Supporting information for Flexible intuitions of Euclidean geometry in an Amazonian indigene group

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Supporting Information

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SI Text

Equations of Planar and Spherical Geometry. ***Triangle completion in planar geometry.*** Consider a triangle ABC on the plane where the angles α and β at the base are known as well as the distance c between the points A and B and where the distances a and b and the angle γ are unknown (Fig. S8).

The angle γ can be derived from the fixed sum of the angles (Eq. S1):

$$\gamma = \pi - (\alpha + \beta). \quad [S1]$$

The lengths of the sides a and b can be extracted from the following equality (Eq. S2):

$$a/\sin(\alpha) = b/\sin(\beta) = c/\sin(\gamma). \quad [S2]$$

Finally, the coordinates x and y of the point C are determined by (Eqs. S3 and S4)

$$x = b\cos(\alpha) \text{ and} \quad [S3]$$

$$y = b\sin(\alpha). \quad [S4]$$

Spherical geometry: definition of straight lines. On a sphere, the straight lines (which in a homogeneous space corresponds to the geodesics or the shortest way between points) are the great equatorial circles. Other circles are not straight lines; they do not constitute the shortest way between two points but rather, curves that constantly bear in one direction. From the point of view of a creature evolving at the surface of a sphere, that creature would need to bend constantly to follow a small circle, much like a creature walking on a plane and bending with a constant angle would be following a circle on that plane. To get an intuitive understanding of the difference between equatorial and smaller circles on a sphere, think of the circles becoming increasingly smaller to the point where they become a very small circle around one of the poles. It should be clear to the reader that such a small circle could not be a straight line. Also, think about flight routes; planes usually follow straight lines along the equatorial circles of the earth. This is why, when traveling across two points of equivalent latitude (like Boston and Barcelona), planes go through places of very different latitudes close to the poles (like Greenland) (Fig. S9).

Triangle completion in spherical geometry. In the following section, we assume that the radius of the sphere is equal to one. Again, we consider a triangle ABC, where the angles at the base α and β are known as well as the distance c between A and B (Fig. S10). The following formulae will enable us to calculate the distances a and b as well as the angle γ.

The angle γ is given by the formula (Eq. S5)

$$\cos(\gamma) = -\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\cos(c). \quad [S5]$$

The length of the sides a and b can be derived from the equality (Eq. S6)

$$\sin(a)/\sin(\alpha) = \sin(b)/\sin(\beta) = \sin(c)/\sin(\gamma). \quad [S6]$$

Finally, the spherical coordinates ϕ and θ of the point C are (Eqs. S7 and S8)

$$\tan(\phi) = \cos(\alpha)\tan(b) \text{ and} \quad [S7]$$

$$\sin(\theta) = \sin(\alpha)\sin(b). \quad [S8]$$

Additional Analyses. ***Lines intuitions test. Data inclusion.*** All of the trials were included in the analyses. For the Mundurucu participants, responses were obtained in 94.8% and 93.4% of the trials in the planar and spherical subtests, respectively. In all other groups, an answer was collected in 100% of the trials. Positive answers were coded as one, negative answers were coded as zero, and nonresponses were coded at chance level (0.5). Responses were collected either verbally (for the Mundurucu, US adults, and French children) or by clicking on one of two buttons on the screen (for the young US children).

Three adult Mundurucu participants were tested two times over two successive field trips in 2006 and 2007. Because the responses were very consistent within participant in the two runs of the task (81.0–95.2% of identical responses), both runs were included in the analyses.

No effect of instruction in the Mundurucu. Within the Mundurucu population, the level of bilingualism and education can vary greatly, especially in the isolated villages where the tests were conducted. This diversity is informative to our research, because it allows us to test for the effects of education while minimizing other environmental differences. In the present sample, all participants were native Mundurucu speakers. Only four of the adults were partially bilingual in Portuguese. Six of the adults and one of the children had received no schooling, and others had received only a few years of education, with considerable variations in its intensity, duration, and content. The first level of schooling typically focuses on basic alphabetization, the second level focuses on basic writing, and the third level focuses on the introduction of basic arithmetical concepts (addition and subtraction). None of the participants had received formal instruction in geometry.

To test for possible effects of education, we followed the strategy used in our past research (1) and separated the participants who had been exposed to some schooling into those with little education (levels 1–2; nine adults and five children) and those with more education (levels 3 or more; seven adults and two children). Only adults were included in analyses on instruction, because there was too little variation in the level of instruction among our child participants. We ran an ANOVA including factors for the level of instruction (no instruction, little instruction, or more instruction), the subtest (planar vs. spherical), and the type of question (correct response identical between the two subtests or distinct across the two subtests), taking the percentage of planar responses as the dependent variable. This analysis identified no effect or interaction involving the level of education ($ps > 0.24$). A similar pattern of performance was observed in all subgroups, even in the least educated participants, with a similar level of percentage of planar responses in all of the questions whose answers fitted the predictions of planar geometry (either on the plane or the sphere), and a smaller percentage of planar responses on the sphere questions had answers that contradicted planar geometry. This pattern elicited effects of the type of subtest and type of question as well as an interaction between these two factors on the percentage of plane responses in all subgroups ($ps < 0.05$) (Figs. S2–S4).

No effect of age in the Mundurucu children. We also looked for potential effects of age on the percentage of plane responses in the groups of Mundurucu, French, and US children. The analyses revealed no effect or interaction involving the age of the Mun-

durucu and US participants (Mundurucu children: $ps > 0.25$; US children: $ps > 0.22$). A marginal effect of age was observed in the group of French children [$F(1,6) = 4.7$, $P = 0.073$], whereas all other interactions involving age were not significant ($Fs < 1$). Globally, the oldest children of the group tended to conform their answers more perfectly to the planar predictions.

No effect of the order of presentation of the subtests. Mundurucu participants, US adults, and French children took part in both the planar and spherical subtests. In contrast, the 5- and 6-y-old children tested in the United States participated only in one of the subtests and therefore, were only introduced to one of the imaginary worlds: either the sphere or the plane. Our results show that only the young US children failed to modulate their responses with the shape considered, which we interpreted as evidence that geometric intuitions are still fragile at ages 5 or 6 y. Here, we consider an alternative explanation: that this difference in performance may stem from the absence of contrasting experience with our planar and spherical worlds.

To address this question, we analyzed the performance of the US adults and French children for potential order effects (the data about order of presentation were not available for the Mundurucu participants). If contrasted experience is necessary, we should see that these participants modulate their responses in the second subtest only and that the responses are uniform in the first subtest across participants tested on the plane and the sphere. An ANOVA with factors for subtest (planar vs. spherical), question type (correct response identical between the two subtests or distinct across the two subtests), subtest order (plane first or sphere first), and age group (adults vs. children) revealed no effect or interaction involving the order factor on the percentage of planar responses (all $ps > 0.13$) in the general analysis and within either group (US adults: $ps > 0.17$; French children: $ps > 0.34$). The plane subtest elicited 98.0% of planar responses when presented first and 97.6% of planar responses when presented second. For the sphere subtest, the percentage of planar responses was 80.6% or 77.8% depending on the order of presentation.

