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# Using classical propositional syntax to model human reasoning

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*Abstract*— There are many formal languages for modelling reasoning. Each captures several properties, but none of them manages to cover all of them. We propose an alternative solution with contextual logic. Its particularity is to remain strictly within monotonic syntactic framework of propositional logic while benefiting from a non-monotonic semantic interpretation function. It verifies a fallibilist and perspectivist behavior. This makes it possible to model and exploit incoherent, epistemic, predicate, and temporal knowledge. The guiding idea of our work is to consider that syntactic production is monotonic. Syntactic interpretation function is classically adequate and complete with respect to syntax. The non-monotonic property of reasoning is captured by a “semantic” interpretation function of knowledge set models.

*Keywords*— Formal Language, Nonmonotonic Logic, Belief Revision, Knowledge Representation, Reasoning, Artificial Intelligence, Mathematical Philosophy, Fallibilism, Perspectivism, Cognitive Sciences

## I. INTRODUCTION

Symbolic Artificial Intelligence is confronted with a major difficulty: to benefit from a unique formalism to model the different modes of human reasoning. Research on this topic has led to a multiplication of formal languages (modal, para-coherent, predicate, multivalued, adaptive, or default logics for examples). Each formalism captures particular properties, without however modelling the many modes of reasoning empirically observed in humans [2].

In this framework, we submit contextual logic  $L_c$  [9]. Its particularity is to keep strictly within the monotonic syntax of the propositional logic  $L_p$ , while benefiting from a non-monotonic semantic interpretation function. It verifies a fallibilist and perspectivist behavior.

After a reminder of the principles of  $L_c$  and its main properties, we present how to use it to model the notions of predicate, temporality and modal knowledge. A paragraph is devoted to the comparative study between  $L_c$  and other formalisms. An example of application is developed. It uses a knowledge base large enough to illustrate the non-monotonic expressiveness of the language. We will conclude our presentation by discussing the relationship between  $L_c$  and some empirical results observed on human reasoning by the cognitive sciences.

In this article, we will use the negation, disjunction, conjunction, implication, and equivalence connectors, denoted  $\neg$ ,  $\vee$ ,  $\wedge$ ,  $\rightarrow$  and  $\leftrightarrow$  respectively.

## II. CONTEXTUAL LOGIC

Let us consider a thought. We perceive it as a whole, in the sense defined by René Descartes:

“By the name of thought, I understand all that is so much in us that we are immediately aware of it”, (translation) R. Descartes [5]

and we describe it in a set of sentences. However, even if this description were ideally complete and perfect, we are

immediately aware that it is not the thought it describes. We model this observation by distinguishing, for a given thought, two notions in the syntax of the language: a unitary sign  $c$ , which symbolize it, and a combination of signs  $f$ , which reproduces the sentences that describe it. This leads to the need to define a relationship between  $c$  and  $f$ . To this end, we consider the following postulate [9]:

**Contextual postulate** Let  $L$  be a formal language with the functions of syntactic production  $\vdash_L$  and the syntactic interpretation  $\models_L$ . A well-formed formula  $f$  of  $L$  is a set of signs that has no meaning. Its meaning is carried by a thought, which is an elementary proposition of  $L$  “which is not pronounced”. For  $c$  symbolizing this thought, the relation between  $c$  and  $f$  is  $c \models_L f$ .

We thus agree with Ludwig Wittgenstein when he states:

“We should not say: The complex sign  $aRb$  says that  $a$  is in the relation  $R$  with  $b$ , but: That  $a$  is in a certain relation  $R$  with  $b$  says that  $aRb$ ”, (translation) L. Wittgenstein [18]

So, the expression  $c \models_L f$ , equivalent to  $\models_L c \rightarrow f$  if  $L$  is the propositional logic  $L_p$ , asserts neither the thought  $c$  nor the sentence  $f$ . It models that the sentence  $f$  expresses the thought  $c$ . The proposition  $c$  respects the syntactic properties of  $L$ .

The application of the postulate to a formalism  $L$  produces the contextualized logic  $L_c$ . By language convention, we call contextual logic, denoted  $L_c$ , the contextualized propositional logic  $L_p$ .

All contextual formulas take a form  $c_i \rightarrow f_i$ , for  $c_i$  a thought (and also an atomic proposition) and  $f_i$  a well-formed formula in the sense of  $L_p$ . No-contextual expressions are acceptable. They allow for  $\vdash_{L_p}$  production and  $\models_{L_p}$  syntactic interpretation but have only meaningless technical behavior, making them useless in the context of the language.

The expressions in  $L_c$  accept a natural order. Formulas consisting of propositions in  $L_p$  are at rank 0. A thought  $c$  of rank  $n$  is expressed by a formula  $f$  of rank at most  $n-1$ .

We will use a classical definition.

**Definition** Let  $E_{L_p}$  be a theory of  $L_p$ , i.e., a set of well-formed formulas of  $L_p$ . A **model** consists in associating to each propositional variable one truth value (true or exclusively false) such that the result verifies the logical constraints expressed by  $E_{L_p}$ .  $E_{L_p}$  is said to be **consistent** if it has at least one model and is said to be **inconsistent** otherwise.

For example, the theory  $E_{L_p} = \{a, b \rightarrow c\}$  is verified by the model  $\{(a, \text{true}), (b, \text{true}), (c, \text{true})\}$ . We deduce that it is consistent in  $L_p$ . As a counter example,  $\{a, \neg a\}$  does not accept a model. So, it is inconsistent.

