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Interpolation of Multi-LPV Control Systems Based on Youla-Kucera Parameterization

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Abstract

This paper presents a methodology which considers an interpolation between Linear Parameter-Varying (LPV) controllers that were designed separately for different objectives. The quadratic stability of the closed-loop system is proved under arbitrary interpolation in terms of a set of Linear Matrix Inequalities (LMIs). The interpolation strategy is based on Youla-Kucera (YK) parameterization. The proposed method can help multi-variable and multi-objective systems, to achieve high performances at different critical situations regardless of the interpolation rate.

Key words: Linear parameter-varying system, Youla parameterization, Gain-scheduling, Linear matrix inequality

1 Introduction And Motivation

The interpolation of several controllers to meet multi-objectives criteria is an important topic in theory and practice. A method has been investigated in the literature to achieve a multi-objective switching Linear Time-Invariant (LTI) controller based on Youla-Kucera (YK) parameterization [5]. It is structured by mapping a set of linear stabilizing controllers onto a Q-based controller. The interest is to parameterize a set of linear stabilizing controllers $K(Q)$ for an LTI system, where each one is parameterized by its corresponding LTI-YK parameter $Q$ [14]. The LTI-YK parameter $Q$ is designed using the doubly coprime factorisation [15]. Using such parameterization for controller interpolation: 1) It allows stable gain-scheduling between unstable controllers [13]; 2) Interpolated controllers can be designed and tuned separately using different techniques ($H\infty$, LQR, PID) [14]; 3) It facilitates adding new parts to an existing system online as Plug&Play control theory [12]; and 4) The closed-loop stability is guaranteed under arbitrary interpolating signals between different stabilizing controllers [5] without requiring a single Lyapunov function.

The YK parameterization has been successfully applied to several domains such as: 1) Noise/vibration control [7]; 2) Interpolation between robust and high performance controllers [4]; and 3) Control systems subjected to convex constraints [3]. A significant literature review of YK work including its recent applications can be found in [10]. In addition, YK parameterisation is used in a rising topic which is autonomous vehicles [11],[9]. [9] uses an LTI-YK control structure to interpolate between fast and slow LTI controllers to optimize a lane change of an autonomous vehicle. However, the study is done on an LTI lateral system with a fixed longitudinal speed of the vehicle.

On the other hand, the control of multi-variable or Linear Parameter-Varying (LPV) systems is studied due to its interest to handle system complexity (as nonlinearities) as well as to bring some adaptive control schemes thanks to the self-scheduling of the controller, see for instance [6] and references therein. Nonetheless, it is today admitted that designing a single LPV controller for various objectives may be conservative [8]. However, while many YK interpolation studies have been concerned with LTI systems, few works have integrated YK concepts to LPV systems. [1] introduces an LPV model which varies with respect to time-varying parameters. It proposes a YK-based gain-scheduling between LTI controllers designed separately at different operating points. Closed-loop quadratic stability and performance are guaranteed at intermediate interpolation points of...
Consider a Multi-Input-Multi-Output (MIMO) LPV Plant and Controllers Description

2 Problem Statement

2.1 LPV Plant and Controllers Description

Consider a Multi-Input-Multi-Output (MIMO) LPV system $G(\rho)$ with $m$ inputs and $p$ outputs and $n_z$ controlled outputs:

$$
\begin{align*}
    x(t) &= A(\rho(t))x(t) + B_1(\rho(t))w(t) + B_{2u}(t) \\
    G(\rho) &= C_1(\rho(t))x(t) + D_{11}(\rho(t))w(t) + D_{12}(\rho(t))u(t) \\
    y(t) &= C_2x(t) + D_{21}w(t)
\end{align*}
$$

where $x(t) \in \mathbb{R}^{n_z}$, $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^m$, $z(t) \in \mathbb{R}^{n_z}$ are the state, output, input, controlled output vectors respectively. $w(t) = \begin{bmatrix} r & n & d \end{bmatrix}^T \in \mathbb{R}^{n_w}$ contains the exogenous inputs of the tracking reference $r$, noise $n$ and input disturbance $d$. $\rho(t) := \rho \in \mathbb{R}^{n_p}$ is a vector of $n_p$ known time-varying parameters.

