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# PHASE DIAGRAM DISTORTION FROM TRAFFIC PARAMETER AVERAGING

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## Abstract

Motorway traffic congestion is a major bottleneck for economic growth. Therefore research on traffic behaviour is being carried out in many countries. Observations and theories of congested traffic, although well describing the free flow phase as an almost straight line in a density-flow or  $(k,q)$ -phase diagram, disagree in the congested flow state. In this paper we investigate the relation between traffic observations and the structure of the phase diagram. It focuses on the way speed observations are averaged, and how this influences the location of the averaged observations in the phase diagram. The analysis implies a phase diagram where the congested phase consists of a straight line. This straight right branch corresponds to upstream moving speed wave regions, called speed waves. The right branch connects the top of the free flow phase line with a point of maximum  $k$  and zero  $q$ . Its slope corresponds to the speed of the upstream moving speed wave, with a value of  $-18\pm 1$  km/h, a value also found by other authors. By simulating traffic on this line, consisting of waves with two different speeds, we show that the location of averaged parameters in the phase diagram lie below this line. We also show that harmonic mean speed is a good indicator for travel time.

## 1. Introduction

Traffic and transport are major factors in modern life, and congestion is an important drawback in economic growth. Hence, many countries financially invest in traffic research and optimisation of traffic control. Since Greenshields introduced a simple model to describe traffic behaviour [Greenshields 1934] almost 75 years of research have generated considerable understanding of traffic in general, and motorway traffic in particular.

Motorway traffic is usually described by three important quantities: flow  $q$  (vehicles per hour per lane), speed  $v$  (km per hour) and density  $k$  (vehicles per km) [Leutzbach 1988]. With any two of these quantities it is possible to describe the traffic state. A common way to do this is by using a phase diagram, where values of the observed or theoretical quantities are plotted. In this paper, we choose  $q$  and  $k$ , and consequently use the flow-density diagram, or  $(k,q)$ -diagram. Such a diagram usually shows traffic in two different phases. One is the free flow phase, represented by an almost

straight line between the origin and a point  $(k_{cap}, q_{cap})$ , in which  $q_{cap}$  equals the maximum flow, also denoted as the road capacity, and  $k_{cap}$  equals the corresponding density. The slope of this line represents the mean speed in free flow,  $v_{free}$ . The other phase corresponds to congested flow, for which  $q < q_{cap}$ ,  $k > k_{cap}$  and  $v < v_{free}$ .

Many different theoretical phase diagrams have been proposed since 1934. The differences between these theories essentially occur in the description of the congested phase. Figure 1 shows some of its empirical and theoretical forms, known from literature [Greenshields 1934, Nagel et al. 2003, Ahn et al. 2004, Wang et al. 2005, Artimi 2007, Bassan and Ceder 2007, Kerner 2007, Leclercq 2007, Schönhof and Helbing 2007].

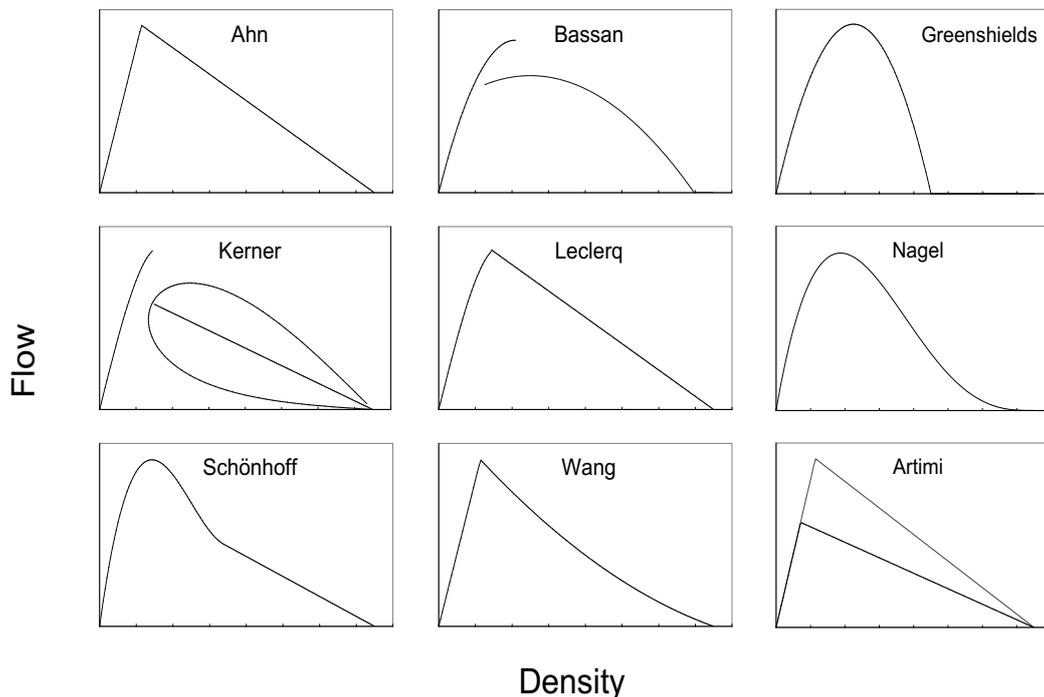


Figure 1. Common  $(k, q)$ -diagrams of motorway traffic literature [Greenshields 1934, Nagel et al. 2003, Ahn et al. 2004, Wang et al. 2005, Artimi 2007, Bassan and Ceder 2007, Kerner 2007, Leclercq 2007, Schönhof and Helbing 2007].

