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The contextual logic

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Abstract

Many researchers in Artificial Intelligence today consider that human reasoning cannot be modelled by “one” formal language. Given its complexity and diversity, observed in the context of work done in cognitive science, it would even be possible that it escapes formal logic. Many convincing arguments have been put forward, and the various failures in this field suffered by logicians during the 20th century reinforce this conclusion. However, failures and arguments are not proof. We show in this article, through a concrete example, that it would be premature to bury the historical ambition of logic. It presents a formalism, noted L_c , which we call contextual logic. This remains strictly and scrupulously within the framework of the syntax of propositional logic. The set of atomic propositions is extended by what we call thoughts. They appear automatically and silently and carry the semantic interpretation function. This is defined by the analysis of the behaviour of the thoughts in the models of the theory. The originality of L_c is to model a totally fallibilistic and perspectivistic reasoning: “Each piece of knowledge is uncertain. Belief is built by aggregating the different justifiable points of view, accepting the possibility that they are incomplete, incorrect, or inconsistent with each other”. In contrast to the principle of proof inherited from mathematics, L_c models that nothing is provable, and adopts the principle of non-refutability to meet the need for decidability. Contextual logic captures notions of inconsistency, predicate, and epistemic modality, accepts abductive and inductive reasoning, and solves belief revision and epistemic rooting problems in a radical way.

Keyword

Artificial Intelligence, Non-monotonic Logic, Belief Revision, Epistemic Rooting, Faillibilism, Perspectivism, Propositional Logic, Paraconsistent Logic, Epistemic Logic, Predicate Logic, Modal Logic

1 Introduction

Formalisms based on the propositional logic L_p classically focus on modelling knowledge that is deemed to be true or false. It is insufficient to model the full diversity and complexity of human reasoning, which obviously also exploits inconsistent or uncertain information.

Another difficulty is to move from a monotonic semantic interpretation (what is supposed to be true at one moment remains true the next moment) to a non-monotonic semantic interpretation (what is supposed to be true at one moment can be false the next moment).

Many propositions have been presented to address these needs: epistemic modal logics, paraconsistent logics,

default logic, intuitionistic logic, adaptive logics, or multivalued logics for example. They each manage to capture different properties. But they have not succeeded in modelling the many modes of reasoning empirically observed in humans (D. Andler [2]).

These formalisms address the problems according to the principle of proof inherited from mathematics. But in the context of Artificial Intelligence, more precisely in the branch of AI which aims to model human reasoning, we explain these failures by postulating that it is strictly based on a principle of non-refutability. To achieve this, we propose to model a faillibilistic reasoning:

We are unable to prove that a piece of knowledge is true. They are all uncertain, and our belief is constructed by identifying those that are justifiable.

Thus, a belief considered justifiable at a given moment can be contradicted and become unjustifiable following the appearance of a new piece of information. We obtain the contextual logic L_c . The exercise leads, mechanically, to a perspectivist reasoning:

Belief is constructed by aggregating some different justifiable points of view, accepting the possibility that they are incomplete, incorrect, or inconsistent with each other.

L_c remains strictly and scrupulously within the monotonic syntax of propositional logic L_p – formulas that are producible at one point in time remain producible, regardless of the new piece of knowledge that arises – and enrich the set of atomic propositions by *silent* propositions. We call them thoughts.

Integrated and consumed automatically, they identify the formulae belonging to the set of knowledge. We use them to define a non-monotonic semantic interpretation function based on the analysis of their behaviour in models of the theory.

We first present the propositional logic to share the vocabulary we use and the associated definitions. The supposed limits of the syntax of L_p are recalled. The principles of L_c and its main properties are described. The assumed limitations of the propositional logic are then revisited.

We show how, while strictly preserving the syntax of L_p , thoughts can be used to model the notions of inconsistent knowledge and predicate. L_c also proposes solutions to the problem of producing a new piece of knowledge by induction and abduction and allows to circumvent the difficulty brought by the exponential complexity of L_p algorithms.

To illustrate our point, we conclude our presentation by developing an example of application of L_c . It calls upon a sufficiently broad knowledge base to demonstrate, through a practical case, the non-monotonic expressiveness of the language and to give meaning to the various technical examples used throughout the text. It shows that belief revision and epistemic rooting are not syntactic problems.

2 The propositional logic

In this article, we refer to several formal languages. We do not detail them in general so as not to make the presentation unnecessarily heavy, inviting the reader to refer to the many documents available on these formalisms.

However, we think it is useful to pause for a moment on the propositional logic L_p . This paragraph does not contain anything new. Its purpose is to share the definitions of the vocabulary and the symbols we use.

The syntax of L_p

The language of the propositional logic L_p is composed of the set P_{L_p} of atomic propositions, the negation connector \neg , the implication connector \rightarrow , and the parenthesis symbols, which are used according to classical mathematical rules. The rules for forming a **well-formed formula** are:

- any atomic proposition is a well-formed formula,
- if f and g are well-formed formulae, then the expressions (f) , $\neg f$ and $f \rightarrow g$ are well-formed formulae,
- a well-formed formula is obtained only by applying the two precedent rules a finite number of times.

Let f , g and h be some well-formed formulae. The following formulae are some **axioms**:

- $f \rightarrow (g \rightarrow f)$
- $(f \rightarrow (g \rightarrow h)) \rightarrow ((f \rightarrow g) \rightarrow (f \rightarrow h))$
- $(\neg f \rightarrow \neg g) \rightarrow ((\neg f \rightarrow g) \rightarrow f)$

These three axioms are sufficient to cover all the axioms of L_p . For example, $f \rightarrow f$ is another axiom, which can be demonstrated from these three using the **theorem formation rules**:

- any axiom is a theorem,
- let f and g be two well-formed formulae. If f and $f \rightarrow g$ are theorems, then g is a theorem (this rule is called the *modus ponens*),
- a theorem can only be obtained by applying the two previous rules a finite number of times.

The statement f is a theorem is denoted $\vdash_{L_p} f$.

A **theory** E_{L_p} is a set of well-formed formulae. The formulae $f \in E_{L_p}$ represent the **hypotheses** of E_{L_p} . A formula f is said **to be provable** in E_{L_p} if, and only if, it can be produced from E_{L_p} by applying the theorem formation rules, for all hypotheses of E_{L_p} behaving as theorems. In this case, f is said a theorem of E_{L_p} , and this is denoted $E_{L_p} \vdash_{L_p} f$.

A theory is said to be **inconsistent** if it produces the negation of a theorem. Otherwise, it is said to be **consistent**.

To simplify the expression of formulae, the language is extended to disjunction (denoted \vee), conjunction (denoted

\wedge) and equivalence (denoted \leftrightarrow) connectors. For f and g two well-formed formulae, they are defined by:

- $f \vee g$ is equivalent to $(\neg f \rightarrow g)$,
- $f \wedge g$ is equivalent to $\neg(f \rightarrow \neg g)$,
- $f \leftrightarrow g$ is equivalent to $(f \rightarrow g) \wedge (g \rightarrow f)$.

A **literal** is an atomic proposition or the negation of an atomic proposition. A **clause** is a disjunction of literals. A formula is said to be in **normal form** if it is a conjunction of clauses.

Any well-formed formula admits a logically equivalent rewriting in normal form (A. Thayse, [23]). For example, the normal form of the formula $((\neg f \rightarrow g) \rightarrow h)$ is $((\neg f \vee h) \wedge (\neg g \vee h))$. P. Siegel [22] proposes a linear complexity process that rewrites any well-formed formula into its normal form.

The semantic of L_p

Classically, the logician's attitude is to consider in formal language only mathematical symbols:

"A formal language is, by definition, a language with only syntax and no semantics" – (translation)

J. Hebenstreit, Encyclopaedia Universalis

What is relevant is the study of the mechanisms and laws of reasoning, modelled by syntactic rules. Any reference to semantic content is discarded. However, it is possible to attribute a meaning to connectors if it is strictly symbolic and univocal.

Let E_{L_p} be a theory of L_p , and f and g two well-formed formulae. The **syntactic interpretation function** of L_p is defined by a function I_{L_p} such that:

- $I_{L_p}(E_{L_p}, f) = \text{true}$ or $I_{L_p}(E_{L_p}, f) = \text{false}$
- If f is a hypothesis of E_{L_p} , then $I_{L_p}(E_{L_p}, f) = \text{true}$

The meaning of the connectors is then defined by:

- $I_{L_p}(E_{L_p}, \neg f) = \text{true}$ if, and only if, $I_{L_p}(E_{L_p}, f) = \text{false}$,
- $I_{L_p}(E_{L_p}, \neg f) = \text{false}$ if, and only if, $I_{L_p}(E_{L_p}, f) = \text{true}$,
- $I_{L_p}(E_{L_p}, f \rightarrow g) = \text{true}$ if, and only if, $I_{L_p}(E_{L_p}, f) = \text{false}$ or $I_{L_p}(E_{L_p}, g) = \text{true}$,
- $I_{L_p}(E_{L_p}, f \rightarrow g) = \text{false}$ if, and only if $I_{L_p}(E_{L_p}, f) = \text{true}$ and $I_{L_p}(E_{L_p}, g) = \text{false}$.

The symbol \models_{L_p} is defined by:

$$E_{L_p} \models_{L_p} f \text{ if, and only if, } I_{L_p}(E_{L_p}, f) = \text{true}$$

The syntactic interpretation of an axiom is always *true*, and L_p is adequate and complete: everything that is produced (using \vdash_{L_p}) is *true* (according to \models_{L_p}), and everything that is *true* is produced:

$$E_{L_p} \models_{L_p} f \text{ if, and only if, } E_{L_p} \vdash_{L_p} f$$

If the constraints of a theory do not allow the *true* or *false* truth value of a formula to be calculated, it is said to have an *unknown* value for this theory. For example, the value of $I_{L_p}(\{a \rightarrow b\}, a)$ is not computable: a can be *true* (in this case, b is *true*) or *false* (in this case, b can be *true* or *false*).

This remark introduces the last definition, with which we close this paragraph. Let E_{L_p} be a theory of L_p . A **model** of E_{L_p} is obtained by associating to each atomic proposition only one truth value (*true* or exclusively *false*) such that the

result verifies the logical constraints expressed by E_{L_p} . E_{L_p} is consistent if it has at least one model. It is inconsistent otherwise.

For example, the theory $\{a \rightarrow b, c\}$ is verified by three models:

$$\begin{aligned} &\{(a, true), (b, true), (c, true)\} \\ &\{(a, false), (b, true), (c, true)\} \\ &\{(a, false), (b, false), (c, true)\} \end{aligned}$$

So, it is consistent. As a counter example, $\{a, \neg a\}$ does not accept a model: if a is assumed to be *true*, $\neg a$ is not verified - and *vice versa*. a is true and false. It is inconsistent.

