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This paper focuses on three key concepts of the anthropological theory of the didactic, after giving an idea of the basic principles of the theory, namely: institution, praxeology and transposition. An understanding of praxeology, being a general model for socially-acknowledged cognitive resources with examples coming from different contexts, is presented. I looked at examples from mathematics and automatic control in the first part and dressmaking in the second part. In both parts, the movements of praxeologies between different institutions and their transpositive effects are discussed. General models of such processes are also provided. I conclude by questioning an anthropology issue of reference to academic mathematics without that of workshop mathematics.

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Introduction: The Anthropological Theory of the Didactic

The anthropological theory of the didactic (hereafter ATD) is at the same time a theory in the scientific usual meaning of the term, that is, an organised body of knowledge, socially legitimated, and the most prominent dimension of a research program in mathematics education developed by a community of researchers, mainly from European and American French-and-Spanish-speaking countries. This program has been initiated by Yves Chevallard in the 1980s with the study of didactic transposition processes (Chevallard 1985, 1989), the anthropological perspective being introduced in 1992 (Chevallard 1992). Although this principle has never been explicitly acknowledged, I dare claim that a socio-cultural conception of humans underpins the ATD. The second point is that the research program focuses on the social aspects of the didactic reality. It is not interested in the individuals who learn or teach some piece of knowledge. The ATD addresses the issues of the social resources and constraints that establish a framework for an individual's didactic activity, as learner, teacher, internship advisor etc. If we consider one knowledge field, this choice is a restriction within education researches related to this very field. Yet, the anthropological approach of the didactic opens up a very large domain to investigate, far beyond the education area in the restricted meaning as usually considered, in accordance with the following reasons. Firstly, transmission of socially produced knowledge characterises human nature, the didactic is everywhere dense in any human society. Secondly the knowledge production is always socially situated; hence, the didactic research must address socio-epistemological issues related to the social footprint on the knowledge at stake in the educative processes. And lastly, as a generalisation of the foregoing, the ATD assumes that what is going on in the classroom between teacher and students is mainly determined by conditions and constraints deriving from various social organisations, from local ones up to society and civilisation. From these principles it derives that some of the ATD key concepts are neither specific to mathematics, nor to education. They especially provide powerful tools to an epistemo-anthropology, in other words, to any epistemological research interested in the issues of production and movement of knowledge within and between various social contexts. I discuss some of them in the following sections: first, the institution and the subject, next the praxeology, and lastly the transposition processes.

The Institutions and the subjects

I define an institution *I* as a stable social organisation that offers a framework in which some different groups of people carry out different groups of activities. These activities are subjected to a set of constraints, - rules, norms, rituals - which specifies the institutional expectations towards the individuals intending to act within the institution *I*. An individual has to satisfy these expectations, at least, to a certain extent depending on the institution. Hence, using the ATD vocabulary, the individual (s/he) is subjected to the institution's expectations and becomes an institutional subject (from Latin *sub-jectus*: literally thrown under). This meaning is very different from the Kantian one, which considers the subject as a responsible agent, usual in several theoretical frameworks that give priority to psychology and individuals, even when, like the cultural historical activity theory (CHAT, Leontiev 1978), they take socio-cultural dimensions into account.

Institutions tend to constrain their subjects but conversely they provide the resources (material and cultural) necessary for activities to take place. Epistemologically, the existence of institutions is an absolute precondition for the development of human culture. They foster collective processes for facing and solving human problems; and they favour the dissemination of inventions/innovations, even when they do not create specific schools for that.

Examples of institutions

Institutions, referring to the production of mathematical knowledge (without claiming to be exhaustive), could be the international mathematics research organisation and its sub-institutions according to the domain or the country; each laboratory, each mathematics journal, each congress.... Institutions referring to mathematics, science and technology education: ICMI and ICME, SAARMSTE, PISA (Program for International Students Assessment), each Department of Education, each National Curriculum, each educational institution, each classroom of a mathematics, science or technology teacher some weeks after school started. With these examples, we can see that various-sized institutions may be concerned with a topic, the more local ones being embedded in and partly determined by some of the more global ones.

In order to diversify the context of these examples, I will draw on Wolf-Michael Roth's research (Roth 2014) about a training program in a Canadian college where students train to become licensed electricians. Let us note that Roth's framework refers to CHAT and to Radford's notions of objectification and subjectification, both approaches focusing on the individual development within socio-cultural contexts. This theoretical choice is common among vocational mathematics education researches (see Educational Studies in Mathematics 86). From the ATD point of view, what are the institutions considered in Roth's analysis of this vocational course? Within the college, mathematics units, science units and shop units; outside, workshops where the students work as apprentice and at a higher level, the Canadian control system of electrical installations, based on the Electrical Code of Canada framing the professional practices. During the program, a student has to become a subject of each of these institutions, that is, has to submit to their specific expectations regarding what students must do and how. Within CHAT framework, Roth addresses the issue of how one individual tackles the experience of crossing boundaries between different socio-cultural contexts. An ATD research would have focused on these contexts considered as some institutions and on the system of constraints and affordances that each one creates for the activities of each category of its subjects (e.g. students and teacher or supervisor). Such a deep exploration of institutions is considered as necessary to address the issue of inter-institutional transitions, in ATD words, boundary crossings in CHAT ones. Without developing further, I suggest that ATD and CHAT are complementary theories, based on a rather similar conception of human, the first one focusing on sociological objects, the second one on psychological ones.

