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Abstract

This paper presents a methodology which considers an interpolation between Linear Parameter-Varying (LPV) controllers that were designed separately for different objectives. The quadratic stability of the closed-loop system is proved under arbitrary interpolation in terms of a set of Linear Matrix Inequalities (LMIs). The interpolation strategy is based on Youla-Kucera (YK) parameterization. The proposed method can help multi-variable and multi-objective systems, to achieve high performances at different critical situations regardless of the interpolation rate.

Key words: Linear parameter-varying system, Youla parameterization, Gain-scheduling, Linear matrix inequality

1 Introduction And Motivation

The interpolation of several controllers to meet multi-objectives criteria is an important topic in theory and practice. A method has been investigated in the literature to achieve a multi-objective switching Linear Time-Invariant (LTI) controller based on Youla-Kucera (YK) parameterization [5]. It is structured by mapping a set of linear stabilizing controllers onto a Q-based controller. The interest is to parameterize a set of linear stabilizing controllers $K(Q)$ for an LTI system, where each one is parameterized by its corresponding LTI-YK parameter Q [14]. The LTI-YK parameter Q is designed using the doubly coprime factorisation [15]. Using such parameterization for controller interpolation: 1) It allows stable gain-scheduling between unstable controllers [13]; 2) Interpolated controllers can be designed and tuned separately using different techniques ($\mathcal{H}_\infty$, LQR, PID) [14]; 3) It facilitates adding new parts to an existing system online as Plug&Play control theory [12]; and 4) The closed-loop stability is guaranteed under arbitrary interpolating signals between different stabilizing controllers [5] without requiring a single Lyapunov function.

The YK parameterization has been successfully applied to several domains such as: 1) Noise/vibration control [7]; 2) Interpolation between robust and high performance controllers [4]; and 3) Control systems subjected to convex constraints [3]. A significant literature review of YK work including its recent applications can be found in [10]. In addition, YK parameterisation is used in a rising topic which is autonomous vehicles [11],[9]. [9] uses an LTI-YK control structure to interpolate between fast and slow LTI controllers to optimize a lane change of an autonomous vehicle. However, the study is done on an LTI lateral system with a fixed longitudinal speed of the vehicle.

On the other hand, the control of multi-variable or Linear Parameter-Varying (LPV) systems is studied due to its interest to handle system complexity (as nonlinearities) as well as to bring some adaptive control schemes thanks to the self-scheduling of the controller, see for instance [6] and references therein. Nonetheless, it is today admitted that designing a single LPV controller for various objectives may be conservative [8]. However, while many YK interpolation studies have been concerned with LTI systems, few works have integrated YK concepts to LPV systems. [1] introduces an LPV model which varies with respect to time-varying parameters. It proposes a YK-based gain-scheduling between LTI controllers designed separately at different operating points. Closed-loop quadratic stability and performance are guaranteed at intermediate interpolation points of

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a convex domain. The parameterized controller is then compared to an LPV controller designed using the polytopic approach. A different concept is proposed in [16] to design an LPV controller based on Youla parameterisation. It extends the coprime factorization of LTI systems to LPV systems and it is proved that any quadratically stabilizing LPV controller can be parameterized, based on Youla concept, providing the closed-loop quadratic stability. Notice that the concept is proved only for observer-based LPV state-feedback control. In [2], a fixed pole assignment application of LPV system is introduced using an LPV Q-based method. Its concept aims to preserve the closed-loop poles at the same location by interpolating between different controllers regardless the variations in the LPV system. Therefore, a unique performance is obtained along the variation of the parameters while keeping stability.

Nevertheless, none of the mentioned works did concern multi-objective controllers, i.e. achieving different performances in the full parameter-varying region. The objective and contribution of this paper is to develop an LPV-YK dynamic output feedback control structure which can interpolate between several LPV controllers, that have been designed to achieve different objectives. Such interpolation scheme aims to improve the closed-loop performance which could achieve both multi-variable and multi-objective requirements.

