec. III D 5).⁴⁵ Thus, it is expected that graphene will tend to bend or
913 wrinkle rather easily in order to relieve in-plane compression.^{143–145} The interaction between
914 the graphene layer and its supporting surface, namely the graphene-substrate adhesion, is
915 therefore an extremely important parameter governing the graphene response to biaxial
916 compression: if the graphene-substrate adhesion is poor, the graphene will not fully follow
917 the substrate’s deformation and instead will slide or wrinkle to reduce its stress. However,
918 the adhesion of graphene to its substrate is an intricate mixture of (a) the interaction energy
919 between carbon and surface atoms, (b) substrate surface roughness, (c) graphene number
920 of layers, (d) commensurability of the graphene and substrate lattices, and (e) the normal
921 force from the PTM, which must modify the effects of (a) to (d). As a consequence, each
922 graphene-substrate system is unique and the amount of strain transferred from the substrate
923 to the graphene layer can only be assessed in a phenomenological way, for example with a
924 “strain transfer efficiency” parameter α .^{38,146} Now to discuss each of these parameters:

925 (a) The strength of the graphene-substrate interaction energy can be assessed from the
926 graphene-substrate distance, which varies greatly from one substrate type to another – as
927 shown in Table I which gives some experimentally measured distances. Graphene deposited
928 on metals tends to form covalent bonds that greatly decrease the graphene-substrate dis-
929 tance. This would correspond to a much increased adhesion energy and forces for peeling
930 off and for sliding. One thus expect the strain transfer efficiency α to be close to 1 in such
931 systems – and $\alpha = 1$ for a copper substrate was indeed observed up to a critical stress of
932 2 GPa.^{34,38} Moreover, the history of the graphene sample plays a huge role in the interaction
933 with its substrate: whether the graphene was transferred or grown on the substrate has
934 a large impact on its adhesion and residual stress. As discussed in Sec. III D 3, epitaxial

935 graphene is under large strain after cooling (hence showing $\alpha = 1$), whereas CVD (usually
936 on Cu) graphene shows the occurrence of ripples to release the stress (resulting in a $\alpha < 1$).
937 An even lower α on Cu is expected if graphene is transferred onto it.

938 (b) When the surface roughness is too high, total unbinding from the substrate surface
939 can occur, at ambient pressure^{145,147} or under high pressure.³⁷ Some substrates show the oc-
940 currence of a critical stress beyond which the strain transfer efficiency is greatly diminished,³⁸
941 which is probably due to a partial unbinding of the graphene layer from its substrate. While
942 not measured experimentally, this critical stress is supposed to be roughness-dependent for
943 a given substrate. The substrate surface roughness also greatly influences the *friction* be-
944 tween the graphene layer and its substrate, which in turns plays a role on the mechanism of
945 the mechanical response of graphene to biaxial compression. When the strain transfer effi-
946 ciency $\alpha < 1$, it means that either (a) graphene slips all over the substrate; or (b) graphene
947 wrinkles, ripples or crumples. Or a combination of (a) and (b) may occur. In the case of a
948 rough substrate with a conforming graphene (as is the case most of the time), case (a) has
949 an extended energy cost because it involves the whole graphene surface, while case (b) only
950 involves the local elastic energy cost of buckling. Depending on the substrate roughness, a
951 varying proportion of slipping and buckling may thus occur. However, the application of a
952 normal force by the hydrostatic pressure to the surface will certainly hinder the formation of
953 wrinkles or ripples. Wrinkles were however observed under hydrostatic pressure (at 4 GPa)
954 even in nano-graphite (~ 30 -layer graphene).¹⁴⁸

955 (c) In the same manner, adhesion to a rough substrate is decreased when the number
956 of graphene layers increases, *i.e.* when the graphene bending rigidity increases.³⁶ With
957 an increasing number of layers, the conformation of graphene to its rough substrate may
958 decrease, resulting in a decreased friction. We may note that in MoS₂, which has a much
959 higher bending stiffness than graphene, a bimodal behavior has been seen when supported
960 on Si/SiO₂. A mixture of regions showing strong adhesion and other regions showing weak
961 adhesion was seen.¹⁴⁹

962 (d) Finally, it was for example shown that for a given substrate such as Co(0001), the
963 graphene-substrate distance decreases when the graphene lattice is matched with the sub-
964 strate lattice,¹⁵⁰ allowing for covalent C-Co bonding. Such an effect therefore plays a large
965 role on the starting *stress+doping* state of graphene. In the following however, we will not
966 consider substrates in which this type of interaction may happen. Pressure-induced doping

967 effects will then be due essentially to interaction with the PTM and will be discussed in
968 the following subsection. One should also note that the (in)commensurability of the lattices
969 should also play a role in the friction between graphene and its substrate.

970 (e) All these parameters may be impacted by the modification of the graphene-substrate
971 distance due to the application of pressure. However, in pressure ranges for which the PTM
972 is liquid (hydrostatic pressure), the graphene Raman response is always linear,^{34,37,38,40,131,133}
973 which tends to show that this effect is limited at pressures below ~ 10 GPa.

974 The mechanical response of graphene to high pressures is usually followed through *in situ*
975 Raman spectroscopy, as stress induces a deformation of the carbon bonds and thus shifts
976 the Raman features such as the G-mode (ω_G) or the 2D-mode (ω_{2D}). But similar shifts can
977 also arise from doping. Graphene is a π -conjugated 2D material, thus its electronic struc-
978 ture is highly sensitive to its environment. This charge sensitivity allows doping through
979 gating or intercalating (“substrate doping”, see *e.g.* Ref. 151 for a review), but it can also
980 occur due to the interaction with polar molecules in the PTM which can be enhanced at
981 high pressure¹³³ (“PTM doping”). Overall, the doping and strain contribution to the ω_G
982 shift can be unravelled by following the slope of $\partial\omega_{2D}/\partial\omega_G$ ^{151,152} (see Fig. 6). Finally, the
983 substrate type and the graphene preparation method (as-prepared, transferred, synthesised,
984 exfoliated...) play a large role in the ω_G frequency^{150,151} and the graphene-substrate dis-
985 tance (Table I). Large variations of the ω_G peak position can be observed for graphene on
986 a substrate according to the crystallographic orientation, matching or random. For exam-
987 ple, while $\omega_G = 1581 \text{ cm}^{-1}$ for free graphene, it redshifts down to 1452 cm^{-1} for oriented
988 graphene on Co(0001), and it can vary between ~ 1550 and 1600 cm^{-1} across a single sam-
989 ple of misaligned graphene on Co(0001).¹⁵⁰ In the case of Co(0001), matching the graphene
990 and substrate lattice orientations allows chemical bonding between the two, resulting in a
991 shortening of the graphene-substrate distance and a significant stretching of the graphene
992 lattice, thus decreasing the ω_G frequency. However, this is peculiar to Co(0001) as, usually,
993 the CVD synthesis of graphene on a metallic substrate results in a compressive stress of the
994 graphene layer, and thus an increased ω_G frequency.¹⁵¹ On the other hand, the incommen-
995 surability of both lattices can result in large local variations of the ω_G position, that can be
996 due to both strain and/or doping.

997 In conclusion, the substrate plays a very important role both at ambient pressure and
998 under high pressure. Before the application of pressure, and together with the history of

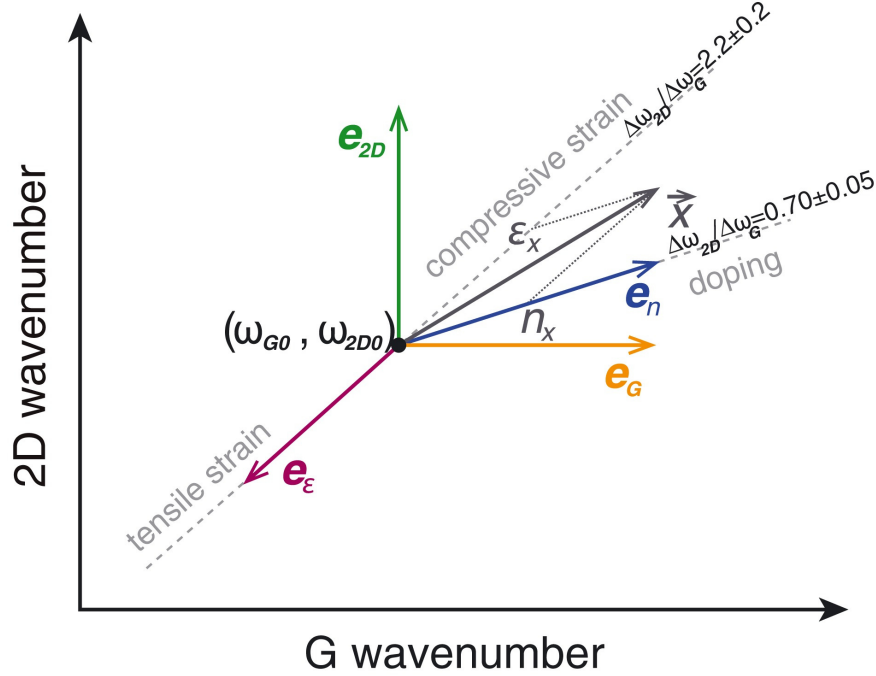


FIG. 6. Lee diagram¹⁵² allowing retrieving the strain and doping (ε_X and n_X) at a point X in the (ω_G, ω_{2D}) space. The origin $(\omega_G, \omega_{2D})_0$ is the reference state at ambient conditions. Reproduced with permission from J. Raman Spectrosc. **49**, 130 (2018).¹⁵¹ Copyright 2018 John Wiley & Sons, Ltd.

