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When praxeologies move from an institution to another: an epistemological approach to boundary crossing

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The issue of vocational mathematics education is commonly approached through a vigotskian lens focusing on the individual development within different socio-cultural contexts: how does one student tackle the experience of crossing boundaries between mathematics, engineering sciences, vocational training? In my presentation, I take the opposite yet complementary point of view, that of the anthropological theory of the didactic, which emphasises the social and institutional dimensions. Drawing on the notion of praxeology as a model for socially acknowledged cognitive resources of institutions, I develop this model to address an epistemological issue: how are mathematical praxeologies transformed when crossing institutional boundaries? Examples refer to mathematics and automatic control.

Introduction: The Anthropological Theory of the Didactic

The anthropological theory of the didactic (hereafter ATD) is at the same time a theory and the most prominent dimension of a research program in mathematics education. This program has been initiated by Yves Chevallard (1985) with the study of didactic transposition processes, the anthropological perspective being introduced in 1992 (Chevallard 1992). A socio-cultural conception of humans underpins the ATD, with a focus on institutions (stable social organisations) as absolute precondition for humanity's development and social activities. Institutions foster collective processes for facing and solving human problems. They favour the dissemination of innovations and more widely provide the necessary resources (material and cultural) for activities to take place. Conversely each institution constrains the different types of activities that it expects people to carry out in the social environment it builds. An individual has to satisfy the institutional expectations, to a certain extent at least, depending on the institution; that is why he is considered as an institutional subject (from Latin sub-jectus: literally thrown under) when acting within this institution. Hence, the ATD considers that human activities are institutionally situated and, consequently, so is knowledge about these activities. When a fragment of social knowledge, produced within a given institution I , moves to another one I_u in order to be used, the ATD's epistemological hypothesis states that such boundary crossing most likely results in some transformations of knowledge, called transpositive effects. Any didactical institution I_d that intends to train students to meet I_u 's demands should be aware of these changes from I to I_u ; otherwise they will leave the full responsibility of knowledge adaptation up to the students. Moreover, let us recall that the specific nature of activities within I_d , under specific constraints, results in other transpositive effects, the so-called didactic transposition.

Praxeology

The key notion of praxeology is the basic unit proposed by ATD to analyse the institutionally acknowledged capitals of practices and knowledge (see Chevallard 2006, p.23). A praxeology entails two interrelated components, *praxis* and *logos*. The practical block (or know-how) associates a type of tasks T and a technique τ . τ is a "way of doing" which is endowed with

certain efficiency for a certain subfield within the set of T tasks. The *logos* block contains two levels: the technology of the technique (θ) gathers the whole rational knowledge referring to the technique; the theory (Θ) is a second level of more general knowledge supporting the technological discourse.

To exemplify the praxeological model and give an idea of its potential to analyse the transpositive effects of boundary crossing, I will consider a mathematical type of tasks encountered in strictly mathematical contexts as well as engineering sciences: *Breaking up a rational function into partial fractions*.

Mathematics praxeologies for breaking up a rational fraction into partial fractions

Let me emphasise that by mathematics praxeology I mean a praxeology that is acknowledged in the current institution of research in mathematics. Hence we will refer in this part to the mathematics' norms for proof. The following technological elements are derived from the analysis of a calculus online textbook¹.

Description of the technique in the general case: (1) Make the denominator monic (leading coefficient 1), and use the Euclidean algorithm to reduce to a problem where the degree of the numerator r is less than the degree of the denominator d . (2) Factorise the denominator as a product of powers of distinct monic irreducible polynomials. (3) Write the fraction as a sum of partial fractions of the form R/Q^k , where Q is one of the irreducible factors, k is at most equal to the multiplicity of Q in d and the degree of R is less than the degree of Q . (4) The coefficients of every R need to be determined. One way of doing this is to take a common denominator, multiply out, equate coefficients and solve the resultant system of equations.

Example: We want to express $\frac{3x+1}{(x-1)^2(x+2)}$ as the sum of its partial fractions $\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$.