This paper develops the work in [16] by proposing a dynamic output-feedback LPV controller (to be parameterized), instead of observer-based state feedback LPV controller, which extends the stability conditions. Moreover, the proposed method extends the work in [1] by adding an exogenous signal (physical-based or parameter-dependent signal) to the model parameters to interpolate between two LPV controllers. This means that new dimensions can be added to the parameter region to incorporate any ad hoc physically-based interpolation. Notice that the addition of such interpolating signal doesn't add any conservatism to the standard LPV problem conservatism. Contrarily, it improves the overall closed-loop performance by choosing the best LPV controller (designed separately) at different situations. As a result, an interpolation scheme is drawn between two LPV controllers based on YK parameterization which guarantees the closed-loop quadratic stability under arbitrary interpolating signal.

2 Problem Statement

2.1 LPV Plant and Controllers Description

Consider a Multi-Input-Multi-Output (MIMO) LPV system $G(\rho)$ with m inputs and p outputs and n_z controlled outputs:

$$G(\rho)\begin{cases}\dot{x}(t) = A(\rho(t))x(t) + B_1(\rho(t))w(t) + B_2u(t)\\ z(t) = C_1(\rho(t))x(t) + D_{11}(\rho(t))w(t) + D_{12}u(t)\\ y(t) = C_2x(t) + D_{21}w(t)\end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^m$, $z(t) \in \mathbb{R}^{n_z}$ are the state, output, input, controlled output vectors respectively. $w(t) = \begin{bmatrix} r & n & d \end{bmatrix}^T \in \mathbb{R}^{n_w}$ contains the exogenous inputs of the tracking reference r, noise n and input disturbance d. $\rho(t) := \rho \in \mathbb{R}^{n_p}$ is a vector of n_p known time-varying parameters.

Here, ρ belongs to a convex polytopic region $\mathcal{P}$ defined by the parameters extremums $[\underline{\rho}, \overline{\rho}]$ as $\mathcal{P} := \mathcal{C}_{\mathcal{O}}\{\nu_1, ..., \nu_{2^{n_p}}\}$, where ν_i represent the vertices of $\mathcal{P}$ $\forall i \in \mathbb{I}[1, 2^{n_p}]$. ρ is then scheduled as: $\rho = \sum_{i=1}^{2^{n_p}} \alpha_i\nu_i$, where $\sum_{i=1}^{2^{n_p}} \alpha_i = 1$, $\alpha_i \geq 0$ $\forall i$. Therefore, the system representation at any operating point $\rho \in \mathcal{P}$ is given as a convex combination of the state-space realizations of the LTI systems given at the vertices ν_i:

$$\left[\begin{array}{c|cc} A(\rho) & B_1(\rho) & B_2 \\ \hline C_1(\rho) & D_{11}(\rho) & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) \left[\begin{array}{c|cc} A_i & B_{1,i} & B_2 \\ \hline C_{1,i} & D_{11,i} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{2}$$

Let $\mathcal{K}$ be a finite set of quadratically stabilizing LPV controllers $K^{(j)}(\rho)$ of $G(\rho)$ that were designed to achieve different objectives and performances. Thus, $\forall j \in \mathbb{I}[0, n_c]$:

$$K^{(j)}(\rho) : \left[\begin{array}{c|c} A_k^{(j)}(\rho) & B_k^{(j)}(\rho) \\ \hline C_k^{(j)}(\rho) & D_k^{(j)}(\rho) \end{array}\right] = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) \left[\begin{array}{c|c} A_{k,i}^{(j)} & B_{k,i}^{(j)} \\ \hline C_{k,i}^{(j)} & D_{k,i}^{(j)} \end{array}\right] \tag{3}$$

where $A_k^{(j)}(\rho) \in \mathbb{R}^{n_k^{(j)} \times n_k^{(j)}}$, $B_k^{(j)}(\rho) \in \mathbb{R}^{n_k^{(j)} \times m_k}$, $C_k^{(j)}(\rho) \in \mathbb{R}^{p_k \times n_k^{(j)}}$ and $D_k^{(j)}(\rho) \in \mathbb{R}^{p_k \times m_k}$.