999 the sample, the substrate determines the reference state of the graphene, *i.e.* the graphene-
1000 substrate distance as well as the initial stress and doping states. During pressure application,
1001 it determines the strain of the graphene layer through the phenomenological parameter α ,
1002 this strain being relative to the initial state. It is inappropriate to consider the evolution
1003 of the Raman G-mode frequency as a function of pressure;³⁸ rather, it should be considered
1004 as a function of the strain, the actual strain depending on the reduction of the substrate
1005 dimensions. Overall, in the simple case where $\alpha = 1$ and *in the absence of doping*, a Raman
1006 G-mode frequency dependence on substrate strain ε of $\frac{\partial \Delta \omega_G}{\partial \varepsilon} = -60 \pm 3 \text{ cm}^{-1}/\%$ is expected.³⁸
1007 Since $\frac{\omega}{\omega_0} = (1 + \varepsilon)^{-2\gamma_{E_{2g}}}$ for in-plane biaxial compression,³⁴ this corresponds to a Grüneisen
1008 parameter $1.8 < \gamma_{E_{2g}} < 2.0$ – in excellent agreement with uniaxial strain experiments³³ and
1009 *ab initio* modeling.¹⁵³ This further confirms the importance of substrate-induced biaxial
1010 strain on the properties of graphene.

1011 2. Role of the PTM

1012 *a. Mechanical response* When the PTM is fluid, it has a much larger compressibility
1013 than the substrate. This, of course, is of no importance while it exerts purely hydrostatic
1014 pressure. However, fluid PTM display large viscosities at pressures in the GPa range, and
1015 local crystallisation of the fluids occurs at the graphene surface.¹⁵⁴ Whatever the state of
1016 the PTM, this configuration results in a biaxial stress field in the system: the top surface
1017 of the graphene layer is in contact with the fluid that applies hydrostatic stress, while
1018 its bottom surface is in contact with the solid substrate which imposes a bi-axial stress
1019 that is determined by the equation of state of the substrate. By symmetry, there are no
1020 shear stresses σ_{xz} and σ_{yz} applied to the graphene, except at the edges and other lateral
1021 inhomogeneities.

1022 In the case of monolayer graphene, the mechanical response of the graphene layer is mainly
1023 governed by the substrate.^{34,36–38,146} In the case of bilayer graphene, that remains so while the
1024 PTM remains liquid¹³³ (however, after solidification of the PTM, a stress gradient between
1025 layers could be measured, as will be discussed in the next section). One might expect, indeed,
1026 that the application of pressure would result in an increased graphene-substrate adhesion by
1027 reducing the graphene-substrate distance; and also that it would prevent the graphene layer
1028 from buckling and hence forming wrinkles or ripples. As mentioned in (b) above, substrate
1029 surface roughness can cause a critical stress above which the graphene layer is not able to
1030 follow the deformation of its substrate.³⁸ So the reality is more complex: despite gigapascals
1031 of pressure applied on the graphene layer, it seems that it is still able to buckle to reduce
1032 stress.¹⁴⁸

1033 To sum up, while the PTM remains liquid, the evidence is that the *mechanical response*
1034 of graphene is fully governed by the adhesion to the substrate. This goes for mono- and
1035 bilayer graphene only; for thicker samples the graphene bending rigidity increases and the
1036 adhesion to the (usually rough) substrate is reduced, leading to a mechanical response closer
1037 to graphite.^{36,146}

1038 *b. Electronic response* Comparing inert PTM (such as Ar, N₂) with polar PTM (such
1039 as 4:1 methanol:ethanol), a significant increase in the pressure coefficient of the Raman G-
1040 band was reported, from ~ 7 to ~ 10 cm⁻¹ GPa⁻¹ for SiO₂/Si substrate), for both mono-
1041 and bilayer graphene.³⁶ This change was however later refuted.^{37,38} Nevertheless, a pressure-

1042 induced decrease of the G-band FWHM was observed when using 4:1 methanol:ethanol
1043 PTM, which was attributed to a doping contribution.³⁶ However, it was not clear whether
1044 this doping remained constant over the whole pressure range.³⁷ A recent study of twisted
1045 isotopically-labelled bilayer graphene in 4:1 methanol:ethanol PTM by Forestier *et al.*¹³³
1046 clarified the doping effect of the PTM. Here, the observation of a difference in the response
1047 of the two layers made it possible to conclude that there was a pressure-induced doping due
1048 to the alcohol PTM: the G-mode of the upper layer in contact with the PTM showed a
1049 larger pressure shift than the lower layer in contact with the substrate, demonstrating the
1050 occurrence of a doping due to the polar PTM which increases with pressure. It is worth
1051 mentioning here that the isotopically-labelled bilayer graphene of this study is made of two
1052 CVD graphene layers transferred on top of each other, resulting in a sample behaving like
1053 two independent layers on top of each other. The difference in isotopic masses decouples the
1054 G-modes of the two layers, so that they can be separately resolved.

1055 C. Graphene sandwiched between two solids

1056 The investigation of the mechanical behavior of graphene monolayers and bilayers inter-
1057 acting with solids under pressure on both sides, as in Fig. 5 (c), constitutes an interesting
1058 route to better understand the mechanical properties of graphene-based nanocomposites.

1059 Under sufficient pressure, any fluid transforms to a solid at ambient temperature. There-
1060 fore, increasing the pressure sufficiently in the two situations discussed in the previous sub-
1061 sections, the systems will evolve to i) graphene between two different solids (Fig. 5 (c.2), for
1062 supported graphene on a substrate) or ii) graphene between two identical solids (Fig. 5 (c.1),
1063 for graphene in suspension). This leads to an asymmetrical or to a symmetrical environment.

1064 Crystallization of the PTM has at least two effects. First, macroscopically, it modifies the
1065 stress field with the appearance of differential strain at the graphene-PTM interface. Second,
1066 microscopically, it creates a periodic potential in interaction with the graphene sheet.

1067 1. *Different solids*

1068 Crystallization of the PTM when compressing a supported graphene leads to an asymme-
1069 try of the environments and of the associated stress fields. Each side of the graphene plane

1070 is in contact with a different solid medium. However, the change of state (fluid to solid) of
1071 the PTM is usually unnoticed in high-pressure Raman spectroscopy on supported monolayer
1072 graphene.¹⁴⁶ The preferred PTM are soft vdW solids (such as argon or nitrogen) and are
1073 considered to provide quasi-hydrostatic conditions even after solidification.¹⁵⁵ The bonding
1074 of such solids only leads to weak vdW type interactions with the graphene. The interactions
1075 between these solids and the graphene can only marginally modify the pressure-induced
1076 behavior which remains dominated by the effect of the substrate.¹⁴⁶

1077 In contrast, crystallization of the PTM is clearly observed by Raman spectroscopy exper-
1078 iments on bilayer graphene where each layer experiences different conditions. The signature
1079 of the solidification is a change in the pressure-dependence of the G-peak position and/or a
1080 change of the width of this peak. The change of slope may be attributed to additional differ-
1081 ential strain components and the broadening is related to inhomogeneity of the stress field.
1082 As the spectroscopic signatures are mainly affected by these external effects, it is difficult
1083 to assess any intrinsic effect, *i.e.* any modification of the elastic properties of graphene.