This vast amount of different theories is related to the complexity of observed traffic behaviour. As illustrated in Figure 2, plots of 1-minute mean flow and mean density data show a broad variety of patterns, depending on the location along the road. This variety is thought to be caused by different geometrical factors [Chung et al. 2007], such as noise [Daganzo 2005], from the effect of upstream queuing [Cassidy 1998] or downstream on- and off-ramps or bottlenecks, lane-changing [Laval 2006] and other specific behaviour, depending on the location or the composition of traffic.

Phase diagrams are based on single location data. An alternative way to visualise traffic behaviour is by using multi-location data. An often-used method is the velocity field diagram. These are plots where the mean speed is shown in colours, in a time-space or  $(s, t)$ -diagram.

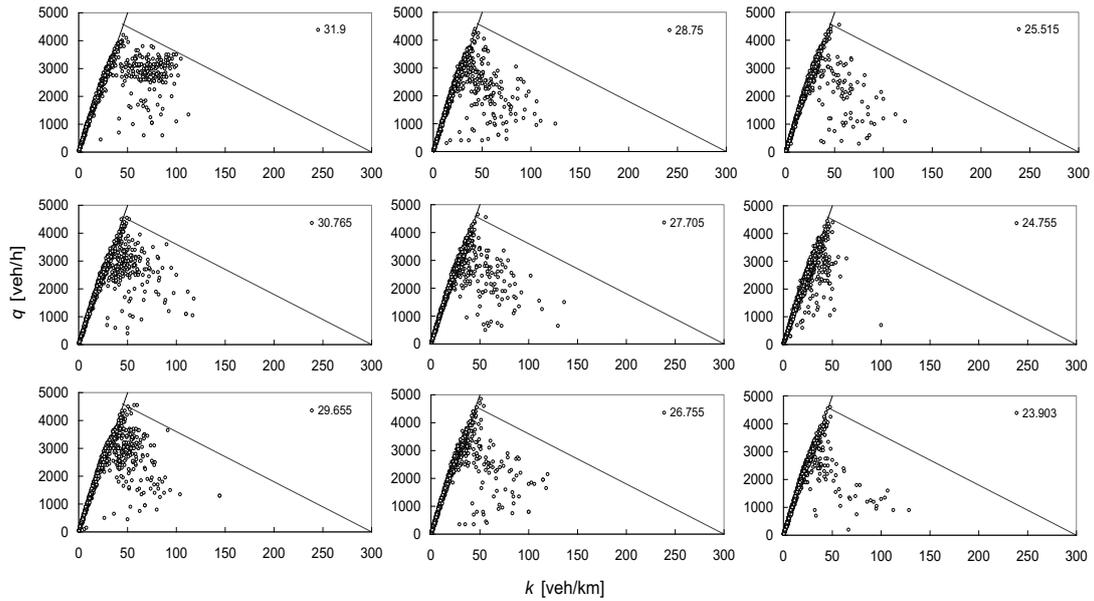


Figure 2. Single location  $(k, q)$ -diagrams of traffic at the Dutch A27 motorway northbound between Knooppunt Hoopolder (km21) and Knooppunt Gorinchem (km36) on 23/5/2007 from 5:45AM to 9:45AM on 23 may 2007, based on 1-minute averaging of speed and flow data according to eqs (7)..(9). The panels correspond to consecutive detection locations along the road, between km31.9, left top row panel, and km 22.9, bottom row right panel.

In a velocity field diagram, congested traffic shows as upstream moving regions of slow traffic, which we will call speed waves. Similar speed waves have been found for American [Bickel et al. 2007], Japanese [Koshi et al. 1983], English [Wang et al. 2005], Dutch [Stipdonk and Postema 2008], German [Bogenberger et al. 2006], and Belgian [Tampère et al. 2007] traffic. Inside the speed waves, the vehicle speed is not uniform. Instead, different regions of constant vehicle speed are represented by parallel strips with the same upstream, negative, wave speed of  $-18 \pm 1$  km/h [Koshi et al. 1983, Kerner 2002, Wang et al. 2005, Bogenberger et al. 2006, Bickel et al. 2007, Schönhof and Helbing 2007, Stipdonk and Postema 2008]. Figure 3 shows a typical example of such a velocity field, taken from a Dutch motorway.

Within speed waves, traffic is repetitiously decelerating, driving slowly and subsequently accelerating, while the corresponding traffic state in the phase diagram moves up and down the right branch. This car following behaviour has been proposed in [Newell 2002]. Figure 4 shows an example of such parallel trajectories and their corresponding velocity field.