3 The limits of L_p

Modelling human reasoning with L_p faces several difficulties. We propose to group them into five themes.

1) A difficulty is to move from a monotonic semantic interpretation (what is supposed to be true at one moment remains true the next moment) to a **non-monotonic semantic interpretation** (what is supposed to be true at one moment can be false the next moment). This topic is covered in paragraph 5.

2) Human reasoning sometimes seems incoherent. But syntactic inconsistency leads to the production of everything and its opposite: whatever f and g two well-formed formulae of L_p , $\{f, \neg f\} \vdash_{L_p} g$. This is **the explosion principle**. It forbids the appearance of a syntactic inconsistency in a theory. This topic is covered in paragraph 6.

3) Human reasoning uses **semantic links between propositions**. But the symmetrical behaviour of connectors prohibits this type of modelling in L_p . Put more explicitly with an example, $f \rightarrow (g \rightarrow h)$ is syntactically equivalent to $g \rightarrow (f \rightarrow h)$: f and g have the same behaviour in the formula, and it is not possible to model a privileged relationship between one of them and h . This topic is covered in paragraph 7.

4) Theory distinguishes three modes of reasoning found in humans: **deduction** (establishing a particular law from general facts), **induction** (establishing a general law from particular facts) and **abduction** (identification of the most likely cause of an observed event). Deduction is formalised by the *modus ponens* rule, but the other two modes escape the formalism of L_p . This topic is covered in paragraph 8.

5) The computational algorithms associated with L_p are of **exponential complexity**. In practice, it takes several seconds to deduce knowledge using a base of a few dozen formulae. This is obviously not acceptable in the context of human reasoning, and more generally in the context of Artificial Intelligence. It requires a good level of responsiveness. This topic is covered in paragraph 9.

The successive failures of logic researchers to solve these problems have led many to conclude that the modelling of human reasoning eludes L_p , and probably logical formalisms more generally (D. Andler [2]). We are going to show in the following paragraphs that this

conclusion is hasty: the syntax of propositional logic is sufficient to resolve these questions.

4 The contextual logic

Let us consider a thought. We perceive it in the sense defined by R. Descartes [4]:

“By the name of thought, I understand all that is so much in us that we are immediately aware of it”
(translation)

and we describe it with a set of sentences. However, even if this description were ideally complete and perfect, we are immediately aware that it is not the thought it describes.

We model this observation by distinguishing two notions in the syntax of the language: a unit sign c , which symbolises a thought, and a combination of signs f , which reproduces the sentences that describe it. This leads to the need to define a relationship between c and f .

To this end, we consider the following postulate [11]:

Contextual postulate Let L be a formal language with the functions of syntactic production \vdash_L and of syntactic interpretation \models_L . A well-formed formula f of L is a set of signs that has no meaning. Its meaning is carried by a thought, which is an atomic proposition of L “which is not pronounced”. For c symbolising this thought, the relation between c and f is $c \models_L f$.

The expression $c \models_L f$ asserts neither the thought c nor the sentence f . It models that the sentence f expresses the thought c . c is an atomic proposition which respects the syntactic properties of L .

For example, consider seven sentences f_1, f_2, \dots, f_7 such that:

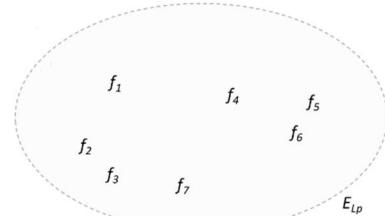


Fig. 1 - A theory of L_p

E_{L_p} admits a syntactic behaviour, but it has no semantic meaning according to the contextual postulate.

To overcome this, we need to consider the thoughts c_1, c_2, \dots, c_7 . $c \models_L f$ is equivalent to $\models_L c \rightarrow f$ if L is L_p . So, after applying the contextual postulate, the set becomes:

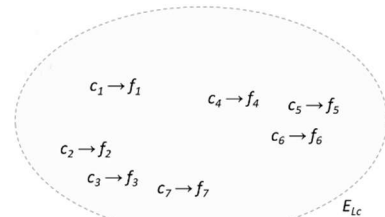


Fig. 2 - A theory of L_c

E_{Lc} models: “each thought c_i is expressed by a sentence f_i ”. We thus agree with L. Wittgenstein when he states [26]:

“We should not say: The complex sign aRb says that a is in the relation R with b , but: That a is in a certain relation R with b says that aRb ”
(translation).

The application of the contextual postulate to a formalism L produces the contextualised logic L . By language convention, we call contextual logic, denoted L_c , the contextualised propositional logic.

Each contextual formula of a theory of L_c takes a form $c \rightarrow f$, for c a thought (and an atomic proposition) and f a well-formed formula in the sense of L_p .

Expressions in L_c accept a natural order:

- an atomic proposition that is not a thought is of rank 0,
- a thought is of rank 1 or higher. We will see later that we propose to automatically handle the assignment of a rank to a thought,
- the rank of a well-formed formula in the sense of L_p is equal to the maximum rank of the atomic propositions (including thoughts) that compose it.

We define a well-formed formula in the sense of L_c to be a formula $c \rightarrow f$, for c a thought of rank n and f a well-formed formula in the sense of L_p of rank m , such that $n > m$. No well-formed expressions are acceptable. They allow for \vdash_{L_p} production and \models_{L_p} syntactic interpretation. However, we only lend them meaningless technical behaviour, which makes them useless in the context of language.

Given the syntax $c \rightarrow f$ of the contextual formulae, the set $\{(c_i, \text{false}), c_i \text{ are the thoughts}\}$ characterises some models that verify any contextual theory. For example, $\{(c_1, \text{false}), (c_2, \text{false}), \dots, (c_7, \text{false})\}$ characterises some models that verify $\{c_1 \rightarrow f_1, c_2 \rightarrow f_2, \dots, c_7 \rightarrow g\}$.

The first consequence is that each contextual theory admits at least one model. So, it is always consistent.

The second consequence is that L_c is unable to produce certainty: any thought is possibly false, and any formula (except for axioms) can be false. In L_c , uncertainty is intrinsically embedded in the syntax. To remedy this problem, we adopt the following principles:

- because any formula can be false, we cannot interrogate a contextual theory with a question such as “Is f true (or false)?”. But we can say: “What can I conclude if I suppose that f is true (or false)?”.

Notation f is called the stimulus. It is possibly empty, and it is denoted S_{Lc} .

- because every thought is possibly false, we propose to relativize the semantic interpretation to the subsets of thoughts identified as *the most relevant*. We cannot conclude that f is true or false, but we can say “ f is true (or false) with respect to the most relevant sets of thoughts”.

For example, let a and b be two atomic propositions of L_p , and c_1 , c_2 and c_3 be three thoughts. Consider the following set:

$$E_{Lc} = \{c_1 \rightarrow a, \\ c_2 \rightarrow \neg a, \\ c_3 \rightarrow b\}$$

We cannot prove that a or b is true or false. But we can say that a is true considering $\{c_1\}$, or that a is false and b is true considering $\{c_2, c_3\}$, etc.

There are many possible combinations, so we should define a method for selecting “the most relevant sets of thoughts”. For this purpose, we need some definitions.

Definitions Let E_{Lc} be a theory of L_c and i and j be 2 integers such that $0 < i \leq j$.

- A set of thoughts is called a **context**.
- A context is said to be **of rank i to j** if all the thoughts in it are of rank i to j . A context of rank i to i is said of rank i .
- A context that is verified by at least one model of E_{Lc} is called a **possible** (or a **consistent**) **context**.
- A context that does not check any model of E_{Lc} is called an **impossible** (or an **inconsistent**) **context**.
- An impossible context is called a **strict impossible context** if each of its strict subsets is possible.
- A possible context that has no strict extension that checks E_{Lc} is called a **maximal context**.
- A possible context is called the **credible context** if it has no join with a strict impossible context and if all its strict extensions have a join with a strict impossible context.

In the following, and in accordance with common practice, we invariably use the notions of conjunction of formulae (for example: $c_1 \wedge c_2$) or of set of formulae (for example: $\{c_1, c_2\}$) to designate the same object. A conjunction of thoughts also means a context.

Example Let a , b and c be three atomic propositions of L_p , and c_1 , c_2 , c_3 , c_4 and c_5 be five thoughts. Consider the following set:

$$E_{Lc} = \{c_1 \rightarrow a, \\ c_2 \rightarrow \neg a, \\ c_3 \rightarrow b, \\ c_4 \rightarrow \neg b, \\ c_5 \rightarrow c\}$$

$c_1 \wedge c_2$ and $c_3 \wedge c_4$ are the only two strict impossible contexts. So, c_5 is the credible context, and there are four maximal contexts: $\{c_1, c_3, c_5\}$, $\{c_1, c_4, c_5\}$, $\{c_2, c_3, c_5\}$ and $\{c_2, c_4, c_5\}$.

We see that, for a given theory, there are possibly several maximal contexts (potentially empty) and a single credible context (potentially empty). They are obtained by calculating the strict impossible contexts in a first step. The different possible combinations of thoughts then produce them.

We use these definitions to define the function that identifies the contexts considered most relevant for semantic interpretation.

Definition Let E_{L_c} be a theory of L_c , S_{L_c} be a stimulus, T_c and T_m be two integers such that $0 < T_c < T_m$, and T be the maximal rank of the thoughts of E_{L_c} . The relevant contexts are defined as follows:

- calculation of the **maximal epistemic contexts**: if $T < T_m$ then there is an empty maximal epistemic context, else calculation on $\{E_{L_c}, S_{L_c}\}$ of the maximal contexts of rank T_m to T ,
- then enrichment of each maximal epistemic context C , by the credible context of rank T_c to T_m-1 on $\{E_{L_c}, S_{L_c}, C\}$.

This defines the set of **epistemic contexts**. It is denoted C_{EL_c, SL_c, T_c, T_m} .

This definition presents the notion of epistemic contexts. They are the *most relevant sets of thoughts*, which meets the need we identified earlier. Other definitions are possible, for example by using the ranks of thoughts more finely. Epistemic contexts are sufficient for the modelling needs presented in this article.

We are now able to define the semantic interpretation function of L_c .

Definition Let E_{L_c} be a theory, S_{L_c} be a stimulus and T_c and T_m be two integers such that $0 < T_c < T_m$. Considering E_{L_c}, S_{L_c}, T_c and T_m , a sentence f , called a **piece of belief**, is said:

- **conceivable** if there is at least one epistemic context C_1 such that $\{E_{L_c}, S_{L_c}, C_1\} \models_{L_p} f$ and there is at least one epistemic context C_2 such that $\{E_{L_c}, S_{L_c}, C_2\} \models_{L_p} \neg f$,
- **credible** if there is at least one epistemic context C_1 such that $\{E_{L_c}, S_{L_c}, C_1\} \models_{L_p} f$ and there is no epistemic context C_2 such that $\{E_{L_c}, S_{L_c}, C_2\} \models_{L_p} \neg f$,
- **improbable** if there is at least one epistemic context C_1 such that $\{E_{L_c}, S_{L_c}, C_1\} \models_{L_p} \neg f$ and there is no epistemic context C_2 such that $\{E_{L_c}, S_{L_c}, C_2\} \models_{L_p} f$,
- **not interpretable** in other cases.