1084 The difference of the applied stress on each side of a twisted bilayer graphene have been
1085 evidenced and quantified by the high-pressure Raman experiment on isotopically labelled
1086 bilayer graphene mentioned in Sec. VB2b. Strain differences up to $\sim 0.1\%$ between the
1087 two graphene layers were observed when applying pressures of up to 10 GPa with nonpolar
1088 solid environments.¹³³

1089 **2. Identical solids**

1090 There are only a few reports of high-pressure experiments on graphene in suspension
1091 above the crystallization pressure of the PTM. It represents a major experimental challenge
1092 to characterize and manipulate a mono- or bi-layer, to load it in a high-pressure cell as
1093 suspended and to follow experimentally the high-pressure behavior across and above the
1094 solidification of the PTM. A particularly important issue is the form of suspension. If the
1095 graphene is freely floating in the PTM, then after solidification it will be subject to the
1096 strain of the PTM as pressure is further increased and the sample volume decreases and
1097 changes shape. However, if the graphene is supported over a trench in a substrate, then
1098 after solidification it will be pressed down into the trench, as the PTM will typically be
1099 much more compressible than the substrate, and this will put the graphene under high

1100 tensile strain.

1101 Table IV includes data from studies before and after solidification of the PTM. Sun *et al.*
1102 dissolved PMMA on which CVD-grown graphene had been transferred.⁴⁰ The solvent (DMF)
1103 was used as the PTM. So this graphene was assumed to be freely-floating. The G-peak pres-
1104 sure coefficient changed at around 2 GPa, from 5.4 cm⁻¹/GPa to 7.5 cm⁻¹/GPa. This
1105 was interpreted as resulting from the adhesion to the more compressible solid PTM, with
1106 the strain transmission effect as for supported graphene. In Tao *et al.*¹³¹ bilayer graphene
1107 suspended on Au grid was sandwiched between ice in different phases. It was not possible
1108 to fit the G-band data evolution using the quadratic term of graphite determined by Han-
1109 fland *et al.*²⁸ Tao *et al.* report a linear G-band evolution with a low pressure coefficient of
1110 3.4 cm⁻¹/GPa up to pressures of 40 GPa¹³¹ without any noticeable variation at the PTM
1111 freezing point. Filintoglou *et al.*³⁷ observed graphene sandwiched in solid fluorinert and
1112 obtained a G-band pressure slope of 5.4 cm⁻¹/GPa. These results, with G-band pressure
1113 coefficients between 3.4 and 7.5 cm⁻¹/GPa, show the extreme sensitivity of graphene to the
1114 nature of the PTM, and the details of the experiment influencing the transmitted strain.
1115 Solidification has a drastic impact for suspended graphene, contrary to the previous case,
1116 supported graphene, for which the solidification has almost no noticeable effect. The high-
1117 pressure mechanical response of graphene can be related to graphene composites. In partic-
1118 ular, and not surprisingly, it has been shown that the level of adhesion between graphene
1119 and polymer matrix is a key factor in the mechanical response of nanocomposites.^{156,157}

1120 After solidification (crystallization or vitrification), the main issue the nature of the ap-
1121 plied stress that includes shear components. However, one may consider the effect of the
1122 increasing pressure on the intrinsic mechanical properties of graphene, especially the c_{33} elas-
1123 tic constant. Increasing interaction between π -orbitals and the surrounding solid media can
1124 strongly modify the π -orbitals and hence perhaps the sp^2 bonds. A detailed quantification
1125 will be introduced in Sec. VIC. This seems to be especially true of polar PTM that ex-
1126 hibit piezo-doping and ultimately the formation of covalent bonds.¹³⁴ In this case, graphene
1127 cannot be considered still as an isolated system under external perturbation.

1128 VI. DRESSED GRAPHENE

1129 A. Functionalisation

1130 Graphene can react with, for instance, oxygen, hydrogen or fluorine. When covalent bonds
1131 develop with other molecules in low proportions we may speak about defective graphene.
1132 When those bonds develop extensively we will prefer to speak about other materials, some
1133 aperiodic, such as graphene oxide, some periodic, such as graphane (hydrogenated graphene
1134 monolayer) or diamane (hydrogenated bilayer of graphene)¹⁵⁸ (see Fig. 7) or fluorinated
1135 single-layer diamond.¹⁵⁹ In these cases there will be either local or extended modifications of
1136 the covalent graphene bonding scheme, with a modification of the associated elastic stiffness
1137 constants. For single-layer graphene oxide, the experimental Young's modulus derived from
1138 AFM measurements is substantially reduced to 207.6 ± 23.4 GPa.¹⁶⁰ As most of these other
1139 materials are new, their properties are derived from *ab initio* calculations.¹⁶¹ For instance
1140 it has been shown by DFT calculations that graphene oxide with increasing proportions
1141 of (-O) or (-OH) displays a progressive elongation of the C-C bond and a softening of the
1142 mechanical properties.¹⁶²

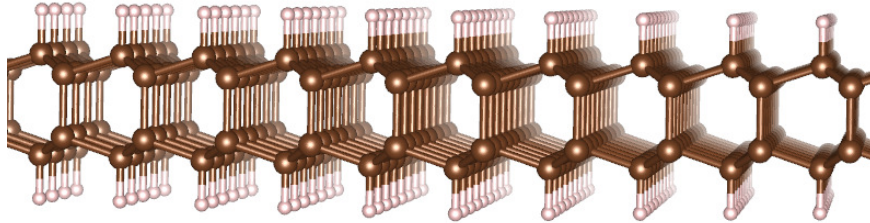


FIG. 7. Diamane structure with sp^3 bonding for C atoms.

1143 B. Derivative geometry

1144 We may also consider making a nanotube by rolling up a graphene sheet until we connect
1145 the edges through covalent bonds. This change in geometry leads to a topologically different
1146 graphene-based system, the carbon nanotube. Of course, the synthesis of carbon nanotubes
1147 does not correspond with this *gedanken* experiment, which, rather, raises the issue whether
1148 carbon nanotubes should be considered to be a material, derived from graphene by the

1149 change in geometry, or to be a structure made of graphene. Many DFT calculations find
1150 that the in-plane Young's modulus of small-diameter carbon nanotubes walls is reduced, due
1151 to the curvature-induced modification of the C-C hybridization. We may then expect that
1152 extensively corrugated, ripple or wrinkled graphene could exhibit an average bonding scheme
1153 differing from flat graphene and hence with different local 2D elastic stiffness constants as
1154 well as the large-scale reduced stiffness due to the corrugation. See Sec. VII for further
1155 discussion of nanotubes.

1156 C. Effect of vdW interactions

1157 Geometry and covalent bonding have an effect on the 2D elastic stiffness constants of
1158 graphene. What about vdW interactions? Consider the difference between graphene in
1159 graphite and in epitaxially grown bilayer graphene, two different cases of vdW graphene
1160 stacking. In graphite the vdW distance between graphene layers is 3.35 Å which may be
1161 considered as the graphene thickness in that particular case (see Sec. II A and Table I). In
1162 bilayer graphene grown on a SiC(0001) surface, the measured graphene-graphene distance is
1163 3.24 Å.¹⁶³ In multilayer graphene grown on an SiC substrate, the distance between graphene
1164 layers is found to be 3.9 Å after the first graphene layer in contact with the SiC substrate.¹⁶⁴
1165 X-ray diffraction measurements have revealed a certain degree of rotational disorder in the
1166 stacking of these graphene layers.¹¹⁶ Do these differences in thickness imply changes in the
1167 C-C sp² bonding?

1168 The thickness of graphene in these different cases is to be related to the extension of its π -
1169 orbitals. This is certainly a point of view in rupture with the Galilean continuum mechanics
1170 approach, but wholly consistent with the modern approach to the radius of atoms. In
1171 vacuum, of course, quantum mechanics tells us that the spatial extension of the π -orbitals
1172 electron cloud is to infinity, like the hydrogen 1s state. Any definition of a finite extension
1173 of an electron orbital in vacuum (such as the Bohr radius) is thus entirely arbitrary. Indeed
1174 such definitions are better described as characteristic lengths (such as the Bohr radius) which
1175 are not arbitrary, but also not obviously the extension or size of the atom. Where the π -
1176 orbitals of a graphene monolayer are delimited by meeting the electronic orbitals of adjacent
1177 materials, their extension is to be defined in just the same way as the size of atoms is defined
1178 (see Sec. II A and V).) In the case of graphite, it is very simple - the distance to the point

1179 between the graphene layers about which the π -electron density is symmetrical. In the case
 1180 of a graphene monolayer with other materials either side, or multilayer graphene in other
 1181 than Bernal stacking, we need to seek criteria (as with the size of atoms in multi-element
 1182 mixtures and compounds) which allow the consistent attribution of a thickness to, e.g. the
 1183 side of a graphene monolayer in contact with another layer in AA or in turbostratic contact,
 1184 and also to the other side in contact with perhaps sapphire. We should certainly define the
 1185 graphene thickness in the asymmetric context as the addition of two different contributions
 1186 on each side of the carbon nuclei.