The absence of order effects argues against the necessity for participants to compare contrasted shapes to form differentiated geometric intuitions about planes and spheres. We cannot exclude that the US children would have drawn different inferences about each shape if they had been presented with both the plane and the sphere; still, the present results show that their intuitions are both less accurate and less flexible than those of older participants.

Potential pragmatic issues. Some participants may have been tempted to use the formulation of the questions to infer the answer expected by the experimenter. In particular, when a question seemed to elaborate on the previous one, participants may have been induced to think that the experimenter was hoping for them to revise their answer. This is the case for questions #3 and #6 ("even if we go very far?") as well as questions #13 and #14 ("can two or more than two lines be drawn through a point?"), #16 ("can two parallels to a given line be drawn through a single point?"), and #18 and #19 ("can two and more than two lines be drawn through two points?"), which all followed the same question asking about the existence of one such line. If a participant had stated that such a line did not exist, he/she might have been induced to revise his/her previous statement when asked about the possibility to draw two or more than two such lines.

However, three arguments suggest that these repetitions cannot by themselves fully explain the performance of the participants.

First, performance was above chance even when these questions were excluded. Accuracy remained above chance in all groups on the plane subset (t tests, $ps < 0.0001$) as well as on the sphere subtest (t tests, $ps < 0.05$), except for the group of US children [plane: $t(15) = 6.5$, $P < 0.0001$; sphere: $t(15) = 1.7$, $P = 0.11$]. Nonetheless, when responses were viewed in terms of percentage of planar responses, these children chose the planar response more often than chance on the sphere subtest [$t(15) = 4.1$, $P = 0.0010$] just as all other groups did ($ps < 0.0001$).

Second, when we restricted analyses to the seven potentially suggestive questions, we observed the expected interaction between condition and question type (same/distinct answers) in all but the US children groups [general ANOVA: interaction condition and question type $F(1,50) = 155.7$, $P < 0.0001$, no triple interaction with group $F < 1$; within-group ANOVAs: interaction condition and question type $ps < 0.01$]. The fact that these questions were given different answers depending on the subtest shows that the participants were not merely responding to a suggestion by the experimenter. The absence of such interaction in the group of US children was in line with their general inability to adapt their answers to the subtest surface.

Third, at two places in the questionnaire, we asked questions following the same progression: "can you place one line going through the center of N villages?"; "can you place two lines going through the center of N villages?"; and "can you place more than two lines going through the center of N villages?" This was asked first about one village and then, about two villages. Depending on the number of villages, we did not expect the same answers: yes/yes/yes vs. yes/no/no. Indeed, the responses followed this pattern (average percentage of yes responses in all but the US children group: 1 point = 99.8%/99.5%/98.6% and 2 points = 96.8%/8.3%/3.9%). In an ANOVA focused on these six questions using the percentage of yes responses as the dependent measure and including factors for the question (one/two/more than two lines) and the number of villages to fit (one or two) together with all factors from other analyses, we observed a significant interaction between the question and the number of villages when all groups (except the group of US children) were included [$F(2,100) = 532.1$, $P < 0.0001$; no interaction with group: $F(6,100) = 1.2$, $P = 0.30$] and within each of these groups (all P values < 0.0001). This would not have been expected if participants responded only to a suggestion, because we would not have observed differentiated answers between the two situations (one or two villages). In the group of US children, the pattern was different: the children answered positively to the possibility of drawing one line (one village: 93.8%; two villages: 87.5%) but at chance for drawing either two (one village: 56.3%; two villages: 43.8%) or more than two lines [one village: 53.1%; two villages: 34.4%; main effect of question: $F(2,60) = 24.7$, $P < 0.0001$; main effect of situation: $F(1,30) = 4.3$, $P = 0.047$; no interaction $F < 1$]. For the present purpose, their responses indeed support the hypothesis that the children were not merely responding to suggestion: first, because we did observe a main effect of the situation (one vs. two villages) and second, because if the children had used the progression of the questions to infer that such lines should exist, then we would have observed an increasing percentage of positive responses. If anything, it was actually declining.

Triangle completion test. Trials inclusion. All trials were included in the analyses of the placement of the intersection point. Given that the angle estimates could have shifted when the point was placed too far from its correct position, in the analyses of the angle estimates, we only included the trials where the intersection point was placed within 2.5 cm (1 in) of the correct position on the screen. This criterion resulted in the inclusion of 69.5% and 71.0% of the trials for the groups of Mundurucu adults and children, respectively, 79.1% of the trials in the group of US adults, 69.0% of the trials in the group of French children, and 41.4% of the trials in the group of the US children. However, the results presented here are virtually unchanged when the totality of the trials are included.

Effect of the type of response on angle estimates. In the first version (2006), we asked participants to give their angle estimates by reproducing them with their hands, and then, we measured the aperture with a goniometer. In the second version (2007), par-

ticipants were asked to give their responses directly by means of a goniometer placed on the table. On average, the angles given by hands were overestimated compared with the angles given on the goniometer [$F(1,22) = 12.4$, $P = 0.0019$] both in adult and child Mundurucu participants (no interaction with age group, $F < 1$). The amount of overestimation did not differ between the planar subtest (8.5°) and the spherical subtest [11.3°; $F(1,25) = 1.5$, $P = 0.23$]. Moreover, the bias observed was consistent across trials, since the responses were highly correlated between the groups tested with their hands or the goniometer [$r^2 = 0.76$, $F(1,45) = 140.2$, $P < 0.0001$, slope 0.95] (Fig. S5). Because the analyses reported in the text rely exclusively on measures of correlation between the responses and the predicted angles, a constant overestimation bias could not affect their outcome, and therefore, the two groups were pooled together in the rest of the analyses.

Parameters influencing angle estimates. In each trial, the display could be described as providing three parameters to the participants: the distance between the two base points of the triangle (c) and the two angles at the base of the triangle (α and β). In planar geometry, the sum of the internal angle of a triangle is always equal to 180° (π radians); therefore, only the sum of the two base angles ($\alpha + \beta$) should have an influence on the angle estimates. In contrast, in spherical geometry, the third angle depends both on this sum ($\alpha + \beta$) and the distance between the two points (c). As a consequence, if the participants give accurate estimates for the angles, we should obtain these effects and interactions:

i) An effect of the subtest (planar vs. spherical) reflecting the fact that the sum of the internal angles of triangles is always larger on a sphere than on a plane.
ii) An inverse correlation with the sum of the two base angles ($\alpha + \beta$).
iii) An interaction between the type of surface and the contribution of the sum of the two base angles, because the contribution of this regressor should be higher in the planar subtest.
iv) An interaction between the distance (c) and the subtest (planar vs. spherical), this parameter has a higher positive contribution in the spherical subtest.
v) An interaction between the distance (c), the sum of the base angles ($\alpha + \beta$), and the subtest (planar vs. spherical), because the effects of $\alpha + \beta$ and c are not additive in the spherical subtest.
vi) As a result of the effect of c and the interaction between $\alpha + \beta$ and c in the spherical subtest, we may observe a main effect of c (positive correlation) and a main interaction between c and $\alpha + \beta$ when both subtests are pooled together.