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \Leftrightarrow 3x + 1 = (A + C)x^2 + (A + B - 2C)x - 2A + 2B + C$$

$$\Leftrightarrow A + C = 0, A + B - 2C = 3, -2A + 2B + C = 1 \Leftrightarrow A = \frac{5}{9}, B = \frac{4}{3}, C = -\frac{5}{9}$$

Several theorems are necessary *to validate this technique*, that is, to prove without further checking that $\frac{3x+1}{(x-1)^2(x+2)} = \frac{5}{9(x-1)} + \frac{4}{3(x-1)^2} - \frac{5}{9(x+2)}$. θ_1 : two rational functions with the same denominator are equal if and only if their numerators are equal; θ_2 : two polynomials are equal if and only if they are of same degree and have the same coefficients; θ_3 : theorem about equality of rational numbers; θ_4 : theorems about equivalent systems of equation. Several mathematical theories, "theory" being used in the usual meaning of the word, are necessary to prove these theorems.

Appraisal of the technique: This technique is tedious in some cases without proper software because there are many coefficients to find. In fact, mathematicians are aware of the heaviness of the technique and look for alternatives. For example, another technique consists in plugging in several appropriate values of x depending on the pole order. The technique is

¹ Partial fractions: an integrationist perspective. Math 153 Section 55. Vipul Naik. University of Chicago. <http://www.indabook.org/d/Integrating-Rational-Functions-by-Partial-Fractions-Using.pdf>

based on necessary conditions, so you have to check the equality, unless you have an existence theorem, deriving from a rather extended part of the polynomials arithmetic theory.

Motivation of one step of the technique: Now, as a transition to the corresponding praxeology in automatics, we can ask the following question: why is it important to make the denominator monic? The fact that linear monic polynomials have 1 as a derivative and that the antiderivative of rational functions $1/(x-a)^k$ is easy to calculate is one motive for the restriction to monic factors.

The automatics praxeology for breaking up a rational fraction into partial fractions

The following example is based on a study of how Laplace transform is taught in an on-line course for higher technicians (see Castela & Romo Vázquez 2011 for more details). Hence the mathematics praxeologies have crossed two boundaries: from mathematics to automatic control and then to an automatic course.

Some elements about the automatics' issues are necessary. The problem at stake is automatic regulation of systems: if a quantity is to be kept constant, an electronic gauge measures its value; when variation is recorded, an appropriate regulation process is triggered to go back to the desired value. The less time needed to get the quantity back to this value, the more efficient the control system. The evolutions of the different systems involved are described by differential equations, turned to algebraic ones by the Laplace transform and easily solved, with a rational fraction $F(p)$ as a solution. To inverse the Laplace transform, the online textbook recommends using a table of Laplace transforms. The type of tasks *Breaking up a rational fraction into partial fractions* appears when complicated $F(p)$ are involved. In what follows, I give an idea of the technique and technology proposed by the textbook.

Description of the technique: Assuming that the mathematical techniques are familiar to the students, the author only specifies that $F(p)$ denominator must be written under the following canonical form $k(1+\tau_1p)(1+\tau_2p) \dots$ with decreasing values of the τ_i . E.g. $3p+2$ is transformed into $2(1+1.5p)$, not into $3(p+2/3)$. This is a significant change to the mathematical technique.

Motivation (raison d'être) of this special factorisation: If $F(p) = \frac{1}{1+1.5p}$, the corresponding original function is $f(t) = K(1-e^{-t/1.5})$. 1.5 is called the time constant of this function. The system reactivity, and therefore its quality, is directly dependent on the higher value of the time constants. Hence, this value must clearly appear in the calculation. This means that the boundary crossing has changed the type of tasks and thus the technique.

Explanation of the relation between time constant and reactivity: if $f(t)$ represents the controlled quantity and K its desired constant value, it is known that after 7τ , here 7×1.5 seconds, the exponential will be equal to 0, that is, considered as negligible in Automatics. Hence, the transitional regime lasts 7×1.5 seconds.

Validation of this claim: $e^{-t/\tau} < 0.01$, hence $t/\tau > 100$, $t > \tau \ln(100) \approx 7\tau$.

What needs does the technology of a technique intend to satisfy?