1187 Practically, the shift of the G-mode frequency under out-of-plane compression is a suitable
 1188 quantity to quantify the weak modification of the in-plane elastic constants by deformation
 1189 of π -orbitals. The G-mode, as mentioned previously, is an in-plane anti-phase vibration
 1190 of C-C atom pairs and is therefore closely related to the in-plane stiffness of graphene and
 1192 graphite. Its eigenvectors (E_{2g}) in graphene and graphite are shown in Fig. 8. The dynamical
 1193 equation of a 1D spring can be written as $Ku = \omega^2 u$, where K is the force constant, u is
 1194 the displacement and ω is the frequency. And it can be extended to 2D for the G-mode of
 1195 graphene:¹⁶⁶

$$\begin{pmatrix} \omega_0^2 & 0 \\ 0 & \omega_0^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \omega^2 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad (11)$$

1196 where the u_1 and u_2 are the relative displacement of the two carbon atoms along the two
 1197 equivalent in-plane directions, as the hexagonal lattice of graphene is isotropic in-plane.
 1198 When an additional graphene layer is added, Eq. 11 becomes¹⁶⁷

$$\begin{pmatrix} \omega_0^2 & 0 & C & 0 \\ 0 & \omega_0^2 & 0 & C \\ C & 0 & \omega_0^2 & 0 \\ 0 & C & 0 & \omega_0^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \omega^2 \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}, \quad (12)$$

1199 where u_3 and u_4 are the displacement of the two carbon atoms in the added layer, and C
 1200 accounts for the interlayer coupling. The longitudinal and transverse modes are not coupled
 1201 due to the hexagonal lattice, hence all the zero elements in the force constant tensor. The
 1202 solutions to the secular equation of Eq. 12 are

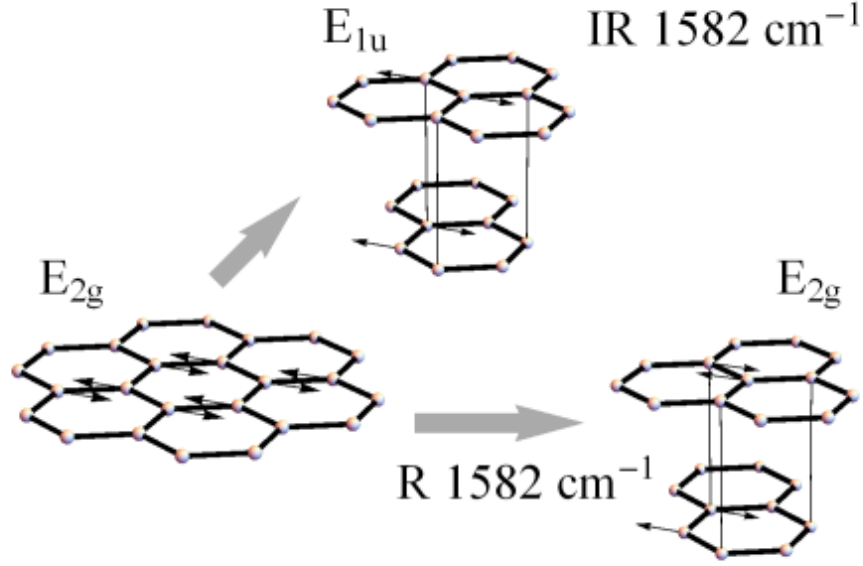


FIG. 8. “Phonon eigenvectors of graphene and graphite. Every phonon eigenvector of graphene gives rise to two vibrations of graphite. For example, the in-phase combination of the two layers for the E_{2g} optical mode of graphene yields $E_{2g} \otimes A_{1g} = E_{2g}$ and the out-of-phase combination $E_{2g} \otimes B_{1u} = E_{1u}$. Next to the graphite modes it is indicated whether they are Raman (R) or infrared (IR) active and the experimentally observed phonon frequencies. The translations of graphite are omitted from the figure.” Reproduced with permission from Phil. Trans. Roy. Soc. A **362**, 2271 (2004).¹⁶⁵ Copyright 2004 The Royal Society.

$$\begin{aligned}\omega_{(1)}^2 &= \omega_0^2 + C \\ \omega_{(2)}^2 &= \omega_0^2 + C \\ \omega_{(3)}^2 &= \omega_0^2 - C \\ \omega_{(4)}^2 &= \omega_0^2 - C\end{aligned}$$

1203 The two different solutions correspond to the E_{1u} and E_{2g} G-modes of graphite, where car-
 1204 bon atoms in both layers in a unit cell vibrate in-plane and in anti-phase, but the vibrations of
 1205 the two layers are in-phase and out-of-phase, respectively. Typical experimentally measured
 1206 values of the graphite E_{1u} and E_{2g} frequencies are 1587 and 1580 cm^{-1} , respectively.^{27,168}
 1207 From the measured frequencies of E_{1u} and E_{2g} of graphite, we can calculate the $\omega_0=1583.5$

1208 cm^{-1} for the G-mode of a graphene plane in graphite – the G-mode frequency of graphene
 1209 should be slightly higher than graphite, even their in-plane stiffnesses are the same.

1210 To quantify the effect of deformation of the π -orbitals on the G-mode frequency, we can
 1211 introduce out-of-plane strain and calculate the shift of G-mode frequency. The off-diagonal
 1212 term C in Eq. 12 for interlayer coupling can be expanded in terms of out-of-plane strain
 1213 ε_{zz} . The diagonal terms can be expanded too, to account for the possible modification of
 1214 the in-plane sp^2 bond stiffness by the compression of the π -orbitals,

$$\begin{pmatrix} \omega_0^2 + A\varepsilon_{zz} & 0 & C + B\varepsilon_{zz} & 0 \\ 0 & \omega_0^2 + A\varepsilon_{zz} & 0 & C + B\varepsilon_{zz} \\ C + B\varepsilon_{zz} & 0 & \omega_0^2 + A\varepsilon_{zz} & 0 \\ 0 & C + B\varepsilon_{zz} & 0 & \omega_0^2 + A\varepsilon_{zz} \end{pmatrix} \begin{pmatrix} u1 \\ u2 \\ u3 \\ u4 \end{pmatrix} = \omega^2 \begin{pmatrix} u1 \\ u2 \\ u3 \\ u4 \end{pmatrix} \quad (13)$$

1215 and the solution to the secular equation is:

$$\begin{aligned} \omega_{(1)}^2 &= \omega_0^2(E_{2g}^{(2)}) + (A + B) \times \varepsilon_{zz} \\ \omega_{(2)}^2 &= \omega_0^2(E_{2g}^{(2)}) + (A + B) \times \varepsilon_{zz} \\ \omega_{(3)}^2 &= \omega_0^2(E_{1u}) + (A - B) \times \varepsilon_{zz} \\ \omega_{(4)}^2 &= \omega_0^2(E_{1u}) + (A - B) \times \varepsilon_{zz} \end{aligned}$$

1216 which indicates an increasing E_{1u} and E_{2g} splitting, and therefore different shift rates with
 1217 pressure of these two modes. Without the contribution from the coupling to adjacent layers,
 1218 the G-mode frequency of graphene under out-of-plane compression is $\omega = \omega_0 + A \times \varepsilon_{zz}$.
 1219 Sun *et al.* calculated the G-mode frequencies of graphite under out-of-plane compression
 1220 by DFT, and introduced a new parameter γ' to compare the contribution from out-of-plane
 1221 strain on with that from in-plane strain.¹⁶⁷

$$\frac{\Delta\omega}{\omega_0} = -\gamma(\varepsilon_{xx} + \varepsilon_{yy}) \mp \frac{1}{2}SDP(\varepsilon_{xx} - \varepsilon_{yy}) - \gamma'\varepsilon_{zz} \quad (14)$$

1222 where γ is the Grüneisen parameter and SDP is the shear deformation potential. The
 1223 values of γ' are -0.0131 and 0.0585 for E_{2g} (GM) and E_{1u} of graphite, respectively, and the
 1224 value for the GM of graphene is in the middle. These values, compared to a typical value
 1225 1.90 of the Grüneisen parameter for the in-plane strain contribution,^{33,62,166} are indeed very
 1226 small. However, both graphene and graphite are about 30 times more compressible out-of-
 1227 plane than in-plane.²⁰ So, under hydrostatic compression, the contribution to the G-mode

1228 frequency from out-of-plane compression, while smaller than the in-plane contribution, is
1229 not negligible.