Here, $\rho$ belongs to a convex polytopic region $\mathcal{P}$ defined by the parameters extremums $[\rho_l, \rho_u]$ as $\mathcal{P} := \{C_0(\nu_1, ..., \nu_{2^n_p}) \mid \nu_i \text{ represent the vertices of } \mathcal{P} \forall i \in \mathbb{I}[1, 2^n_p] \}$, $\rho$ is then scheduled as: $\rho = \sum_{i=1}^{2^n_p} \alpha_i \nu_i$, where $\sum_{i=1}^{2^n_p} \alpha_i = 1$, $\alpha_i \geq 0 \forall i$. Therefore, the system representation at any operating point $\rho \in \mathcal{P}$ is given as a convex combination of the state-space realizations of the LTI systems given at the vertices $\nu_i$:

$$
\begin{bmatrix}
    A(\rho) & B_1(\rho) & B_2 \\
    C_1(\rho) & D_{11}(\rho) & D_{12} \\
    C_2 & D_{21} & 0
\end{bmatrix} = \sum_{i=1}^{2^n_p} \alpha_i(\rho) \begin{bmatrix}
    A_i & B_{1,i} & B_2 \\
    C_{1,i} & D_{11,i} & D_{12} \\
    C_2 & D_{21} & 0
\end{bmatrix}
$$

Let $\mathcal{K}$ be a finite set of quadratically stabilizing LPV controllers $K^{(j)}(\rho)$ of $G(\rho)$ that were designed to achieve different objectives and performances. Thus, $\forall j \in \mathbb{I}[0, n_c]$:

$$
\begin{bmatrix}
    A_k^{(j)}(\rho) & B_k^{(j)} \\
    C_k^{(j)}(\rho) & D_k^{(j)}(\rho)
\end{bmatrix} = \sum_{i=1}^{2^n_p} \alpha_i(\rho) \begin{bmatrix}
    A_k^{(j)}(\rho) & B_k^{(j)} \\
    C_k^{(j)}(\rho) & D_k^{(j)}(\rho)
\end{bmatrix}
$$

where $A_k^{(j)}(\rho) \in \mathbb{R}^{n_{k,i} \times n_{k,i}}$, $B_k^{(j)}(\rho) \in \mathbb{R}^{n_{k,i} \times m_k}$, $C_k^{(j)}(\rho) \in \mathbb{R}^{p_k \times n_{k,i}}$ and $D_k^{(j)}(\rho) \in \mathbb{R}^{p_k \times m_k}$.

**Notation 2.1** For the rest of the paper, the subscript $i$ of a matrix (resp. system) denotes the LTI matrix (resp. system) at its corresponding vertex $\nu_i$ of the polytope $\mathcal{P}$. The superscript $(j)$ denotes the $j^{th}$ controller in the set of already designed LPV controllers $\mathcal{K}$.

2.2 Coprime Factorization

Using the YK parameterisation concept, the plant model $G(\rho)$ and the controllers $K^{(j)}(\rho)$ given at each vertex $\nu_i$ of $\mathcal{P}$ (namely $G_i$ and $K^{(j)}_i$), can be factorized using the...
The structural stability of the closed-loops ($K_i$ factors are computed at each vertex ($\nu_i$), $\nu_i, \nu_j$), the coprime factors are computed such that $M_i, N_i, \tilde{M}_i, \tilde{N}_i, U_i^{(j)}, V_i^{(j)}, \tilde{V}_i^{(j)}, \tilde{U}_i^{(j)} \in \mathbb{R}^{\infty}$ (proper, stable and rational) [14] and satisfying the following Bezout Identity:

$$
\begin{bmatrix}
\tilde{V}_i^{(j)} - \tilde{U}_i^{(j)} \\
-M_i \ N_i \ V_i^{(j)}
\end{bmatrix} =
\begin{bmatrix}
M_i U_i^{(j)} \\
N_i V_i^{(j)}
\end{bmatrix} \begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix}
$$