(Castela & Romo Vázquez 2011) analyses the Laplace transform chapter in one mathematics and two automatics textbooks from tertiary vocational courses for engineers and higher technicians. The first one, in a classical mathematics style, is focused on the comprehensive accurate presentation of concepts, theorems and proofs. The Laplace transform technique to solve non-linear differential equations is alluded to, without any examples related to engineering sciences. As shown in the above example, the automatics textbooks are very different. They give a lower priority to mathematical proofs and instead, they develop another kind of knowledge about techniques, strongly correlated with the vocational context. Actually there are many things to know about Laplace transform and the derived techniques, but all these technological elements satisfy diverse needs. Drawing on the aforementioned textbooks, Castela and Romo Vázquez (2011, pp. 88-90) differentiate six of them: *describing* the technique, *validating* it i.e. proving that this technique produces what is expected from it, *explaining* the reasons why this technique is efficient (knowing about causes), *motivating* the different gestures of the technique (knowing about objectives), *making it easier* to use the technique and *appraising* it (with regard to the field of efficiency, to the using comfort, relatively to other available techniques). Such technological elements are present in both previous examples of mathematics and automatics praxeologies. This list should not be taken as exhaustive. For instance, I currently consider one more need: *controlling* the technique implementation by the individuals, that is making sure they have correctly used the technique.

The technological analysis as a relevant tool for transition issues

This approach is an incentive for vocational institutions to analyse the nature and extent of the transpositive effects on mathematics praxeologies within the scientific and professional fields included in their curriculum. As seen in the example above, each component of the praxeology may change or develop for rational reasons that take into account the specific conditions of activities. Educational institutions should consider the motives and legitimacy of these evolutions.

Furthermore, the analysis grid of the technological component is also relevant when the issue of transition to advanced mathematics is addressed. Advancing in mathematics not only consists in learning new theories, it also means facing tasks that get closer to the mathematicians' activities. According to the ATD, any human activity contains elements of genericity. Hence, mathematics researchers, even if they have to be creative, also draw on previously developed praxeologies with a technological component that satisfies practical needs within problem solving and generally derives from experiencing the technique implementation, in other words, not from a mathematical theory. Most of this practical part of mathematics praxeologies is not taught. Yet students need such knowledge, in France at least as of high school (for detailed argumentation, see Castela 2009). So the responsibility for building this practical knowledge lies on the students, and this may be an important cause of failure. Therefore, the challenge for tertiary education would be to organise the students' training to praxeological development from their own mathematical experiences, especially for maths majors.

Modelling the praxeological inter-institutions dynamics

In the foregoing, we have considered one praxeology produced by a research institution in mathematics. The online textbook designed by a college lecturer to teach this praxeology reveals that the technology of the technique contains fragments of knowledge substantiated by mathematical proofs deriving from mathematical theories, as well as other practical elements, empirically developed by mathematicians as they use the considered technique. This part of the technology, being very much linked to the concrete conditions of mathematicians' activities, may appear within a mathematical education setting. However, it will be generally considered of low interest by mathematicians when the dissemination of the mathematical praxeologies to other non educative institutions is at stake. This hypothesis sustains the following modelling of the transpositive effects on a praxeology produced by a research institution and crossing a boundary, that is to say moving from one institution to another in order to be used or taught.

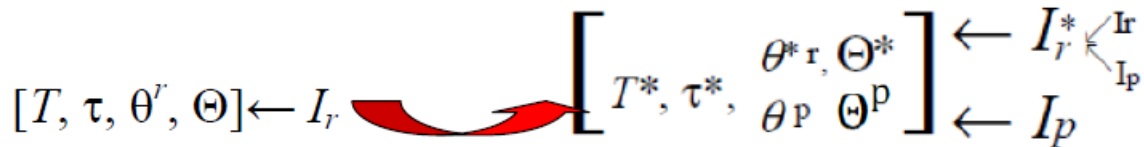


Figure 1. From I_r to I_p , the transposition model

Let us consider the different symbols in this figure which generalises the mathematics case.

The original praxeology

I_r is a research institution, namely an institution socially in charge of producing new praxeologies to address certain types of tasks and organising systematic processes of validation in order to substantiate their legitimacy and institutionalisation. The arrow between the praxeology and the institution represents these institutional processes, which have both an epistemological dimension and a social one.