1230 Three useful points can be summarised from the above discussion: first, deformation
1231 of π -orbitals can modify the in-plane bond stiffness by a non-negligible amount, especially
1232 when the vdW interaction between graphene and the surrounding medium increases under
1233 compression; second, although the G-mode is a good measure of in-plane stiffness, its fre-
1234 quencies in graphene and graphite are slightly different even if the in-plane stiffnesses are
1235 the same; and third, the shift rate of the monolayer graphene G-mode with pressure should
1236 be higher than that graphite, because for the graphene G-mode, there is no impact from the
1237 coupled vibration in adjacent layers.

1238 Returning to the issue of different vdW interactions with different media at ambient
1239 pressure, some authors have proposed that the graphene layers in multilayer graphene on
1240 4H-SiC(0001) behave as monolayer graphene¹¹⁶ due to the predominant rotational stacking
1241 faults weakening the graphene-graphene interaction. Hence, following the same principle of
1242 thickness definition, the flat turbostratic (free-standing) graphene thickness will be 3.9 Å,
1243 *i.e.* $\sim 6.5\%$ greater than the vdW graphite distance.

1244 We may then conclude that different schemes of vdW stacking lead to changes in the
1245 π -orbitals. If we need to consider effects on the 2D elastic stiffness constants of the π -
1246 orbitals, then they should be considered as affected by graphene stacking schemes. The
1247 effect of such changes on the sp^2 bonds are from deformed π -orbitals modifying the in-plane
1248 C-C sp^2 bonds. The weak modifications of the C-C sp^2 bonds by the π -orbitals can be
1249 quantified in detail as above. In particular, as graphene is little affected by weak van der
1250 Waals interactions or geometry variations with curvatures less than about $\sim 1\text{ nm}^{-1}$, the
1251 electronic structure bonding scheme determining the 2D elastic constants of graphene is
1252 preserved. Flat or weakly bent graphene in vacuum or in graphite or in single-wall and
1253 multi-wall nanotubes may be then considered as having the same 2D elastic constants as
1254 graphite.

1255 VII. CARBON NANOTUBES: PROPERTIES OF GRAPHENE

1256 Carbon nanotubes, particularly single-walled and double-walled carbon nanotubes, are
1257 interesting structures in their own right (but that is outside the scope of this review). They

1258 provide perhaps the only way in which graphene can be studied free-standing and in vacuum
 1259 - graphene with nothing touching either side and nothing to constrain it in-plane either.
 1260 Closed nanotubes provide the opportunity to study graphene with vacuum inside and other
 1261 media outside, while open-ended nanotubes provide other possibilities. Most important, and
 1262 our focus here, they can reveal aspects of the mechanical properties of graphene, such as its
 1263 bending stiffness, that are difficult or impossible to study in other forms of graphene. Many
 1264 of the opportunities to learn about graphene by studying nanotubes have not yet been fully
 1265 exploited, as we shall see below.

1266 Most of the work we discuss depends on Raman spectroscopy, observing the G-mode and
 1267 the other graphene/graphite phonons, but crucially the radial breathing mode (RBM) which
 1268 has no equivalent in graphite.

1269 A. G-mode in nanotubes

1270 The G-mode has the inconvenience that it is hard to resolve the contributions of nan-
 1271 otubes of different diameters and chiralities. For that reason, much more attention has been
 1272 paid to the RBM (Sec. VII B).

1273 The G-mode frequency is sensitive to confinement effects, dynamical effects, and curva-
 1274 ture, which lift the degeneracy to give G^+ and G^- peaks.¹⁶⁹ In Piscanec *et al.*,¹⁶⁹ the effect
 1275 of curvature is deduced from the difference between experimental phonon wavenumbers and
 1276 calculations and obeys $\Delta\omega = -\zeta \times d^{-2}$. Values are given for the TO mode (circumferential
 1277 motion) with $\zeta = 25.16 \text{ cm}^{-1}\cdot\text{nm}^2$ and LO modes (axial motion) with $\zeta = 12.0 \text{ cm}^{-1}\cdot\text{nm}^2$.
 1278 Up to now, there is no direct calculation of the curvature effect. However, it can be esti-
 1279 mated from a simple continuum model. We consider the thickness of the graphene h_G and
 1280 treat the electrons on both sides (i for inner and o for outer) in a first approximation as a
 1281 continuous medium. The tension (o) and compression (i) are opposite but not equal, giving
 1282 a strain at the centre. Energy equilibrium gives $\epsilon_i^2 \times (d/2 - h_G/4) = \epsilon_o^2 \times (d/2 + h_G/4)$.
 1283 For a curved plate, we have: $\epsilon_o = h_G/2d$. So we deduce the average strain for the center
 1284 to be $\langle \epsilon \rangle = (\epsilon_i + \epsilon_o)/2 = h_G^2/(8d^2)$. Using the strain coefficient $-57 \text{ cm}^{-1}/\%$,¹⁷⁰ we find
 1285 $\Delta\omega_{TO} = -80.0 \times d^{-2}$. This gives the correct dependence on d but too large a value for ζ .
 1286 However, a homogenous medium is not a satisfactory model for graphene (*c.f.* the Yacobson
 1287 paradox, Sec. II A). Refining the model as a structure, (σ bonds at the centre, and π for

1288 the inner and outer material), the relation is the same but the coefficient is different. From
 1289 energy equilibrium, we have $\epsilon_\sigma = \sqrt{\frac{c_{11}^\sigma(h_G-h_\sigma)}{c_{11}^\pi h_\sigma}} \times \epsilon_\pi = \sqrt{\frac{c_{11}^\sigma(h_G-h_\sigma)}{c_{11}^\pi h_\sigma}} \times \epsilon_\pi = 0.170 \times \epsilon_\pi$ leading
 1290 to $\Delta\omega_{TO} = -13.6 \times d^{-2}$, which is in a more reasonable range and $\epsilon_\sigma = \frac{2.38 \times 10^{-3}}{d^2}$, to be
 1291 compared with $\frac{1.6 \times 10^{-3}}{d^2}$ from calculation.¹⁷¹

1292 B. RBM in nanotubes

1293 In the RBM fundamental, all atoms move radially together. The restoring force is
 1294 straightforwardly due to c_{11} , and the frequency depends inversely on the diameter, and to a
 1295 much lesser extent on the chirality, in the 100-300 cm^{-1} region of the spectrum. Moreover,
 1296 tunable excitation picks out those tubes that are resonant with the excitation wavelength.
 1297 Consequently, very detailed studies of the RBM have been reported.¹⁷²

1298 Simple models can account for this mode using continuous mechanics or atomic descrip-
 1299 tions. We start by supposing the nanotube wall to consist of a 2D sheet of continuum
 1300 material with the 2D graphene elastic constants $c_{11}^{2D} = c_{11}d = 372 \text{ Nm}^{-1}$ and $c_{12}^{2D} = c_{12}d =$
 1301 47 Nm^{-1} . In the RBM motion, the wall has tangential strain but no axial strain (the RBM
 1302 frequency is too high to induce any axial motion), so the relevant elastic stiffness constant
 1303 is c_{11}^{2D} . In contrast, the approach of Mahan,¹⁷³ modelling with a three-dimensional isotropic
 1304 plate, invokes not only c_{11} but also c_{12} , which is incorrect. The potential energy per unit
 1305 length of tube at the extreme of a sinusoidal motion $r = A \cos \omega t$ is

$$U_{max} = \frac{1}{2} c_{11}^{2D} \epsilon^2 \times 2\pi R_C = \pi R_C c_{11}^{2D} \frac{A^2}{R_C^2} \quad (15)$$

1306 while the kinetic energy at the center of the motion is

$$E_{max} = \frac{1}{2} mA^2\omega^2 = \frac{1}{2} A^2\omega^2 \times 2\pi R_C N m_0 \quad (16)$$