Praxeology

Coherently with the foregoing, ATD is interested in the processes and products of what we may consider as the institutional cognition, that is to say, in how institutions develop their socially acknowledged capitals of practices and knowledge. In other words, the aforementioned epistemo-anthropology is a subfield of the ATD research program. The key notion of praxeology is the basic unit proposed by this theory to analyse the institutional cognition.

> What exactly is *a* praxeology? [...] one can analyse any human doing into two main, interrelated components: *praxis*, i.e. the practical part, on the one hand, and *logos*, on the other hand. [logos refers to human thinking, rational discourse]. How are *P* [Praxis] and *L* [Logos] interrelated within the praxeology [*P*/*L*], and how do they affect one another? The answer draws on one of the fundamental principle of ATD [...] according to which no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such as an explanation or justification may be cast. *Praxis* thus entails *logos* which in turn backs up *praxis*. For *praxis* needs support – just because, in the long run, no human doing goes unquestioned.[...] Following the French anthropologist Marcel Mauss (1872-1950), I will say that a praxeology is a "social idiosyncrasy", that is, an organised way of doing and thinking contrived in a given society. (Chevallard 2006, p.23)

The practical block (or know-how) associates a type of tasks T and a technique τ. τ is a "way of doing" which is endowed with certain efficiency for a certain subfield within the set of T tasks.

> The *logos* block contains two levels of description and justification of the *praxis*. The first level is called a "technology", using the etymological sense of "discourse" (*logos*) of the technique (*technè*). The second level is simply called the "theory" and its main function is to provide a basis and support of the technological discourse. (Bosch & Gascón 2014, p. 68)

This four components model is usually represented as follows: $[T, \tau, \theta, \Theta]$, θ being the technology of τ, Θ the theory supporting θ.

To exemplify the praxeological model, I will consider a mathematical type of tasks we can meet in strictly mathematical contexts as well as in engineering sciences. This type of tasks is the following one: *Breaking up a rational function into partial fractions*. Let us note that, except when I give an example of effective calculation, everything below belongs to the logos block, mostly to the technology.

Mathematics praxeologies to break up a rational fraction into partial fractions

Description of the technique in the general case: (1) Make the denominator monic (leading coefficient 1), and use the Euclidean algorithm to reduce to a problem where the degree of the numerator r is less than the degree of the denominator d. (2) Factorize the denominator as a product of powers of distinct monic irreducible polynomials. (3) Write the fraction as a sum of partial fractions of the form R/Q^k where Q is one of the irreducible factors, k is at most equal to the multiplicity of Q in d and the degree of R is less than the degree of Q. (4) At first every R is unknown. The coefficients need to be determined. One way of doing this is to take a common denominator, multiply out, equate coefficients and solve the resultant system of linear equations.

The fact that every rational function may be written as such a sum of partial fractions, uniquely determined, is a theorem. Even for rational functions on the fields of real or complex numbers, the proof needs many results as the fundamental theorem of algebra, division in the ring of polynomials, Bezout theorem, that is, the polynomials arithmetic theory. Moreover, mathematical induction is necessary. This is a great part of *the praxeology theory*. But this non-constructive theorem does not provide a technique to determine the polynomials R. However, step 4 will do since we know that the system has a unique solution.

Example: We want to express $\frac{3x+1}{(x-1)^2(x+2)}$ as the sum of its partial fractions $\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$.

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{x+2} \Leftrightarrow 3x+1 = A(x-1)(x+2)+B(x+2)+C(x-1)^2 \Leftrightarrow (*)$$

$$(*) \Leftrightarrow 3x+1 = (A+C)x^2+(A+B-2C)x-2A+2B+C \Leftrightarrow \begin{cases} A+C=0 \\ A+B-2C=3 \\ -2A+2B+C=1 \end{cases} \Leftrightarrow \begin{cases} A=5/9 \\ B=4/3 \\ C=-5/9 \end{cases}$$

Two theorems validate this technique, that is, prove without further checking that $\frac{3x+1}{(x-1)^2(x+2)} = \frac{5}{9(x-1)}+\frac{4}{3(x-1)^2} - \frac{5}{9(x+2)}$. θ_1: two rational functions with the same denominator are equal if and only if their numerators are equal. θ_2: two polynomials are equal if and only if they have same degree and same coefficients.