$\{E_{L_c}, S_{L_c}, C \in C_{EL_c, SL_c, T_c, T_m}\}$ is said a **semantic perspective**.

This definition presents the basic semantic interpretation function of L_c . It can be enriched, for example by distinguishing true formulae in all semantic perspectives. This version is sufficient for the modelling needs presented in this article.

Example Let a and b be two atomic propositions of L_p , c_1 and c_2 be two thoughts of rank 1, and c_3 and c_4 be two thoughts of rank 2. Consider the following set:

$$E_{L_c} = \{c_1 \rightarrow a, \\ c_2 \rightarrow \neg b, \\ c_3 \rightarrow c_1, \\ c_4 \rightarrow \neg c_1\}$$

Let $T_c=1$ and $T_m=2$, and we consider the stimulus is empty. $\{c_3, c_4\}$ is incoherent, so $\{c_3\}$ and $\{c_4\}$ are the two maximal contexts of rank 2. Let's add to each the credible context of rank 1 associated with it to calculate the two epistemic contexts. We obtain:

- $\{c_3, c_1, c_2\}$. The associated semantic perspective says that a is true and b is false,

- $\{c_4, c_2\}$. The associated semantic perspective says that b is false.

So, according to the semantic vocabulary of L_c , a is credible and b is improbable.

The semantic interpretation function has a mathematical definition and is therefore rigorous. This is not compatible with our natural language habits. Therefore, we will allow ourselves some linguistic shortcuts, for example:

- a context is said to be a belief or a piece of knowledge, and *vice versa*,
- a semantic perspective is said to be a perspective,
- a conceivable expression is said to be true and false, or possible,
- a credible expression is said to be true, conceivable, or possible,
- and an improbable expression is said to be false, conceivable, incredible, or impossible.

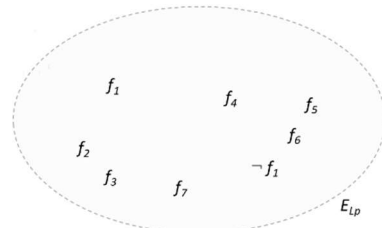
We will use them in a way that does not create confusion.

5 The properties of L_c

We now present the properties of L_c . We remain on a technical observation and not discuss their relevance.

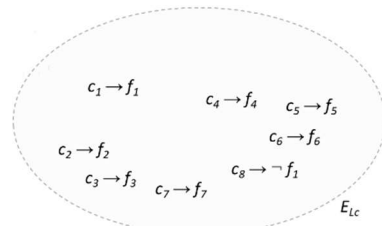
Indeed, each property echoes philosophical concepts and deserves a dedicated article. The debates are rich, and there are as many defenders as detractors. The interested reader will easily find in-depth presentations of these topics in the literature. We do not bring new philosophical elements to enrich these exchanges – only a few practical findings that we will share in paragraphs 6 to 9.

To facilitate the understanding of what is to come, we propose to illustrate the principles of L_c with some small diagrams. Consider the following set:



Pic. 3 - A syntactically inconsistent theory of L_b

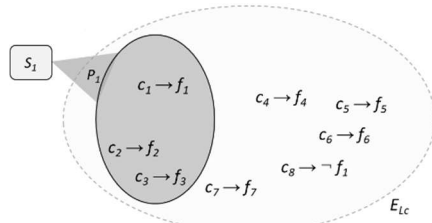
It is syntactically inconsistent, and it has no semantic meaning according to the contextual postulate. Let us apply it by enriching the set of atomic propositions with the thoughts $c_1, c_2, c_3, c_4, c_5, c_6, c_7$ and c_8 such that:



Pic. 4 - A semantically inconsistent theory of L_c

The resulting set E_{L_c} is a kind of dictionary of thoughts: each thought c_i is expressed by a formula f_i . It is syntactically consistent even though the definitions may be semantically inconsistent with each other. This result is obtained at the cost of an absolute uncertainty: all thoughts are potentially false.

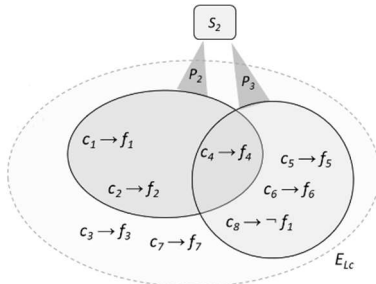
To analyse a situation (formalised by a stimulus), L_c identifies the set of relevant thoughts concerning it.



Pic. 5 – The semantic interpretation of the stimulus S_1

In this example (presented for illustrative purposes only), the stimulus S_1 generates the semantic perspective P_1 . Its epistemic context is $\{c_1, c_2, c_3\}$. c_1, c_2 and c_3 are the most relevant thoughts considering S_1 .

Assume another stimulus S_2 :



Pic. 6 – The semantic interpretation of the stimulus S_2

S_2 is seen through two mutually incoherent semantic perspectives P_2 and P_3 .

The syntax production function is monotonic

L_c respects the syntax of the propositional logic and is therefore syntactically monotonic: whatever f and g are contextually well-formed formulae, if a theory E_{L_c} produces f then $\{E_{L_c}, g\}$ produces f . Note that the syntactic interpretation function is mechanically also monotonic.

The semantic interpretation function is non-monotonic

L_c decorrelates the syntactic interpretation function from the semantic interpretation function. Syntax produces a set of formulae according to the rules of the contextualised formalism. Semantics then provides an interpretation by analysing the behaviour of the thoughts in the models of the theory. They are considered as they are produced by the syntax of L_p . We do not employ the concept of extension sometimes used by non-classical formalisms.

The models of a theory can change if a new piece of knowledge is introduced. So, $C_{E_{L_c}, T_c, T_m}$ must be recalculated in this case, and L_c has a non-monotonic semantic: considering the same stimulus S_{L_c} , a formula f can be credible considering $\{E_{L_c}, S_{L_c}\}$ and incredible

considering $\{E_{L_c}, g\}$, for g a new piece of knowledge. We present some examples of use in the following paragraphs.

The formalism adopts a de dicto behaviour

De dicto and *de re* are two locutions that distinguish two modalities of statements and the reasoning behind them. *De dicto* means in Latin *about what is said*, and *de re* means *about the fact*. In L_c , a piece of knowledge ($c \rightarrow f$) does not express a fact, but models that the thought c is expressed by the sentence f .

Contextual logic thus distinguishes between the thought c , which is the whole that one wishes to express, and the sentence f , which is the way it is said. The reasoning then exploits the logical relations that appear in the sentence which is said.

The semantic is faillibilistic

Non-monotonicity is a matter of completeness of knowledge: a belief that is true in one state may be false in an enriched state. Faillibilism (K. Popper [18]) is a more radical philosophical principle. It assumes that absolute knowledge is impossible: all belief can, at any time, be questioned – and possibly contradicted.

The syntax of L_c is that of L_p . It is therefore based on axiomatic principles which it considers as absolute. However, a consequence of the contextual postulate is that every proposition (which is not an axiom) is possibly false. L_c thus proposes the paradox of relying on a syntax considered as absolutely certain to model knowledge interpreted semantically as absolutely uncertain.

To avoid this, the solution is to consider that what is not explicitly false is credible and will remain so until it is explicitly contradicted or challenged. We illustrate this with some examples which we develop in the following paragraphs.

The semantic is perspectivistic

Perspectivism (F. Kaulbach [10]) refers to philosophical doctrines that defend the idea that our perception of reality is composed of the sum of the perspectives we have on it.

In L_c , the semantic interpretation is obtained by considering the interpretations, possibly contradictory, of each epistemic context: belief is not the consequence of a global point of view built on the whole of thoughts, but the juxtaposition of several points of view built from distinct subsets of thoughts each considered to be relevant.

Atomic propositions of L_p are attributes and not assertions

In the most adopted mathematical approach, an atomic proposition is an assertion apprehended in its content. Considering a theory, its semantic interpretation admits a truth value: it is true or false (or possibly another value in the case of multi-valued formalisms).

In L_c , it is not possible to deduce that a proposition is true or false according to L_p . A proposition (or a formula) can only be interpreted in relation to a set of thoughts, called a context. *It characterises it*. This is a mechanical consequence of the application of the contextual postulate.

So, in L_c , a proposition is not an assertion in the strict sense of the term. It must be understood as a characteristic, or an attribute, of the context or of the stimulus.

Consider, for example, the sentence:

“If Tweety is a bird, then it flies”.

Its modelling in predicate logic can be:

$$\text{Bird}(\text{Tweety}) \rightarrow \text{Fly}(\text{Tweety})$$

In L_c , this assertion is modelled by:

$$\{c_1 \rightarrow (\text{Tweety} \rightarrow \text{Bird}), c_2 \rightarrow (\text{Bird} \rightarrow \text{Fly})\}$$

which allows for several readings - for example: attributes *Bird* and *Fly* are attributes of the stimulus *Tweety* if we consider the context $\{c_1, c_2\}$.

Reasoning is introspective

We end this paragraph with probably the most important property of the contextual logic. By distinguishing expression and thought, and by modelling a relationship between them, the contextual postulate brings a capacity for introspective reasoning to the formal language: the thoughts can reason about themselves using the constraints carried by the sentences that express them.

It thus brings formal language closer to natural language: formalism is used as a means of expression. The properties carried by the syntax of language model the reasoning mechanisms of thought (J. Fodor [6]). They generate, by opportunity, the ability to reason about what is said.

We have finished with the presentation of L_c . The following sections show how to use its properties to provide answers to the four last difficulties identified in paragraph 3 (the first being covered as we have just observed).

Considering the definition of epistemic contexts, T_c and T_m can theoretically take any value. According to the work of J. Pitrat [17], there are probably cognitive thresholds limiting the capacities of human reasoning (see paragraph 9). In the rest of this document, we use the thresholds 2 and 3, which are sufficient to cover the expected level of expressiveness expected in this article.

And by writing convention, we now note c_{ij} the thoughts. i singularizes the atomic proposition and j indicates its rank.

6 Modelling an inconsistent information

For example, consider a set of L_p 's propositions $\{a, b, c\}$ and let be the following set:

$$E_{L_p} = \{a \rightarrow b, \\ a \rightarrow \neg b, \\ c, \\ a\}$$

It is inconsistent because $E_{L_p} \vdash_{L_p} b \wedge \neg b$. According to the explosion principle, whatever f a well-formed formula, $E_{L_p} \vdash_{L_p} f$. This is not acceptable.