1307 where $N = 3.8 \times 10^{19}$ is the number of carbon atoms of mass m_0 in a unit area of graphene.
 1308 Equating U_{max} and E_{max} , and rearranging, we have

$$\omega_{RBM} = \frac{1}{R_C} \sqrt{\frac{k_{11}}{N m_0}} \equiv \frac{235}{d(nm)} \text{cm}^{-1} \quad (17)$$

1309 where the diameter $d = 2R_C$, in excellent agreement with experiments.¹⁷⁴ At much higher
 1310 frequency, 1590 cm^{-1} , the G-mode phonon corresponds to atomic motion in antiphase. To

1311 relate the RBM to the G-mode considering atomic motion, a 1D model (atoms equispaced
 1312 around a circle) has been proposed by Gerber *et al.*¹⁷⁵ leading to :

$$\omega_{RBM} = \frac{a_0}{d} \omega_G = \frac{0.142 \text{nm} \times 1590 \text{cm}^{-1}}{d(\text{nm})} = \frac{226}{d(\text{nm})} \text{cm}^{-1} \quad (18)$$

1313 in excellent agreement with Eq. 17. Considering a 2D system, the equations are the same
 1314 because the E_{2g} G-mode is doubly degenerate in the plane allowing the basis to be aligned
 1315 with the chiral vector, which is the circumference. The approximation here is that the
 1316 G-mode degeneracy is not lifted by the curvature.

1317 For nanotubes in a medium (e.g. a liquid or nanotube bundles) the vdW interaction leads
 1318 to an upshift of the RBM frequencies above the values of Eq. 17 by some $10 - 20 \text{ cm}^{-1}$.
 1319 The stiffening of the vdW interaction under pressure is largely responsible for the further
 1320 increase of RBM frequencies under pressure.¹³⁶

1321 The phonon spectrum of nanotubes includes also the soft modes, in the $10 - 100 \text{ cm}^{-1}$
 1322 spectral region, which are the higher-order modes of a series in which the RBM is the zeroth
 1323 member. Unlike the RBM, the soft modes depend on the bending stiffness D . The n^{th} soft
 1324 mode has $2n + 2$ nodes around the circumference of the tube. They should soften under
 1325 pressure, and would go to zero frequency at the collapse pressure. However, like the RBM,
 1326 they are raised in frequency by the vdW interaction with the PTM, and the increase in this
 1327 interaction with pressure actually results in the soft modes stiffening instead of softening
 1328 under pressure.¹⁷⁶

1329 C. SWCNTs under pressure

1330 The pressure dependence of single-walled nanotubes provides two opportunities at least
 1331 to learn about graphene. The diameters of nanotubes are usually given as defined by the
 1332 nuclear positions, [in contrast to taking the outside diameter over the electron orbitals](#). That
 1333 is what the general formula $d = a_0 \sqrt{n^2 + mn + m^2}$ gives, and that is the diameter usually
 1334 considered when analysing the response of nanotubes to high pressure, for example, the rate
 1335 of shift of the G-mode and RBM phonon frequencies under pressure. If the pressure P were
 1336 applied at the radius $r = d/2$, pressure coefficients of, for example, the G-mode phonon would
 1337 be expected to be approximately r/h times the graphite or graphene pressure coefficients,
 1338 where h is the relevant graphene thickness (see Sec. II A). (A full analysis would take into

1339 account the differing tangential and axial stresses in a tube under pressure, respectively Pr
 1340 and $Pr/2$.) However, if the graphene has a thickness h and the pressure is applied at a
 1341 radius $r + h/2$, the stresses on the sp^2 bonds will be greater and the pressure coefficients
 1342 correspondingly increased. How much they are increased, however, depends critically on the
 1343 mechanical properties of graphene and their response to pressure and bending. We are not
 1344 aware of a full analysis along these lines of the nanotube pressure coefficients. The situation
 1345 is further confused by the stiffening of the RBM mode which is largely due to the increasing
 1346 vdW interaction between the PTM and the nanotube,¹³⁶ and also by any effects of the PTM
 1347 on the graphene as discussed in the previous section, which may account for the different
 1348 pressure coefficients reported for nanotubes in different PTM.^{177–180}

1349 D. SWCNT Collapse

1350 At sufficiently high pressures or large diameters, nanotubes collapse. Collapse of nan-
 1351 otubes under high pressure has also been challenging, not least because, apparently as ob-
 1352 vious a case of Euler buckling as the collapse of pillars, the collapse of tubes under external
 1353 hydrostatic pressure is mathematically intractable. Many experimental observations have
 1354 been interpreted as corresponding to collapse, at a very wide range of pressures, and fitted to
 1355 a variety of theoretical equations. A complete solution for the simple (ideal) elastic ring was
 1356 reported only in 2011.¹⁸¹ Good agreement with the Levy-Carrier formula for a thin-walled
 1357 tube, $P_C = \frac{3D}{R^3}$ where D is the bending stiffness was confirmed, and the collapse to a peanut
 1358 shape above P_C was found to be quite slow, complete only at about $1.5P_C$.¹⁸² Torres-Dias
 1359 *et al.* reported that this fitted experimental data for the quenching of the RBM for a range
 1360 of SWCNT diameters, giving an estimate of $D = 1.7$ eV.⁴⁵

1361 Some caveats must be mentioned. First, the effects of the thickness h need to be known
 1362 and taken into account, as for the pressure coefficients. Also, it is clear that vdW inter-
 1363 actions will reduce the collapse pressure, so that SWCNTs tubes above about 4 nm will
 1364 spontaneously collapse (the same physics as the folding of Sec. III D 6). This scarcely affects
 1365 smaller tubes where the bending energies involved are very much greater. Then, for diam-
 1366 eters below about 1 nm, Torres-Dias *et al.*⁴⁵ reported that the collapse pressure is reduced
 1367 below the Levy-Carrier formula. This is quite a strong effect, observed experimentally and
 1368 in theoretical modelling,⁴⁵ and also previously noticed by Elliott *et al.*¹⁸³ It extrapolates to

1369 $P_C = 0$ for a diameter of about 0.4 nm, not much smaller than the smallest nanotubes ever
1370 reported.¹⁸⁴ One source of this behaviour could be softening of the bending potential with
1371 angle. Another source is the reduction in the Euler buckling load even of straight pillars
1372 when the compliance is discretised rather than continuous. This effect is reported by Carter
1373 et al,⁷⁷ and discussed in Sec. III D 5. Both of these explanations remain to be analysed in
1374 detail.

1375 When the diameter of a SWCNTs is large enough, *i.e.* above about 5 nm, a spontaneous
1376 collapse occurs. This leads to a cross-section in the form of a dogbone or peanut with a
1377 twist along the axis¹⁸⁵ if the SWCNT is free (in liquid or gas for example). The cavities of
1378 the edges have a diameter of the order of C_{60} fullerene, like the cavities of folded graphene
1379 (Section III D 6) while the stacking depends on the chirality.¹⁸⁶ Indeed, the phenomenon is
1380 very closely related to folding, with the same balance between adhesion energy and bending
1381 stiffness. Like folding, it has not been fully exploited to refine our knowledge of these two
1382 important mechanical parameters of graphene.

1383 Del Grande *et al.*¹⁸⁷ note that the energy barrier for a circular tube to collapse is many eV,
1384 so it should not be possible for it to happen through thermal activation – but once initiated
1385 at one point in the tube, the collapse will readily propagate along the tube. This is not an
1386 uncommon situation in condensed matter physics. It may be compared with the initiation
1387 of plastic deformation in a perfect crystal, where the activation energy for the creation of
1388 a dislocation is very high, or with boiling in a pure liquid, where the activation barrier for
1389 the formation of a bubble is very high. In these examples as in doubtless many others, it is
1390 a local defect, impurity, or perturbation that breaks the impasse. Del Grande *et al.* suggest
1391 that the collapse of large SWCNTs is likely to be produced by small mechanical stresses
1392 that naturally occur during synthesis. The nanoscale force required to initiate collapse, to
1393 bypass the energy barrier, is about 5 nN¹⁸⁷ which is easily achievable in AFM compression
1394 experiments.¹⁸⁸

1395 E. DWCNTs coefficients under pressure

1396 While the mechanical behaviour of SWCNTs under pressure is reasonably well understood
1397 as described in the previous section, double-walled nanotubes provide further opportunities
1398 - and challenges - to better understand the mechanical properties of graphene. Again, these

1399 opportunities come from the pressure coefficients of the phonon modes while the tubes
1400 remain circular, and then from the collapse pressures and modes of collapse. Many papers
1401 report pressure coefficients of the Raman G-modes of the outer and inner walls of DWCNTs
1402 which are not dissimilar. Yet it is hard to understand how the external pressure may be
1403 transmitted to the inner tube, given the enormous anisotropy of the graphite elastic stiffness
1404 tensor. Moreover, the sum of the reported pressure coefficients of the inner and outer tube
1405 is usually considerably in excess of the coefficient of an empty outer tube, *i.e.* an SWCNT of
1406 the same diameter. Yet the load on the walls of the inner and out tubes should sum to the
1407 load of an SNCNT of the same diameter, and so therefore should the pressure coefficients.