Appraisal of the technique: This technique τ_1 is obviously tedious in some cases with many coefficients to find, if you do not have any software to solve the system. Indeed, mathematicians are aware of this heaviness of the "equating coefficient-wise technique" and look for other techniques. For example, in the following excerpt of a calculus on-line textbook[1], we find an appraisal of τ_1 when the denominator is a product of *n* linear terms together followed by the presentation of another technique τ_2:

> For $n \leq 2$, this procedure [τ1] is possible and quick. However, for $n \geq 3$ the procedure becomes messy because we first need to do a lot of tedious term multiplication to find coefficients, and then we need to solve a tedious system of linear equations. [...] The approach we will now use is based on the key idea that *if two polynomials are equal, their values at every number are equal.* (ibid, p.7)

Using τ_2 on the example gives the following: $3x + 1 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$; so plugging in the values *x*= 1 and *x*= -2, we get every term but one equal to 0 (this point is important to understand *as a motivation of the values choice*), so that we have 4=3*B* and -5= 9*C*. To get *A*, there is no special value that would eliminate *B* and *C*, we choose *x*=0 and get 1= -2A+2B+C. Hence A = 5/9.

According to the on-line textbook, this technique is clearly "preferable (from a speed perspective) when $n \geq 3$" in the case of linear factors. In a French on-line text-book[2], we find another case of *appraisal of a technique* τ_3, using a combination of variable substitution and polynomial long division: "

> This method is especially interesting when there is a pole of high order (≥ 4) and few other factors *B*(*x*) [the denominator] or when the pole is around 0 from the beginning. (p.29) (the translation is mine)

A missing motivation of one step of the technique: Now, as a transition to the corresponding praxeology in automatics, we can ask a question: why is it important to make the denominator monic? In the aforementioned Chicago text-book we find no explicit motivation of this choice. The author only argues that: "Every non-zero polynomial can be expressed as a constant multiple of a monic polynomial. Since constants can be pulled out of integration and differentiation problems" (p. 2), there is no loss of generality by restricting to monic polynomials. Indeed, the fact that linear monic polynomials have 1 as a derivative and hence that the antiderivative of rational functions $1/(x-a)^k$ is easy to calculate is one motive of the restriction to monic factors. It remains implicit in both text-books, although well known among mathematicians.

The automatics praxeology to break up a rational fraction into partial fractions

The following example is from the PhD study carried out by A. Romo Vázquez (2009) and supervised by M. Artigue and C. Castela. This research addressed the issue of the mathematical preparation of engineers. The central part of the research was twofold, consisting of an analysis of engineering projects on the one hand (practical activity carried out as part of the training of engineers at one French University Vocational Institute) and of Engineering Sciences and Mathematics courses on the other hand. Due to the central role played by the Laplace transform[3] in one of the projects, the courses

[1] Partial fractions: an integrationist perspective. Math 153 Section 55. Vipul Naik. University of Chicago.

[2] Analyse 2, M. Hasler, Université d'Antille-Guyane

[3] The Laplace transform is a widely used integral transform in mathematics with many applications in physics and engineering. It is a linear operator of a function *f*(*t*) with a real argument *t* ($t \geq 0$) that transforms *f*(*t*) to a function *F*(*p*) with complex argument *p*. The most important result is that it transforms *f'*(*t*) to *p*.*F*(*p*).

study focused on this notion and compared the way it was taught in textbooks of different institutions (2nd and 3rd university years), one written by a mathematics lecturer, the other two by automatics lecturers, one being an on-line course for higher technicians[1]. The example we consider now is from the latter (see Castela & Romo Vázquez 2011for a detailed study).

Some explanations about the automatics issues are necessary. The problem at stake is automatic regulation of systems (e.g., heating installation with controlled temperature): if a quantity must be kept constant, an electronic gauge measures its value; when some variation is recorded, an appropriate regulation process is triggered to come back to the desired value. The less time is necessary to get the quantity back to this value, the more efficient is the control system. The temporal evolutions of the different systems involved are described by differential equations (linear ones in the considered textbook), turned to algebraic ones by the Laplace transform and easily solved, with a rational fraction $F(p)$ as a solution. At last you have to get back to the temporal function, that is, to inverse the Laplace transform. The on-line textbook recommends using a table of Laplace transforms, especially adapted to automatics requirements. The type of tasks *Breaking up a rational fraction into partial fractions* appears when complicated rational fraction $F(p)$ are involved. In what follows, I give an idea of the praxeology (technique and technology) proposed by the textbook.

Description of the technique: In fact, the author assumes that the mathematical techniques are familiar to the students. The only point that he specifies is that $F(p)$ denominator must be written under the following canonical form $k(1+\tau_1 p)\,(1+\tau_2 p)\,(1+\tau_3 p)\ldots$ with decreasing values of the τ_i. For instance, $3p+2$ is transformed into $2(1+1.5p)$ and not into $3(p+2/3)$. This is a significant change to the original mathematical technique.

Motivation (raison d'être) of this special factorisation: If $F(p) = \frac{1}{1+1.5p}$, the corresponding original function is $f(t) = K(1-e^{-t/1,5})$. 1,5 is called the time constant of this function. The system reactivity, hence its quality, is directly dependent on the time constants τ_i, more precisely on the higher value; therefore, this value must appear clearly in the calculation.