Notation 2.1 *For the rest of the paper, the subscript i of a matrix (resp. system) denotes the LTI matrix (resp. system) at its corresponding vertex ν_i of the polytope $\mathcal{P}$. The superscript (j) denotes the j^{th} controller in the set of already designed LPV controllers $\mathcal{K}$.*

2.2 Coprime Factorization

Using the YK parameterisation concept, the plant model $G(\rho)$ and the controllers $K^{(j)}(\rho)$ given at each vertex ν_i of $\mathcal{P}$ (namely G_i and $K_i^{(j)}$), can be factorized using the

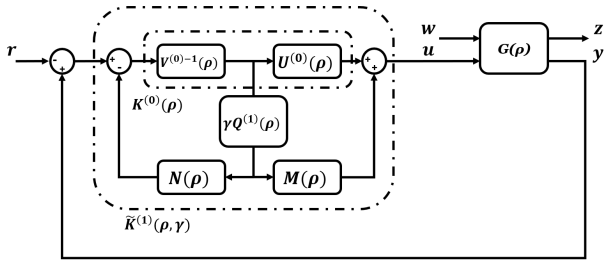

Fig. 1. YK control structure

doubly coprime factorisation as $\forall i, j$:

$$\begin{aligned} G_i &= N_i M_i^{-1} = \tilde{M}_i^{-1}\tilde{N}_i \quad \forall i \\ K_i^{(j)} &= U_i^{(j)} V_i^{(j)-1} = \tilde{V}_i^{(j)-1}\tilde{U}_i^{(j)} \quad \forall i, \forall j \end{aligned} \tag{4}$$

Thus $\forall i, \forall j$, the coprime factors are computed such that M_i, N_i, $\tilde{M}_i$, $\tilde{N}_i$, $U_i^{(j)}$, $V_i^{(j)}$, $\tilde{U}_i^{(j)}$, $\tilde{V}_i^{(j)} \in \mathcal{RH}_\infty$ (proper, stable and rational) [14] and satisfying the following *Bezout Identity*:

$$\begin{bmatrix} \tilde{V}_i^{(j)} & -\tilde{U}_i^{(j)} \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} \begin{bmatrix} M_i & U_i^{(j)} \\ N_i & V_i^{(j)} \end{bmatrix} = \begin{bmatrix} M_i & U_i^{(j)} \\ N_i & V_i^{(j)} \end{bmatrix} \begin{bmatrix} \tilde{V}_i^{(j)} & -\tilde{U}_i^{(j)} \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{5}$$

At each vertex ν_i, all the factorized LTI controllers $K_i^{(j)}$ stabilize G_i (proof in [14]). To perform a stable coprime factorisation, polytopic LPV state-feedback controllers $F_g(\rho)$ and $F_k^{(j)}(\rho)$ are designed to quadratically stabilize $G(\rho)$ and $K^{(j)}(\rho)$ respectively $\forall j$. This ensures the structural stability of the closed-loops $(A_i + B_2 F_{g,i})$ and $(A_{k,i}^{(j)} + B_{k,i}^{(j)} F_{k,i}^{(j)})$ which are Hurwitz $\forall i, j$. The coprime factors are computed at each vertex ν_i using the state-space representations written in (6)-(7).

$$\begin{bmatrix} M_i & U_i^{(j)} \\ N_i & V_i^{(j)} \end{bmatrix} : \left[\begin{array}{cc|cc} A_i + B_2 F_{g,i} & 0 & B_2 & 0 \\ 0 & A_{k,i}^{(j)} + B_{k,i}^{(j)} F_{k,i}^{(j)} & 0 & B_{k,i}^{(j)} \\ \hline F_{g,i} & C_{k,i}^{(j)} + D_{k,i}^{(j)} F_{k,i}^{(j)} & I & D_{k,i}^{(j)} \\ C_2 & F_{k,i}^{(j)} & 0 & I \end{array}\right] \tag{6}$$

$$\begin{bmatrix} \tilde{V}_i^{(j)} & -\tilde{U}_i^{(j)} \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} : \left[\begin{array}{cc|cc} A_i + B_2 D_{k,i}^{(j)} C_2 & B_2 C_{k,i}^{(j)} & -B_2 & B_2 D_{k,i}^{(j)} \\ B_{k,i}^{(j)} C_2 & A_{k,i}^{(j)} & 0 & B_{k,i}^{(j)} \\ \hline F_{g,i} - D_{k,i}^{(j)} C_2 & -C_{k,i}^{(j)} & I & -D_{k,i}^{(j)} \\ C_2 & -F_{k,i}^{(j)} & 0 & I \end{array}\right] \tag{7}$$