E_{L_p} has no meaning according to the contextual postulate. Let us now place ourselves in the contextual logic framework. Considering the set of thoughts $\{c_{10,2}, c_{20,2}, c_{30,2}, c_{40,2}\}$, we assume the following theory:

$$E_{L_c} = \{c_{10,2} \rightarrow (a \rightarrow b), \\ c_{20,2} \rightarrow (a \rightarrow \neg b), \\ c_{30,2} \rightarrow c, \\ c_{40,2} \rightarrow a\}$$

Note that thoughts name formulas, which will allow us to carry out introspective reasoning. E_{L_c} is consistent, and there is an incoherence between the three thoughts $c_{10,2}$, $c_{20,2}$, and $c_{40,2}$ because:

$$\{E_{L_c}, c_{10,2}, c_{20,2}, c_{40,2}\} \vdash_{L_p} b \wedge \neg b$$

$\{c_{10,2}, c_{20,2}, c_{40,2}\}$ is a strict impossible context. So, $\{c_{30,2}\}$ is the only epistemic context. If the stimulus is empty, we obtain one perspective which says $\{c\}$, and $\{a, c\}$ if the stimulus is $\{a\}$.

This is a first result showing the possibility of exploiting inconsistent beliefs in L_c . The solution is to get around the problem by considering that the thoughts $c_{10,2}$, $c_{20,2}$ and $c_{40,2}$ are not credible because they produce an inconsistency.

We now want to address this inconsistency, by modelling that $a \rightarrow b$ (i.e., the thought $c_{10,2}$) is not always true - or, put differently, is sometimes true and sometimes false. Let's use two new thoughts, $c_{11,3}$ and $c_{12,3}$:

$$E_{L_c} = \{c_{10,2} \rightarrow (a \rightarrow b), \\ c_{11,3} \rightarrow c_{10,2}, \\ c_{12,3} \rightarrow \neg c_{10,2}, \\ c_{20,2} \rightarrow (a \rightarrow \neg b), \\ c_{30,2} \rightarrow c, \\ c_{40,2} \rightarrow a\}$$

Considering a is the stimulus, let's calculate the epistemic contexts. There are 2 maximum contexts of rank 3: $\{c_{11,3}\}$ and $\{c_{12,3}\}$. Let us extend each of them to their associated credible contexts of rank 2:

- considering $\{E_{L_c}, a, c_{11,3}\}$, $\{c_{20,2}, c_{40,2}\}$ is the only strict impossible context, so $\{c_{10,2}, c_{30,2}\}$ is the credible context of the rank 2 in this case,
- considering $\{E_{L_c}, a, c_{12,3}\}$, $\{c_{10,2}\}$ is the only strict impossible context, and $\{c_{20,2}, c_{30,2}, c_{40,2}\}$ is the credible context in this.

In fine, considering the stimulus $\{a\}$, we obtain two epistemic contexts:

- $\{c_{11,3}, c_{10,2}, c_{30,2}\}$ which says $\{a, b, c\}$ is true,
- $\{c_{12,3}, c_{20,2}, c_{30,2}, c_{40,2}\}$ which says $\{a, \neg b, c\}$ is true.

Taking a as the stimulus, we conclude that c is credible (or true), and that b is conceivable (or true and false). The formalism does this by modelling an epistemic information: the belief $a \rightarrow b$ (i.e., $c_{10,2}$) is true (what $c_{11,3}$ formalises) and false (what $c_{12,3}$ formalises).

We have used a single contradiction $\{c_{11,3}, c_{12,3}\}$ to illustrate our point. If multiple contradictions (two contradictions $\{c_{x1,3}, c_{x2,3}\}$ and $\{c_{y1,3}, c_{y2,3}\}$ for example), the different cases are managed on the maximal epistemic contexts ($\{c_{x1,3}, c_{y1,3}\}$, $\{c_{x1,3}, c_{y2,3}\}$, $\{c_{x2,3}, c_{y1,3}\}$, and $\{c_{x2,3}, c_{y2,3}\}$ with the example).

We obtain by combination the set of relevant perspectives. This is illustrated in the example that we develop at the end of this article.

Comparison with other formalisms

In this section, we point out the major gaps in the treatment of inconsistent or incomplete information between L_c and other non-classical formalisms.

The first and, from our point of view, the main difference is that contextual logic strictly respects the syntax of propositional logic. Contrary to what is commonly shared, it is not necessary to add new connectors or to modify the syntactic rules of L_p to model a notion of inconsistency. But there is not just this gap.

Paraconsistent and multivalued logics aim to tolerate inconsistencies by escaping the principle of explosion. The approach, theorised by J. Lukasiewicz [13], is either to weaken Aristotle's principles to limit the inferential capacities of language or to add a third truth value to indicate that the piece of knowledge concerned is both true and false.

L_c addresses the issue of uncertainty and inconsistency through its perspectivistic property: it models that something is simultaneously true according to some thoughts and false according to others. L_c is therefore not a paraconsistent or a multivalued formalism: it strictly preserves the syntax of L_p . Therefore, it does not escape the principle of explosion. If one retains a reference context that syntactically produces $f \wedge \neg f$, then it produces any belief g whatsoever.

Inconsistency is accepted in the semantic interpretation of L_c . It remains non-tolerable in its syntax.

Another difference between L_c and other non-classical formalisms is its faillibilistic property: noting that there is no certainty, it takes as credible what is not explicitly false. In other words, it takes as true everything that is possible and not clearly impossible. This property gives L_c a particular behaviour, which does not allow it to fully capture modal logics or default logics, for example.

Default logic is proposed by R. Reiter [20]. To reason with uncertain information, he extends production rules by expressions of the form $(a : b / c)$ which read:

If a is true and if b is possible then c is produced

In L_c , the thought that b is possible “generates the thought b ”. However, related to R. Reiter's syntax, L_c 's expressiveness is limited to normal default rules, of the form $(a : c / c)$ [11] (the link with a normal default is that “ b is included in c ”).

Modal epistemic logics extend the expressiveness of languages by adding a new connector for reasoning about the quality of the interpretative value. The most widely used epistemic modal connector is the alethic connector \Box . $\Box f$ usually expresses that f is necessary, and its dual $\neg\Box\neg f$, denoted $\Diamond f$, that f is possible. The language relies on the semantics of possible worlds of S.A. Kripke [12] to benefit from a syntactic interpretation function of \Box .

We have proposed a relationship between modal epistemic logics and L_c [11]. This requires an evolution of the definition of epistemic context, using ranks to capture the imbrications of the monadic connector (rank i for \Box , rank $i+1$ for $\Box\Box$, etc.). It models the sets $\{\Box f\}$ and $\{\Diamond f, \Diamond\neg f\}$, but the set restricted to $\{\Diamond f\}$ is interpreted as

$\{\Box f\}$: f is considered necessary if the possibility of its opposite is not explicitly expressed.

The behaviour of L_c is equivalent to adding a default rule to the K system: $\{\Diamond f : \Box f / \Box f\}$. In the framework of Kripke's semantics, this expression can be understood as:

If I know an accessible world in which f is true, and if I do not know an accessible world in which $\neg f$ is true, then I consider that f is true in all accessible worlds

Paraconsistent, default and modal epistemic logics deal with the issue of incoherence by evolving the syntactic capabilities of the language. D. Batens proposes another approach [3]. He considers that there are several reasoning strategies, and that the solution consists in choosing the one that is best adapted to the situation. These are adaptive logics. For example, let be the following set of formulae:

$$E_{La} = \{\neg p, \\ \neg q, \\ p \vee q, \\ p \vee r, \\ q \vee r\}$$

It is incoherent, and therefore explosive in the context of propositional logic because $\{\neg p, \neg q\}$ contradicts $p \vee q$. If one adopts a strategy favouring reliable reasoning, it is not possible to deduce r : it would be unwise to conclude anything using the first three formulae. However, if we choose a strategy that minimises abnormalities, and assume that at least two of the first three formulae are true, then r is produced.

In contextual logic, the set becomes:

$$E_{Lc} = \{c_{1,2} \rightarrow \neg p, \\ c_{2,2} \rightarrow \neg q, \\ c_{3,2} \rightarrow p \vee q, \\ c_{4,2} \rightarrow p \vee r, \\ c_{5,2} \rightarrow q \vee r\}$$

Assume that the stimulus is empty. $\{c_{4,2}, c_{5,2}\}$ is the reference context because $\{c_{1,2}, c_{2,2}, c_{3,2}\}$ is a minimal impossible context. As far as we know, r is not interpretable. Using epistemic contexts that retain the maximal credible contexts at rank 2 is therefore a prudent strategy.

So, L_c is not an adaptive logic. Both formalisms have the capacity to adapt their semantic interpretation to local characteristics: L_c chooses to use or not a thought depending on the stimulus. But its principle is not to adapt its reasoning according to the typology of the situation. It uses a unique analysis strategy, based on the definition of epistemic contexts.

We end this comparative section with the circumscription logic of J. McCarthy [14]. It consists in extending the set of atomic propositions by some atomic propositions that indicate the epistemic character of a formula. For example, “ $a \rightarrow b$ is true except in atypical cases” is modelled by $((a \rightarrow b) \vee abnormal)$.

The atomic proposition *abnormal* carries the exceptional behaviours when needed. The models of the

theory are then analysed to select those that minimise the abnormalities.

This approach, which consists in seeking a solution by enriching the set of atomic propositions and then analysing the models of the theory, is most certainly the closest to ours. We have shown that contextual logic can capture its expressive capacity by adapting the definition of epistemic contexts to meet the minimality criterion [11].

However, beyond this result, the choice to minimise abnormalities seems reasonable but can easily be questioned with use cases. This difficulty is shared with adaptive logics, or more generally with the concept of epistemic rooting proposed by P. Gardenfors and D. Makinson [7].

Indeed, these methods suppose the existence of an order relation (on pieces of knowledge or on reasoning strategies) which would oversee selecting the information in case of incoherence. In L_c , syntactic consistency is guaranteed. It is therefore not necessary to manage this in the formalism.

7 Modelling a predicate information

L_p sees a proposition as a whole, which is given a universal value. It is necessary to decompose this whole when we wish to use a singular value. To this end, predicate logic meets this need by allowing the desired relationship to be modelled directly in the elementary proposition.

It then becomes possible to model that *Socrates is a man*, and to deduce that *Socrates is mortal* because *a man is mortal*:

$$\begin{aligned} & \text{Man}(\text{Socrates}) \\ & \text{Man}(\text{Socrates}) \rightarrow \text{Mortal}(\text{Socrates}) \end{aligned}$$

This syllogism uses the link between *Man* and *Socrates* to deduce the association with *Mortal*. In this context, G. Frege [5] theorised the notion of universal quantifier.