Given the syntax  $c_i \rightarrow f_i$  of the contextual formulas, the set  $\{(c_i, \text{false}), c_i \text{ are the thoughts}\}$  characterize some models that verify any contextual theory. The first consequence is

that a contextual theory is always consistent: it admits at least one model.

The second consequence is that contextual logic is unable to express certainty. Put differently, in  $L_c$ , any thought is possibly false, and, as a direct effect of the application of the form  $c_i \rightarrow f_i$ , any formulae  $f_i$  can be true or false: there is no certainty in  $L_c$ . Uncertainty is intrinsically embedded in the syntax.

To remedy this problem,  $L_c$  adopts the following principles:

- Since any formula can be true or false, there is no need to interrogate a contextual theory with a question such as “Is  $f$  true or false?”. The solution adopted is to ask the question in the form: “What can I conclude if I suppose that  $f$  (or  $\neg f$ ) is true?”.

**Notation**  $f$  is called the stimulus. It is possibly empty and is denoted  $S_{L_c}$ .

- Since every thought is potentially false, we propose to relativize the semantic interpretation to the subset of thoughts identified as the most relevant. We cannot say “ $f$  is true (or false)”, but we can say “ $f$  is true (or false) with respect to this subset of thoughts”.

We understand that sets of thoughts will play an important role in semantics of  $L_c$ , and that we need to define a method for selecting “relevant thoughts”. For this we need some definitions.

**Definitions** Let  $E_{L_c}$  be a theory of  $L_c$  and  $i$  and  $j$  be 2 integers such that  $0 < i \leq j$ .

A conjunction (or a set) of thoughts is called a **context**.

A context is said to be **of rank  $i$  to  $j$**  if all the thoughts in it are of rank  $i$  to  $j$ . A context of rank  $i$  to  $i$  is said of rank  $i$ .

A context that is verified by at least one model of  $E_{L_c}$  is called a **possible context** (or a consistent context).

A context that does not check any model of  $E_{L_c}$  is called an **impossible context** (or an inconsistent context).

An impossible context is called a **strict impossible context** if each of its strict subsets is possible.

A possible context that has no strict extension that checks  $E_{L_c}$  is called a **maximum context**.

A possible context is called **the credible context** if it has no join with a strict impossible context and if all its strict extensions have a join with a strict impossible context.

**Example** Let  $E_{L_c}$  be a theory and  $C_1$  and  $C_2$  the only two strict inconsistent contexts, i.e.,  $E_{L_c} \vdash_{L_p} \neg C_1$  and  $E_{L_c} \vdash_{L_p} \neg C_2$  and each of their strict subsets is possible.

- The credible context is the largest set of thoughts that does not have a join with  $C_1$  or  $C_2$ ,
- A maximal context is composed of the complete set of thoughts from which we remove one thought common to  $C_1$  or  $C_2$ , or one thought from  $C_1$  and one different thought from  $C_2$ .

We note that, for a given theory, there are possibly several maximal contexts (potentially empty) and a single credible context (potentially empty), and that they are obtained by computing the inconsistent contexts.

We have seen that it is not possible in  $L_c$  to conclude that a formula  $f$  is true or false. The way to do so is to define its semantic interpretation in relation to a context (a set of thoughts). A function using these definitions then identifies the contexts considered relevant for the semantic interpretation.

**Definition** Let  $E_{L_c}$  be a theory of  $L_c$ ,  $S_{L_c}$  be a stimulus and  $T_c$  and  $T_m$  be two integers such that  $0 < T_c < T_m$ . The relevant contexts are defined as follows:

- calculation of the maximal contexts of rank  $T_m$  and above on  $\{E_{L_c}, S_{L_c}\}$ . This defines a set of contexts  $\{C_k\}$ ,
- then enrichment of each maximal context  $C_k$ , by the credible context of rank  $T_c$  to  $T_m-1$  on  $\{E_{L_c}, S_{L_c}, C_k\}$ .

This defines the set of **epistemic contexts**. It denoted  $C_{E_{L_c}, S_{L_c}, T_c, T_m}$ .  $\{E_{L_c}, S_{L_c}, C \in C_{E_{L_c}, S_{L_c}, T_c, T_m}\}$  is said the **set of semantic perspectives**.

The interpretation is now performed by considering the productions of each perspective.

**Definition** Let  $E_{L_c}$  a theory,  $S_{L_c}$  a stimulus and  $T_c$  and  $T_m$  be two integers such that  $0 < T_c < T_m$ . An expression  $f$  is:

- **possible** if it is true from at least one perspective and false from at least one other perspective,
- **credible** if it is true from at least one perspective and false from no perspective,
- **incredible** if it is false from at least one perspective and true from no perspective,
- **not interpretable** in other cases.

This definition presents the basic semantic interpretation function of  $L_c$ . It can be made more complex [9], for example by distinguishing true formulas in all perspectives, or by using thought ranks to identify semantics according to pairs  $(T_c, T_m)$  and  $(T_c', T_m')$ . We use this, for example, to study the relationship between contextual semantics and the concept of possible worlds [10]. We will discuss this point in a later paragraph.

**Example** By writing convention, we will henceforth note  $c_{ij}$  the thoughts.  $i$  singularizes the proposition and  $j$  indicates its rank.