2.3 The LPV-YK Control Structure

Fig.1 shows the proposed structure of the dynamic interpolation between two dynamic output-feedback LPV controllers $K^{(0)}(\rho)$ and $K^{(1)}(\rho) \in \mathcal{K}$. Notice that, as explained above, $N(\rho)$, $M(\rho)$, $U^{(0)}(\rho)$, $V^{(0)-1}(\rho)$ and $Q^{(1)}(\rho, \gamma)$ are computed from the convex combination of their corresponding LTI systems at the vertices of $\mathcal{P}$. $K^{(0)}(\rho)$ is chosen to be the nominal controller, where it can be designed to achieve high robustness. The remaining controllers (such as $K^{(1)}(\rho)$) can be designed to deal with higher performances or critical situations. Then, $\forall i$, $K_i^{(1)}$ is written in terms of its corresponding YK parameter $Q_i^{(1)}$ represented as:

$$Q_i^{(1)} : \left[\begin{array}{c|c} A_{q,i}^{(1)} & B_{q,i}^{(1)} \\ \hline C_{q,i}^{(1)} & D_{q,i}^{(1)} \end{array}\right] \tag{8}$$

Following Fig. 1, in this work, an external signal $\gamma(t)$ multiplying the output of the designed $Q_i^{(1)}$, i.e. its output matrices are written as $C_{q,i}^{(1)}(\gamma) = \gamma(t) C_{q,i}^{(1)}$ and $D_{q,i}^{(1)}(\gamma) = \gamma(t) D_{q,i}^{(1)}$ obtaining $Q_i^{(1)}(\gamma) = \gamma Q_i^{(1)}$ (see Fig. 1). γ could be any continuous or discontinuous switching signal which interpolates between $K^{(0)}(\rho)$ and $K^{(1)}(\rho)$. This complete interpolated LPV controller is referred to as the parameterized controller $\tilde{K}^{(1)}(\rho, \gamma)$. The state-space representation of $Q^{(1)}(\rho, \gamma)$ is represented as:

$$Q^{(1)}(\rho, \gamma) : \left[\begin{array}{c|c} A_q^{(1)}(\rho) & B_q^{(1)}(\rho) \\ \hline C_q^{(1)}(\rho, \gamma) & D_q^{(1)}(\rho, \gamma) \end{array}\right] \tag{9}$$

where $A_q^{(1)} \in \mathbb{R}^{n_q^{(1)} \times n_q^{(1)}}$. The interpolated LPV-YK controller $\tilde{K}^{(1)}(\rho, \gamma)$ is computed as:

$$\begin{aligned} \tilde{K}^{(1)}(\rho,\gamma) &= (U^{(0)}(\rho) + M(\rho)Q^{(1)}(\rho,\gamma))(V^{(0)}(\rho) + N(\rho)Q^{(1)}(\rho,\gamma))^{-1} \\ &= (\tilde{V}^{(0)}(\rho) + Q^{(1)}(\rho,\gamma)\tilde{N}(\rho))^{-1}(\tilde{U}^{(0)}(\rho) + Q^{(1)}(\rho,\gamma)\tilde{M}(\rho)) \end{aligned} \tag{10}$$

Its state space matrices are written as:

$$\begin{aligned} \tilde{A}_k^{(1)}(\rho,\gamma) &= \begin{bmatrix} A(\rho) + B_2 F_g(\rho) - B_2 D_q^{(1)}(\rho,\gamma) C_2 & -B_2 D_q^{(1)}(\rho,\gamma) F_k^{(0)}(\rho) & B_2 C_q^{(1)}(\rho,\gamma) \\ -B_k^{(0)}(\rho) C_2 & A_k^{(0)}(\rho) & 0 \\ -B_q^{(1)}(\rho) C_2 & -B_q^{(1)}(\rho) F_k^{(0)}(\rho) & A_q^{(1)}(\rho) \end{bmatrix} \\ \tilde{B}_k^{(1)}(\rho,\gamma) &= \begin{bmatrix} B_2 D_q^{(1)}(\rho,\gamma) & B_k^{(0)}(\rho) & B_q^{(1)}(\rho) \end{bmatrix}^T \\ \tilde{C}_k^{(1)}(\rho,\gamma) &= \begin{bmatrix} F_g(\rho) - (D_k^{(0)}(\rho) + D_q^{(1)}(\rho,\gamma)) C_2 & C_k^{(0)}(\rho) - D_q^{(1)}(\rho,\gamma) F_k^{(0)}(\rho) & C_q^{(1)}(\rho,\gamma) \end{bmatrix} \\ \tilde{D}_k^{(1)}(\rho,\gamma) &= D_k^{(0)}(\rho) + D_q^{(1)}(\rho,\gamma) \end{aligned} \tag{11}$$