The fact that regions with constant vehicle speed all move upstream with the same wave speed is of consequence to the phase diagram. Kinematic wave theory [Logghe and Immers 2008] dictates that such traffic must correspond to a congested phase that appears as a straight line in a  $(q, k)$ -diagram, its slope equal to the upstream speed of the speed waves. Therefore, the observed speed waves should correspond to a straight right branch in a triangular phase diagram as depicted in Figure 1, top left panel, or stated by

$$q = v_{\text{jam}} ( k - k_{\text{jam}} ), \quad (1)$$

where the constant  $k_{jam}$  is the maximum density at zero speed and flow, and  $v_{jam}$ , the negative slope of this line, is the negative wave speed.

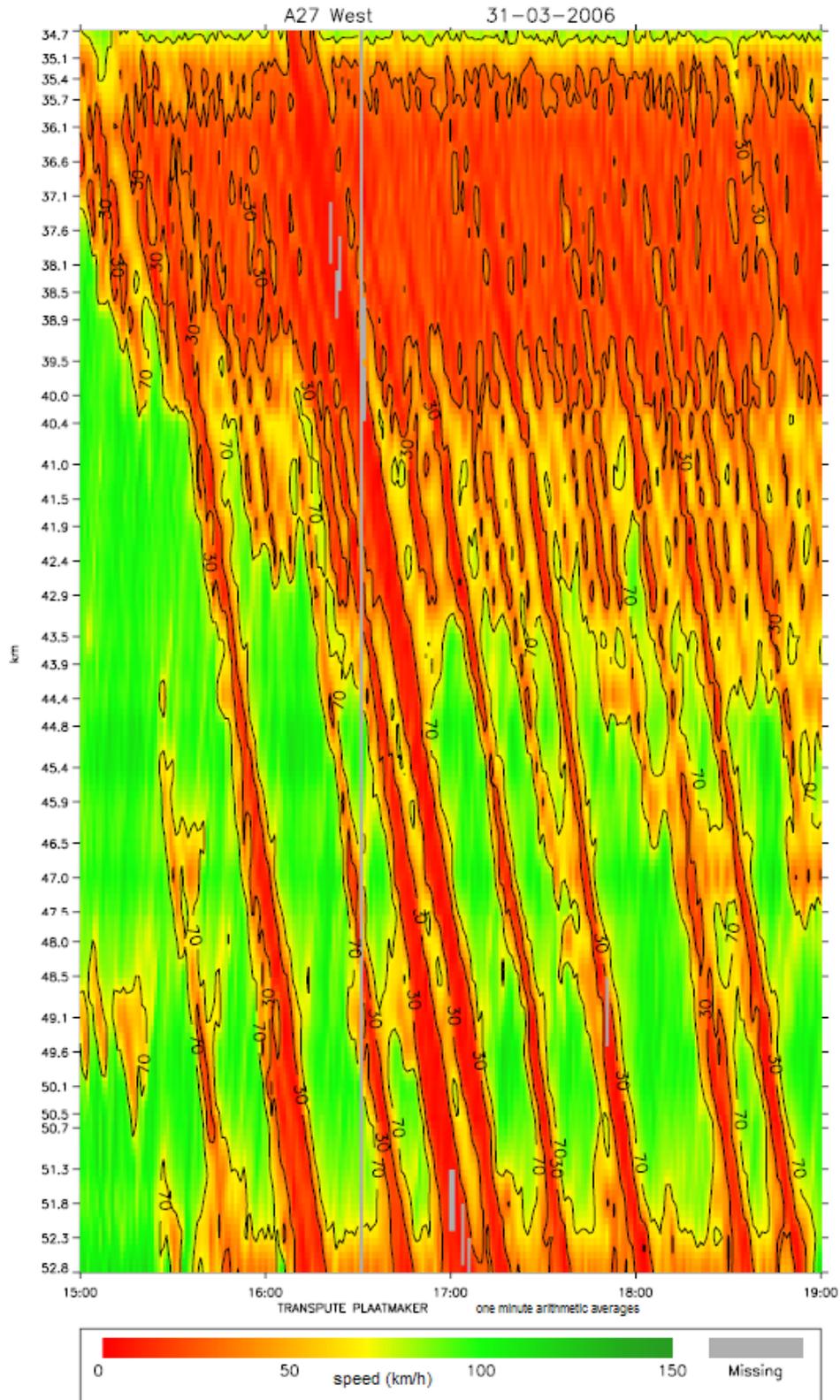


Figure 3. Traffic velocity field measured at the Dutch A27 motorway southbound between Lexmond (km52.8) and Avelingen (km34.7) on Friday 31/3/2006 from 3:00 PM to 7:00PM. Colours correspond to 1-minute arithmetically averaged speeds. Contours denote constant speed of 30 km/h or 70 km/h. [Oostveen and Kijk in de Vegte 2006].