Let be a set of  $L_p$ 's propositions  $\{a, b, c\}$  and a set of thoughts  $\{c_{10,1}, c_{20,1}, c_{30,1}, c_{11,2}, c_{12,2}\}$ . We consider the following theory:

$$E_{L_c} = \left\{ \begin{array}{l} c_{10,1} \rightarrow (a \rightarrow b), \\ c_{11,2} \rightarrow c_{10,1}, \\ c_{12,2} \rightarrow \neg c_{10,1}, \\ c_{20,1} \rightarrow (a \rightarrow \neg b), \\ c_{30,1} \rightarrow c \end{array} \right\}$$

We suppose  $T_c=1$  and  $T_m=2$  and  $\{a\}$  is the stimulus. So, there are 2 maximum contexts of rank 2:  $\{c_{11,2}\}$  and  $\{c_{12,2}\}$ .

We also have:

$$\{c_{10,1}, c_{20,1}, a\} \vdash_{L_p} b \wedge \neg b$$

So:

- $\{c_{20,1}\}$  is the only strict impossible context considering  $\{E_{L_c}, a, c_{11,2}\}$ , so  $\{c_{10,1}, c_{30,1}\}$  is the credible context of the rank 1 in this case,
- $\{c_{10,1}\}$  is the only strict impossible context considering  $\{E_{L_c}, a, c_{12,2}\}$ , and  $\{c_{20,1}, c_{30,1}\}$  is the credible context in this.

In fine, we obtain 2 semantic perspectives:

- $\{E_{L_c}, a, (c_{11,2}, c_{10,1}, c_{30,1})\}$  which produces  $\{a, b, c\}$ ,
- $\{E_{L_c}, a, (c_{12,2}, c_{20,1}, c_{30,1})\}$  which produces  $\{a, \neg b, c\}$ .

We conclude that, taking  $a$  as the stimulus,  $c$  is true (or credible), and that  $b$  is true and false (or possible).

We note that a perspective is always unitarily consistent, but an expression can be true and false from two different perspectives.

Considering the definition of epistemic contexts,  $T_c$  and  $T_m$  can theoretically take any value. In this example we have used the levels 1 and 2. In the rest of this document, we use the thresholds 2 and 3, which are sufficient to cover the expected level of expressiveness expected in this article.

To conclude this presentation, let note that  $L_c$  brings a reflexive capacity to reasoning by modelling a relation between the thought  $c$  and the sentence  $f$  which expresses it.

### III. PROPERTIES OF $L_c$

The properties of  $L_c$  will be presented briefly. We indicate them here as consequences of the contextual postulate. It is not the purpose of this article to discuss their relevance. These concepts have as many defenders as detractors in mathematical philosophy. The debates, very rich and always topical, would deserve a dedicated article.

#### The syntax production function is monotonic

$L_c$  respects the syntax of propositional logic and is therefore syntactically monotonic: whatever  $f$  and  $g$  are contextually well-formed formulas, if a theory  $E_{L_c}$  produces  $f$  then  $\{E_{L_c}, g\}$  produces  $f$ .

Note that the syntactic interpretation function is mechanically also monotonic.

#### The semantic interpretation function is non-monotonic

We note that  $C_{E_{L_c}, T_c, T_m}$  must be recalculated if a new knowledge is introduced. So  $L_c$  has a non-monotonic semantic: considering the same stimulus  $S_{L_c}$ ,  $f$  can be credible considering  $\{E_{L_c}, S_{L_c}\}$  and incredible considering  $\{(E_{L_c}, g), S_{L_c}\}$ .

#### The semantic is fallibilist

Fallibilism [15] is a philosophical principle that assumes that absolute knowledge is probably impossible: all belief can, at any time, be questioned – and possibly contradicted.

We note that the syntax of  $L_c$  is based on that of  $L_p$ . It is therefore based on axiomatic principles which it considers as absolute. However, a consequence of the contextual postulate is that every proposition is possibly false.  $L_c$  thus proposes the paradox of relying on a syntax considered as absolutely true to model knowledge interpreted semantically as absolutely uncertain.

To avoid this, the solution is to consider that what is not explicitly false is credible and will remain so until it is explicitly contradicted or challenged.

We will illustrate this with some examples which we will develop in the following paragraphs.

#### The semantic is perspectivist

Perspectivism [8] refers to philosophical doctrines that defend the idea that our perception of reality is composed of the sum of the perspectives we have on it.

In  $L_c$ , the semantic interpretation is obtained by considering the interpretations, possibly contradictory, of each epistemic context: truth is not the consequence of a global point of view built on the whole of thoughts, but the juxtaposition of several points of view from distinct subsets of thoughts.

#### Propositions are attributes, not assertions

In the most adopted mathematical approach, a proposition is an assertion apprehended in its content. Considering a theory, its semantic interpretation admits a truth value: it is true, or false, or perhaps contradictory, etc.

In  $L_c$ , a proposition is not considered true or false. This is a mechanical consequence of the application of the contextual postulate. The expression  $c \rightarrow f$  does not say that  $c$  or  $f$  is true or false. It says that the thought  $c$ , considered as an atomic proposition, is expressed by the formula  $f$ . Consequently, a proposition (or a formula) is assumed to be credible in relation to a set of thoughts, called a context. It characterizes it.

So, in  $L_c$ , a proposition is not an assertion in the strict sense of the term. It must be understood as a characteristic, or an attribute, of the context.

Consider, for example, the sentence “If Tweety is a bird, then it flies”. Its modelling in predicate logic can be:

$$Bird(Tweety) \rightarrow Flying(Tweety)$$

In  $L_c$ , the precedent assertion is modelled by:

$$\begin{aligned} c_{1j} &\rightarrow (Tweety \rightarrow Bird) \\ c_{2j} &\rightarrow (Bird \rightarrow Flying) \end{aligned}$$

which allows for several readings, for example: attributes  $Bird$  and  $Flying$  are attributes of the context  $\{c_{1j}, c_{2j}\}$  if we consider the stimulus  $Tweety$ .