To test for the presence of these effects, we used a general linear model including regressors for the distance between the two points, the sum of the base angles, and a factor representing the type of surface (planar vs. spherical subtest). In both Mundurucu adults and children as well as in US adults, we observed all of the effects listed as *i–v* [all $P < 0.0031$ except in the effect of subtest in Mundurucu children: $F(1,1) = 89.9$, $P = 0.067$; the triple interaction between the type of surface, $\alpha + \beta$, and c in Mundurucu children: $F(1,7) = 6.2$, $P = 0.042$; the interaction between c and the type of surface in US adults: $F(1,16) = 8.3$, $P = 0.011$]. In addition, we observed most of the effects listed as *vi* [$P < 0.0096$ except for the interaction between $\alpha + \beta$ and c in the Mundurucu adults: $F(1,20) = 3.6$, $P = 0.072$; in the Mundurucu children: $F < 1$].

In the group of French children, these effects were weaker. We observed principally an inverse correlation with the sum of the base angles [$F(1,2) = 22.6$, $P = 0.042$], which interacted with the subtest [$F(1,4) = 15.7$, $P = 0.017$; effects *ii* and *iii*]. In addition, the French children showed a tendency to take the length of the base into account as well [effect of c: $F(1,3) = 6.3$, $P = 0.087$; effect *vi*]. Their responses tended to be larger in the spherical than in the planar subtest [$F(1,1) = 43.8$, $P = 0.095$; effect *i*], and a marginal triple interaction was also observed [$F(1,7) = 4.1$, $P = 0.082$; effect *v*]. Both other interactions (interaction *iv* and the interaction between c and the base angles in *vi*) were not significant (*ps* > 0.15).

In a general analysis including two factors for culture and age, we observed an interaction between the cultural group and the effects *i*, *iii*, and *iv* (*ps* < 0.0041) as well as with the effects in *vi* ($P = 0.0015$ and 0.072), reflecting the fact that the control groups from the United States and France showed a lesser modulation of responses between the planar and spherical subtests (converging analyses shown in the main text); the responses of these two groups were globally attracted to the planar predictions. In contrast, the factor for age group had an influence only on the effect of subtest [$F(1,29) = 4.6$, $P = 0.040$; all other interactions *ps* > 0.15].

As for the group of younger children tested in the United States, in contrast with all other groups, only a main effect of the distance (c) was observed [$F(1,16) = 29.8$, $P < 0.0001$]. Contrary to the mathematical predictions, this effect did not interact with the type of surface ($F < 1$), the sum of the base angles did not have any influence on the angle estimates ($F < 1$), and this sum did not interact with the distance or the type of subtest (all *Fs* < 1). Young children, therefore, have a poor understanding of angles: the angles that they produced reflected exclusively the distance between the two base points (the longer this distance, the larger the angle produced) and were not modulated by the type of surface.

Correlations between the responses and the predicted values. Individual estimates for the position of the intersection (*x* and *y* coordinates) and the angle at the intersection were compared with the values predicted by planar and spherical geometry. To correct for the differences in scale across trials, the *x* and *y* coordinates of the intersection were normalized by the apparent distance between the two base villages in each trial. Table S1 reports the outcome of these correlations as average r^2 and β across individuals. In each group, we also tested whether the predictor contributed significantly to the responses by comparing the β values against 0.

The results of these correlations for the angle estimates of the young US children could seem surprising at first (better correlation with the predictions on the sphere than on the plane subtest). However, remember that the children base their angle estimates exclusively on the distance separating the two base points and not on the base angles; indeed, this distance has an effect on the angle only in the equations of spherical geometry but not in the equations of planar geometry. Therefore, the significant correlation obtained for the angle responses in the case of the sphere seems rather fortuitous and needs not be taken as indicating a better understanding of angles on the sphere than on the plane.

No effect of the level of instruction in Mundurucu participants. To evaluate the potential impact of education level, we introduced a factor describing the education level of the participants (no schooling, little schooling, or more schooling). Again, only adults were included in this analysis, because there was little variability in the level of education of the children. For each participant, using a multiple regression analysis, the angle estimates were fitted with two regressors corresponding to the predictions of the planar and spherical equations, respectively. The difference between the contribution of these regressors was submitted to an ANOVA with two factors for the education level (between subject) and the type of surface tested (within-subject: plane vs. sphere). As reported in the text, the ANOVA yielded a main effect of the type of surface [$F(1,19) = 287.7$, $P < 0.0001$], corresponding to the fact that the responses on the plane were

better predicted by the planar equations, whereas the responses on the sphere were better predicted by the spherical equations. The analysis revealed that the level of instruction had no effect on the regression coefficients ($ps > 0.20$); in particular, the modulation of the contribution of the regressors by the type of surface was present at all levels of instruction ($ps < 0.001$).

As an illustration, Figs. S2–S4 show the performance of the Mundurucu adults separated by education level.

No effect of age in Mundurucu children. We ran additional analyses testing for effects of age in the children tested in the Amazon (ages 7–13 y), France (ages 7–13 y), and the United States (ages 5.0–6.9 y). The analysis of the relative contribution of the planar and spherical predictors confirmed that only the older children modulated their angle responses depending on the type of surface (Mundurucu and French children: $ps < 0.05$; US children: $Fs < 1$). No effect of age was observed within the group of Mundurucu children, although their ages spanned a large interval ($Fs < 1$ on both analyses). In the French children, a marginal interaction between age and condition was observed [$F(1,6) = 3.9$, $P = 0.095$], which was mainly because of the contribution of the youngest child (7 y), who behaved more similarly to the group of 5- to 6-y-olds than to her older peers, especially on the sphere. The main effect of age was not significant [$F(1,6) = 2.6$, $P = 0.16$]. In the young US children, no effect of age was found ($F < 1$). Note that the absence of an effect of age in the US children could be expected given that the performance is at chance level for the group.

No effect of the order of presentation of the subtests. The US adults, French children, and Mundurucu participants took part in both the planar and spherical subtests, and the order of presentation of the subtest was randomized. In the analyses of the contribution of the planar and spherical regressors, the order of presentation of the test did not affect the results (all $ps > 0.39$). This observation was true of each of the groups (all $ps > 0.20$). Since the Mundurucu participants and the French children had already been familiarized to the plane and the sphere in the line intuitions test, the absence of order effect may be explained by their familiarity with both of these shapes; still, the absence of order effects argues that (*i*) the participants produced differentiated responses even during the first subtest and (*ii*) it was not necessary for the participants to compare the two shapes to give differentiated angle estimates.