As a classical example of use:

$$\forall x \text{Man}(x) \rightarrow \text{Mortal}(x)$$

which reads: whatever x is, if x is a man then x is mortal. We note two enrichments with respect to the native modelling capabilities of the propositional logic:

- the notion of the universal quantifier \forall . We will come back to the subject of universal connectors at the paragraph 11,
- the possibility of breaking down an atomic proposition of L_p into several singular instances. In our example, the proposition *he is a man* is modelled by using two distinct units, *Man* and x . The expression $\text{Man}(x)$ creates a syntactic relationship, which formalises a semantic link, between these two-unit elements. x then allows to create a semantic link between the two propositions $\text{Man}(x)$ and $\text{Mortal}(x)$.

We have seen in paragraph 3 that it is not possible to model a relation between two atomic propositions in L_p because of the symmetric behaviour of connectors. We now present how this point can be solved in L_c .

Suppose in L_p the set of propositions $\{a, b, c, d, e, f\}$ and the following theory:

$$E_{L_p} = \{a \rightarrow b, \\ c \rightarrow d, \\ a \rightarrow \neg c, \\ c \rightarrow (a \rightarrow e), \\ c \rightarrow (a \rightarrow f)\}$$

We want to model that $a \rightarrow e$ is a predicate of c – i.e., c is the subject of $a \rightarrow e$. The problem is that the formula $c \rightarrow (a \rightarrow e)$ is syntactically equivalent to $a \rightarrow (c \rightarrow e)$. Now, consider the following set in L_c :

$$E_{L_c} = \{c_{10,2} \rightarrow (a \rightarrow b), \\ c_{20,2} \rightarrow (c \rightarrow d), \\ c_{30,2} \rightarrow (a \rightarrow \neg c), \\ c_{40,2} \rightarrow (c \rightarrow c_{41,1}), \\ c_{41,1} \rightarrow (a \rightarrow e), \\ c_{50,2} \rightarrow (c \rightarrow (a \rightarrow f))\}$$

$c_{40,2}$ and $c_{41,1}$ model the predicative piece of knowledge. They break the syntactic symmetry and introduce two new pieces of information: the predicate is syntactically distinguished by the thought $c_{41,1}$, and $c_{40,2}$ says that $c_{41,1}$ is true if the subject c is true. It is important to note that $c_{41,1}$ is of rank 1, so, by definition, it does not participate in the calculation of epistemic contexts. It only appears later, in the contexts in which $c_{40,2}$ is true.

Let $c \wedge a$ be the stimulus. $\{c_{10,2}, c_{20,2}, c_{40,2}, c_{50,2}\}$ is its epistemic context. The perspective associated says $\{c, a, b, d, e, f, c_{41,1}\}$. The presence of $c_{41,1}$ indicates that $a \rightarrow e$ is a predicate, but the relationship with its subject c is not apparent. It can be found by applying the following method:

- calculate the perspectives of the stimulus c . It says $\{c, \neg a, d, c_{41,1}\}$. The predicate $c_{41,1}$ is obtained by $c_{40,2}$,
- calculate the perspectives of the stimulus a . It says $\{a, \neg c, b\}$. The thought $c_{41,1}$ is not syntactically produced,
- apply the predicates associated with each perspective to the other perspectives. This produces $\{e\}$ by applying $c_{41,1}$ to the perspective of a . e is associated with the stimulus c associated with the applied predicate,
- and finally, calculate the perspectives of the stimulus $c \wedge a$. This produces $\{f\}$.

If the thought of a predicate appears in a perspective, then it expresses that its subject is its stimulus. Unlike in predicate logic, the notion of predicate is not carried by the syntax of L_c . It is interpreted from the semantics of perspectives. This method extends the consumption of epistemic contexts by a recursive function:

The semantics of a complex universe is obtained by crossing the semantics of the objects that compose it.

The semantics of a universe composed of three objects A , B and C can only be partially obtained by analysing the perspectives of $A \wedge B \wedge C$. To obtain a complete perception, we must analyse $\{A\}$, $\{B\}$, $\{C\}$, $\{A, B\}$, $\{A, C\}$, $\{B, C\}$ and $\{A, B, C\}$ separately, and cross-reference the properties associated with these seven different objects.

We call this method the **generalised contextual semantics**. It allows exhaustively capturing the characteristics of each object, of each combination of objects, and to calculate cross-predictive inferences. It seems to produce redundancies. We have not studied whether technical optimizations are possible.

8 Modelling the imagination

There is a long established and commonly accepted point that contextual logic visualises well. Consider the following set of knowledge:

$$E_{L_c} = \{c_1 \rightarrow a, \\ c_2 \rightarrow (a \rightarrow b)\}$$

The application of the *modus ponens* leads to the production of the formula $(c_1 \wedge c_2) \rightarrow b$. We see that the “theorem” b is not a new piece of knowledge. L_c formalizes and preserves the information that b is the result of two existing thoughts. It is the consequence of its introspective capacity to reason.

In fact, the *modus ponens* rule is incapable of producing anything new. This finding sums up the limit of the principle of proof: it only generates what is already known. This is the essential condition to guarantee the maintenance of the syntactic coherence of a theory of L_p – and, more generally, of a mathematical theory.

This observation led early on to the need to identify the mechanics of human imagination. Reflections on this subject resulted in the theorization of two concepts: abduction and induction.

Abduction (identification of the most likely cause of an observed event) has been formalised by C.S. Peirce [24]. In *common sense* logic, it can be expressed as follows:

If $a \rightarrow b$ and b are both true and if a is possible then a is true

For example:

If I see something flying and if it could be a bird, then I guess it is a bird because birds fly”.

This conclusion is often true, but it can sometimes be false.

Induction (establishing a general law from particular facts) has been identified and studied in a logical approach since Aristotle. In *common sense* logic, it can be expressed as follows:

If $a \rightarrow b$ and $a \rightarrow c$ are both true then $b \rightarrow c$ is true.

For example:

If it rains, then there are clouds, and if it rains, then the sky is grey, so, if there are clouds, then the sky is grey.

This production rule seems interesting. Unfortunately, Aristotle showed that this is not correct and can lead to false conclusions:

The donkey, the mule and the horse live long; they are animals without gall; therefore, animals without gall live long”.

In fact, any inductive or abductive production is potentially false. It's a direct consequence of the imagination. The

explosion principle makes it impossible to model them in the context of L_p .

In L_c , syntactic consistency is guaranteed as soon as the new piece of knowledge is associated with a new thought (see pictures 3 and 4), even if it is completely meaningless. This solves the problem, and we propose the following principles:

Definitions

Production by abduction *If b is the stimulus and is credible, if $a \rightarrow b$ is a belief that describes a thought belonging to an epistemic context in which b is true, and if a is not improbable or conceivable, then a new thought $c_{i,2}$ expressed by a is produced.*

Production by induction *If a is the stimulus and is credible, if $a \rightarrow b$ and $a \rightarrow c$ are two beliefs that describe two thoughts belonging to an epistemic context in which a is true, and if b and c are not improbable or conceivable, then a new thought $c_{i,2}$ expressed by $b \rightarrow c$ is produced.*

We assume that the production by induction or abduction is initiated by a stimulus and is conditional on the absence of contradiction of the thought produced in the associated perspectives.

Given its fallibilistic property, L_c will consider the new thought introduced by abduction or induction to be possible. Its appearance in E_{L_c} leads to a recalculation of the semantic interpretation function, in accordance with the non-monotonic character of L_c . The relevant sets of contexts are then subject to change. The new thought, imagined the moment before, may eventually turn out to be finally not credible.

So, the questions of abduction or induction can be modelled in L_c . Within the context of contextual logic, they are not linked to a possible limitation of formalism.

The definition that we propose would need to be deepened by studies associating the cognitive sciences. We will not comment on them anymore. It would be possible to study their mathematical properties, and to illustrate them with use cases, but this would not validate their relevance. None can be representative of the complexity of human reasoning.

Turing's test [25] is sometimes criticised. The arguments advanced by its detractors are understandable – and, according to the contextual approach, it would be impossible to contradict them definitively: nothing is provable. Anyway, no algorithm has yet managed to pass it. However, we consider with Alan Turing that only a behavioural test carried out “blind” and based on a substantial body of knowledge would make it possible to form a relevant opinion.

This remark introduces the subject of algorithmic complexity.

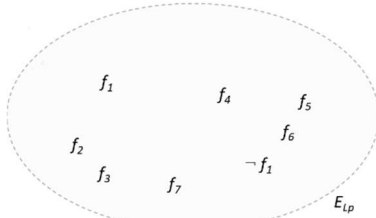
9 The algorithmic complexity

We use the well-known algorithms of L_p , which are of exponential complexity.

But let us return to our objective. It is not to model a formal mathematical language, but to capture human reasoning through a mathematical formalism. In this

context, the question becomes: why does algorithmic complexity cause a problem in the use of a formal language in Artificial Intelligence?

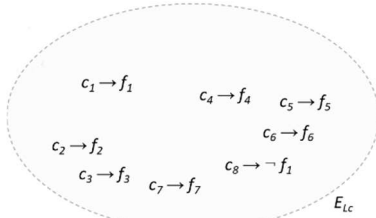
A first need is to integrate a new piece of knowledge. For example:



Pic. 7 – The new information $\neg f_1$ introduces a syntactically inconsistent in E_{Lp}

With $\neg f_1$, L_p faces the problem of the epistemic rooting [7]: what to do? What to choose?

In fact, this set makes no sense according to the contextual postulate. If we apply it, E_{Lp} becomes:

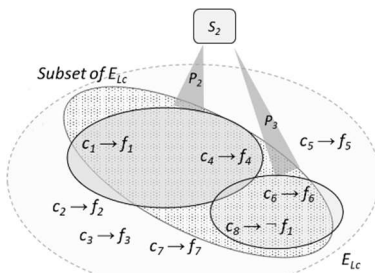


Pic. 8 – The new thought c_8 introduces a semantically inconsistent in E_{Lc}

The arrival of new information is no longer a problem. The contextual postulate guarantees the maintenance of the syntactic consistency of the database, whatever the new thought integrated. The topic of epistemic rooting is resolved, and the first need is therefore covered.

The second need is to be able to analyse the information, and its semantic impact. L_c must calculate inconsistent contexts, which requires the use of the algorithms of L_p . The application of the semantic interpretation function on a few dozen formulae will therefore lead to a response time problem. But it is possible to take advantage of the properties of L_c to concentrate the semantic analysis on a few selected pieces of knowledge.

Let us illustrate our point by taking the example of picture 6. We consider that the semantic interpretation is only performed using a subset of E_{Lp} . We obtain:



Pic. 9 – The semantic interpretation function is calculated with a subset of E_{Lc}

The semantic result is incorrect (see the gap with the picture 6). However, we agree with D. Battens [3]:

“Humans decide on provisional and fallible intuitions, even in logical matters”.