#### Brief discussion of these properties

The proposed languages are usually either completely monotonic (syntactically and semantically) or completely non-monotonic (syntactically and semantically). Contextual logic proposes a formalism that is syntactically monotonic and semantically non-monotonic.

Given the principles of adequacy (what is produced is true) and completeness (what is true is produced), a completely non-monotonic language produces formulas only if nothing contradicts this production. This leads to the need to use recursive calculation algorithms. By maintaining syntactic monotonicity, contextual logic avoids this difficulty.

Classically, formalisms focus on modelling knowledge that is deemed true. To deal with inconsistencies and

semantic uncertainties, they are enriched to express levels of necessity (epistemic modal logics), quantification (predicate logics), inconsistency (para-consistent logics), new rules of syntactic production (such as default logic), multiple interpretation strategies (adaptive logics), or multi-valued interpretation (multivalued logics).

These logics and contextual logic model reasoning according to completely different conceptions: demonstrative behavior for them (this formula is demonstrated considering this theory) and fallibilist and perspectivist behavior for  $L_c$  (considering this theory, this formula is credible because it is not incredible, or it can be true and false depending on how you look at it).

This is one possible answer to the concept of *non-demonstrative inference*, a property of human reasoning that emerges from empirical work carried out by researchers in cognitive science [2].

We will now present how to use  $L_c$  to model predicate, temporal, and epistemic information.

#### IV. MODELLING A PREDICATE KNOWLEDGE

$L_p$  sees a proposition as a whole, to which a universal value is given. It is then necessary to decompose this whole when we wish to use a singular value. To this end, predicate logic meets this need by introducing a notion of quantification. It then becomes possible to model that *All men are mortal*, and to deduce that *Socrates is mortal* because *He is a man*. This syllogism uses the link between *Man* and *Socrates* to deduce the association with *Mortal*. In this context, Gottlob Frege [7] theorized the notion of universal quantifier. As a classical example of use :

$$\forall x \text{ Man}(x) \rightarrow \text{Mortal}(x)$$

which reads: *Whatever x is, if x is a man then x is mortal*. We note two enrichments with respect to the native modelling capabilities of propositional logic:

- a notion of universal quantifier. In paragraph VII we will return to this point,
- the possibility of breaking down a proposition into several singular instances. In our example, *Men are mortal* is modelled by using two distinct units, *Man* and  $x$ . The expression  $\text{Man}(x)$  creates a syntactic relationship, which formalizes a semantic link, between these two-unit elements.

It is not possible to model a relation between two atomic propositions in  $L_p$  because the symmetric behavior of the connectors. Put more explicitly,  $f \rightarrow (g \rightarrow h)$  is equivalent to  $g \rightarrow (f \rightarrow h)$ :  $f$  and  $g$  have the same behavior in the formula, and neither has a privileged relationship with  $h$ .

We will present how this point can be solved in  $L_c$ . Suppose in  $L_p$  the set of propositions  $\{a, b, c, d, e, f, g\}$  and the following theory:

$$E_{L_p} = \left\{ \begin{array}{l} a \rightarrow b, \\ c \rightarrow a, \\ d \rightarrow a, \\ d \rightarrow \neg b, \\ a \rightarrow e, \\ f \rightarrow (c \rightarrow g) \end{array} \right\} \quad (pk)$$

$(pk)$  indicates a particular knowledge: we want to model that  $c \rightarrow g$  is a predicate of  $f$  - to express, for example: "The cat (denoted  $f$ ) attacks (denoted  $g$ ) the sparrow (denoted  $c$ )".

The formula  $f \rightarrow (c \rightarrow g)$  is syntactically equivalent to  $c \rightarrow (f \rightarrow g)$ , which says: the sparrow attacks the cat – which does not mean the same thing.

Now, consider the following set in  $L_c$ :

$$E_{L_c} = \left\{ \begin{array}{l} c_{10,2} \rightarrow (a \rightarrow b), \\ c_{20,2} \rightarrow (c \rightarrow a), \\ c_{30,2} \rightarrow (d \rightarrow a), \\ c_{40,2} \rightarrow (d \rightarrow \neg b), \\ c_{50,2} \rightarrow (a \rightarrow e), \\ c_{60,1} \rightarrow (c \rightarrow g), \\ c_{61,2} \rightarrow (f \rightarrow c_{60,1}), \\ c_{62,3} \rightarrow c_{61,2}, \\ c_{63,3} \rightarrow \neg c_{61,2} \end{array} \right\}$$

This transformation requires some comments:

- there are many syntactic ways of modelling a body of knowledge in  $L_c$ . This raises the open question of learning. We present the one that seems most relevant based on our studies to date,
- according to the contextual postulate, thoughts are propositions that are not pronounced: natural language is only expressed directly through propositions of  $L_p$  (i.e., of rank 0). Therefore, the transformation from  $E_{L_p}$  to  $E_{L_c}$  is based on an algorithm (which is presented in paragraph VIII). It offers an automatic translation to the  $c_{ij}$ .