Mundurucu terms with geometrical content. The following list of words, established with the help of several informants who were native speakers of Mundurucu, updates the list published in the supplementary material of ref. 2. A more complete list, including terms related to arithmetical and temporal properties, can be downloaded from P.P.'s webpage: http://www.pierrepica.com/publications/.

Figures. All these terms can be applied to plane or nonplane surfaces (such as painted bodies) and therefore, are not restricted to the domain of planar geometry.

iroyruy'at: spherical object and by extension, circle (as a 2D projection of a spherical object)
iwaketkut'at: imperfect rounded object and by extension, circular shape or curved square (as a 2D projection of an imperfect rounded object)

Points, lines, and angles.

koap: beginning
cũg: straight (ahead)
ibucũg: curve, line (literally, his straight finger)
etako: side
kadi: border, side [literally, bank (of a river)]
ya: corner (literally, shoulder)
ta rũg: sharp point (by extension, angle)

Proportions.

i'in: part of
i'in pũg: one part of
ipidase: center, middle, half (literally, on the mouth/skin of the earth/ground; according to Mundurucu cosmology, the world is flat with the earth/ground as the middle layer)
ipidasesu: quasi in the middle

Directions and axes.

kaxi jem ap: sun/moonrise (literally, the place from which the sun/moon rises)
kaxi a cap: sun/moonset (literally, the place at which the sun/moon declines)
kaxicũg: vertical/straight position of the sun/moon (with respect to the speaker; by extension, zenith)
kaxijodek: secondary axis of declension of the sun/moon (with respect to the speaker; literally, secondary, slanted position of the sun/moon)
kaxicerere: slanted axis of declension of the sun/moon (with respect to the speaker; literally, slanted position of the sun/moon)
kaxipitẽg: setting sun/moon (literally, sun/moon near the ground)
yabicũg: main rigid axis (of an object) or vertical/horizontal axis (with respect to the speaker; literally, straight at the line of his/its shoulder)
yabijodek: secondary rigid axis (internal to an object or defining an object's position with respect to the speaker)
yabicerere: slanted rigid axis (internal to an object or defining an object's position with respect to the speaker)
wetabiju: one's view (literally, at my eyes and by extension, horizon)

Translation of the Lines Intuitions Test. ***Introduction.*** Plane: This is a place where the land is flat. It goes on forever.
Ijoce kake ipi cũg at. Yabi be gũ iat.
Sphere: This is a place where the land is round (like a ball).
Ijoce kake ipi iroyruy'at.
Both: There are paths. Paths are all very straight.
Kake e. E soat em cũg at.
Paths go straight, always in front of them.
Soat em eyũ cewap.
Here is a small part of a path.
Ijoce kake e pũg ma i e it ma.
In fact, it continues on this side and on this side (showing the right and left parts of the screen).
Iceman xep xep kadiwi e jodiku jodiwiku'i.
There are villages. They are very small.
Kake ijoce ãgokayũ. I ka i it'at ka.
Paths go right through the center of them.
Ãgoka pidase o muyku e kap ap.
Paths continue straight in front of them.
Soat em cewap e cu cum.
Questions. Slide 1. Let us approach to see it (better).
Gâ'a ajojo cexewi.
Here are two paths on this flat land.
Ijoce, xep xep e ipi cũg at.
Plane: Remember that the land is completely flat
Gebum cuy ipi cũg at kay
Sphere: Remember that the land is completely round
Gebum cuy ipi iroyruy' at kay
Both: and that the paths are very straight
E cũg at e
and go on for very long.
Ixeyũ wuy jijã ip cũ cũm.
Question 1 (showing right side): Will they cross on this side?

Jewebotbot tu ip jodi etako ku?
Question 2 (showing left side): Will they cross on this side?
Jewebotbot tu ip bodi etako ku exe?
Question 3: Even if they go very far?
Ace sũ e dobuxik tu ixe wũy bit?
Slide 2. How about these two paths?
Ijoce kake xep xep wara'at e?
Question 4 (showing left side): Will they cross on this side?
Jewebotbot tu ip jodi etako ku?
Question 5 (showing right side): Will they cross on this side?
Jewebotbot tu ip jodi etako ku?
Question 6: Even if you go very far?
Ace sũ e dobuxik tu ixe wũy bit?
Slide 3. Here are two paths.
Xepxep e omuy ku.
The top one can turn!
Ak i korerẽn i at'e'dak!
Question 7: Can you turn this path (pointing to the top line) so that it crosses the other path on both sides, both there (pointing to the right) and here (pointing to the left)?
I'ebut tu ka o e yabi kadi e mukorerẽn am wara'at e kay. Jodi ku, bodi dak?
Question 8: Can you turn this path (pointing to the top line) so that it never crosses the other path on both sides, neither here (point to the right) nor here (point to the left)?
I'ebut tu ka o'e e abi kadi e mukorerẽn ũm am wara'at' ekay gũ. Bodi ku, jodiku'i?
Slide 4. Here are two paths.
Xep xep e omuy ku.
Question 9: Can you place another path that crosses these two paths?
I'ebut tu kuka oe'e wara'at e mugeam xep xep e ayũ mudek'am?
Question 10: Can you place another path so that it crosses one of these paths (but not the other)?
I'ebut tu kuka o'e pũg e dobuxik am?
Question 11: Can you place another path so that it cross either of these paths?
I'ebut tu kuka o'e eyũ dobuxik ũm am?

Slide 5. Here is one village.
Ijoce kake pũgka ãgoka.
Question 12: Can you place a path so that it passes through the center of the village?
I'e mugebut tu e ãgoka pidase?
Question 13: Could you place two such (straight) paths through the center of the same village?
O'e du kuka ãgoka pidase xep xep e muge am?
Question 14: Could you place even more of this type?
O en ma du e muge am koap at puxim?
Slide 6. Question 15: Can you place a straight path through the center that will never cross the other path?
Pũg e omuyku cũg at e ãgoka pidase at e, wara'at e mudek ũm am?
Question 16: Can you place two such paths?
O'en ma du e muge am xep xep em ãgoka pidase?
Slide 7. Here are two villages.
Ijoce kake xep xepka ãgoka.
Question 17: Could you place one path so that it passes through the center of the two villages?
I'e mugebut tu e xep xep ãgoka pidase?
Question 18: Could you place two such (straight) paths through the center of the two villages?
O'e du kuka ãgokayũ pidase xep xep e muge am?
Question 19: Could you place even more straight paths of this type (all of which running through the center of the two villages)?
O en ma du e muge am koap at puxim?
Slide 8. Here are three villages.
Ijoce kake ebapũgka ãgokayũ.
Question 20: Could you place a (straight) path so that it passes through the center of all three villages?
O'edu kuka ebapũgka ãgokayũ pidase e muge am?
Slide 9. Here are another group of three villages more distant from each other.
Ijoce ebapũg ãgokayũ wũy wũy jewekawi.
Question 21: Could you place a (straight) path so that it passes through the centers of all three villages ?
Ie mugebut tu kuka o'e pũg e ebapũgka ãgokayũ pidase?