Explanation of the relation between time constant and reactivity: if $f(t)$ represents the controlled quantity and K its desired constant value, it is known that after 7τ, here 7×1.5 seconds, the exponential will be equal to 0, that is, considered as negligible in Automatics. Hence the transitional regime lasts $7\times1,5$ seconds.

Validation of this claim: $e^{-t/\tau}<0.01$, hence (the textbook does not use $\Leftrightarrow$) $e^{t/\tau}>100$, $t > \tau \ln(100) \approx 7\,\tau$

What needs does the technology of a technique intend to satisfy?

As mentioned previously, (Romo Vázquez 2009) analyses the Laplace transform chapter in one mathematics and two automatics textbooks from tertiary vocational courses for engineers and higher technicians. The first one, in a classical mathematics style, is focused on the comprehensive accurate presentation of concepts, theorems and proofs. The Laplace transform technique to solve non-linear differential equations is alluded to, without any examples related to engineering sciences. As shown in the above example, the automatics textbooks are very different. They give a lower priority to mathematical proofs and instead, they develop another kind of knowledge about techniques, strongly correlated with the vocational context. Actually there are many things to know about Laplace transform and the derived techniques but all these technological elements satisfy diverse needs. Drawing on the aforementioned textbooks, Romo Vázquez and I have differentiated six of them: *describing* the technique, *validating* it i.e. proving that this technique produces what is expected from it, *explaining* the reasons why this technique is efficient (knowing about causes), *motivating* the different gestures of the technique (knowing about objectives), *making* it *easier* to use the technique and *appraising* it (with regard to the field of efficiency, to the using comfort, relatively to other

[1] http://public.iutenligne.net/automatique-et-automatismes-industriels/verbeken

available techniques). Such technological elements are present in both previous examples of mathematics and automatics praxeologies. This list should not be taken as exhaustive. For instance, drawing on some other researches (e.g. Covián Chávez 2013, addressing land surveyor training), I currently consider one more need: *controlling* the technique implementation. Even if a mathematical proof justifies that the technique, when it works and is adequately implemented, produces the expected solution, an individual may make errors when using it. The institution where this individual works needs that he or some supervisor can verify the process.

This analysis grid of the praxeology technological component has been developed within vocational contexts. It is in full convergence with results obtained by most of researches addressing the issue of occupational mathematics (as recent examples, see *Educational Studies in Mathematics* 86). But it is not only relevant for this kind of context. I use it to describe knowledge involved in mathematical problem solving, for mathematicians as well as students. From the ATD point of view, problems, even research ones, contain elements of genericity, that is, may be related in some way to others already encountered and solved problems. This means that mathematics researchers, even if they need much inventiveness, draw also on previously developed praxeologies. In research journals as well as in scholarly books, the praxis part of praxeologies is often peripheral and the technological one focuses on the validating need, satisfied by mathematical proofs; sometimes causes are explained, sometimes they are not (think of analytical proofs for geometrical theorems). According to the contemporary mathematics epistemology, these technologies directly derive from theories. Yet I claim that other knowledge about mathematical techniques are necessary to use them efficiently and that this knowledge is produced and disseminated in the local mathematics research institutions, such as research teams, laboratories, seminars. This knowledge appears in some tertiary textbooks (see the aforementioned example of mathematics praxeology), not all. This part of the technologies satisfies practical needs within problem solving and generally derives from experiencing the technique implementation, that is to say, not from a mathematical theory. Hence this is an empirical kind of knowledge.

Most of this practical part of mathematics praxeologies is not taught. Yet students need such knowledge, in France at least from high school (for detailed argumentation, see Castela 2004, 2009). So the responsibility for building this practical knowledge on their exercise and problem-solving experiences rests on the students. But some sociological and didactic French researches have given solid evidence that this requirement has a strong differentiating effect, students from disadvantaged backgrounds being largely unaware of the learning objective of exercises and problems. Hence, it is up to the mathematics teachers to introduce, in the classroom, the idea of practical mathematics knowledge and to develop their students' skill to build by themselves such knowledge. This means that, at least within teacher training institutions, the pre-service secondary school teachers should work on examples of mathematics praxeologies with practical as well as theoretical technology as fully institutionally acknowledged mathematics. That is why, working myself in a teacher training institution, I give specific importance to the analysis grid of the praxeological technology developed with Romo Vázquez.

When a praxeology moves from a research institution to another one: The transpositive effects.

We are now going to work on the original ATD symbolic representation $[T, \tau, \theta, \Theta]$ for a praxeology, in order to incorporate the institutional cognitive process in the model and to make it visible in the diagram. A praxeology is an institutional idiosyncrasy, that is, an organised way of doing and thinking acknowledged by this institution as legitimate. The anthropology of the institutional cognition I have previously called epistemo-anthropology is not only interested in praxeologies developed by a given institution I but also by the production and legitimating processes in this institution. Institution and processes are therefore added in the following diagram: $[T, \tau, \theta, \Theta]\leftarrow I$.