The next step proposes a gain-scheduling/interpolation between the designed LPV controllers to achieve different performances depending on the situations. Knowing that $Q^{(1)}(\rho, \gamma)$ is quadratically stable by construction (see later in the proof of Theorem 3.1), it is worth mentioning that:

- for $\gamma = 0$, $\tilde{K}^{(1)}(\rho, \gamma = 0) = K^{(0)}(\rho)$
- for $\gamma = 1$, $\tilde{K}^{(1)}(\rho, \gamma = 1) = K^{(1)}(\rho)$

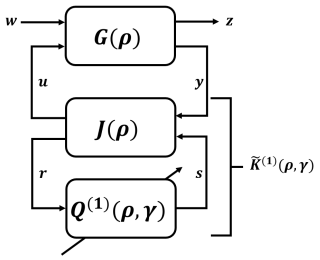

Fig. 2. YK general configuration

3 Main Results

Based on the statements on LPV concepts and YK parameterization, a quadratically stable interpolation procedure between two LPV controllers is formulated. A non-minimal state-space realization of each LPV controller is computed based on Linear Matrix Inequality (LMI) optimization problem [5]. A gain-scheduling signal γ is included to interpolate between both controllers.

Theorem 3.1 *Consider an LPV plant $G(\rho)$ (2), and given a set of LPV controllers $\mathcal{K}$ (3) that quadratically stabilizes $G(\rho)$. Let us choose any two controllers $K^{(0)}(\rho)$ and $K^{(1)}(\rho) \in \mathcal{K}$, where $K^{(0)}(\rho)$ represents the nominal controller. Then, the YK-parameterized LPV controller $\tilde{K}^{(1)}(\rho,\gamma)$ (11) (see Fig. 1) stabilizes $G(\rho)$ for any continuous/discontinuous signal $\gamma \in [0,1]$, if there exist symmetric, positive definite matrices $X_g \in \mathbb{R}^{n_x \times n_x}$, $X_k \in \mathbb{R}^{n_k^{(0)} \times n_k^{(0)}}$, and matrices W_i and V_i such that:*

$$A_i X_g + X_g A_i^T + B_2 W_i + W_i^T B_2^T < 0 \qquad (12)$$

$$A_{k,i}^{(0)} X_k + X_k A_{k,i}^{(0)T} + B_{k,i}^{(0)} V_i + V_i^T B_{k,i}^{(0)T} < 0 \qquad (13)$$

for all $i = 1, ..., 2^{n_p}$.

PROOF. According to YK parameterisation concept, $\forall \rho \in \mathcal{P}$, the LPV parameterized controller $\tilde{K}^{(1)}(\rho,\gamma)$ can be formulated as a Linear Fractional Transformation (LFT) system [14], i.e. $\tilde{K}^{(1)}(\rho,\gamma) = \mathcal{F}_l(J(\rho), Q^{(1)}(\rho,\gamma))$ (see Fig. 2). Where, $J(\rho)$ and $Q^{(1)}(\rho,\gamma)$ are written as a convex combination of their corresponding LTI systems (14)-(15) at the vertices ν_i of $\mathcal{P}$.