1. Dehaene S, Izard V, Spelke E, Pica P (2008) Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science* 320:1217–1220.

2. Dehaene S, Izard V, Pica P, Spelke E (2006) Core knowledge of geometry in an Amazonian indigene group. *Science* 311:381–384.

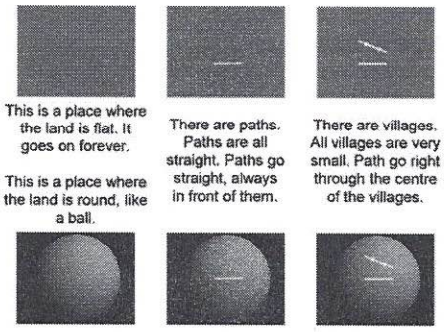


Fig. S1. Excerpts from the text and illustrations introducing notions of plane, sphere, straight line, and dot to the participants. To improve visibility, the segments drawn here are longer than on the real stimuli.

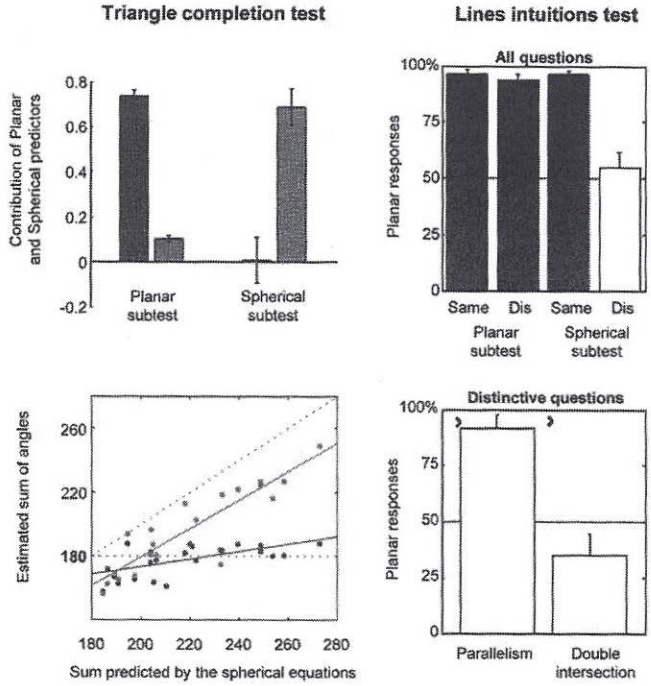


Fig. S2. Performance of noneducated Mundurucu adults. (*Left*) Performance on the triangle completion test. *Upper* shows the weight attributed to the planar (blue) and spherical (red) regressors in a multiple regression analysis for the responses at the planar and spherical subtests. *Lower* shows the sum of the internal angles of the estimated triangle (sum of the two given angles and the angle response) across trials for the planar (blue) and spherical (red) subtests. The dotted lines indicate the prediction of the planar (blue; flat) and spherical (red) equations. The plain lines correspond to a linear fit of the data. (*Right*) Responses to the lines intuitions test. *Upper* shows the percentage of plane responses in both subtests for the questions that call for the same answer on both the plane and the sphere (Same) and the questions that call for distinct answers (Dis). The bar in white corresponds to the questions where the theoretical answer contradicts planar geometry: the planar response is incorrect in this case. For all other bars, the planar answer is correct. *Lower* shows the percentage of planar responses for the critical questions pertaining to the existence of parallel lines (three questions) or double intersections between straight lines (five questions). The bars correspond to the percentage of planar responses for the spherical subtest (the planar response is incorrect in this case). For reference, the percentages of planar responses to the same questions asked in the planar subtest are indicated by arrows on the left of each bar.

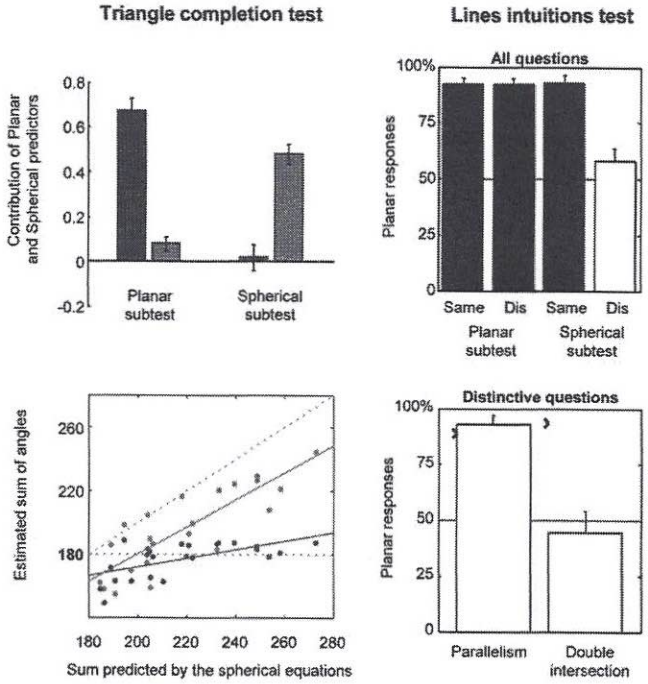


Fig. S3. Performance of Mundurucu adults with some education. (*Left*) Performance on the triangle completion test. *Upper* shows the weight attributed to the planar (blue) and spherical (red) regressors in a multiple regression analysis for the responses at the planar and spherical subtests. *Lower* shows the sum of the internal angles of the estimated triangle (sum of the two given angles and the angle response) across trials for the planar (blue) and spherical (red) subtests. The dotted lines indicate the prediction of the planar (blue; flat) and spherical (red) equations. The plain lines correspond to a linear fit of the data. (*Right*) Responses to the lines intuitions test. *Upper* shows the percentage of plane responses in both subtests for the questions that call for the same answer on both the plane and the sphere (Same) and the questions that call for distinct answers (Dis). The bar in white corresponds to the questions where the theoretical answer contradicts planar geometry: the planar response is incorrect in this case. For all other bars, the planar answer is correct. *Lower* shows the percentage of planar responses for the critical questions pertaining to the existence of parallel lines (three questions) or double intersections between straight lines (five questions). The bars correspond to the percentage of planar responses for the spherical subtest (the planar response is incorrect in this case). For reference, the percentages of planar responses to the same questions asked in the planar subtest are indicated by arrows on the left of each bar.