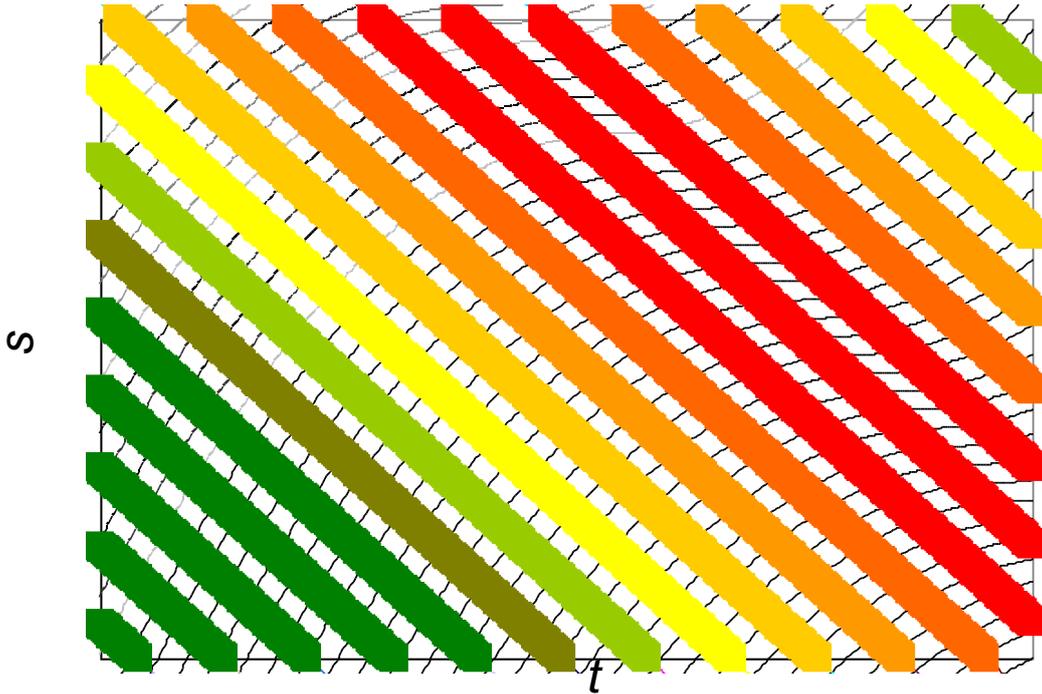


Figure 4. Parallel trajectories, shown as thin curved lines, and their corresponding velocity field, where the colours correspond to those in Figure 3.

Paradoxically, such a straight right branch is seldom observed from double loop detector data. By contrast, for single loop detector data the corresponding flow–occupancy diagrams are found to be triangular, but only if data points all stem from homogeneous traffic [Cassidy 1998].

In [Stipdonk and Postema 2008] we explained the mathematical background underlying this paradox. In this paper, we briefly restate this explanation. After assuming a simple form of congested flow, where waves of only two different speeds occur, we calculate the effect of averaging congested flow. Furthermore, we compare mean speeds as experienced by drivers with the averaged speeds as measured by detectors.

## 2. Measuring traffic parameters

The time headway  $\tau$  of a vehicle is measured using single or double induction loop detectors. The flow of this vehicle follows from

$$q = \tau^{-1}, \quad (2)$$

which is related to the space headway  $\sigma$  of the vehicle through

$$\sigma = v \tau, \quad (3)$$

which follows from simple classic theory of kinematics for bodies moving with constant speed. If speed  $v$  is known, eq. (3) can be used to calculate space headway  $\sigma$ , whereupon the vehicle density is derived from

$$k = \sigma^{-1}. \quad (4)$$

Obviously, flow, speed and density are related through

$$q = v k. \quad (5)$$

With double loop detectors  $v$  is measured directly. With single loop detectors this is not possible. However, from the occupancy  $f$ , defined by the fraction of time the induction loop is occupied, the density is found, assuming that the mean observed vehicle length  $L$  is constant. Occupancy  $f$  and vehicle length  $L$  are related through

$$f = L (L + \sigma)^{-1}. \quad (6)$$

Thus, single loop detectors are used to directly observe an estimate of  $k$ , apart from an unknown constant involving vehicle length. From this mean  $k$ , an estimation of mean  $v$  can be calculated [Bickel et al. 2007].

Eq. (5) holds for individual vehicles that pass a fixed detector at some point along the road. It also holds for mean values of  $q$ ,  $v$  and  $k$  for sets of vehicles, as long as the speed of these vehicles is uniform. If speed is not uniform, mean speed can be consistently defined using eq. (5) and values of mean flow and mean density [Logghe and Immers 2008].