$$J_i = \left[\begin{array}{cc|cc} A_i + B_2 F_{g,i} & 0 & 0 & B_2 \\ -B_{k,i}^{(0)} C_2 & A_{k,i}^{(0)} & B_{k,i}^{(0)} & 0 \\ \hline F_{g,i} - D_{k,i}^{(0)} C_2 & C_{k,i}^{(0)} & D_{k,i}^{(0)} & I \\ -C_2 & -F_{k,i}^{(0)} & I & 0 \end{array}\right] \qquad (14)$$

$$Q_i^{(1)}(\gamma) = \left[\begin{array}{cc:c|c} A_i + B_2 D_{k,i}^{(1)} C_2 & B_2 C_{k,i}^{(1)} & B_2[D_{k,i}^{(1)} - D_{k,i}^{(0)}] F_{k,i}^{(0)} - B_2 C_{k,i}^{(0)} & B_2[D_{k,i}^{(1)} - D_{k,i}^{(0)}] \\ B_{k,i}^{(1)} C_2 & A_{k,i}^{(1)} & B_{k,i}^{(1)} F_{k,i}^{(0)} & B_{k,i}^{(1)} \\ \hdashline 0 & 0 & A_{k,i}^{(0)} + B_{k,i}^{(0)} F_{k,i}^{(0)} & B_{k,i}^{(0)} \\ \hline \gamma[D_{k,i}^{(1)} C_2 - F_{g,i}] & \gamma C_{k,i}^{(1)} & \gamma[(D_{k,i}^{(1)} - D_{k,i}^{(0)}) F_{k,i}^{(0)} - C_{k,i}^{(0)}] & \gamma[D_{k,i}^{(1)} - D_{k,i}^{(0)}] \end{array}\right] \qquad (15)$$

At each vertex ν_i, the closed-loop system $CL_i(\gamma)$ is derived from the LFT interconnection between G_i and $\tilde{K}_i^{(1)}(\gamma)$. The closed-loop state matrix $A_{cl}(\rho,\gamma) = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) A_{cl,i}(\gamma)$ is quadratically stable if there exist a symmetric, positive definite, constant matrix X_{cl} such that:

$$X_{cl} A_{cl}(\rho,\gamma) + A_{cl}^T(\rho,\gamma) X_{cl} < 0 \ \ \forall \gamma \qquad (16)$$

Now , let $T = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ I & -I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}$ be a state transformation matrix which is applied to $CL_i(\gamma)$ without changing its input-output nature, then

$$\bar{A}_{cl,i}(\gamma) = T A_{cl,i}(\gamma) T^{-1} = \left[\begin{array}{cc:c:c} A_i + B_2 F_{g,i} & B_2 C_{q,i}^{(1)} & -B_2(F_{g,i} - (D_{k,i}^{(0)} + D_{q,i}^{(1)}(\gamma)) C_2) & B_2(C_{k,i}^{(0)} - D_{q,i}^{(1)} F_{k,i}^{(0)}) \\ 0 & A_{q,i}^{(1)} & B_{q,i}^{(1)} C_2 & -B_{q,i}^{(1)} F_{k,i}^{(0)} \\ \hdashline 0 & 0 & A_i + B_2 D_{k,i}^{(0)} C_2 & B_2 C_{k,i}^{(0)} \\ 0 & 0 & B_{k,i}^{(0)} C_2 & A_{k,i}^{(0)} \end{array}\right]. \qquad (17)$$

Due to the block-triangular form of $\bar{A}_{cl}(\rho,\gamma)$ (17) , (16) is satisfied if the following equations hold (check *Lemma 2* in [16]):

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho)(Y_g(A_i + B_2 F_{g,i}) + (A_i + B_2 F_{g,i})^T Y_g) < 0 \qquad (18)$$

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho)(Y_q A_{q,i}^{(1)} + A_{q,i}^{(1)T} Y_q) < 0 \qquad (19)$$

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho)(Y_0 \mathcal{A}_i^{(0)} + \mathcal{A}_i^{(0)T} Y_0) < 0 \qquad (20)$$

where $Y_g \in \mathbb{R}^{n_x \times n_x}$, $Y_q \in \mathbb{R}^{n_q^{(1)} \times n_q^{(1)}}$ and $Y_0 \in \mathbb{R}^{(n_x + n_k^{(0)}) \times (n_x + n_k^{(0)})}$ are symmetric, positive definite, parameter-invariant matrices, with $X_{cl} = T^T \operatorname{diag}(Y_g, Y_q, Y_0)\, T$, and

$$\mathcal{A}_i^{(0)} = \begin{bmatrix} A_i + B_2 D_{k,i}^{(0)} C_2 & B_2 C_{k,i}^{(0)} \\ B_{k,i}^{(0)} C_2 & A_{k,i}^{(0)} \end{bmatrix} \qquad (21)$$