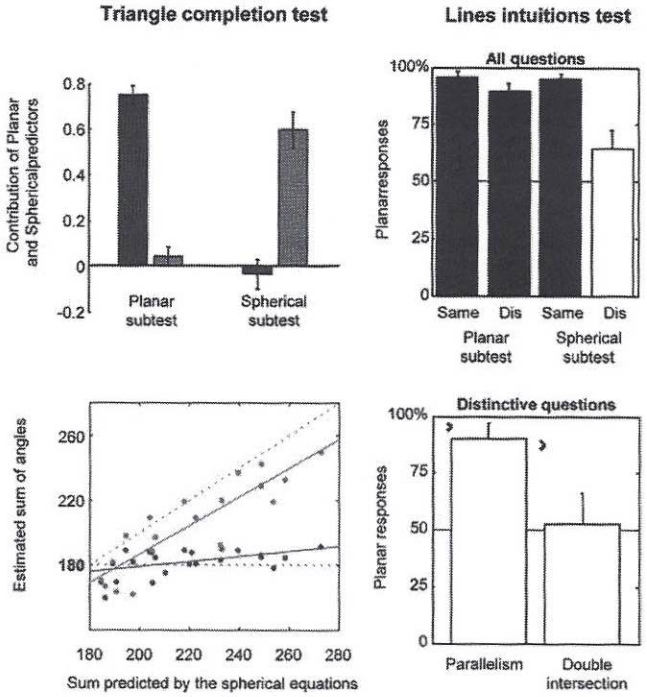


Fig. S4. Performance of the most educated Mundurucu adults. (*Left*) Performance on the triangle completion test. *Upper* shows the weight attributed to the planar (blue) and spherical (red) regressors in a multiple regression analysis for the responses at the planar and spherical subtests. *Lower* shows the sum of the internal angles of the estimated triangle (sum of the two given angles and the angle response) across trials for the planar (blue) and spherical (red) subtests. The dotted lines indicate the prediction of the planar (blue; flat) and spherical (red) equations. The plain lines correspond to a linear fit of the data. (*Right*) Responses to the lines intuitions test. *Upper* shows the percentage of plane responses in both subtests for the questions that call for the same answer on both the plane and the sphere (Same) and the questions that call for distinct answers (Dis). The bar in white corresponds to the questions where the theoretical answer contradicts planar geometry: the planar response is incorrect in this case. For all other bars, the planar answer is correct. *Lower* shows the percentage of planar responses for the critical questions pertaining to the existence of parallel lines (three questions) or double intersections between straight lines (five questions). The bars correspond to the percentage of planar responses for the spherical subtest (the planar response is incorrect in this case). For reference, the percentages of planar responses to the same questions asked in the planar subtest are indicated by arrows on the left of each bar.

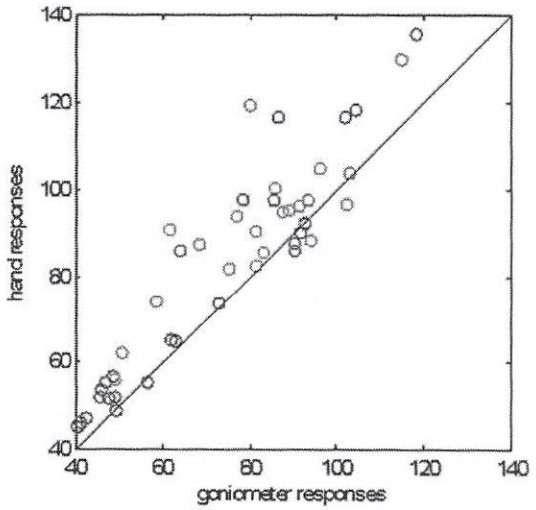


Fig. S5. Comparison of the angle estimates in the groups responding with the goniometer (x axis) or their hands (y axis) across the different trials. The blue dots represent the trials of the planar subtest, and the red dots represent the trials of the spherical subtest. The plain line indicates the diagonal ($x = y$). Angles were systematically overestimated in the group responding with their hands compared with the group responding with the goniometer. This systematic deviation emerged probably from the measurement of the aperture of the hands (for example, if the measurement was taken close to the finger tip of curved fingers).

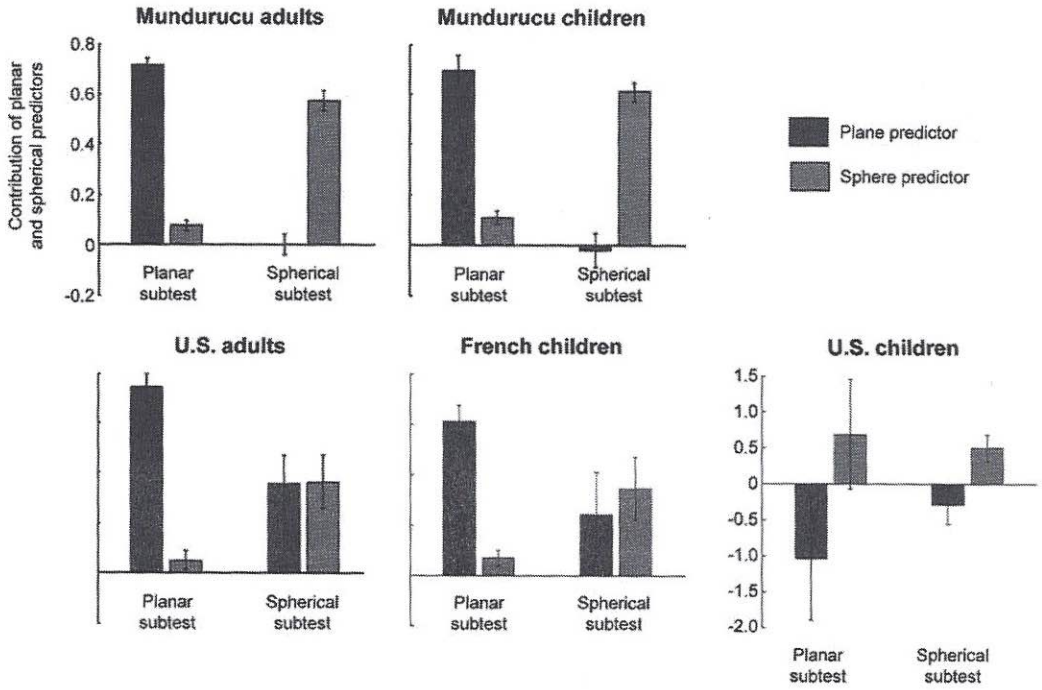


Fig. S6. Weights attributed to the planar (blue) and spherical (red) regressors in a multiple regression analysis performed over angle estimates.