### 3. Averaging traffic parameters

In practice, traffic is treated as a stochastic process, where for individual vehicles  $i$ , values of  $q_i$ ,  $k_i$ , and  $v_i$  have to be averaged to be interpreted as traffic parameters [Rakha et al. 2007]. For  $q$  this is done harmonically, by counting the number of vehicles  $N$  that pass the detector during some fixed time  $T$ , and calculating mean flow  $q_m$  using

$$q_m = N T^{-1} = N (\sum_i q_i^{-1})^{-1} = N (\sum_i \tau_i)^{-1}. \quad (7)$$

Therefore,  $q_m$  is not calculated directly from individual  $q_i$ , but averaged harmonically. Mean speed  $v_m$  is usually calculated arithmetically, using

$$v_m = N^{-1} \sum_i v_i. \quad (8)$$

Finally, the value for  $k_m$  is found using  $q_m$  from eq. (7),  $v_m$  from eq. (8), and their relation from eq. (5):

$$k_m = q_m v_m^{-1}. \quad (9)$$

Arithmetic mean speed is commonly used in practise. In theoretical models, harmonic mean speed  $v_H$  is also used:

$$v_H = (N^{-1} (\sum_i v_i^{-1}))^{-1}. \quad (10)$$

For double loop detectors,  $q_m$  and  $v_m$  are calculated from directly measured  $q_i$  and  $v_i$  using eqs (7) and (8), while mean density  $k_m$  is derived from  $q_m$  and  $v_m$

using eq. (5). However, when single loop detectors are used and  $k$  is derived from occupancy  $f$ , a different value  $k_a$  is calculated using

$$k_a = N^{-1} \sum_i k_i . \quad (11)$$

Under free flow conditions, the differences in  $v_i$  of vehicles that pass a detector pair during one minute, are negligible. The same holds for individual values of  $q_i$ . The effect of averaging under free flow conditions is merely a matter of cancelling out extreme values. In congestion, for example, vehicles can decelerate from 100 km/h to stand still in less than 30 seconds, while flow decreases from values near  $q_{cap}$  to zero. Thus, the effect of averaging traffic quantities in congestion more strongly depends on the actual distribution of individual values of speed and flow than in a free flow situation. Consequently, the calculated values for  $k_m$  from eq. (5) depend on the correlation between  $q$  and  $v$ , given by eq. (1) in the congested phase.

#### 4. Averaging traffic parameters in a simulated congested phase

To study the effect of averaging traffic parameters in the congested phase, we simulate a congested phase where  $q$  depends linearly on  $k$ , following eq. (1). We choose typical values of  $v_{jam} = -18$  km/h and  $k_{jam} = 150$  veh/km/lane. We simulate 1 minute averages.

There are infinitely many ways traffic can accelerate and decelerate, and thus move up and down the congested phase line. We chose a simplified example, in which traffic alternates between two speeds, neglecting the effect of acceleration and deceleration, and assuming instantaneous speed change. We assume vehicles to move in speed waves with constant speed of either  $v_1$  or  $v_2$ . Every vehicle that enters a wave with speed  $v_1$  maintains this speed during an interval  $T^*_1$ , and then changes to speed  $v_2$ , which is maintained during  $T^*_2$ . We call these speed waves the  $v_1$ -wave and the  $v_2$ -wave. The speed waves move upstream with negative speed  $v_{jam}$ . The border between these waves are denoted the  $v_1$ - $v_2$ -transition and the  $v_2$ - $v_1$ -transition.

As a reference to the roadside measurements, we first calculate mean speed as experienced by the drivers. If drivers maintain speed  $v_1$  during  $T^*_1$ , and then speed  $v_2$  during  $T^*_2$ , their mean speed  $v_d$  equals

$$v_d = (T^*_1 v_1 + T^*_2 v_2) (T^*_1 + T^*_2)^{-1}. \quad (12)$$

If  $T^*_1$  and  $T^*_2$  are equal,  $v_d$  is the arithmetic average of  $v_1$  and  $v_2$ . Mean speed, derived from detectors should preferably lead to comparable values as obtained from eq. (11).

The intervals  $T^*_1$  and  $T^*_2$  during which vehicles drive at speeds  $v_1$  and  $v_2$  are shorter than the intervals during which these speeds are measured with the detectors,  $T_1$  and  $T_2$  respectively. This is because, during the passage of a vehicle through a speed wave, the speed wave moves upstream with  $v_{jam}$ . The detector measures traffic with speed  $v_1$  during a time  $T_1$ , which equals

$$T_1 = T^*_1(1 - v_1 v_{jam}^{-1}), \text{ or } T^*_1 = T_1(1 - v_1 v_{jam}^{-1})^{-1}. \quad (13)$$

This can be understood from a hypothetical measurement of a speed wave. We start a measurement at  $t = 0$ , the moment that a  $v_2$ - $v_1$ -transition passes a detector. Thus, speed changes from  $v_2$  to  $v_1$ . The vehicle that at that time passes the detector, accelerates instantaneously to speed  $v_1$ . It maintains this speed during an interval  $T^*_1$ . At  $t_1 = T^*_1$ , the vehicle has travelled a distance  $v_1 T^*_1$ , where it decelerates from  $v_1$  to  $v_2$ . This means that at that moment it traverses through a  $v_1$ - $v_2$ -transition which, from that point will move upstream and reach the detector at time  $t_2$ , where  $t_2 - t_1 = -v_1 T^*_1 v_{jam}^{-1}$ , where  $v_{jam}$  is negative. The detector measures vehicles at speed  $v_1$  between  $t_0$  and  $t_2$ , which equals an interval  $T_1 = t_2 - t_0 = -v_1 T^*_1 v_{jam}^{-1} + v_1 T^*_1$ . Eq. (13) follows from this equality.