Inequality (18) can be reformulated by $Y_g = X_g^{-1}$ which

leads to (12) when choosing $W(\rho) = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) F_{g,i} X_g$. Regarding (15), the state matrix of the LPV-YK parameter $A_q^{(1)}(\rho) = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) A_{q,i}^{(1)}(\rho)$ is quadratically stable if there exist symmetric, positive definite, constant matrices $P_1 \in \mathbb{R}^{(n_x+n_k^{(1)})\times(n_x+n_k^{(1)})}$ and $P_2 \in \mathbb{R}^{n_k^{(0)}\times n_k^{(0)}}$ such that:

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho)(P_1 \mathcal{A}_i^{(1)} + \mathcal{A}_i^{(1)T} P_1) < 0 \tag{22}$$

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho)(P_2(A_{k,i}^{(0)}+B_{k,i}^{(0)}F_{k,i}^{(0)})+(A_{k,i}^{(0)}+B_{k,i}^{(0)}F_{k,i}^{(0)})^T P_2) < 0 \tag{23}$$

being,

$$\mathcal{A}_i^{(1)} = \begin{bmatrix} A_i + B_2 D_{k,i}^{(1)} C_2 & B_2 C_{k,i}^{(1)} \\ B_{k,i}^{(1)} C_2 & A_{k,i}^{(1)} \end{bmatrix} \tag{24}$$

The condition in (22) is verified given that $K^{(1)}(\rho)$ quadratically stabilizes $G(\rho)$. Moreover, the inequality (23) satisfies (13) by choosing $P_2 = X_k^{-1}$ and $V(\rho) = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) F_{k,i}^{(0)} X_k$. Thus, $A_q^{(1)}(\rho)$ is quadratically stable and (19) is verified. Finally, (20) is fulfilled given that $K^{(0)}(\rho)$ quadratically stabilizes $G(\rho)$. □

After verifying the conditions of the theorem, it is worth mentioning that they depend only on the nominal LPV controller $K^{(0)}(\rho)$ and not on the other designed LPV controllers given the considered YK parameterization.

The following brief example presents the objective of the proposed approach in improving the closed-loop performance. Given two LPV controllers that are designed separately achieving distinct performance specifications: 1) $K^{(0)}(\rho)$ for noise rejection with slow tracking capabilities; and 2) $K^{(1)}(\rho)$ for fast tracking capabilities but without respecting the robust margin. Let us construct a YK-based interpolation scheme $\tilde{K}^{(1)}(\rho,\gamma)$ as shown in Fig. 1. As a result, multiple closed-loop performances are obtained thanks to the variation of the interpolating signal $\gamma(t)$. At an instant $t = t_k$:

(1) if $\gamma(t_k) = 0$, $\tilde{K}^{(1)}(\rho, \gamma(t_k) = 0) \equiv K^{(0)}(\rho)$, the closed-loop performance achieves only noise rejection
(2) if $\gamma(t_k) = 1$, $\tilde{K}^{(1)}(\rho, \gamma(t_k) = 1) \equiv K^{(1)}(\rho)$, the closed-loop performance include only tracking capabilities
(3) if $0 < \gamma(t_k) < 1$, $\tilde{K}^{(1)}(\rho, \gamma(t_k))$ interpolates between both controllers to obtain a percentage of noise rejection performance and a percentage of tracking capabilities.

An example is shown in [9] on how to choose γ according to the required control objectives.

4 Conclusion

This work has proposed a new LPV YK-based method to design an interpolation between two dynamic output-feedback LPV controllers. An external signal is introduced to the LPV-YK parameter, which can be used to incorporate any ad-hoc physically-based interpolation. As a result, a YK-based interpolation scheme is drawn between two LPV controllers while guaranteeing the closed-loop quadratic stability under arbitrary interpolating signal. This approach improves the performance for LPV complex systems that need to deal with various objectives and situations. As a future work, the LPV-YK control scheme will be extended to more than two controllers. In addition, application of the presented results and possible extension for discrete-time systems will be explored.

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