$c_{60,1}$ ,  $c_{61,2}$ ,  $c_{62,3}$  and  $c_{63,3}$  model predicative knowledge. The semantic interpretation is obtained in several stages:

- the first step interprets  $f$  and  $c$  as separate stimuli. We get two perspectives. The one that carries  $f$  uses  $c_{61,2}$ , to associate  $c \rightarrow g$  (i.e.,  $c_{60,1}$ ) with it, and thus memorize that  $c \rightarrow g$  is true in models verifying  $f$ ,
- the second step consists in crossing the semantics of the two perspectives on the models that verify  $c_{62,3}$ . The resultant products the proposition  $g$  and retains  $c_{61,2}$  which traces the relation between  $c \rightarrow g$  and  $f$ .

We now understand how  $L_c$  proposes to model a predicate. It uses the notion of thought for this.  $c_{60,1} \rightarrow (c \rightarrow g)$  syntactically associates  $c$  and  $g$  by  $c_{60,1}$ . This association is preserved, and is interpreted as predicate information of  $f$  in the expression  $c_{61,2} \rightarrow (f \rightarrow c_{60,1})$ .  $c_{60,1}$  generate the necessary asymmetry for the syntax to model the predicate relation.

The semantic interpretation we have just described extends the consumption of epistemic contexts by a recursive function (presented in paragraph VIII):

*The semantics of  $\{f, c\}$  is not obtained by computing the semantic of  $\{f \wedge c\}$ , but by crossing the semantics of  $\{f\}$  and  $\{c\}$ .*

This method enriches the semantic reading of the models of a contextual theory by allowing the modelling and exploitation of the notion of predicates.

## V. TEMPORAL LOGIC

A theoretical subject seems to us important to be able to model the human reasoning within the framework of an approach in Artificial Intelligence. It is about capturing the notion of temporality in formal languages.

To achieve this, we propose to exploit the predicative property of  $L_c$  by using two atomic propositions of  $L_p$  (i.e., of rank 0), *Future* and *Past*. Consider the following expressions:

$$\begin{aligned} f \rightarrow (\text{Future} \rightarrow g) & \quad (pk) \\ f \rightarrow (\text{Past} \rightarrow g) & \quad (pk) \\ \neg \text{Future} \vee \neg \text{Past} & \end{aligned}$$

The first (respectively second) expression can be read: “If  $f$  is true, then  $g$  is true in the Future (respectively Past) of  $f$ ”.

This modelling makes it possible to reason using a vision of “simplified time”, broken down into three blocks: *Past*, *Future* and *Present* (by default). We are studying this model to investigate the possibility of capturing a notion of linear temporality (e.g., if  $f$  is true, then  $g$  is true in the future of  $f$ , and  $h$  is true in the future of  $g$ ) close to that of propositional temporal logic (PTL). For example, the notion of “Always” could be modelled by the following expression:

$$f \rightarrow (\text{Future} \rightarrow f) \quad (pk)$$

The exercise requires an evolution of the semantic interpretation function to go beyond the “simplified time” view. We have not yet managed to identify the right definition.

## VI. MODELLING AN EPISTEMIC MODAL KNOWLEDGE

Let in  $L_p$  the set of propositions  $\{a, b, c, d, e\}$ , and the following theory:

$$E_{L_p} = \left\{ \begin{array}{l} a \rightarrow b, \\ c \rightarrow a, \\ d \rightarrow a, \\ d \rightarrow \neg b, \\ a \rightarrow e, \\ f \rightarrow (c \rightarrow g) \end{array} \right\} \quad (em) \quad (pk)$$

(*em*) indicates a particular knowledge that cannot be modelled in  $L_p$ : the belief  $a \rightarrow b$  is not always true. It is a modal knowledge. For example, we want it to be true for  $c$  (allowing, from  $c$ , to produce  $b$ ), and invalid for  $d$  ( $d$  produces  $\neg b$  and must not produce  $b$ ). Let consider the following set:

$$E_{L_c} = \left\{ \begin{array}{l} c_{10,2} \rightarrow (a \rightarrow b), \\ c_{11,3} \rightarrow c_{10,2}, \\ c_{12,3} \rightarrow \neg c_{10,2}, \\ c_{20,2} \rightarrow (c \rightarrow a), \\ c_{30,2} \rightarrow (d \rightarrow a), \\ c_{40,2} \rightarrow (d \rightarrow \neg b), \\ c_{50,2} \rightarrow (a \rightarrow e), \\ c_{60,1} \rightarrow (c \rightarrow g), \\ c_{61,2} \rightarrow (f \rightarrow c_{60,1}), \\ c_{62,3} \rightarrow c_{61,2}, \\ c_{63,3} \rightarrow \neg c_{61,2} \end{array} \right\}$$

The algorithm for this transformation is presented in paragraph VIII.  $c_{11,3}$  and  $c_{12,3}$  model the modal

knowledge we wish to express. It is a generic transformation: it does not depend on  $c$  or  $d$ . The meaning that can be given to  $c_{11,3}$  is that  $a \rightarrow b$  is, and to  $c_{12,3}$  that  $a \rightarrow b$  is not.

The semantic interpretation (the algorithm is presented in paragraph VIII) produces  $\{a, b, e\}$  for the stimulus  $c$ , and  $\{a, \neg b, e\}$  for the stimulus  $d$ :

- the maximal contexts of rank 3 identify the combinations of thoughts allowed by the modal knowledge. In the example, we get  $\{c_{11,3}\}$ , which verifies  $a \rightarrow b$ , and  $\{c_{12,3}\}$ , which says nothing about  $a \rightarrow b$ ,
- each maximal context is then enriched with its associated credible context on rank 2. Now  $c_{1,2}$ ,  $c_{5,2}$  and  $c_{6,2}$  are inconsistent with  $d$ :

$$\{c_{10,2}, c_{30,2}, c_{40,2}, d\} \vdash_{L_p} b \wedge \neg b$$

So  $c_{30,2}$  and  $c_{40,2}$  are rejected from models verifying  $\{c_{11,3}, d\}$  (because  $c_{11,3} \rightarrow c_{10,2}$ ), but is found in  $\{c_{12,3}, d\}$  (because  $c_{12,3} \rightarrow \neg c_{10,2}$ , which solves the contradiction).