As  $v_1$  and  $v_2$  are unequal, this means that if  $T_1$  and  $T_2$  are equal,  $T^*_1$  and  $T^*_2$  are not.

The arithmetic mean speed  $v_m$ , measured during  $T_1 + T_2$  with the detector pair, is calculated by averaging the speeds of every vehicle that passes the detector during  $T_1 + T_2$ . The numbers of vehicles  $N_1$  and  $N_2$  passing in the  $v_1$ -wave and the  $v_2$ -wave are given by

$$\begin{aligned} N_1 &= T_1 q_1; \\ N_2 &= T_2 q_2. \end{aligned} \tag{14}$$

The values of  $q_1$  and  $q_2$  measured by the detector must meet eq. (1), combined with eq. (5). Substitution of eq. (5) into eq. (1) gives

$$q = -v_{jam} v k_{jam} (v - v_{jam})^{-1} \tag{15}$$

With eq. (13),  $q_1$  and  $q_2$  as measured with the detectors, can be calculated. Thus, for the arithmetic mean speed  $v_m$  as measured by the detectors we find

$$v_m = (T_1 q_1 v_1 + T_2 q_2 v_2) (T_1 q_1 + T_2 q_2)^{-1}. \tag{16}$$

The mean flow  $q_m$ , simulated for these two speed waves follows from eq. (7) as

$$q_m = (N_1 + N_2) (T_1 + T_2)^{-1}. \tag{17}$$

Consequently, mean density  $k_m$  follows from  $v_m$ ,  $q_m$  and eq. (9).

In [Stipdonk and Postema 2008] we proposed alternative expressions for mean flow and density, that we denote  $q_a$  and  $k_a$ . We recommend arithmetic averaging of individual flow and density, as opposed to the common reciprocal or harmonic averaging. We suggest

$$\begin{aligned} q_m &= N^{-1} \sum_i q_i; \\ k_m &= N^{-1} \sum_i k_i \end{aligned} \tag{18}$$

for  $N$  consecutive vehicles with index  $i$ . Application of eq. (18) to the  $v_1$ -wave and the  $v_2$ -wave provides alternative values for mean flow  $q_a$  and density  $k_a$ .

$$\begin{aligned} q_a &= (N_1 q_1 + N_2 q_2) (N_1 + N_2)^{-1}; \\ k_a &= (N_1 q_1 v_1^{-1} + N_2 q_2 v_2^{-1}) (N_1 + N_2)^{-1}. \end{aligned} \quad (19)$$

These values lead to points of  $(q_a, k_a)$  on the congested phase line given by eq. (1). Although they are intuitively illogical, they result in a point in the phase diagram that is consistent with the straight congested phase line describing the individual waves.

Analogous to eq. (5), average speed  $v_a$  is found from:

$$v_a = q_a k_a^{-1} = (N_1 q_1 + N_2 q_2) (N_1 q_1 v_1^{-1} + N_2 q_2 v_2^{-1})^{-1} \quad (20)$$

Analogous to eq.(10), harmonic mean speed of the simulated congested phase  $v_h$  is given by

$$v_h = (N_1 v_1^{-1} + N_2 v_2^{-1})^{-1} (N_1 + N_2). \quad (21)$$

## 5. Examples of averaged traffic parameters in the simulated congested phase

In Table 1, we give three examples of averaging congested traffic. Each example consists of alternating fast ( $v_1$ -wave) and slow ( $v_2$ -wave) waves that pass the detector in detector intervals of 30 s ( $T_1$  and  $T_2$  both equal 30 s). The corresponding values of  $T^*_1$  and  $T^*_2$ , the intervals during which individual vehicles drive with  $v_1$  and  $v_2$ , deviate from these detector-intervals, depending on  $v_1$  and  $v_2$ . These values are given, and also the mean speed  $v_d$  as experienced by the driver. Below we give the number of vehicles, the flow and the density of each speed wave as it passes the detector. With these values we give first the common results for mean flow, speed and density, and then the values we proposed in the previous paragraph.

In the first example, we choose a fast wave with  $v_1 = 90$  km/h and a slow wave with  $v_2 = 1$  km/h. This example is to simulate traffic that is (almost) at rest between waves at high speed. As traffic in rest cannot be measured by detectors ( $N_2 = 0$  if  $v_2 = 0$ ), we chose  $v_2 = 1$  km/h instead of  $v_2 = 0$ .

In this example  $v_d$  appears to be only 14 km/h, whereas  $v_m$  at the detector suggests the speed to be 85 km/h. This is because only moving vehicles are counted and measured by the detector. In this example, only 1 slow vehicle is detected, as opposed to 19 fast vehicles. In case of a complete stand still, the measured speed would equal the speed of the fast wave. The slow wave would go unnoticed, except for one very high time headway.