(Romo Vázquez 2009) considers four institutions: two scientific research institutions, in mathematics $[I_r(M)]$ and in automatic control $[I_r(AC)]$ and the related teaching institutions in different vocational courses $[E(M), E(AC)]$. The social responsibility of the first two is to produce new praxeologies in their own field, with a particular emphasis on systematic validation according to each field's specific scientific epistemology. In the small case of the *Breaking up into partial fractions* praxeology ("Bup" praxeology in the following) we have considered here and more generally for the differential equations solving praxeologies related to Laplace transform, $I_r(M)$ nowadays acts as a praxeology producer, even if Heaviside's operational calculus has been a precursor, acknowledged by the electromagnetism research institution at the end of the 19th century, if not by the mathematics one. $I_r(AC)$ uses $I_r(M)$*'s* praxeologies but the movement from the mathematics institution to the automatic control one changes the praxeologies, this is the phenomenon of transposition that Yves Chevallard has introduced at first in the case of didactic transposition (1985, 1989). Then the movement goes on from $I_r(AC)$ to $E(AC)$, with new possible transpositive effects, due to the fact that students are beginners and to the working context perspective. We have seen that in the "Bup" praxeology, the type of tasks and the technique have changed, due to the requirement about the time constants. If we consider the technology, we may assume that there is no change regarding the mathematical theoretical validation of the technique itself. But we find new elements coming from the fact that the mathematical task has been embedded in an automatic control type of tasks.

The diagram in Figure 1 gives a general representation of the possible transpositive effects when a praxeology $[T, \tau, \theta^r, \Theta]$ produced by a research institution I_r moves to be used in an institution I_u. I_u may be an educational institution working with the praxeology to teach it. What does this diagram say? At first, the asterisk expresses that every component of the original praxeology may evolve. This transformation is an object of institutional transactions completed in a specific institution I^*_r, created and controlled by I_r and I_u. I^*_r is more or less vanishing, the transactions are more or less difficult and controversial, depending on several factors: the extent of the transformations, the distance between the two institutional epistemologies (e.g., I_r *is* mathematics and I_u is an experimental science), the importance for I_u that I_r validates the new technique (e.g., I_u is a profession as nursing with high security requirements), the importance for I_r that the transposed praxeology be not too far from the original one (it is frequent that when I_u is an education institution, mathematicians have a critical look on what is taught). At last, this diagram says that a practical technology θ^p is developed and acknowledged by I_u on specific empirical bases.

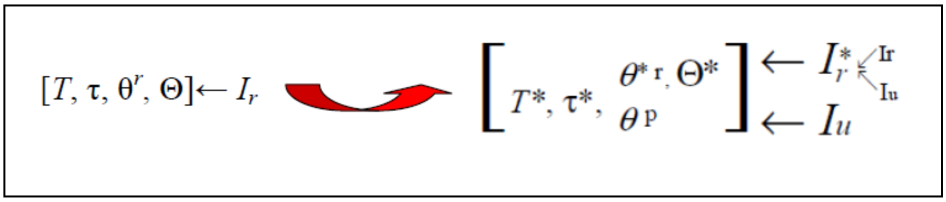

Figure 1. From I_r to I_u, the transpositive effects model

(Romo Vázquez 2009) shows that the transactions between institutions about the logos block $[\theta^r, \Theta]$ of the Laplace transform praxeologies may result in different forms of transposition according to the considered vocational training institution. The crucial issue that underpins the transpositive choices is whether or not engineers or higher technicians should know the most possible about the mathematical validation and explanation of the techniques that they will be supposed to use. In other words, does the training institution try to reduce the presence of black boxes or not? If so, these praxeologies are taught by a mathematics lecturer, if not they may be directly included in an engineering science course. Romo Vázquez meets the first case in a very high level engineering school; as aforementioned, $E(M)$ textbook is focused on the mathematical theory, exhaustively presenting proofs. In this case, transposition totally remains under the mathematics' epistemological control; changes in $[\theta^r, \Theta]$ are limited. The two $E(AC)$ textbooks provide examples of the second case; they come from different vocational training institutions with emphasis on the occupational competencies. The analysis (Castela & Romo Vázquez 2011) reveals that the $[\theta^r, \Theta]$ block is partially vanishing: the mathematical proofs of some theorems (mainly the easiest ones, with low theoretical needs) are presented; for other theorems, the proof is omitted but the textbook refers to its existence as a mathematical guarantee; at last, some claims should need validation but their problematic dimension is completely hidden. The counterpart to these choices relative to the mathematical component of the technologies is that in $E(M)$ textbook, the practical component is totally missing, while it is especially developed in $E(AC)$ textbooks, with emphasis on the technological elements satisfying motivation and appraisal needs.

An epistemo-anthropological approach of custom dressmaking in Argentina

Up till now, we have considered some mathematical praxeologies, 'mathematical' being understood as produced and acknowledged by mathematics research viewed as an institution and we have addressed the issue of how they change when moving to other institutions in order to be used. In what follows, I intend to show that the praxeological and institutional approach is also relevant for occupational contexts, even with few school mathematics involved. I draw on a research realised in Argentina by C. Elguero (Elguero 2009, Castela & Elguero 2013) in the custom dressmaking context.