1408 Experimentally, pressure transmitted to an inner nanotube can be monitored by the
1409 upshift of the GM or RBM. The former should be less dependent on the PTM as the upshift
1410 is from the C-C bond stiffening under pressure, whereas the latter is from the increasing
1411 interaction of a tube with its surroundings. Consequently, it is easier to describe the GM
1412 pressure coefficient. The experimental challenge to monitor pressure by the GM frequency
1413 is the assignment of the GM to tubes of a specific diameter and chirality, and, further, to
1414 distinguish inner and outer tubes in DWCNTs. While the RBM is diameter-dependent,
1415 tubes of different diameters have very similar GM frequency, if not the same, at ambient
1416 condition. [An ideal situation would be to have only one RBM and its corresponding G-mode](#)
1417 [dominating the spectrum \(either due to a special sample containing only one chirality, or](#)
1418 [having only one chirality in resonance at a specific laser excitation\)](#). Many factors can add to
1419 the complication of the situation: 1) common CNT samples contain tubes of many different
1420 chiralities; 2) many more than one chirality can be in resonance or close to the resonance
1421 condition; 3) while the outgoing laser energy is only shifted by 10 – 20 meV for the RBM,
1422 the difference is 200 meV for the GM, making it possible that the GM is in resonance with
1423 the outgoing laser while its corresponding RBM is far from the resonance condition; 4)
1424 for DWCNTs, one has to further assign Raman peaks to the outer or the inner tube, and
1425 the interaction between inner and outer tubes modifies the Kataura plot,¹⁷². In particular,
1426 Hirschmann *et al.*¹⁸⁹ showed that the wall-to-wall distance between inner and outer tubes in
1427 DWCNTs increases with increasing tube diameters, which makes the RBM upshifts of the
1428 inner tubes from intertube interaction no longer a constant, as most earlier work supposed.
1429 This requires further caution on the assignment of RBMs to inner tubes, but can be used
1430 to refine our calculations if needed.

1431 Early studies on DWCNTs under pressure observed at least two components in a GM
1432 profile, shifting with pressure at different rates. It is tempting to assign these two components
1433 to outer and inner tubes for two reasons, one is that stress transmitted to the inner tube
1434 should be lower than hydrostatic pressure, resulting in two different responses to pressure,
1435 and the other is that outer (or inner) tubes in resonance at the same condition can have very
1436 close diameters and they should response similarly to pressure. Among various work, the
1437 GM pressure coefficients can be different, as different tubes are in resonance; they can be
1438 either PTM dependent, as PTM modifies the transition energy of CNTs, making different
1439 tubes in resonance, or PTM independent as no other charility in those samples is available
1440 near the resonance condition. The results of these high pressure study on DWCNTs are
1441 summarised in Table V below.

TABLE V. Experimental shift rates of DWCNTs GM with pressure.

Outer tube ($\text{cm}^{-1}\text{GPa}^{-1}$)	Inner tube ($\text{cm}^{-1}\text{GPa}^{-1}$)	PTM	Laser Excitation (nm)	Reference and notes
9.6	6.4	paraffin oil	514	Ref. 190
8.4	5.5	NaCl	514	Ref. 190
5.8	3.3	methanol-ethanol	633	Ref. 191
6.9	4.1	Oxygen	633	Ref. 191
8.6	5.1	Argon	633	Ref. 191
5.5	4.3	methanol-ethanol	514	Ref. 192

1442
1443

1444 In this section we have seen that much could be learned about the mechanical proper-
1445 ties of graphene from further experimental and theoretical work on nanotubes, particularly
1446 under pressure. The major obstacle is that the RBM modes are highly resonant and the
1447 resonances shift with pressure. For the G-mode, this means that as pressure increases, dif-
1448 ferent diameter tubes (which are not well-resolved) may dominate the Raman spectrum at
1449 different pressures.¹⁹³ For the RBM the tubes of different diameters and chiralities are well-
1450 resolved. However, both for identifying or for choosing which tubes are observed, and for
1451 tracking given tubes over a substantial pressure range, tunable Raman excitation is needed
1452 (*e.g.* a dye laser or Ti-sapphire laser), together with a tunable Raman spectrometer. The
1453 number of laboratories worldwide with such equipment, and also a high-pressure capability,

1454 is small indeed.

1455 VIII. MODELS FOR THE MECHANICS OF GRAPHENE

1456 Mechanical properties, more perhaps than any other properties of matter, invite the
1457 construction of models, for purposes ranging from visualisation, through understanding, to
1458 prediction. We comment briefly here on what can be suitably expressed by or learnt from
1459 different models, starting from the simplest. Continuum models for graphene have been
1460 considered. The flat plate of isotropic material and a thickness chosen to give the right
1461 bending stiffness was mentioned in Section II A. While useful for considering the behaviour
1462 of graphene as a beam, plate or shell, the model does not attempt to replicate the thickness
1463 a_{33} or the out-of-plane compressibility c_{33} of graphite. For that reason a continuum model
1464 was considered in which the nuclei and the sp^2 bonding orbitals were treated as an infinitely
1465 thin sheet with the 2D c_{11} and c_{12} values of graphene, sandwiched between two layers of soft
1466 material modelling the pi-orbitals.³⁹ This model could replicate the thickness, the bending
1467 stiffness and the out-of-plane compressibility by a suitable choice of an anisotropic 3D elastic
1468 tensor for this soft material. The model is suitable for considering the behaviour of, for
1469 example, nanotubes under pressure (see above, Sec. VII C). For example, it can be used to
1470 consider questions such as the radius at which a nanotube is loaded by external pressure. It
1471 is also a model that can be readily discretised as a ball-and-spring model. In this case, the
1472 nuclei are the balls, the sp^2 bonds are springs (2D stretching potentials) between them and
1473 the pi-orbitals are springs that terminate on a point that is not an atom. These points could
1474 also be joined by springs that give the bending stiffness.³⁹ That introduces the question,
1475 whether this is the physically-realistic representation of the origin of the bending stiffness of
1476 graphene, or whether it actually arises from the torsional stiffness of the sp^2 bonds through
1477 4-atom potentials in-plane. A comparison with the torsional vibrational modes of ethane
1478 (sp^3) and ethylene (sp^2) could be useful here,

1479 At this point we are approaching the valence force field (VFF, Keating) models¹⁹⁴ and
1480 those used in MD and Monte Carlo simulation. Keating, however, used only two-atom
1481 (stretch) and three-atom (angular) springs, or interactions, with nearest neighbours only to
1482 model the elastic constants c_{ij} and internal strains of, *e.g.* silicon. Keating did not consider
1483 phonon frequencies. It is in considering the phonons as well as the elastic constants that

1484 Keating models tend to break down.

1485 In MD simulation, Ref. 26 used two-atom interactions with atoms out to the fifth-nearest
1486 neighbours to model the phonon frequencies and the phonon dispersions. The necessity
1487 to include the fifth-nearest neighbours is demonstrated by investigating the origin of the
1488 GM frequency. Consideration of the 2D bulk modulus, or area modulus A , gives the force
1489 constant of the C-C bonds as 748 N/m, or $46.7 \text{ eV}/\text{Å}^2$, from the experimental values of elastic
1490 constants of graphite.²⁰ We assume that the GM frequency comes only from the nearest C-C
1491 stretching, and we can obtain the GM frequency as 1450 cm^{-1} . The gap to the experimental
1492 value of about 1580 cm^{-1} can be filled by other contributions beyond the nearest neighbour.
1493 Quantifying these contributions requires more information than the frequency of LO at Γ
1494 point (G-Mode)²⁷ in the phonon dispersion relation of graphite. We have already given an
1495 example that one has to include the second nearest neighbour out-of-plane interaction to
1496 describe the separation of E_{1u} and E_{2g} . It was found that up to the fourth nearest neighbour
1497 interaction has to be included to well fit the dispersion from Γ point to M (especially the
1498 initial increase of E_{1u} frequency from Γ point),⁴¹ obtained by inelastic neutron scattering.⁴²
1499 A further fifth nearest neighbour interaction was included to fit more recent in-elastic x-ray
1500 data, which gave a finer description of the local minimum of TO at K point.²⁶ Surprisingly,
1501 the empirical force constant model including up to the fifth nearest neighbour that fits well
1502 the experimental data of the full phonon dispersion of graphite (as shown in Fig. 9), gives a
1503 force constant for the nearest neighbour C-C stretching, as small as $25.88 \text{ eV}/\text{Å}^2$, indicating
1504 that about half of the contributions to the C-C vibrational frequency in graphite come from
1505 other interactions than the nearest C-C stretching. This is truly unexpected, yet explaining
1506 the existing data best.