		Units	Example 1		Example 2		Example 3	
Congested phase parameters	$v_{jam}$	km/h	-18		-18		-18	
	$k_{jam}$	veh/km	150		150		150	
Chosen speed wave parameters	$v_1, v_2$	km/h	<b><math>v_1</math>-wave</b>	<b><math>v_2</math>-wave</b>	<b><math>v_1</math>-wave</b>	<b><math>v_2</math>-wave</b>	<b><math>v_1</math>-wave</b>	<b><math>v_2</math>-wave</b>
	$T_1, T_2$	s	90	1	72	3	60	6
Driver interval and speed	$T'_1, T'_2$	s	5	28	6	26	7	23
	$v_d$	km/h	14		16		19	
Calculated speed wave parameters	$N_1, N_2$	veh	19	1	18	3	17	6
	$q_1, q_2$	veh/h	2250	142	2160	386	2077	675
	$k_1, k_2$	veh/km	25	142	30	129	35	113
Common detector averages	$q_m$	veh/h	1196		1273		1376	
	$k_m$	veh/km	14		21		29	
	$v_m$	km/h	85		62		47	
Averages proposed in this paper	$q_a$	veh/h	2125		1891		1733	
	$k_a$	veh/km	32		45		54	
	$v_a$	km/h	66		42		32	
	$v_h$	km/h	14		16		19	

Table 1 Traffic parameters for three examples of simulated congested traffic, consisting of speed waves, with speeds alternating between  $v_1$  and  $v_2$ .

The value for harmonic mean density  $k_m$  as calculated using eq. (9), comes out to be 14 veh/km, which is in strong contrast with the fact that in this example, the corresponding densities of each separate wave are 25 veh/km and 142 veh/km. A mean value of 14 veh/km therefore is misleadingly low, as it is even less than the density in the fast wave. Such a value is suggesting free flow. Also, the resulting point  $(q_m, k_m) = (1196 \text{ veh/h}, 14 \text{ veh/km})$  is far away from the congested phase line given by eq. (5).

Our proposed values for  $(q_a, k_a)$ , *i.e.* (2125 veh/h, 32veh/km) do lie on the congested phase line. These values, as calculated with eq. (18) are the arithmetical averages of the values of  $q$  and  $k$  in each wave, weighted with the number of vehicles detected. Thus, both mean flow and mean density come out much higher than the common value. The corresponding value of  $v_a = 66 \text{ km/h}$  calculated from  $q_a$  and  $k_a$  and eq. (5), is still much higher than  $v_d = 14 \text{ km/h}$ . Thus, for a realistic estimation of travel times,  $v_a$  cannot be used. For that purpose the harmonic speed  $v_H = 14 \text{ km/h}$  happens to be a very good alternative.

The other two examples strengthen the evidence from the first example. In both cases the common traffic parameters  $(q_m, k_m)$  do not lie on the congested phase line, whereas  $(q_a, k_a)$  does. In both cases the resulting densities are extremely low as compared to the densities of each wave separately, even less than the lesser of the two separate values. As for the mean speed: in both examples the harmonic speed equals the driver mean speed exactly. The other values for mean speed are higher, leading to underestimation of travelling time.

It can be shown that the equations of  $v_d$  and  $v_h$  are equivalent, by substituting eq. (14) in (12) to get

$$T^*_1 = T_1 q_1 v_1^{-1} k_{jam}^{-1}, \quad (22)$$

and substitution of eq. (22) in eq. (11) shows that  $v_d = v_h$ .

## 6. Discussion and Conclusions

The common way to average flow, density and speed of motorway traffic generates traffic parameters that obscure the traffic behaviour in congested traffic. When individual vehicles in congested traffic show in the  $(q, k)$  phase diagram as a straight line with negative slope  $v_{jam}$ , their averages do not lie on this line. This effect was shown to occur even in a simple example of simulated congestion. In this simulated example, traffic alternates between two deterministic speed waves of constant speed, both on a straight right branch of the  $(q, k)$ -phase diagram given by eq. (1), where every wave passed the detector in 30 s. This effect of averaging seriously interferes with the interpretation of traffic data.

In reality, traffic is certainly more complex, even if the congested phase were well described by eq. (1). If, for example, the two speed waves would not take 30 s each to pass the detector, but a different time instead, this would lead to averaged traffic parameters that varied strongly in time, suggesting complex dynamical behaviour of points of  $(q_m, k_m)$  in the phase diagram. Also, there is no reason to assume periodicity in the passing of waves with different speed. Moreover, traffic will not jump from one speed to another instantaneously, but instead change speed more or less continuously. A further complexity is that vehicles do not all have a deterministic and identical relation between flow and density, but show a stochastic variation instead. Therefore, mean speed, flow and density are even harder to predict.

However, we have shown that even in our very simplified case of deterministic congested traffic, the  $(q, k)$ -phase diagram of averaged traffic parameters doesn't show the expected triangular shape. Averaged parameters are misleading, even without all possible complicating factors that characterise real traffic.