We will consider two institutions: One is a system developed by an experienced acknowledged tailor H. Zampar, whose pattern drafting techniques are disseminated all over Argentina through numerous books, a website and a web of training schools. The second one is Gladys's workshop; Gladys being a confirmed master 'craftswoman' is charged with the on-the-job training of several apprentices. Roughly, what are the main problems that dressmakers tackle? Firstly, making a piece of clothing, that is, a spatial complicated object, from a flat raw material; hence they have to draft the flat patterns from which they will draw the spatial form. Secondly, tailoring the pattern to the customer's morphology, knowing that for some practical reasons, some of the necessary measurements are not taken on the customer's body; they need therefore to infer the missing measurements from those they have. Zampar's system provides techniques to solve these problems for any piece of clothing, for man, woman and child. Gladys uses some of them but she also chooses techniques from several other well-known systems, thus building her own mixed system that she teaches to her apprentices.

Examples of pattern drafting praxeologies

The following example is drawn from a Zampar's book (2003) for children dressmaking. The type of tasks is *Drafting the back pattern for a shirt*. We consider only a few steps of the technique.

The first step is to draw a rectangle with following measurements: Height= Back length+2cms, Width= ¼(bust circumference+4cms). The coefficient 1/4 is partly explained in an online course written by another expert: human body is modelled by a cylinder, its right and left parts are considered symmetrical. This should give a coefficient ½ but in Zampar's book, for a child shirt, back and front too are assumed symmetrical. Zampar insists on the *raisons d'être* of the additional 4 and 2 cms: the first ones provide ease for breathing and the second ones anticipate the future child's growth.

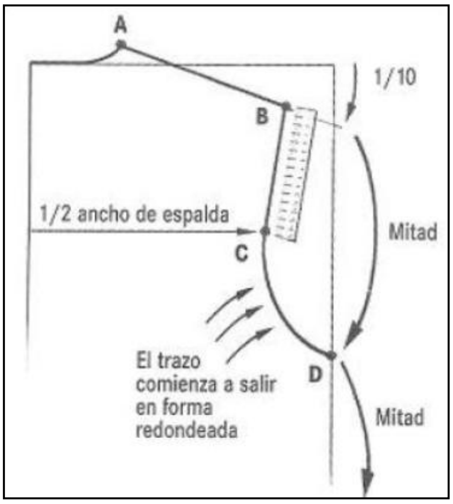

Figure 2. Back shirt pattern for child

The second step is to divide the rectangle in three horizontal unequal parts, the upper one with a height of one tenth of the basic rectangle height. Zampar does not explain why he takes this coefficient, nor does he for virtually all relations he uses in the different praxeologies. Yet some scarce ones derive from geometrical reasons. For instance, the point A in Figure 2 is obtained reporting one sixth of the neck circumference. Within Elguero's investigation, no one in the workshops could explain this formula. However this comes from an approximation of the relation between the circumference of a circle and its radius, with $\pi \approx 3$. In both institutions we consider, this mathematically validated formula is not differentiated from the other ones.

The second type of tasks follows the first one; it is *Drafting the sleeve pattern for a shirt*, in the same context of child dressmaking. We focus on one step: on the basic rectangle, drawing an upper horizontal rectangle (A in Figure 3-left) in which will be further drafted the sleeve head (Figure 3-right). Zampar's technique for this rectangle height (*medida 3*) is to take 2/3.BD where BD is measured on the back pattern (Figure 2), that is, not on the customer's body. Gladys uses this technique. When asked to explain the formula, Gladys and her assistant answer that they have never addressed such issue. Gladys adds: "Es una fórmula para que la manga te salga con la medida justa[1]". (Elguero 2009, p. 91). She does not know about the causes of this relation. Yet she knows about the objectives as we can see in the following:

> Gladys: Yo antes enseñaba la manga de otra manera [ese refers to another system]. Pero por mi escuela yo tengo que actualizar con todo lo nuevo que sale en moldería y buscar lo más práctico para las alumnas. La manga que yo enseño, no requiere que vos tomes la medida del contorno de sisa en el cuerpo, sino que medís directamente en el molde de espalda, entonces es más exacta, porque es una línea lo que medís…Las instrucciones son mucho más sencillas que las que enseñan en otros sistemas[2]. (Elguero 2009, p. 90)