We constantly make decisions based on approximate reasoning, even though we have in memory all the information necessary to solve the problem.

In the context of Artificial Intelligence, this second need can therefore be covered if we accept that human reasoning is imperfect. In this case, it is sufficient to define a function that selects, for a given stimulus, a “relevant” subset of knowledge. The semantic interpretation is computed only on this subset, and not on the whole knowledge base.

To achieve this, we propose to exploit the results obtained by cognitive science. L_c is well suited to model human memory (D. Norman [15]). Indeed, the guarantee of syntactic consistency allows a safe distinction between long-term and short-term memory. This makes it possible to integrate into the algorithm cognitive thresholds from cognitive science research:

- the minimal change in thoughts between t_n and t_{n+1} ,
- the evolution criterion, which favours the use of the most recent or “primitive” thoughts,
- the technical incapacity threshold, which limits the number of thoughts that can be used simultaneously,
- the semantic thresholds: J. Pitrat shows that a human is not able to reason on more than four levels of meta-knowledge [17] (this threshold limits the number of different ranks of thoughts that can be used simultaneously),
- the threshold of proportionate reasoning: when confronted with a stimulus, the objective is not to perform the best theoretical analysis, but to reach a level of analysis sufficient to cause a reaction.

We see a three-level architecture emerging:

- a long-term memory (LTM), in which knowledge is stored as it arrives. It is E_{Lc} ,
- a short-term memory (STM) fed by a function that selects some pieces of knowledge from the long-term memory using cognitive thresholds. It is the subset of E_{Lc} , on which the semantic interpretation is performed,
- and a working memory (WM), which carries the computational requirements for calculating the semantic interpretation of the content of short-term memory - in particular the set of formulas obtained by saturation of the STM.

This is a very simple first approach. It does not incorporate the latest discoveries in this area. Our presentation is illustrative, the principle can be enriched. We regularly update the algorithms we develop to test their behavioural relevance against the various tests and practical cases identified by cognitive science.

10 An example of application

After presenting the theoretical principles of contextual logic, we propose to develop an example of application to

clarify our purpose, and to illustrate the knowledge modelling capabilities of L_c . To do this, we use the example of the bird Tweety, a classical case study in the literature on non-monotonicity and belief revision.

Example Consider the following knowledge, which we call the E_{NL} (for Natural Language) set:

Birds and felines are animals ^(01 and 02). *Birds are not felines* ⁽⁰³⁾. *Animals are diurnal* ⁽⁰⁴⁾. *Diurnal animals are not nocturnal* ⁽⁰⁵⁾. *Birds fly* ⁽⁰⁶⁾. *They are insectivorous* ⁽⁰⁷⁾ and *gregarious* ⁽⁰⁸⁾. *Felines are carnivorous* ⁽⁰⁹⁾ and *solitary* ⁽¹⁰⁾. *Solitaires are not gregarious* ⁽¹¹⁾. *Insectivores are not carnivorous* ⁽¹²⁾. *Swallows, sparrows, ostriches, and owls are birds* ^(13, 14, 15 and 16). *Swallows are not sparrows* ⁽¹⁷⁾, *ostriches* ⁽¹⁸⁾, or *owls* ⁽¹⁹⁾. *Sparrows are not ostriches* ⁽²⁰⁾ or *owls* ⁽²¹⁾. *Ostriches are not owls* ⁽²²⁾. *Ostriches do not fly* ⁽²³⁾. *Owls are solitary* ⁽²⁴⁾, *nocturnal* ⁽²⁵⁾, *carnivorous* ⁽²⁶⁾, and *insectivorous* ⁽²⁷⁾. *Cats and lions are felines* ^(28 and 29). *Cats are not lions* ⁽³⁰⁾. *Cats are nocturnal* ⁽³¹⁾. *Lions are gregarious* ⁽³²⁾. *Carnivores are hunters* ⁽³³⁾. *Herbivores are prey for hunters* ⁽³⁴⁾. *Hunters attack prey* ⁽³⁵⁾. *If the prey is larger than the hunter, the latter does not attack* ⁽³⁶⁾. *Ostriches are larger than cats* ⁽³⁷⁾ and *owls* ⁽³⁸⁾.

E_{NL} contains a lot of inconsistent, epistemic, and predicative information - for example: birds fly and do not fly, birds are usually insectivores and sometimes carnivorous, and hunters attack prey (and not the reverse). This knowledge is deemed to escape the syntax of propositional logic. We are however going to show that L_p is sufficient to model and exploit them.

In a first time, we propose to translate it into the syntax of L_p by the following formulae.

01	$Bird \rightarrow Animal$	
02	$Feline \rightarrow Animal$	
03	$Bird \rightarrow \neg Feline$	
04	$Animal \rightarrow Diurnal$	(em)
05	$Diurnal \rightarrow \neg Nocturnal$	
06	$Bird \rightarrow Fly$	(em)
07	$Bird \rightarrow Insectivore$	(em)
08	$Bird \rightarrow Gregarious$	(em)
09	$Feline \rightarrow Carnivore$	
10	$Feline \rightarrow Solitary$	(em)
11	$Gregarious \rightarrow \neg Solitary$	
12	$Insectivore \rightarrow \neg Carnivore$	(em)
13	$Swallow \rightarrow Bird$	
14	$Sparrow \rightarrow Bird$	
15	$Ostrich \rightarrow Bird$	
16	$Owl \rightarrow Bird$	
17	$Swallow \rightarrow \neg Sparrow$	
18	$Swallow \rightarrow \neg Ostrich$	
19	$Swallow \rightarrow \neg Owl$	
20	$Sparrow \rightarrow \neg Ostrich$	
21	$Sparrow \rightarrow \neg Owl$	
22	$Ostrich \rightarrow \neg Owl$	
23	$Ostrich \rightarrow \neg Fly$	
24	$Owl \rightarrow Solitary$	
25	$Owl \rightarrow Nocturnal$	
26	$Owl \rightarrow Carnivore$	
27	$Owl \rightarrow Insectivore$	
28	$Cat \rightarrow Feline$	
29	$Lion \rightarrow Feline$	
30	$Cat \rightarrow \neg Lion$	
31	$Cat \rightarrow Nocturnal$	
32	$Lion \rightarrow Gregarious$	
33	$Carnivore \rightarrow Hunter$	

34	$Herbivore \rightarrow (Hunter \rightarrow Prey)$	(pk)
35	$Hunter \rightarrow (Prey \rightarrow Attack)$	(em) and (pk)
36	$Hunter \rightarrow (Prey \rightarrow (Larger \rightarrow \neg Attack))$	(pk)
37	$Ostrich \rightarrow (Cat \rightarrow Larger)$	(pk)
38	$Ostrich \rightarrow (Owl \rightarrow Larger)$	(pk)

We have introduced two new pieces of information, denoted by (em) and (pk). The explanations will come later. For the time being, we assume that there is a learner and an accompanying instructor. Let us consider that the knowledge is entered in the order in which it appears, according to the following algorithm:

```

For each formula  $f_i$ 
  If  $f_i$  has a subject, then PK
  Else
    Creating the thought  $c_{i,0.2}$ 
    Creating the formula  $c_{i,0.2} \rightarrow f_i$ 
    If there is a contradiction, then EM

```

The sentences are processed one after the other.

01	$c_{01,0.2} \rightarrow (Bird \rightarrow Animal)$
02	$c_{02,0.2} \rightarrow (Feline \rightarrow Animal)$
03	$c_{03,0.2} \rightarrow (Bird \rightarrow \neg Feline)$
04	$c_{04,0.2} \rightarrow (Animal \rightarrow Diurnal)$
05	$c_{05,0.2} \rightarrow (Diurnal \rightarrow \neg Nocturnal)$
06	$c_{06,0.2} \rightarrow (Bird \rightarrow Fly)$
07	$c_{07,0.2} \rightarrow (Bird \rightarrow Insectivore)$
08	$c_{08,0.2} \rightarrow (Bird \rightarrow Gregarious)$
09	$c_{09,0.2} \rightarrow (Feline \rightarrow Carnivore)$
10	$c_{10,0.2} \rightarrow (Feline \rightarrow Solitary)$
11	$c_{11,0.2} \rightarrow (Gregarious \rightarrow \neg Solitary)$
12	$c_{12,0.2} \rightarrow (Insectivore \rightarrow \neg Carnivore)$
13	$c_{13,0.2} \rightarrow (Swallow \rightarrow Bird)$
14	$c_{14,0.2} \rightarrow (Sparrow \rightarrow Bird)$
15	$c_{15,0.2} \rightarrow (Ostrich \rightarrow Bird)$
16	$c_{16,0.2} \rightarrow (Owl \rightarrow Bird)$
17	$c_{17,0.2} \rightarrow (Swallow \rightarrow \neg Sparrow)$
18	$c_{18,0.2} \rightarrow (Swallow \rightarrow \neg Ostrich)$
19	$c_{19,0.2} \rightarrow (Swallow \rightarrow \neg Owl)$
20	$c_{20,0.2} \rightarrow (Sparrow \rightarrow \neg Ostrich)$
21	$c_{21,0.2} \rightarrow (Sparrow \rightarrow \neg Owl)$
22	$c_{22,0.2} \rightarrow (Ostrich \rightarrow \neg Owl)$
23	$c_{23,0.2} \rightarrow (Ostrich \rightarrow \neg Fly)$
24	$c_{24,0.2} \rightarrow (Owl \rightarrow Solitary)$

For each sentence, the system asks if there is a subject, and the instructor's answer is no. The formula is integrated, and the system tested its semantic consistency. Until formula 23, there is no question.

The formula 24 is then integrated, and a potential inconsistency is detected: owls are birds ⁽¹⁶⁾, birds are gregarious ⁽⁰⁸⁾, owls are solitary ⁽²⁴⁾ and gregarious is not solitary ⁽¹¹⁾. So, owls do not exist, or they are solitary and not solitary. The instructor is questioned, and has three possible answers:

- he does not know, or he considers it is normal: the semantic inconsistency is accepted, and the system moves on to the next step. Note that it is the maintenance of syntactic consistency that allows this,
- he indicates that one of the pieces of knowledge involved in the inconsistency is false. In this case, the system applies the EM module to the indicated formula,
- he indicates that one of the pieces of knowledge involved in the inconsistency is true and false. In this case, birds are gregarious ⁽⁰⁸⁾ is sometimes true

and sometimes false. Then the system applies the EM module to the indicated formula.