This research institution may be a scientific one or a technical one. But the category is much extended: the crucial point is that I_r is not directly interested in addressing tasks of the type T . For instance, in the French IREM (*Instituts de Recherche sur l'Enseignement des Mathématiques*), teachers meet to develop collective thinking, design teaching sequences they implement in their classes, assess and if necessary consider afresh. They take some distance from their daily teaching practices and assume the role of researcher.

It should be underlined that the validation processes depend on the specific paradigm of I_r . In the case of mathematics, the technology is proved by demonstrations, sustained by theories that are assessed by the mathematics community. But we know that in physics, the relation between claims and theories is very different, not to mention human sciences and mathematics education.

Finally, let us note that the symbol I_r is a simplification. In fact, a whole hierarchy of embedded institutions are involved in the research activities: laboratories, mathematics journals, congresses, etc.

The transposed praxeology

An institution, some subjects of which have to address tasks of the type T , imports the original praxeology produced and warranted by I_r . In Figure 1, this institution is represented by the symbol I_p underlining the fact that this institution has only pragmatic relations to the praxeology and its development. This I_p (or hierarchy of embedded institutions) could be a research institution of the same domain, using the technique to address other types of tasks (e.g. breaking up a rational fraction to integrate it), or of another domain (e.g. Automatic in the first part of this text). It could also be a professional institution or an educative institution (about didactic transposition, see Chevallard 1985, 1989).

The asterisks express the idea that every component of the original praxeology may evolve. This transformation is an object of institutional transactions completed in a specific institution I_r^* , created and controlled by I_r and I_p . I_r^* is more or less vanishing, the transactions are more or less difficult and controversial, depending on several factors: the extent of the transformations (e.g. no transformation if I_p is a mathematics laboratory), the distance between the two institutional epistemologies (e.g., I_r is mathematics and I_p is an experimental science), the importance for I_p that I_r validates the new technique (e.g., if I_p is a profession such as nursing, with high security requirements, it will be of great importance for this professional institution that, the technique remains valid despite the changes), and the importance for I_r that the transposed praxeology not be too far from the original one (in France, it is common for mathematicians to have a critical look on what is taught when I_p is an educational institution).

At last, this diagram says that a practical technology θ_p is developed and acknowledged by I_p on specific empirical bases, possibly sustained by a discourse of second level, which according to the ATD is considered as a theory. Hence the symbol Θ^p represents a true oxymoron, a pragmatic theory. Such object has not yet been thoroughly investigated within ATD framework; Castela and Elguero (2013) suggest that the technologies of the validation praxeologies developed in the institution I_p could contribute to such theory. However, the extent of this theoretical discourse depends very much on the institution's nature and may be quite limited. Let us notice that this model puts forward, not only the institutional knowledge formulated in the technological and theoretical discourse, but also all the social processes of validation and acknowledgment (represented by the arrows) which represent objects of interest for an anthropological epistemology of institutional praxeologies. For instance, in his PhD dissertation, Morel (2013) shows that during the second half of the 18th century, the mining administration in Saxony creates a mining academy in Freiberg to train the mine officials and imposes, from 1797, that a mathematics teacher becomes in charge of a geometrical course for surveyors (*Markscheider*), whereas before, qualified surveyors were responsible of the teaching of the practical praxeologies for mining, within the training program. Morel substantiates that, at that time, the change does not take place smoothly. Such an educative decision from this professional institution is a social acknowledgement of the geometrical praxeologies produced by mathematicians. We may assume that organising the dissemination of a given praxeology relating to a type T for the subjects concerned by T is one of the more primitive and more frequent ways of institutionally acknowledging the legitimacy of this very praxeology.