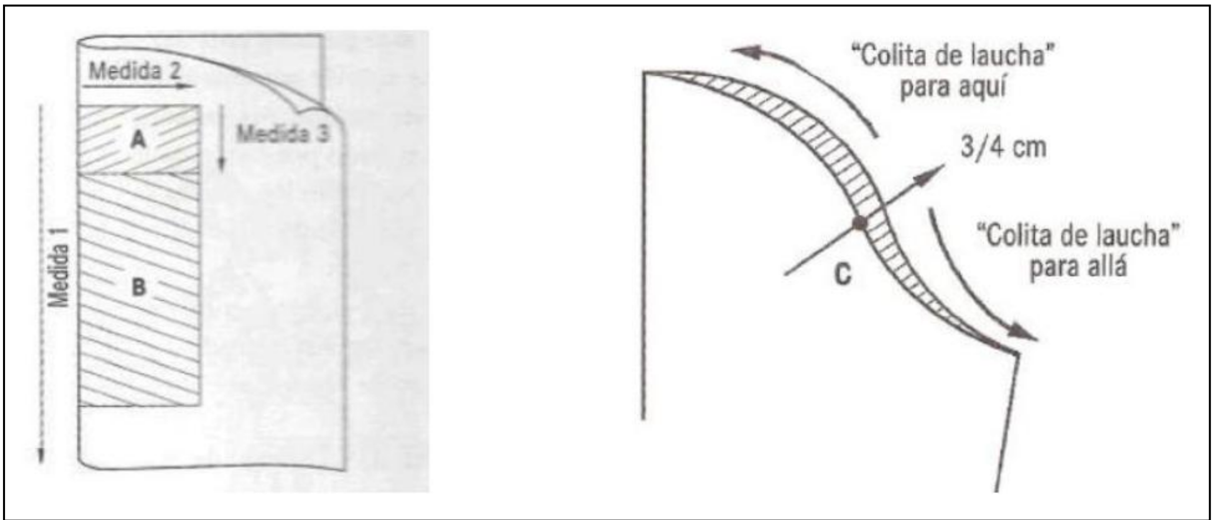

Figure 3. Drafting the sleeve head on the basic rectangle

We see that Gladys's training responsibility leads her to appraise the different techniques for the same type of tasks on criteria of simplicity and accuracy, that is to say, of ergonomics, efficiency and perhaps ease of teaching/learning. This appraisal motivates a change of technique. It should be noted that she too proposes an explanation of the improved accuracy obtained by this technique: measuring a segment on the pattern is more accurate than measuring a circumference on the customer's body. So

[1] It is a formula for the sleeve leaves you with just the right measure. (our translation)

[2] I previously taught sleeve [she refers to another system] otherwise. But at my school I have to update with anything new that comes in dressmaking and seek the most practical for students. The sleeve I teach, does not require that you take the armhole circumference in the body, but you measure directly into the back pattern then it is more accurate because it is a line which you measure ... The instructions are much easier teaching than in other systems. (our translation)

we note that Gladys has chosen to adopt Zampar’s technique after using another one, she uses it and teaches it as it stands, and develops her own technology that we assume she passes on to her apprentices.

Movements of praxeologies between professional institutions

It is time now to address the issue of whether the model of praxeological movements and transposition effects presented in Figure 1 is relevant in the case of the aforementioned pattern drafting praxeologies.

At first we focus on Zampar’s system. Are Zampar’s praxeologies transposed forms of praxeologies produced by research institutions? What would be the research field of such institutions? Possible answers would be ergonomics, anthropometry, applied anthropology. Castela & Elguero (2013) give some specific examples of laboratories collaborating with dressmaking industries. But there is not the least sign of a contact between such a laboratory and H. Zampar for the pattern drafting techniques. With respect to mathematics, the scarce types of tasks involved are to draft a cylinder pattern and to calculate the radius of a circle from its circumference’. We should consider therefore Zampar’s system as a professional dressmaking institution (I_p) with praxeology producing activities.

How do Zampar and his collaborators produce the techniques and relations they use? This issue has remained unaddressed, Elguero’s research being not interested in this level of dressmaking institutions that produce and disseminate pattern drafting praxeologies. If we consider the model $[T, \tau, \theta, \Theta]\leftarrow I_p$, we see that, although Zampar’s books give access to the praxis and technological components of the praxeologies, the production and legitimating processes represented by the arrow remain fully unexplored as well as the institutional theory supporting these processes. I should here emphasise that in such context, the ATD concept of theory does not refer to the organised set of knowledge considered by the scientific meaning of the word. Θ is the institutional knowledge that supports the technology θ, or rather, the technologies of several praxeologies. For instance, the theory of Zampar’s praxeologies should express the technology of the techniques which are used to produce and legitimate the pattern drafting techniques and their related technologies. If this pattern drafting system is totally produced and empirically validated in the course of Zampar’s and collaborators’ dressmaking work, how do they manage to do so? How do they explain that it is possible? If not, that is if they sometimes have a specific activity, away from any relation with customers and from any economical preoccupations, focused on the invention and systematic validation of new praxeologies, what can they tell us about this kind of dressmaking research? I propose to represent these two configurations as follows, with the first case on the left, the second on the right, I_p^r being a research institution, ‘offshoot’ from I_p:

$$\left[T, \tau, \theta^p, \Theta\right] \leftarrow I_p \qquad \left[T, \tau, \begin{matrix}\theta^r \\ \theta^p\end{matrix}, \Theta\right] \begin{matrix}\leftarrow I_p^r \\ \leftarrow I_p\end{matrix}$$