The EM module is:

```

if  $f_i$  is of type  $em$ 
  if  $f_i$  is false
    Creating the thoughts  $c_{i2,3}$ 
    Creating the formula  $c_{i2,3} \rightarrow \neg c_{i0,2}$ 
    // The combinatorics on the maximal contexts
    // of  $T_m$  means that  $c_{i0,2}$  will no longer be
    // retained in the epistemic contexts
  else if  $f_i$  is true and false
    Creating the thoughts  $c_{i1,3}$  et  $c_{i2,3}$ 
    Creating the formula  $c_{i1,3} \rightarrow c_{i0,2}$ 
    Creating the formula  $c_{i2,3} \rightarrow \neg c_{i0,2}$ 

```

We have previously identified by (em) the formulae that are affected by this module. We will only use the “true and false” last case which is the most interesting. Let's continue the treatment. We obtain:

- 01 $c_{010,2} \rightarrow (Bird \rightarrow Animal)$
- 02 $c_{020,2} \rightarrow (Feline \rightarrow Animal)$
- 03 $c_{030,2} \rightarrow (Bird \rightarrow \neg Feline)$
- 04 $c_{040,2} \rightarrow (Animal \rightarrow Diurnal)$
 $c_{041,3} \rightarrow c_{040,2}$
 $c_{042,3} \rightarrow \neg c_{040,2}$
- 05 $c_{050,2} \rightarrow (Diurnal \rightarrow \neg Nocturnal)$
- 06 $c_{060,2} \rightarrow (Bird \rightarrow Fly)$
 $c_{061,3} \rightarrow c_{060,2}$
 $c_{062,3} \rightarrow c_{060,2}$
- 07 $c_{070,2} \rightarrow (Bird \rightarrow Insectivore)$
 $c_{071,3} \rightarrow c_{070,2}$
 $c_{072,3} \rightarrow \neg c_{070,2}$
- 08 $c_{080,2} \rightarrow (Bird \rightarrow Gregarious)$
 $c_{081,3} \rightarrow c_{080,2}$
 $c_{082,3} \rightarrow \neg c_{080,2}$
- 09 $c_{090,2} \rightarrow (Feline \rightarrow Carnivore)$
- 10 $c_{100,2} \rightarrow (Feline \rightarrow Solitary)$
 $c_{101,3} \rightarrow c_{100,2}$
 $c_{102,3} \rightarrow \neg c_{100,2}$
- 11 $c_{110,2} \rightarrow (Gregarious \rightarrow \neg Solitary)$
- 12 $c_{120,2} \rightarrow (Insectivore \rightarrow \neg Carnivore)$
 $c_{121,3} \rightarrow c_{120,2}$
 $c_{122,3} \rightarrow \neg c_{120,2}$
- 13 $c_{130,2} \rightarrow (Swallow \rightarrow Bird)$
- 14 $c_{140,2} \rightarrow (Sparrow \rightarrow Bird)$
- 15 $c_{150,2} \rightarrow (Ostrich \rightarrow Bird)$
- 16 $c_{160,2} \rightarrow (Owl \rightarrow Bird)$
- 17 $c_{170,2} \rightarrow (Swallow \rightarrow \neg Sparrow)$
- 18 $c_{180,2} \rightarrow (Swallow \rightarrow \neg Ostrich)$
- 19 $c_{190,2} \rightarrow (Swallow \rightarrow \neg Owl)$
- 20 $c_{200,2} \rightarrow (Sparrow \rightarrow \neg Ostrich)$
- 21 $c_{210,2} \rightarrow (Sparrow \rightarrow \neg Owl)$
- 22 $c_{220,2} \rightarrow (Ostrich \rightarrow \neg Owl)$
- 23 $c_{230,2} \rightarrow (Ostrich \rightarrow \neg Fly)$
- 24 $c_{240,2} \rightarrow (Owl \rightarrow Solitary)$
- 25 $c_{250,2} \rightarrow (Owl \rightarrow Nocturnal)$
- 26 $c_{260,2} \rightarrow (Owl \rightarrow Carnivorous)$
- 27 $c_{270,2} \rightarrow (Owl \rightarrow Insectivore)$
- 28 $c_{280,2} \rightarrow (Cat \rightarrow Feline)$
- 29 $c_{290,2} \rightarrow (Lion \rightarrow Feline)$
- 30 $c_{300,2} \rightarrow (Cat \rightarrow \neg Lion)$
- 31 $c_{310,2} \rightarrow (Cat \rightarrow Nocturnal)$
- 32 $c_{320,2} \rightarrow (Lion \rightarrow Gregarious)$
- 33 $c_{330,2} \rightarrow (Carnivore \rightarrow Hunter)$

We come to the formula number 34: $Herbivore \rightarrow (Hunter \rightarrow Prey)$. Before integration, the system asks if it contains a subject. The answer is yes, for $Herbivore$. The PK module is then applied:

```

if  $f_i$  is of type  $pk$ 
  //  $f_i$  is a clause, so it is of type  $g \rightarrow h$ 
  // with  $g$  is a conjunction of literals
  // and  $h$  is a disjunction of literals
  //  $g$  is indicated by the instructor
  Creating the thoughts  $c_{i0,2}$  et  $c_{i3,1}$ 
  Creating the formula  $c_{i0,2} \rightarrow g \rightarrow c_{i3,1}$ 
  Creating the formula  $c_{i3,1} \rightarrow h$ 

```

We have previously identified by (pk) the formulae that are affected by this module. After application to the whole of E_{NL} , we obtain:

- 01 $c_{010,2} \rightarrow (Bird \rightarrow Animal)$
- 02 $c_{020,2} \rightarrow (Feline \rightarrow Animal)$
- 03 $c_{030,2} \rightarrow (Bird \rightarrow \neg Feline)$
- 04 $c_{040,2} \rightarrow (Animal \rightarrow Diurnal)$
 $c_{041,3} \rightarrow c_{040,2}$
 $c_{042,3} \rightarrow \neg c_{040,2}$
- 05 $c_{050,2} \rightarrow (Diurnal \rightarrow \neg Nocturnal)$
- 06 $c_{060,2} \rightarrow (Bird \rightarrow Fly)$
 $c_{061,3} \rightarrow c_{060,2}$
 $c_{062,3} \rightarrow c_{060,2}$
- 07 $c_{070,2} \rightarrow (Bird \rightarrow Insectivore)$
 $c_{071,3} \rightarrow c_{070,2}$
 $c_{072,3} \rightarrow \neg c_{070,2}$
- 08 $c_{080,2} \rightarrow (Bird \rightarrow Gregarious)$
 $c_{081,3} \rightarrow c_{080,2}$
 $c_{082,3} \rightarrow \neg c_{080,2}$
- 09 $c_{090,2} \rightarrow (Feline \rightarrow Carnivore)$
- 10 $c_{100,2} \rightarrow (Feline \rightarrow Solitary)$
 $c_{101,3} \rightarrow c_{100,2}$
 $c_{102,3} \rightarrow \neg c_{100,2}$
- 11 $c_{110,2} \rightarrow (Gregarious \rightarrow \neg Solitary)$
- 12 $c_{120,2} \rightarrow (Insectivore \rightarrow \neg Carnivore)$
 $c_{121,3} \rightarrow c_{120,2}$
 $c_{122,3} \rightarrow \neg c_{120,2}$
- 13 $c_{130,2} \rightarrow (Swallow \rightarrow Bird)$
- 14 $c_{140,2} \rightarrow (Sparrow \rightarrow Bird)$
- 15 $c_{150,2} \rightarrow (Ostrich \rightarrow Bird)$
- 16 $c_{160,2} \rightarrow (Owl \rightarrow Bird)$
- 17 $c_{170,2} \rightarrow (Swallow \rightarrow \neg Sparrow)$
- 18 $c_{180,2} \rightarrow (Swallow \rightarrow \neg Ostrich)$
- 19 $c_{190,2} \rightarrow (Swallow \rightarrow \neg Owl)$
- 20 $c_{200,2} \rightarrow (Sparrow \rightarrow \neg Ostrich)$
- 21 $c_{210,2} \rightarrow (Sparrow \rightarrow \neg Owl)$
- 22 $c_{220,2} \rightarrow (Ostrich \rightarrow \neg Owl)$
- 23 $c_{230,2} \rightarrow (Ostrich \rightarrow \neg Fly)$
- 24 $c_{240,2} \rightarrow (Owl \rightarrow Solitary)$
- 25 $c_{250,2} \rightarrow (Owl \rightarrow Nocturnal)$
- 26 $c_{260,2} \rightarrow (Owl \rightarrow Carnivorous)$
- 27 $c_{270,2} \rightarrow (Owl \rightarrow Insectivore)$
- 28 $c_{280,2} \rightarrow (Cat \rightarrow Feline)$
- 29 $c_{290,2} \rightarrow (Lion \rightarrow Feline)$
- 30 $c_{300,2} \rightarrow (Cat \rightarrow \neg Lion)$
- 31 $c_{310,2} \rightarrow (Cat \rightarrow Nocturnal)$
- 32 $c_{320,2} \rightarrow (Lion \rightarrow Gregarious)$
- 33 $c_{330,2} \rightarrow (Carnivore \rightarrow Hunter)$
- 34 $c_{340,2} \rightarrow (Herbivore \rightarrow c_{343,1})$
 $c_{343,1} \rightarrow (Hunter \rightarrow Prey)$
- 35 $c_{350,2} \rightarrow (Hunter \rightarrow c_{353,1})$
 $c_{351,3} \rightarrow c_{350,2}$
 $c_{352,3} \rightarrow \neg c_{350,2}$
 $c_{353,1} \rightarrow (Prey \rightarrow Attack)$
- 36 $c_{360,2} \rightarrow (Hunter \rightarrow c_{363,1})$
 $c_{363,1} \rightarrow (Prey \rightarrow (Larger \rightarrow \neg Attack))$
- 37 $c_{370,2} \rightarrow (Ostrich \rightarrow c_{373,1})$
 $c_{373,1} \rightarrow (Cat \rightarrow Larger)$
- 38 $c_{380,2} \rightarrow (Ostrich \rightarrow c_{383,1})$
 $c_{383,1} \rightarrow (Owl \rightarrow Larger)$

We have converted E_{NL} into a set E_{Lc} that is fully compatible with the syntax of propositional logic. It is L_p consistent. Here are some examples obtained by applying the generalised contextual semantic and considering that there is no cognitive threshold:

- if the stimulus is *Bird*: birds are animals, diurnal, gregarious, insectivore, and fly,
- if the stimulus is *Swallow*: swallows are birds, animals, diurnal, gregarious, insectivore, and fly,
- if the stimulus is *Ostrich*: ostriches are birds, animal, diurnal, gregarious, insectivore, and do not fly,
- if the stimulus is *Owl*: owls are birds, animals, nocturnal, solitary, carnivorous, and fly,
- if the stimulus is *Cat*: cats are feline, animals, diurnal, carnivorous and solitary,
- if the stimulus is *Lion*: lions are feline, animals, diurnal, carnivorous and gregarious,
- if the stimulus is $\{Cat, Sparrow\}$: the sparrow is attacked,
- if the stimulus is $\{Cat, Owl, Sparrow, Ostrich\}$: the sparrow is in a bad way, but the ostrich can go about its business,
- and if the stimulus is $\{Lion, Ostrich\}$: the ostrich would have some reason to be worried.