As an example of application, this allows to model:

- sparrows (denoted  $c$ ) fly (denoted  $b$ ), because sparrows are birds (denoted  $a$ , so  $c \rightarrow a$ ) and birds fly (i.e.,  $a \rightarrow b$ ),
- ostriches (denoted  $d$ ) are birds ( $d \rightarrow a$ ) that do not fly ( $d \rightarrow \neg b$ ). The knowledge that birds fly ( $a \rightarrow b$ ) is discarded from contexts carrying ostriches because of the contradiction *flying* and  $\neg$ *flying* that it generates.

Ostriches inherit all the characteristics associated with birds except the attribute *flying*. And, unless explicitly known otherwise, any other type of bird (e.g., a swallows) is treated like sparrows, and associated with the same attributes.

## VII. GAP BETWEEN $L_c$ AND OTHERS FORMALISMS

Fallibilism and perspectivism properties are probably the main features of  $L_c$  as a formal language. They lead to substantial differences in the way  $L_c$  treats knowledge compared to other formalisms.

We present below those we have identified. Our study is not exhaustive. There are many formalisms. All of which are probably relevant to the modelling they wish to do. We have selected some of them because they are sometimes general, and most often because they have the advantage of being more familiar to us than the others.

### Para-consistent logics

Classical formalisms assume an assumption: syntactic rules are autonomous from the meanings of propositions. Combined with Aristotle's three principles (excluded third ( $a$  is true or false), non-contradiction ( $a$  is not both true and false), and identity ( $a$  is  $a$ )), it leads to the principle of explosion: syntactic inconsistency leads to the production of everything and its opposite: whatever  $f$  and  $g$  two assertions,  $f \wedge \neg f \vdash_{L_p} g$ .

Para-consistent logics aim to tolerate inconsistencies by escaping the principle of explosion. The common approach is to weaken Aristotle's principles to limit the inference capabilities of language [11]. The path taken by  $L_c$  is to keep

Aristotle's principles, and to circumvent the initial hypothesis: elementary propositions are all equal with respect to syntax, *but not with respect to semantics*. This uses the distinction that is made in the set of atomic propositions between thoughts and atomic propositions of  $L_p$ .

$L_c$ 's perspectivist approach allows it to consider that a proposition can be true from one perspective and false from another, and that both perspectives (the reference contexts) are true simultaneously.

It is therefore possible to capture in  $L_c$  the modelling of the claim that cheap horses are expensive, since they are rare, and that what is rare is expensive – and yet not expensive, since they are cheap.

$L_c$  is therefore not a para-consistent formalism. It can say that  $f$  is true *and* false in relation to two distinct contexts (i.e., possible according to the semantic vocabulary of  $L_c$ ), but it does not escape the principle of explosion. If one retains a reference context that syntactically produces  $f \wedge \neg f$ , then it will produce any belief  $g$  whatsoever. Inconsistency is accepted in the semantics of  $L_c$ , it remains non-tolerable in its syntax.

Further information

In general, non-contextual logics model that  $f$  is true or that  $f$  is false. In  $L_c$ , these two expressions read: *I think  $f$  is true* (implied by one epistemic context) and *I think  $f$  is false* (implied by another context).

It is therefore possible to model *I think that  $f$  is not true*, and *I think that  $f$  is not false*, which do not say quite the same thing, or *I do not think that  $f$  is true*, which expresses even different information.

So  $L_c$  offers an alternative to the problem of double negation, allowing one to express *I think  $f$  is not true* and, at the same time, *I do not think  $f$  is false*. It is the meaning of  $c'$  in the set  $\{c \rightarrow f, c' \rightarrow \neg c\}$ : it assumes the possibility of  $\neg f$  without expressing it.

### Modal epistemic logic

To reason about the quality of the interpretation value, modal approaches extend the expressiveness of languages by adding a new connector. The most widely used epistemic modal connector is the alethic connector  $\Box$ .  $\Box f$  usually expresses that  $f$  is necessary, and its dual  $\neg\Box\neg f$ , denoted  $\Diamond f$ , that  $f$  is possible. The language relies on the semantics of possible worlds [10] to benefit from an interpretation function.

Contextual logic retains the syntax of propositional logic and focuses the level of credibility on the interpretation function by exploiting a classification of elementary propositions (the ranks of thoughts). In the context of the  $K$ -system, we propose the following relationship between the two formalisms:

*Let  $f$  be a formula. Let  $i$  be the highest rank of the elementary propositions that compose it.  $f$  is said to be possible (in the modal sense) if there exist two thoughts  $c_{1,i+1}$  and  $c_{2,i+1}$  such that  $c_{1,i+1} \models f$  and  $c_{2,i+1} \models \neg f$  and is necessary if  $c_{1,i+1}$  exists and  $c_{2,i+1}$  does not.*

This translation requires the use of an enriched semantic interpretation function using thought ranks to capture  $\Box\Box f$  for example [9].