Figure 4. Professional institutions as praxeology producers

Let us now consider Gladys’s workshop, another professional institution i_p at a more local level. i_p demonstrates some independence from Zampar’s system, choosing techniques from several well known books. In the examples, the chosen techniques are used as they stand. It seems that the issue of their validation is not raised again by Gladys. Although (Elguero 2009) does not provide any explicit evidence of such a hypothesis, I will assume that Gladys believes in Zampar’s guarantee. In other words, i_p’s theory should express some principle like: when a technique τ comes from an institution I_p with acknowledged professional expertise, we take the τ validity for granted. This may be compared with the ‘black-boxing’ phenomenon that we have encountered in the automatic control textbook, where mathematical proofs are sometimes only alluded to. To this, Gladys could possibly add that throughout her long workshop experience with Zampar’s techniques, she has never faced any problem. This means that the theoretical component changes from I_p to i_p; likewise, as some technological elements of the original praxeology may vanish in the workshop when others are added, in relation with the local working conditions, especially with the training educational context. The diagram in Figure 5 represents a general model for praxeological movements between professional institutions with different status within the professional field, assuming that every component may possibly change:

$$\left[T, \tau, \theta^p, \Theta\right] \leftarrow I_p \longrightarrow \left[T^*, \tau^*, \theta^{*p}, \Theta^*\right] \leftarrow i_p$$

Figure 5. Transpositive effects from one professional institution to another

This model aims at generality; however, it is influenced by the dressmaking example. It should at least consider, as does Figure 1, the possibility that I_p be concerned with what happens to its praxeological production when disseminated in the workshops i_p. In other words, this is a proposal that should be adapted to other contexts of investigation.

Conclusion: Towards an anthropology of mathematics

If we look back on the path taken in this text, we should see that the first group of examples considers academic mathematics as a reference. The addressed issue is the following: What does a mathematical praxeology become when used by other scientific or occupational domains? In other words, which mathematical praxeologies are embedded in the praxeologies of these domains? This is a classical approach in mathematics education. What is the meaning of the word ‘mathematics’ in that case? Mathematics is a body of knowledge, or preferably of praxeologies, produced through specific research activities; both, knowledge and activities, are acknowledged as mathematics by international mathematics institutions. Mathematics is at the same time a body of knowledge, a field of activities and an institution. This looks very much like a closed world. When someone of this world, that is, a mathematician, begins to investigate on mathematics education, especially but not only in vocational education, he needs tools to distance himself with the ‘*alma mater’*. Since the beginning, this has been Chevallard’s objective with the anthropological theory of the didactic. I contend that the work I have presented here around the notion of praxeology provides a powerful tool to investigate the mathematics dimension of human social activities in any context, without referring to academic mathematics.

Indeed, in the second group of examples, we have some evidence that the dressmakers’ pattern drafting techniques only scarcely draw on mathematical ones. Of course, we have not considered here the calculations with rationale numbers involved in the formulas: this was the central issue addressed by Elguero’s investigation (2009); Castela & Elguero (2013) analyse in detail the related dressmakers ‘praxeologies in relation to the mathematical usual ones. But, in the present text, I have intentionally focused on the modelling part of the techniques. These techniques provide solution to a very broad class of problems related to the plane development of spatial surfaces. As a mathematics teacher with a rather high level (up to PhD in mathematics), I never met any consistent mathematical theory solving this problem for the complex forms involved in dressmaking. Perhaps such theory does exist, but it is obvious that dressmakers, independently from the scholarly mathematics institutions, have produced knowledge about these problems and found solutions. Will we consider this knowledge and these solutions as mathematics? The answer is clearly no, if we take academic mathematics as a reference. Yet, producing a plane development of a spatial surface has certainly some mathematical dimension, since such problems are considered for polyhedrons. Hence, we should consider at least dressmakers as providers of solutions to mathematical problems; why not as workshop mathematicians? This is the kind of issues that anthropology of the mathematics should address to deconstruct the whole concept of mathematics. The contemporary anthropology no longer considers human evolution as a long path culminating in the occidental human being; an anthropological approach to the mathematics should also get rid of any reverence vis-a-vis academic mathematics considered as one of the highest achievement of mankind. It is a condition for addressing afresh the issue of what is mathematics, a condition that is not always satisfied by the researchers referring to ethnomathematics.

This anthropology of the mathematics should investigate social practices without too narrow restrictions on what is an interesting object. That is why I consider the praxeological model as previously presented as an interesting tool. It highlights dimensions of the institutional cognition that would be neglected otherwise, especially when the reference to acknowledged mathematics is too strong. Such a research program is directed towards epistemological and anthropological goals, intending to unearth the diversity of human mathematics praxeologies. And, as a conclusion, I would like to emphasise the fact that I consider as not at all obvious the question of whether any such "ethno-praxeologies" should be incorporated into school curricula (about this discussion, see Pais 2011).

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