1507 Despite contributing only half to the G-mode frequency, the nearest C-C stretching is
1508 expected to contribute dominantly to the upshift of G-mode with pressure from Pauli exclu-
1509 sion, which is a measure of C-C bond anharmonicity. The evolution of the phonon dispersion
1510 relation in graphite would not only verify or dispute the small value of the nearest C-C force
1511 constant, but also quantify the anharmonicity, when it becomes available. We should point
1512 out that in addition to in-plane contributions, the GM frequency can also be modified by
1513 deformed π -orbitals from out-of-plane, as discussed in section VIB.

1514 It is clear that Keating potentials are not generally capable of representing both the
1515 elastic constants and the phonon frequencies, and certainly fail in this regard for graphene.

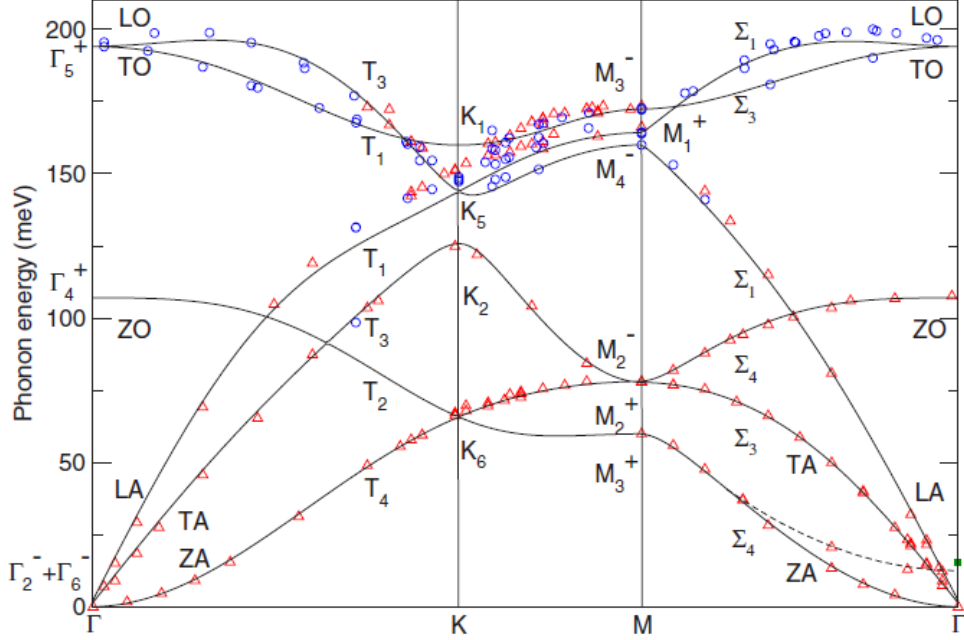


FIG. 9. Phonon dispersion of graphite from inelastic x-ray scattering (symbols). Triangles are from Ref. 26. Circles are from Ref. 195. Squares are INS data from Ref. 42. Solid lines are the fit of experimental data by a force constant model including up to the fifth-nearest neighbor. The dashed line is a quadratic extrapolation of the data. Reproduced with permission from Phys. Rev. B **76**, 035439 (2007).²⁶ Copyright (2007) The American Physical Society.

1516 It is not clear from the literature what the simplest models are that can do this, nor if such
 1517 a model is physically interpretable (two-atom interactions between fifth-nearest neighbours
 1518 do not have any obvious physical interpretation). On the other hand, VFF models continue
 1519 to be developed and exploited for various purposes, *e.g.* thermomechanical properties¹⁹⁶ and
 1520 phonon frequencies.¹⁹⁷

1521 Usually, molecular dynamics modelling uses optimized reactive potentials (multi-body
 1522 potentials such as the second generation reactive empirical bond order (REBO) potential of
 1523 Brenner¹⁹⁸ or the recent implementation of the ReaxFF potential¹⁹⁹) which allow the study
 1524 of mechanical properties at the nanoscale such as fracture. An example is the finding that
 1525 fracture requires a force of about 8 nN per C=C bond.²⁰⁰ This is equivalent to about 90
 1526 GPa uniaxial stress and compares with the 130 GPa strength reported by Lee *et al.*²⁵ in
 1527 AFM nanoindentation (see also Fig. 3 (a)).

1528 Turning to quantum-mechanical models, the simple tight-binding description is used for

1529 obtaining the electronic structure²⁰¹ but is not suitable for mechanical properties.

1530 DFT provides models that can be made to replicate experimental data excellently.²²
1531 However, apart from the maps of electron density, there is little in DFT output that can
1532 assist a physical understanding of the predicted properties. It could be said that in graphene
1533 research as in high-pressure research, the greatest value of DFT is that it can tell us what
1534 happens under experimental conditions that are not (yet) accessible to experimenters. For
1535 that reason we discuss it under different headings above. An example in graphite is the
1536 determination of c_{13} (Sec. III A, see also Sec. IV F).

1537 IX. CONCLUSIONS

1538 This paper reviews the mechanical properties of graphene, both those that are expected
1539 to be similar to graphite and those expected to be different from graphite – and anomalies.
1540 Graphene is commonly called a 2D material, which implies a thickness tending to zero.
1541 However, the π -electrons above and below the 2D plane of carbon nuclei extend the electron
1542 density of monolayer graphene into the third dimension, perpendicular to the 2D plane. For
1543 example, we can define a vdW thickness of graphene, 3.35 Å, which is the experimentally
1544 measured spacing of graphene layers in graphite. One key conclusion is that, far from being
1545 a 2D material, graphene has a well-defined 3D structure, which may be modelled in various
1546 ways to help understand its mechanical properties. That is not to say that it cannot display
1547 2D physics, much as can a 100 Å quantum well – which has a 3D physical structure of *e.g.*
1548 GaAs sandwiched between GaAlAs. Following from that, those of its mechanical properties
1549 which are related to those of graphite are indeed very similar, if not identical.

1550 Without neighboring layers, unsupported graphene is not mechanically stable and has
1551 intrinsic ripples. The low bending stiffness further promotes the formation of ripples, making
1552 them common in graphene samples. It also contributes to the softening of the ZA phonon
1553 dispersion, resulting in a negative thermal expansion (although, again, it is not clear whether
1554 this is significantly different from that of graphite). Properties such as the out-of-plane
1555 stiffness, though expected to be similar to graphite, require indirect approaches to define
1556 and to quantify.

1557 Due to the small sample size of exfoliated graphene – at least out-of-plane – experiments
1558 to measure many of its mechanical properties requires special design. In addition, the

1559 environment surrounding graphene adds further complexity to the interpretation of these
1560 experimental data, from determining factors as substrates transferring strain to graphene,
1561 to subtle modification by influencing the π -orbital distribution.

1562 There are many derivative structures from graphene, in a way making the extraordinary
1563 properties of graphene tunable. They can also be used to help understand the properties
1564 of graphene. Among those, measurements on carbon nanotubes in some circumstances give
1565 the most accurate values for mechanical properties of graphene, perhaps even better than
1566 measurements on graphene itself, as nanotubes can be self-supporting, free-standing, and
1567 stable, thus excluding many of those complexities.

1568 Finally, composite materials in which matrix material is reinforced mechanically by the
1569 inclusion of graphene flakes are perhaps one of the most exciting applications in which the
1570 mechanical properties of graphene are central. Many other so-called 2D materials are also
1571 used, and their mechanical properties are often less well characterised than those of graphite.
1572 One may expect them to be related to the corresponding bulk materials much as graphene
1573 is related to graphite.

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1578 **DATA AVAILABILITY**

1579 Data sharing is not applicable to this article as no new data were created or analyzed in
1580 this review.

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