It models the set  $\{\Diamond f, \Diamond\neg f\}$  by  $\{c_{1,i+1} \models f, c_{2,i+1} \models \neg f\}$ , but the set restricted to  $\{\Diamond f\}$  (translation  $\{c_{1,i+1} \models f\}$ ) is interpreted as  $\{\Box f\}$ .

$L_c$  is therefore less expressive than modal language. This is a consequence of its fallibilistic character: if there is a reasoning that justifies the possibility of  $f$ , then  $f$  is considered credible if the possibility of its opposite is not explicitly expressed. Provided that this behavior is acceptable,  $L_c$ 's contribution to modal formalisms is its ability to model epistemic information within the framework of propositional syntax.

### Default logic

Default logic, proposed by Raymond Reiter [17], extends production rules by expressions of the form:

$$(a : b / c)$$

which read: “if  $a$  is true and nothing contradicts  $b$  then  $c$  is produced”. We have seen in the previous example that  $L_c$  allows the modelling and exploitation of rules with exception. However, related to Raymond Reiter's syntax,  $L_c$ 's expressiveness is limited to rules of the form [9]:

$$(a : b \wedge c / b \wedge c)$$

adding  $a \rightarrow b \wedge c$  to all extensions (in Reiter's sense) that allow it. We are therefore back to the rules of so-called normal default. With this translation, the Reiter's extension is enriched by  $b$ .

Take the following example of Reiter's theory:

$$\{a, (a : b / c)\}$$

$b$  is possible, so, the Reiter's extension is  $\{a, c\}$ . In  $L_c$ , we obtain the perspective,  $\{a, b, c\}$ . The explanation of these results is the same as for modal languages: expressing a possibility (in this case “ $b$  is possible”) leads to considering it credible if the possibility of its opposite is not explicitly expressed.

Now, if we consider the following example:

$$\{a, (a : b / c), (a : \neg b / d)\}$$

$b$  and  $\neg b$  are possible, so, the Reiter's extension is  $\{a, c, d\}$ . In  $L_c$  we obtain two perspectives,  $\{a, b, c\}$  and  $\{a, \neg b, d\}$ :  $a, c$  and  $d$  are credible, and  $b$  is explicitly possible (i.e., true, and false).

### Adaptative logic

To solve the various problems encountered by classical logics, Diderick Batens [3] proposes to consider that there are several reasoning strategies. It would be advisable to select the one best adapted to the knowledge base. Consider, for example, the following set of formulas:

$$E_{La} = \{\neg p, \neg q, p \vee q, p \vee r, q \vee r\}$$

It is incoherent, and therefore explosive in the context of propositional logic. If one adopts a strategy favoring reliable reasoning, it is not possible to deduce  $r$ : it would be unwise to conclude anything using the first three formulas. However, if we adopt a strategy that minimizes abnormalities, and assume that at least two of the first three formulae are true, then  $r$  is produced.

In contextual logic, the set becomes:



$$E_p = \left\{ \begin{array}{l} c_{1,1} \models \neg p, \\ c_{2,1} \models \neg q, \\ c_{3,1} \models p \vee q, \\ c_{4,1} \models p \vee r, \\ c_{5,1} \models q \vee r \end{array} \right\}$$

Assume that the stimulus is empty.  $\{c_{1,1}, c_{2,1}, c_{3,1}\}$  being a minimal impossible context,  $\{c_{4,1}, c_{5,1}\}$  is the reference context. As far as we know,  $r$  is not interpretable. Using epistemic contexts that retain the maximal credible contexts at rank 1 is therefore a prudent strategy. Let's modify the definition of epistemic contexts and adopt the maximum possible contexts. In this case, we obtain three reference contexts:

- $\{c_{1,1}, c_{2,1}, c_{4,1}, c_{5,1}\}$ , that produces  $\{\neg p, \neg q, r\}$ ,
- $\{c_{1,1}, c_{3,1}, c_{4,1}, c_{5,1}\}$ , that produces  $\{\neg p, q, r\}$ ,
- $\{c_{2,1}, c_{3,1}, c_{4,1}, c_{5,1}\}$ , that produces  $\{p, \neg q, r\}$ .

With this definition of the reference contexts,  $r$  is credible.

However,  $L_c$  is not an adaptive logic:

- Both formalisms share the ability to adapt the semantic interpretation to local characteristics:  $L_c$  chooses to use or not to use knowledge depending on the stimulus,
- But its principle is not to adapt the definition of its reasoning in the face of a contradiction. It uses a single inference strategy, based on reference contexts, whose definition is definitively established according to general theoretical criteria.

In the above example, a possible solution to retain the original definition of epistemic contexts would be to model, by a meta-thought, the strategic choice that at least two of the thoughts  $c_{1,1}$ ,  $c_{2,1}$  or  $c_{3,1}$  is true. This meta-thought could be:

$$\begin{array}{l} c_{6,3} \rightarrow (c_{1,1} \wedge c_{2,1}) \\ c_{7,3} \rightarrow (c_{1,1} \wedge c_{3,1}) \\ c_{8,3} \rightarrow (c_{2,1} \wedge c_{3,1}) \end{array}$$

In this case, the semantic function will maximize the possibilities, by considering the contexts  $(c_{1,1}, c_{2,1})$ ,  $(c_{1,1}, c_{3,1})$  and  $(c_{2,1}, c_{3,1})$  separately to conclude that  $r$  is credible.

In  $L_c$ , the choice of the reasoning strategy is not carried by the formalism. It is carried by the knowledge (or not) of complementary information that the agent uses (or not).