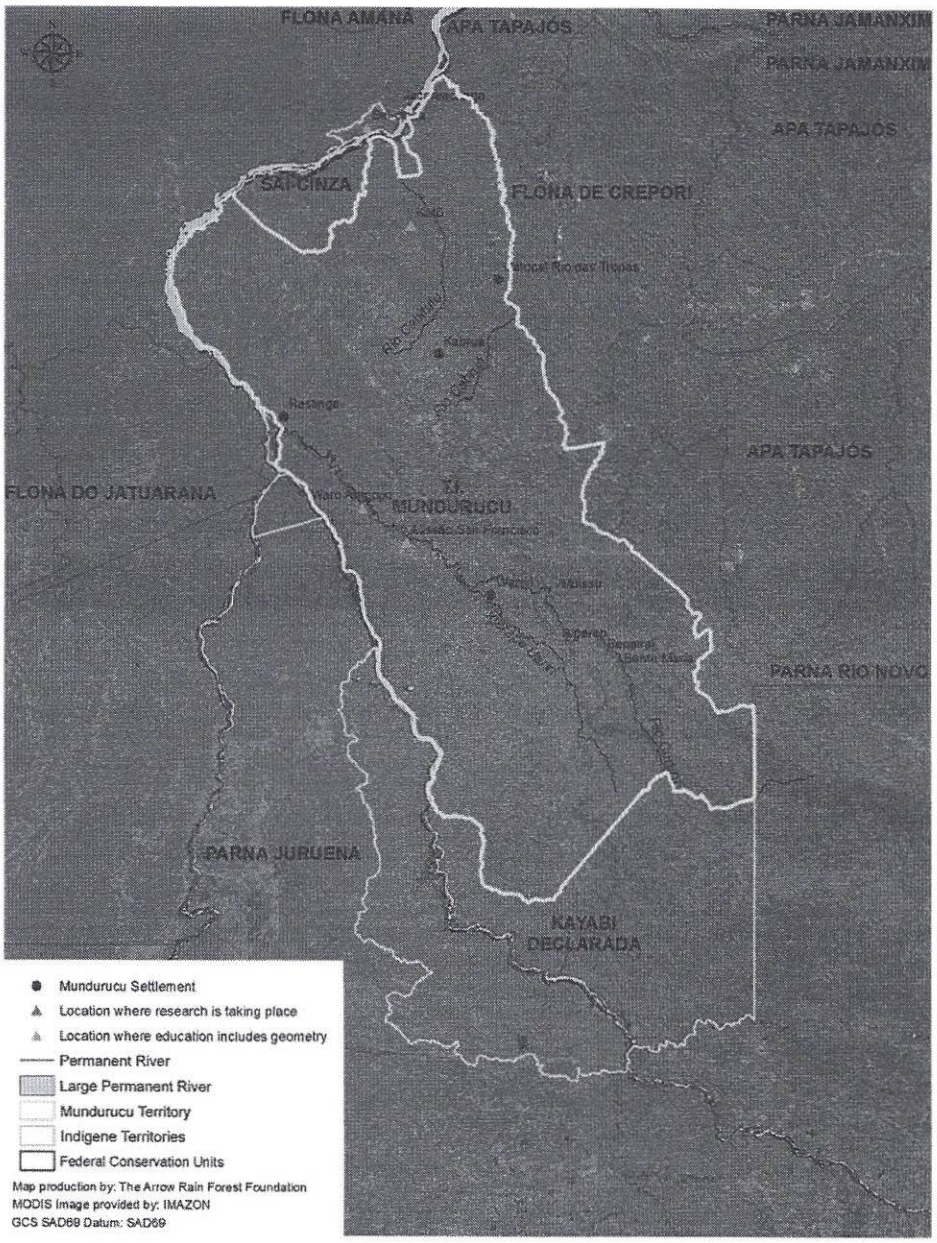


Fig. S7. Simplified map of the Mundurucu territory showing locations where the research was conducted. The village Waro Amnopu is the Mundurucu post of the FUNAI (Fundação do Indio).

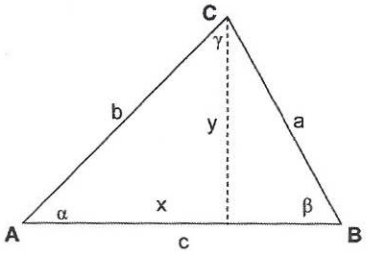


Fig. S8. Triangle on a plane.

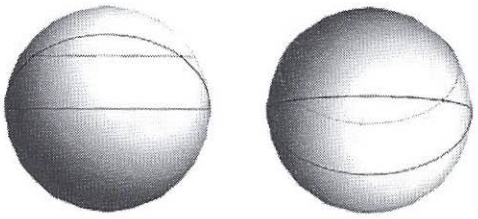


Fig. S9. A sphere with its horizontal equator (red horizontal) and a circle parallel to this great circle (yellow). Two points are marked on this smaller circle, and the straight great circle passing through these two points is drawn in red. (*Right*) The same sphere is presented from a different view, highlighting the difference in length between the lines connecting these two points along the red and yellow circles.

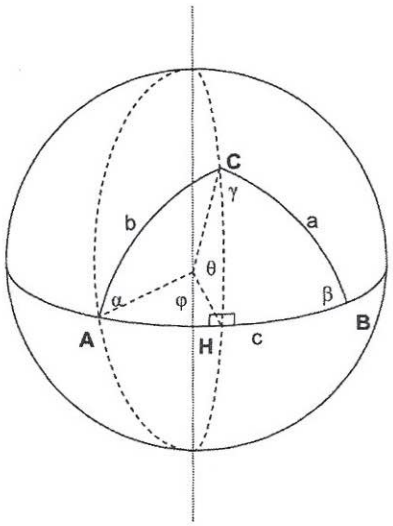


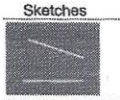
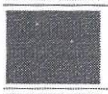
Fig. S10. Triangle on a sphere.

Table S1. Comparison of the estimates for the position of the intersection (*x* and *y* coordinates) and the angle at the intersection with the values predicted by planar and spherical geometry