At each vertex $\nu_i$, all the factorized LTI controllers $K_i^{(j)}$ stabilize $G_i$ (proof in [14]). To perform a stable coprime factorisation, polytopic LPV state-feedback controllers $F_{g,i}$ and $F_{k,i}$ are designed to quadratically stabilize $G_i$ and $K_i^{(j)}$ respectively $\forall i,j$. This ensures the structural stability of the closed-loops ($A_i + B_{g,i} F_{g,i}$) and ($A_i^{(j)} + B_{k,i}^{(j)} F_{k,i}^{(j)}$) which are Hurwitz $\forall i,j$. The coprime factors are computed at each vertex $\nu_i$ using the state-space representations written in (6)-(7).

$$
\begin{bmatrix}
M_i U_i^{(j)} \\
N_i V_i^{(j)}
\end{bmatrix} =
\begin{bmatrix}
A_i + B_{g,i} F_{g,i} & 0 \\
0 & A_i^{(j)} + B_{k,i}^{(j)} F_{k,i}^{(j)}
\end{bmatrix}
\begin{bmatrix}
B_{g,i} & 0 \\
0 & B_{k,i}^{(j)}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\tilde{V}_i^{(j)} - \tilde{U}_i^{(j)} \\
-M_i \ N_i \ V_i^{(j)}
\end{bmatrix} =
\begin{bmatrix}
A_i + B_{g,i} F_{g,i} C_2 B_2 C_2^{(j)} A_i^{(j)} - B_2 B_{k,i}^{(j)} \\
F_{g,i} - D_{k,i}^{(j)} C_2 - C_2^{(j)} A_i^{(j)} & 0 & B_{k,i}^{(j)}
\end{bmatrix}
\begin{bmatrix}
B_{g,i} & 0 \\
0 & B_{k,i}^{(j)}
\end{bmatrix}
$$

The LPV-YK Control Structure

Fig.1 shows the proposed structure of the dynamic interpolation between two dynamic output-feedback LPV controllers $K^{(0)}(\rho)$ and $K^{(1)}(\rho) \in K$. Notice that, as explained above, $N(\rho), M(\rho), U^{(0)}(\rho), V^{(0)}(\rho), V^{(0)}(\rho)$ and $Q^{(1)}(\rho)$ are computed from the convex combination of their corresponding LTI systems at the vertices of $P$. $K^{(0)}(\rho)$ is chosen to be the nominal controller, where it can be designed to achieve high robustness. The remaining controllers (such as $K^{(1)}(\rho)$) can be designed to deal with higher performances or critical situations. Then, $\forall i$, $K_i^{(1)}$ is written in terms of its corresponding YK parameter $Q_i^{(1)}$ represented as:

$$
Q_i^{(1)} : \begin{bmatrix}
A_{q,i}^{(1)} & B_{q,i}^{(1)} \\
C_{q,i}^{(1)} & D_{q,i}^{(1)}
\end{bmatrix}
$$

Following Fig. 1, in this work, an external signal $\gamma(t)$ multiplying the output of the designed $Q_i^{(1)}$, i.e. its output matrices are written as $C_{q,i}^{(1)}(\gamma) = \gamma(t) C_{q,i}^{(1)}$ and $D_{q,i}^{(1)}(\gamma) = \gamma(t) D_{q,i}^{(1)}$ obtaining $Q_i^{(1)}(\gamma) = \gamma Q_i^{(1)}$ (see Fig. 1), $\gamma$ could be any continuous or discontinuous switching signal which interpolates between $K^{(0)}(\rho)$ and $K^{(1)}(\rho)$. This complete interpolated LPV controller is referred to as the parameterized controller $\bar{K}^{(1)}(\rho)$. The state-space representation of $Q_i^{(1)}(\rho, \gamma)$ is represented as:

$$
Q^{(1)}(\rho, \gamma) : \begin{bmatrix}
A_{q,i}^{(1)}(\rho) & B_{q,i}^{(1)}(\rho) \\
C_{q,i}^{(1)}(\rho, \gamma) & D_{q,i}^{(1)}(\rho, \gamma)
\end{bmatrix}
$$

where $A_{q,i}^{(1)} \in \mathbb{R}^{n_{q,i} \times n_{q,i}}$. The interpolated LPV-YK controller $\bar{K}^{(1)}(\rho, \gamma)$ is computed as:

$$
\bar{K}^{(1)}(\rho, \gamma) = (U^{(0)}(\rho) + M(\rho) Q^{(1)}(\rho, \gamma)(V^{(0)}(\rho) + N(\rho) Q^{(1)}(\rho, \gamma))^{-1}
$$

$$
= (U^{(0)}(\rho) + Q^{(1)}(\rho, \gamma) N(\rho))^{-1}(U^{(0)}(\rho) + Q^{(1)}(\rho, \gamma) M(\rho))
$$

Its state space matrices are written as:

$$
\begin{bmatrix}
A_{i}^{(1)}(\rho, \gamma) - A_{i}^{(0)}(\rho) - B_{i}^{(1)}(\rho, \gamma) C_{i}^{(0)}(\rho) C_{i}^{(1)}(\rho, \gamma) - B_{i}^{(1)}(\rho, \gamma) C_{i}^{(1)}(\rho, \gamma) C_{i}^{(1)}(\rho, \gamma) C_{i}^{(1)}(\rho, \gamma) C_{i}^{(1)}(\rho, \gamma)
\end{bmatrix}
$$

The next step proposes a gain-scheduling/interpolation between the designed LPV controllers to achieve different performances depending on the situations. Knowing that $Q^{(1)}(\rho, \gamma)$ is quadratically stable by construction (see later in the proof of Theorem 3.1), it is worth mentioning that:

- for $\gamma = 0$, $\bar{K}^{(1)}(\rho, \gamma = 0) = K^{(0)}(\rho)$
- for $\gamma = 1$, $\bar{K}^{(1)}(\rho, \gamma = 1) = K^{(1)}(\rho)$
3 Main Results

Based on the statements on LPV concepts and YK parameterization, a quadratically stable interpolation procedure between two LPV controllers is formulated. A non-minimal state-space realization of each LPV controller is computed based on Linear Matrix Inequality (LMI) optimization problem [5]. A gain-scheduling signal γ is included to interpolate between both controllers.

**Theorem 3.1** Consider an LPV plant $G(\rho)$ (2), and given a set of LPV controllers $K(\rho)$ (3) that quadratically stabilizes $G(\rho)$. Let us choose any two controllers $K(0)(\rho)$ and $K(1)(\rho) \in K$, where $K(0)(\rho)$ represents the nominal controller. Then, the YK-parameterized LPV controller $\tilde{K}(\rho,\gamma)$ (11) (see Fig. 1) stabilizes $G(\rho)$ for any continuous/discontinuous signal $\gamma \in [0,1]$, if there exist symmetric, positive definite matrices $X_\gamma \in \mathbb{R}^{n_x \times n_x}$, $X_k \in \mathbb{R}^{n_{o_k} \times n_{o_k}}$, and matrices $W_i$ and $V_i$ such that:

$$A_i X_\gamma + X_\gamma A_i^T + B_k W_i + V_i^T B_k^T < 0 \quad (12)$$

$$A_{k,i}^{(0)} X_k + X_k A_{k,i}^{(0)T} + B_{k,i}^{(0)} V_i + V_i^T B_{k,i}^{(0)T} < 0 \quad (13)$$

for all $i = 1,\ldots,2^{n_p}$.