Other dynamics

So far, we have only considered a specific case, namely a praxeology produced by a research institution, moving to another institution in order to be used or taught. Yet, even with the very broad meaning given to the notion of research institution, we cannot assume that the praxeological production is exclusively operated in such institutions. Occupational and, more broadly, social settings may develop their own original praxeologies, $[T, \tau, \theta^p, \Theta^p]$, within the empirical context of working and of social life. These praxeologies move to other institutions in order to be used with possible transpositive effects. Another possibility is that a research institution, created by the occupational one or not, investigates $[T, \tau, \theta^p, \Theta^p]$ with the objective of improving it and more systematically substantiating its validity. As an example, refer to Castela and Elguero (2013) who examined the case of custom dressmaking in Argentina, with various sized institutions involved. We won't detail this situation any further within the limits of this text.

Conclusion

In this paper, I have addressed the issue of transitions between secondary and tertiary education and between general and professional oriented programs from an epistemological point of view. This proposal is centred on the notion of praxeology, an ATD key concept, with two directions: a grid to investigate the technology of a technique, based on an analysis of the different needs created by the technique used in a given institutional context and a model for the transpositive effects of inter-institutional praxeological dynamics. I think that the first tool is especially interesting for mathematics majors' higher education. In fact, it introduces an organisation of mathematical knowledge that gives as much importance to types of problems and techniques as to concepts and theories. It also acknowledges the necessity of some mathematical practical knowledge as a component of the mathematicians' expertise. Hence, it may support the design of modules aiming at training the students not only to solve problems, but also to draw fragments of this practical knowledge from their experience as well as from the proofs given in lectures.

Furthermore, I believe that the second part of the text is a relevant tool for addressing the issue of choosing the appropriate mathematics for professional oriented higher education. To tackle this problem, mathematicians need to take some distance with their own culture, with their mathematical *alma mater*, as Chevallard is wont to say. They have to reconsider the following questions: which mathematical praxeologies are useful for such engineering or professional domains? What needs would be satisfied? Which discourse makes the mathematical technique intelligible? This is actually an epistemological investigation that we consider as a prerequisite to the design of mathematics syllabi for professional training programs. Mathematics researchers and lecturers are too often not aware of the necessity and of the complexity of such an investigation; they are not necessarily prepared for it by their mathematics education. This should be accomplished collectively with researchers and professionals of the domains using the mathematics at stake in the program. I assume that the text's proposals could first introduce this epistemological problematic and then support the investigations.

References

- Castela, C. (2009). An anthropological approach to a transitional issue: analysis of the autonomy required from mathematics students in the French Lycée. *NOMAD (Nordisk Matematikk Didaktikk)*, 14(2), 5-27.
- Castela, C. & Romo Vázquez, A. (2011). Des mathématiques à l'automatique : étude des effets de transposition sur la transformée de Laplace dans la formation des ingénieurs. *Recherches en Didactique des Mathématiques*, 31(1), 79-130.
- Castela, C. & Elguero, C. (2013). Praxéologie et institution, concepts clés pour l'anthropologie épistémologique et la socioépistémologie. *Recherches en Didactique des Mathématiques*, 33 (2), 79-130.
- Chevallard, Y. (1985). *La transposition didactique, du savoir savant au savoir enseigné* (2nd ed. 1991). Grenoble : La Pensée Sauvage.
- Chevallard, Y. (1989). On didactic transposition theory: Some introductory notes. In Proceedings of the international symposium on selected domains of research and development in mathematics education (pp. 51-62).
- http://yves.chevallard.free.fr/spip/spip/IMG/pdf/On_Didactic_Transposition_Theory.pdf
- Chevallard, Y. (1992). Fundamental concepts in didactics: perspectives provided by an anthropological approach. In R. Douady & A. Mercier (Eds.) *Research in Didactique of Mathematics, selected papers* (pp.131-167). Grenoble: La Pensée Sauvage.
- Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.) *Proceedings of the IV Congress of the European Society for Research in Mathematics Education* (pp. 21-30). Barcelona: FUNDEMI-IQS.
- Morel, T. (2013). *Mathématiques et politiques scientifiques en Saxe (1765-1851) : institutions, acteurs et enseignements*. Thèse de doctorat de l'Université de Bordeaux.
- Morel, T (2015). Usefulness and practicability of Mathematics: German Mining Academies in the 18th century. Preprint of the Max-Planck Institute for the History of Science, « The Making of Useful Knowledge », à paraître.