11 Some points of clarification and opening

In the previous paragraph, we have certainly not answered all the questions that our presentation raises. But there are also some topics that we have deliberately skimmed over so as not to make our remarks totally indigestible.

We are not going to develop them, for the same reason. But, for readers who wish to explore these concepts in more depth, we offer to list them below, in a thought-provoking format.

Informatic tool

In paragraph 10, the different steps are described in detail, which probably makes reading tedious at times. The goal is to allow the readers to control and to reproduce the process if they wish.

We use a classical propositional logic solver to calculate the minimum inconsistent contexts, and a classical combinatorial algorithm using the minimum inconsistent contexts to calculate the epistemic contexts.

These tools are common and very easily accessible. We hold those which we developed at the disposal of the readers who would like it.

Use case

We use the example of Tweety, which may seem simplistic or even naive. Its first interest is to be understandable by all, while illustrating all the theoretical problems of non-monotonicity and of belief revision.

Any other subject could have done the trick. The syntactic and semantic rules used were defined by the theory, and we did not integrate any behavioural rules specific to the subject.

Modelled information

While strictly preserving the syntax of the propositional logic, L_c allows us to define a semantic function on the models of the theories of L_p . It exploits:

- semantically incoherent pieces of knowledge:
Birds fly. Ostriches are birds. Ostriches do not fly
(Thoughts $c_{060,2}$, $c_{150,2}$ and $c_{230,2}$)
- predicative pieces of knowledge:
Herbivores are prey for hunters
(Thoughts $c_{340,2}$ and $c_{343,1}$)
- and epistemic modal pieces of knowledge:
Birds are generally insectivorous
(Thoughts $c_{070,2}$, $c_{071,3}$ and $c_{072,3}$)

The example contains several different cases to illustrate their use thanks to the combinatorics calculated for the identification of reference contexts. And we have adjoined a case which simultaneously uses a predicate and an epistemic modality:

Hunter \rightarrow (Prey \rightarrow Attack)
(Thought $c_{350,2}$)

It is false if the prey is large (thought $c_{360,2}$).

The notion of subject

We do not use the notion of subject on all formulas. In absolute terms, this should have been done in the first sentence. But the result would have been unreadable, without any contribution to what we wished to demonstrate.

In fact, this remark is not insignificant. It raises some questions as: are we saying the same thing with $a \rightarrow b$ and $\neg b \rightarrow \neg a$? The question is about the meaning of connectors. It has been extensively studied, notably by J. Lukaszewicz [13].

Without elaborating on the subject, let us point out to readers interested in this topic that L_c proposes an answer. $c \rightarrow f$ says that if c is true then f is true. But if c is false, then f can be true or false. The fact that a thought is false does not imply that the expression that describes it is false in the sense of syntactic interpretation.

Thoughts

They are indeed *silent* atomic propositions. They appear completely automatically in the syntax. In the algorithms of paragraph 10, the communication interface with the instructor does not see them, and only acts through the propositions of L_p .

Thought carries semantics. Any other proposition is a set of meaningless signs. This conception individualises semantics: the same expression is associated with the thoughts specific to everyone. They can therefore have different meanings for each other.

Our approach questions the semantic principles theorised by G. Frege, who propose to associate each atomic proposition with a precise and unambiguous meaning.

Belief revision

The properties of L_c significantly simplify the belief revision: it is only done by adding some new pieces of knowledge, without ever modifying the knowledge already entered in the system.

In case of error, it is possible to “cancel” a knowledge $c_{i,2} \rightarrow f$ by creating a thought $c_{ix,3}$, and integrating the formula $c_{ix,3} \rightarrow \neg c_{i,2}$. The combinatorics on the maximal contexts of T_m means that $c_{i,2}$ is not retained in the epistemic contexts. The reader has probably noted that this “cancellation” is systematic only if there is no cognitive threshold.

Epistemic rooting

There is no subject of epistemic rooting at the syntactic level: semantically incoherent information is memorised as it arrives. Incidentally, the system registers that it is “*true and false*” if this information is transmitted to it by the instructor.

We mentioned in paragraph 6 that the principle of epistemic rooting is not useful in the context of contextual logic. However, experimental studies on human reasoning have shown that it responds to a behavioural reality.

In L_c , the purpose of epistemic rooting is not to answer a problem of modelling inconsistency. It finds its operational reality in cognitive thresholds (see paragraph 9). We propose it is the mechanical solution that nature has put in place in response to the problem posed by the calculation limits imposed by physics.

Normal form

In paragraph 10, we have chosen to associate a new thought with each clause of the normal form of the theory. This has an impact on the semantic interpretation: for the same stimulus, the interpretation of the set $\{c_1 \rightarrow f, c_2 \rightarrow g\}$ may be different from that which would be obtained on the set $\{c \rightarrow f \wedge g\}$.

Our choice has no theoretical basis. It is pragmatic: the conversion of a set of formulae into its normal form is achieved by a linear algorithm, and each clause identifies a unique formula and therefore a unique thought. We have chosen the simplest technical solution.

Epistemic context

Concerning the semantic interpretation function, we propose the definition of epistemic contexts to identify the relevant sets of thoughts. Other definitions are possible.

Another method, perhaps more purist, would have consisted in giving the different possible definitions and comparing their mathematical properties. It requires a lot of methodical work that we have not done.

Universal connector

A difference between predicate logic and contextual logic is that the latter has no universal quantifier.

In fact, L_c natively models a form of quantifier. For example, assume a knowledge base consisting of a single piece of information: $\{c \rightarrow (Bird \rightarrow Fly)\}$. This says that *Birds fly*. Given this piece of knowledge, it would be the same to say *All bird fly*.

L_c is fallibilistic. So, asserting that a state is universal does not make sense: a state is universal as long as it is not contradicted. In L_c , the notion of universality cannot be carried by a syntactic connector. It is deduced from the semantic interpretation: a thought c is interpreted as universal long as there does not appear a thought c' expressed by $\neg c$.

In the same vein, epistemic modalities are not expressed by a dedicated connector, but are deduced from a semantic interpretation.

Birds fly is modelled by $c_i \rightarrow (Bird \rightarrow Fly)$. Two thought $c_{i1,3}$ and $c_{i2,3}$ such that $c_{i1,3} \rightarrow c_i$ and $c_{i2,3} \rightarrow \neg c_i$ indicates that *Birds fly in general*. Indeed, they identify two types of epistemic contexts: one ($c_{i1,3}$) in which *Birds fly*, and the second ($c_{i2,3}$) in which *nothing is said about whether birds fly or not*.

Stimulus

Contextual logic introduces the notion of stimulus. Its concept is necessary to generate a possible reaction. We use it in the different examples as an imposed external event f . Relations with facts are carried by thoughts, according to an anchoring principle that remains to be defined.

This remark indirectly raises the question of the role of emotions. This comment refers to the many ongoing debates around the notion of strong Artificial Intelligence (see J. Searle [21]).

Stimulus can also be used to analyse a hypothesis in a form $c \rightarrow f$. The semantic interpretation will then look for the different perspectives that verify or not verify c . This is the principle we use in induction or abduction production. The *imagined* thought is syntactically and automatically produced, and just as automatically generates the production of a new semantic interpretation.

Natural language

A formal language seems to be a language “like any other” as soon as it is used in its contextual form. The ability to reason takes a back seat, as an opportunity effect of the properties of the language. We are currently studying the work of J. Piaget [16] to challenge this intuition.

Algorithm complexity

We propose to use some results from cognitive science to get around the problem of algorithmic complexity. In parallel, we are working on a technical solution that uses the properties of Horn clauses (A. Horn [9], used for example by A. Colmerauer and his teams [8] for the Prolog programming language).

We have indicated that we transform the formulae to their normal form, and then consider the individual clauses. Each has a form $f \rightarrow g$, for f a conjunction of positive literals and g a disjunction of positive literals. We propose to decompose $f \rightarrow g$ by a set of formulae $f \rightarrow a_i$, for a_i each literal of g .

Thus, a formula like $a \rightarrow (b \vee c)$ is expressed by two distinct knowledges $c_1 \rightarrow (a \rightarrow b)$ and $c_2 \rightarrow (a \rightarrow c)$. The sense of this operation is:

If a is true and if $a \rightarrow (b \vee c)$ is true, then b and c are, considered individually, true or conceivable unless explicitly contradicted.

This amounts to a form of weakening of the disjunction principle. Its formal application requires a prior in-depth study. We analyse the level of loss of expression that this induces to evaluate its semantic relevance in the context of human reasoning.

Its technical contribution would be considerable: at the cost of a linear multiplication of the size of the theory, we would benefit from a resolution algorithm of polynomial complexity – *versus* the exponential complexity of the original algorithms of L_p .

Sub-symbolic AI

We have dealt with the subject of inconsistency in the context of symbolic languages. It is shared with Sub-symbolic AI.

Being able to establish that two strictly incompatible events have a probability of occurrence of 49% and 51% respectively makes it possible to refine the understanding of a situation. But it is not inconsistent. This occurs when these two events each have a probability of occurrence greater than 50% for example. This case is not possible in a statistical model, which is based on a strict mathematical principle of proof.

It may seem paradoxical, but the fallibilistic notion cannot be modelled in statistics. Indeed, it does not consist in asserting that an event is $x\%$ possible. It consists in asserting and exploiting at the same time the fact that this probability itself is fallible.

This is the reason why we believe that Sub-symbolic AI is not enough and needs Symbolic AI to cover all possible reasoning done by a human.

12 Conclusion

Contextual logic is obtained by applying the contextual postulate on propositional logic. It proposes to formalise a relation between thoughts and languages. This brings the possibility of modelling an introspective reasoning.

The principle of proof is the foundation of mathematical philosophy. But introspective reasoning automatically generates the inability to access certainty. Faced with the need for decidability, L_c uses the principle of non-refutability, which is backed by fallibilism and perspectivism.

These are some old and still open subjects (H. Albert [1] and W. Quine [19] for example). Our work reconciles the different theories, by an answer based on an absolutely uncertain semantic modelled by an absolutely certain syntax.

The historical ambition of logic is to model the process of human reasoning. Contextual logic models inconsistent and predicative information, absorbs imaginary thoughts, and solves belief revision and epistemic rooting problems in a radical way by staying scrupulously and strictly within the syntax of propositional logic.

In the context of Artificial Intelligence, this result invites us to relaunch (see paragraph 11), around expert systems, work in natural language and in strong AI, by combining studies carried out around cognitive sciences.

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