Current traffic theory is based on measured traffic parameters that are averaged so as to give distorted values during congestion. Even if the congested phase is not conform eq. (1), averaging traffic parameters of accelerating or decelerating traffic give rise to serious deformations of the phase diagram, hampering the correct interpretation of the data and theories of traffic flow. Moreover, the current method to obtain averaged speed results in an overestimation of its actual value as experienced by the drivers. For the purpose of estimating mean speed, harmonic mean speed is an alternative which, in the simulated congested phase described here is shown to be exact.

In countries where averaging is carried out in road computers, traffic management systems and research are based on distorted congestion data.

We recommend that the current way to average traffic parameters be altered or extended with the averages proposed in this paper, if averaging is unavoidable. Furthermore, we recommend the introduction of occupancy  $f$  to measure and average density directly, instead of through speed.

Finally, we recommend that harmonic mean speed of individual vehicles be used to estimate travel times. However, if speed is zero, vehicles either stand still on top of a detector, or next to it. In the former case, an extremely low speed is detected, whereas in the latter case a very long time headway is detected, causing numerical instabilities for the calculated harmonic mean speed. Hence, care should be taken that  $v_H$  computations are corrected for these effects to prevent singularities.

## 7. References

Ahn, S., Cassidy, M. J., and Laval, L. (2004). Verification of a simplified car-following theory, **Transportation Research Part B**, **38** (5): 431-440.

Artimy, M. (2007) Local density estimation and dynamic transmission-range assignment in vehicular ad hoc networks, **IEEE Transactions on Intelligent Transportation Systems**, **8** (3) 400-412.

Bassan, S. and Ceder, A. (2007) Calibrated time-dependent two-regime traffic flow models, **Traffic Engineering and Control**, **48** (5) 228-234.

Bickel, P., Chen, C., Kwon, J., Rice, J., Van Zwet, E., and Varaia, P. (2006) Measuring Traffic, **Statistical Science**, **22** (4), 581-597.

Bogenberger, K., Bertini, R., Boice, S. (2006) Analytische Methoden zur Interpretation von Verkehrsdaten. **Straßenverkehrstechnik**, (9), 515-524.

Cassidy, M. J. (1998) Bivariate relations in nearly stationary highway traffic, **Transportation Research Part B**, **32** (1) 49-59.

Chung, K., Rudjanakanoknad, J., and Cassidy, M. J. (2007) Relation between traffic density and capacity drop at three freeway bottlenecks, **Transportation Research Part B**, **41** (2) 82-95.

Daganzo, C. F. (2005) A variational formulation of kinematic waves: solution methods, **Transportation Research Part B**, **39** (2) 934-950.

Greenshields, B. N. (1934) A study of traffic capacity, **Proceedings of the 14th Annual Meeting of the Highway Research Board**, 448-474.

Kerner, B. S. (2002) Empirical features of congested patterns at highway bottlenecks, **Transportation Research Record** **1802**, 145-154.

Kerner, B. S. (2007) Control of spatiotemporal congested traffic patterns at highway bottlenecks, **IEEE Transactions on Intelligent Transportation Systems**, **8** (2) 308-320.

Koshi, M., Iwasaki, M., and Ohkura, I. (1983) Some findings and an overview on vehicular flow characteristics, **Proceedings of the 8th International Symposium on Transportation and Traffic Flow Theory**, 403-426.

Laval, J. A. and Daganzo, C. F. (2006) Lane-changing in traffic streams, **Transportation Research Part B**, **40** (3) 251-264.

Leclercq, L. (2007) Bounded acceleration close to fixed and moving bottlenecks, **Transportation Research Part B**, **41** (3) 309-319.

Leutzbach, W. (1988) *Introduction to the Theory of Traffic Flow*, Springer-Verlag, Berlin.

Logghe, S. and Immers, L. H. (2008) Multi-class kinematic wave theory of traffic flow, **Transportation Research Part B**, **42** (6) 523-541.

Nagel, K., Wagner, P., and Woesler, R. (2003) Still flowing approaches to traffic flow and traffic jam modelling, **Operations Research**, **51** (5) 681-710.

Newell, G. F. (2002) A simplified car-following model: a lower order model, **Transportation Research Part B**, **36** (3) 195-205.

Oostveen, A. J. and Kijk in de Vegte, N. (2006) *Fileoverzicht rijkswegen Zuid-Holland 2006; Rijkswaterstaat Zuid-Holland, Rotterdam*, Transpute, Amersfoort.

Rakha, H., Pecker, C. C., and Cybis, H. B. B. (2007) Calibration procedure for Gipps' car-following model, **Transportation Research Record** **1999**, 115-127.

Schönhof, M. and Helbing, D. (2007) Empirical features of congested traffic states and their implications for traffic modelling, **Transportation Science**, **41** (2) 135-166.

Stipdonk, H. and Postema, M. (2008) On the congested motorway traffic paradox, submitted.

Tampère, C., Stada, J., and Immers, B. (2007) Een methodiek voor het vaststellen van de kwetsbare wegvakken in een wegennetwerk, **Tijdschrift voor Vervoerswetenschap**, **43** (3) 26-36.

Wang, J., Liu, R., Montgomery, F. (2005) Car-following model for motorway traffic, **Transportation Research Record** **1934**, 33-42.