**Proof.** According to YK parameterisation concept, $\forall \rho \in \mathcal{P}$, the LPV parameterized controller $\tilde{K}(\rho,\gamma)$ can be formulated as a Linear Fractional Transformation (LFT) system [14], i.e. $\tilde{L}(\rho,\gamma) = F_{\gamma}(J(\rho),Q(\rho,\gamma))$ (see Fig. 2). Where, $J(\rho)$ and $Q(\rho,\gamma)$ are written as a convex combination of their corresponding LTI systems (14)-(15) at the vertices $\nu_i$ of $\mathcal{P}$.

$$J_i = \begin{bmatrix} A_i + B_2 D_{k,i}^{(0)} C_2 & B_2 C_{k,i}^{(0)} \hline 0 & 0 & B_2 \end{bmatrix}$$

$$Q_i^{(1)}(\gamma) = \begin{bmatrix} A_i + B_2 D_{k,i}^{(0)} C_2 & B_2 C_{k,i}^{(0)} \hline 0 & 0 & B_2 \end{bmatrix}$$

At each vertex $\nu_i$, the closed-loop system $CL_{\nu_i}(\gamma)$ is derived from the LFT interconnection between $G_i$ and $\tilde{K}_i^{(1)}(\gamma)$. The closed-loop state matrix $A_{cl}(\rho,\gamma) = \sum_{i=1}^{2^{n_p}} \alpha_i(\rho) A_{cl,i}(\gamma)$ is quadratically stable if there exist a symmetric, positive definite, constant matrix $X_{cl}$ such that:

$$X_{cl} A_{cl}(\rho,\gamma) + A_{cl}^T(\rho,\gamma) X_{cl} < 0 \quad (16)$$

Now, let $T = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}$ be a state transformation matrix which is applied to $CL_{\nu_i}(\gamma)$ without changing its input-output nature, then

$$\tilde{A}_{cl,i}(\gamma) = T A_{cl,i}(\gamma) T^{-1} = \begin{bmatrix} A_i + B_2 F_{g,i} & B_2 C_{k,i}^{(0)} & B_2 C_{k,i}^{(0)} & B_2 C_{k,i}^{(0)} \\ 0 & A_i + B_2 F_{g,i} & B_2 C_{k,i}^{(0)} & B_2 C_{k,i}^{(0)} \\ 0 & 0 & A_i + B_2 F_{g,i} & B_2 C_{k,i}^{(0)} \\ 0 & 0 & 0 & A_i + B_2 F_{g,i} \end{bmatrix} \quad (17)$$

Due to the block-triangular form of $\tilde{A}_{cl}(\rho,\gamma)$ (17), (16) is satisfied if the following equations hold (check Lemma 2 in [16]):

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho) (Y_g(A_i + B_2 F_{g,i}) + (A_i + B_2 F_{g,i})^T Y_g) < 0 \quad (18)$$

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho) (Y_q A_q^{(i)} + A_q^{(i)T} Y_q) < 0 \quad (19)$$

$$\sum_{i=1}^{2^{n_p}} \alpha_i(\rho) (Y_0 A_0^{(i)} + A_0^{(i)T} Y_0) < 0 \quad (20)$$

where $Y_g \in \mathbb{R}^{n_x \times n_x}$, $Y_q \in \mathbb{R}^{n_q \times n_q(i)}$ and $Y_0 \in \mathbb{R}^{(n_x+n_{o_k}) \times (n_x+n_{o_k})}$ are symmetric, positive definite, parameter-invariant matrices, with $X_{cl} = T^T \text{diag}(Y_g,Y_q,Y_0) T$, and

$$A_{i}^{(0)} = \begin{bmatrix} A_i + B_2 D_{k,i}^{(0)} C_2 & B_2 C_{k,i}^{(0)} \\ 0 & 0 & B_2 \end{bmatrix} \quad (21)$$