	Planar subtest			Spherical subtest		
	x coordinate	*y* coordinate	Angle	*x* coordinate	*y* coordinate	Angle
Mundurucu adults	$r^2 = 0.65$ $\beta = 0.60$ $t(21) = 9.0$ ($P < 0.0001$)	$r^2 = 0.94$ $\beta = 0.84$ $t(21) = 49.2$ ($P < 0.0001$)	$r^2 = 0.85$ $\beta = 0.75$ $t(21) = 29.1$ ($P < 0.0001$)	$r^2 = 0.60$ $\beta = 0.45$ $t(21) = 9.7$ ($P < 0.0001$)	$r^2 = 0.80$ $\beta = 0.71$ $t(21) = 26.6$ ($P < 0.0001$)	$r^2 = 0.62$ $\beta = 0.57$ $t(21) = 18.4$ ($P < 0.0001$)
Mundurucu children	$r^2 = 0.76$ $\beta = 0.63$ $t(7) = 7.9$ ($P < 0.0001$)	$r^2 = 0.95$ $\beta = 0.85$ $t(7) = 25.6$ ($P < 0.0001$)	$r^2 = 0.81$ $\beta = 0.75$ $t(7) = 14.7$ ($P < 0.0001$)	$r^2 = 0.60$ $\beta = 0.49$ $t(7) = 6.8$ ($P = 0.0003$)	$r^2 = 0.83$ $\beta = 0.74$ $t(7) = 22.5$ ($P < 0.0001$)	$r^2 = 0.63$ $\beta = 0.60$ $t(7) = 17.5$ ($P < 0.0001$)
US adults	$r^2 = 0.89$ $\beta = 1.0$ $t(18) = 17.5$ ($P < 0.0001$)	$r^2 = 0.96$ $\beta = 0.95$ $t(18) = 33.8$ ($P < 0.0001$)	$r^2 = 0.80$ $\beta = 0.77$ $t(18) = 14.8$ ($P < 0.0001$)	$r^2 = 0.71$ $\beta = 0.78$ $t(18) = 16.8$ ($P < 0.0001$)	$r^2 = 0.85$ $\beta = 0.88$ $t(18) = 25.4$ ($P < 0.0001$)	$r^2 = 0.37$ $\beta = 0.62$ $t(18) = 11.2$ ($P < 0.0001$)
French children	$r^2 = 0.84$ $\beta = 0.85$ $t(7) = 10.1$ ($P < 0.0001$)	$r^2 = 0.95$ $\beta = 0.82$ $t(7) = 27.8$ ($P < 0.0001$)	$r^2 = 0.81$ $\beta = 0.65$ $t(7) = 10.7$ ($P < 0.0001$)	$r^2 = 0.67$ $\beta = 0.64$ $t(7) = 4.9$ ($P = 0.0017$)	$r^2 = 0.70$ $\beta = 0.76$ $t(7) = 23.2$ ($P < 0.0001$)	$r^2 = 0.49$ $\beta = 0.56$ $t(7) = 19.4$ ($P < 0.0001$)
US children	$r^2 = 0.48$ $\beta = 0.47$ $t(9) = 6.4$ ($P = 0.0001$)	$r^2 = 0.74$ $\beta = 0.61$ $t(9) = 17.3$ ($P < 0.0001$)	$r^2 = 0.47$ $\beta = -0.30$ $t(9) = -0.8$ ($P = 0.45$)	$r^2 = 0.28$ $\beta = 0.26$ $t(9) = 2.6$ ($P = 0.029$)	$r^2 = 0.63$ $\beta = 0.68$ $t(9) = 8.9$ ($P < 0.0001$)	$r^2 = 0.18$ $\beta = 0.27$ $t(9) = 2.8$ ($P = 0.021$)

Individual estimates were correlated with the values predicted by the equations of planar and spherical geometry. To correct for the differences in scale across trials, the *x* and *y* coordinates of the intersection were normalized by the apparent distance between the two base villages in each trial. The table reports the outcome of these correlations as average r^2 and β across individuals. In each group, we also tested whether the predictor contributed significantly to the responses by comparing the β-values against zero.

Table S2. Responses of all groups to the lines intuitions test

Sketches	Questions	Phase/sphere				
		Mundurucu adults	Mundurucu children	US adults	French children	US children
	Do the lines cross on the small-angle side? (right side)	95.5%/100.0%	100.0%/100.0%	100.0%/100.0%	100.0%/100.0%	
	Do the lines cross on the large-angle side?*†	2.3%/22.7%	0.0%/25.0%	0.0%/56.3%	0.0%/37.5%	
	Would they cross on the large-angle side if going very far?*†	18.2%/67.0%	12.5%/81.3%	0.0%/87.5%	0.0%/62.5%	
	Do the lines cross on the small-angle side? (left side)	100.0%/100.0%	100.0%/87.5%	100.0%/100.0%	100%/100.0%	75.0%/81.3%
	Do the lines cross on the large-angle side?*†	4.5%/45.5%	0.0%/62.5%	0.0%/87.5%	0.0%/37.5%	0.0%/25.0%
	Would they cross on the large-angle side if going very far?*†	4.5%/71.6%	0.0%/75.0%	6.3%/87.5%	0.0%/62.5%	0.0%/18.8%
	Can a line be made to cross another line at two different places?*†	11.4%/70.5%	12.5%/62.5%	0.0%/81.3%	0.0%/37.5%	37.5%/43.8%
	Can a line be made to never cross another line?*‡	88.6%/89.8%	93.8%/93.8%	100.0%/93.8%	100.0%/87.5%	56.3%/62.5%
	Can a line be made to cross two other parallel-looking lines?	90.9%/100.0%	100.0%/100.0%	100.0%/100.0%	100.0%/100.0%	56.3%/75.0%
	Can a line cross one of two parallel-looking lines but not the other?	11.4%/13.6%	18.8%/12.5%	6.3%/25.0%	12.5%/12.5%	37.5%/31.3%
	Can a line be made to never cross two other parallel-looking lines?*‡	88.6%/88.6%	100.0%/87.5%	100.0%/100.0%	100.0%/87.5%	75.0%/56.3%
	Can one line be drawn through a point?	100.0%/98.9%	100.0%/100.0%	100.0%/100.0%	100.0%/100.0%	100.0%/87.5%
	Can two lines be drawn through a point?	100.0%/100.0%	93.8%/100.0%	100.0%/100.0%	100.0%/100.0%	37.5%/75.0%
	Can more than two lines be drawn through a point?	97.7%/95.5%	100.0%/100.0%	100.0%/100.0%	100.0%/100.0%	37.5%/68.8%
	Can a line be drawn through a point and never cross another line?*‡	100.0%/96.6%	100.0%/100.0%	93.8%/100.0%	100.0%/87.5%	62.5%/68.8%
	Can two such lines be drawn?	25.0%/22.7%	31.3%/18.8%	0.0%/6.3%	25.0%/12.5%	31.3%/56.3%
	Can a line be drawn through two points?	97.7%/86.4%	100.0%/100.0%	100.0%/100.0%	100.0%/100.0%	87.5%/87.5%
	Can two such lines be drawn?	11.4%/0.0%	0.0%/6.3%	6.3%/6.3%	25.0%/25.0%	37.5%/50.0%
	Can more than two such lines be drawn?	1.1%/0.0%	0.0%/0.0%	0.0%/0.0%	25.0%/25.0%	18.8%/50.0%
	Can a line be drawn through three nonaligned points?	2.3%/11.4%	0.0%/0.0%	0.0%/6.3%	0.0%/0.0%	6.3%/25.0%
	Can a line be drawn through the two poles and a third point?*	11.4%/65.9%	0.0%/81.3%	0.0%/25.0%	0.0%/12.5%	18.8%/37.5%

The table lists the questions in the order that they appeared in the test together with the sketches presented to the participants and the percentage of positive (yes) responses in each group of participants. The same sketches were presented in both planar and spherical subtests, except for the last question.
*Questions that call for different answers in planar and spherical geometry.
†Questions about intersections occurring in both directions.
‡Questions about parallelism.