Inequality (18) can be reformulated by $Y_g = X_{cl}^{-1}$ which
leads to (12) when choosing $W(\rho) = \sum_{i=1}^{2^p} \alpha_i(\rho)F_{i,g}X_g$. Regarding (15), the state matrix of the LPV-YK parameter $A_i^{(1)}(\rho) = \sum_{i=1}^{2^p} \alpha_i(\rho)A_i^{(1)}(\rho)$ is quadratically stable if there exist symmetric, positive definite, constant matrices $P_1 \in \mathbb{R}^{(m+n_k)\times (m+n_k)}$ and $P_2 \in \mathbb{R}^{n_k \times n_k}$ such that:

$$\sum_{i=1}^{2^p} \alpha_i(\rho)(P_1A_i^{(1)} + A_i^{(1)T}P_1) < 0$$

(22)

$$\sum_{i=1}^{2^p} \alpha_i(\rho)(P_2(A^{(0)}_{k,i} + B^{(0)}_{k,i}F^{(0)}_{k,i}) + (A^{(0)}_{k,i} + B^{(0)}_{k,i}F^{(0)}_{k,i})^TP_2) < 0$$

(23)

being,

$$A_i^{(1)} = \begin{bmatrix}
A_k + B_2D_{k,i}C_2 & B_2C_{k,i} \\
B_{k,i}^T & A_{k,i}^{(1)}
\end{bmatrix}$$

(24)

The condition in (22) is verified given that $K^{(1)}(\rho)$ quadratically stabilizes $G(\rho)$. Moreover, the inequality (23) satisfies (13) by choosing $P_2 = X_k^{-1}$ and $V(\rho) = \sum_{i=1}^{2^p} \alpha_i(\rho)F^{(0)}_{k,i}X_k$. Thus, $A_i^{(1)}(\rho)$ is quadratically stable and (19) is verified. Finally, (20) is fulfilled given that $K^{(0)}(\rho)$ quadratically stabilizes $G(\rho)$. □

After verifying the conditions of the theorem, it is worth mentioning that they depend only on the nominal LPV controller $K^{(0)}(\rho)$ and not on the other designed LPV controllers given the considered YK parameterization.

The following brief example presents the objective of the proposed approach in improving the closed-loop performance. Given two LPV controllers that are designed separately achieving distinct performance specifications: 1) $K^{(0)}(\rho)$ for noise rejection with slow tracking capabilities; and 2) $K^{(1)}(\rho)$ for fast tracking capabilities but without respecting the robust margin. Let us construct a YK-based interpolation scheme $\tilde{K}^{(1)}(\rho, \gamma)$ as shown in Fig. 1. As a result, multiple closed-loop performances are obtained thanks to the variation of the interpolating signal $\gamma(t)$. At an instant $t = t_k$:

1. if $\gamma(t_k) = 0$, $\tilde{K}^{(1)}(\rho, \gamma(t_k) = 0) \equiv K^{(0)}(\rho)$, the closed-loop performance achieves only noise rejection
2. if $\gamma(t_k) = 1$, $\tilde{K}^{(1)}(\rho, \gamma(t_k) = 1) \equiv K^{(1)}(\rho)$, the closed-loop performance includes only tracking capabilities
3. if $0 < \gamma(t_k) < 1$, $\tilde{K}^{(1)}(\rho, \gamma(t_k))$ interpolates between both controllers to obtain a percentage of noise rejection performance and a percentage of tracking capabilities.

An example is shown in [9] on how to choose $\gamma$ according to the required control objectives.

4 Conclusion

This work has proposed a new LPV YK-based method to design an interpolation between two dynamic output-feedback LPV controllers. An external signal is introduced to the LPV-YK parameter, which can be used to incorporate any ad-hoc physically-based interpolation. As a result, a YK-based interpolation scheme is drawn between two LPV controllers while guaranteeing the closed-loop quadratic stability under arbitrary interpolating signal. This approach improves the performance for LPV complex systems that need to deal with various objectives and situations. As a future work, the LPV-YK control scheme will be extended to more than two controllers. In addition, application of the presented results and possible extension for discrete-time systems will be explored.

References