### Predicate logic

A difference between predicate logic and contextual logic is that this latter has no quantifier. The formula:

$$\forall x \text{ Bird}(x) \rightarrow \text{Flying}(x)$$

is therefore not capturable in  $L_c$  and become:

$$c_{i,j} \models \text{Bird} \rightarrow \text{Flying}$$

We use contextualized propositional logic. It would be possible to use contextualized predicate logic. The exercise may be interesting mathematically. Predicate logic is a very powerful tool, which has proved itself in mathematics.

But we assume that the principle of quantification is contrary to the fallibilist conception of human reasoning. Indeed, it supposes that our belief at a given moment is built on what we know at that moment. It can be questioned at any

time. On the other hand, from our point of view, new information does not modify the knowledge we have: the new knowledge is just added in the set of knowledge and modifies the semantic interpretation we have. These statements are evidently open to discussion. But if we accept them according to an absolute application of fallibilism, then nothing can be universal.

In fact,  $L_c$  natively models a form of quantifier. For example, assume a knowledge base consisting of a single piece of information:

$$E_{L_c} = \left\{ c_{i,j} \models \text{Bird} \rightarrow \text{Flying} \right\}$$

This information says that *Bird fly*. But, given this body of knowledge, it would be the same to say *All birds fly*. In fact, whatever  $x$ , if we consider that  $x$  is a bird:

$$E_{L_c} = \left\{ \begin{array}{l} c_{i,j} \models \text{Bird} \rightarrow \text{Flying}, \\ c_{k,j} \models x \rightarrow \text{Bird} \end{array} \right\}$$

then we conclude that  $x$  flies. Now suppose we receive the information that *Not all birds fly* - that is, formally in the sense of  $L_c$ , that the thought  $c_{i,j}$  can be true or false. The knowledge base is enriched, and becomes:

$$E_{L_c} = \left\{ \begin{array}{l} c_{i,j} \models \text{Bird} \rightarrow \text{Flying}, \\ c_{k,j} \models x \rightarrow \text{Bird} \\ c_{i1,1} \models c_{i,j} \\ c_{i2,1} \models \neg c_{i,j} \end{array} \right\}$$

We also conclude that  $x$  flies. It is a consequence of the fallibilism behavior of  $L_c$ . To say that  $x$  does not fly, it is necessary to introduce this information explicitly:

$$E_{L_c} = \left\{ \begin{array}{l} c_{i,j} \models \text{Bird} \rightarrow \text{Flying}, \\ c_{k,j} \models x \rightarrow \text{Bird} \\ c_{i1,1} \models c_{i,j} \\ c_{i2,1} \models \neg c_{i,j}, \\ c_{k2,j} \models x \rightarrow \neg \text{Flying} \end{array} \right\}$$

It appears that, in  $L_c$ , the notion of universality is not carried by a syntactic quantifier. It is deduced from the semantic interpretation: a state is universal as long as it is not explicitly contradicted. Finally, let us note that  $L_c$  allows us to define and reason about theoretical sets, for example: if I assume that all birds fly (that is, in the sense of  $L_c$ , if I consider the stimulus  $c_{i1,1}$ ), what can I conclude?

The critique of the universal quantifier just mentioned can be extended to epistemic modal connectors. If we consider that human reasoning is strictly and absolutely fallibilistic, then the syntax of formal language, because it carries the logic of thought [6], must not contain any element modelling a notion of absolute. This assertion is highly debatable and is still much discussed in mathematical philosophy (see [1] and [16] for example).

Pending definitive argumentation one way or the other, we have chosen to remain within the framework of contextualized propositional logic, and not to use contextualized predicate logic or contextualized modal logic for example. This theoretical position is obviously open, and it may lead our work to a dead end. But it seems interesting to us to go to the end of the technical analysis of the hypothesis.



## VIII. EXAMPLE OF AN APPLICATION

In this paragraph, we present an example of application. Our objective is certainly not to propose a definitive argument, but we hope that it will clarify our presentation and illustrate the capabilities of  $L_c$  for knowledge modeling.

Beyond the school cases used to present contextual logic, let consider a more substantial body of knowledge to illustrate the global behavior of  $L_c$  and the possibility of expression that it offers.

**Example** Consider the following knowledge:

*Birds and felines are animals* <sup>(01 and 02)</sup>. *Birds are not felines* <sup>(03)</sup>. *Animals are generally diurnal* <sup>(04)</sup>. *Diurnal animals are not nocturnal* <sup>(05)</sup>. *Birds usually fly* <sup>(06)</sup>. *They are generally insectivorous* <sup>(07)</sup> and *gregarious* <sup>(08)</sup>. *Felines are carnivorous* <sup>(09)</sup> and *usually solitary* <sup>(10)</sup>. *Solitaires are not gregarious* <sup>(11)</sup>. *Insectivores are generally not carnivorous* <sup>(12)</sup>.

*Swallows, sparrows, ostriches, and owls are bird* <sup>(13, 14, 15 and 16)</sup>. *Swallows are not sparrows* <sup>(17)</sup>, *ostriches* <sup>(18)</sup>, or *owls* <sup>(19)</sup>. *Sparrows are not ostriches* <sup>(20)</sup> or *owls* <sup>(21)</sup>. *Ostriches are not owls* <sup>(22)</sup>. *Ostriches do not fly* <sup>(23)</sup>. *Owls are solitary* <sup>(24)</sup>, *nocturnal* <sup>(25)</sup>, *carnivorous* <sup>(26)</sup>, and *insectivorous* <sup>(27)</sup>.

*Cats and lions are felines* <sup>(28 and 29)</sup>. *Cats are not lions* <sup>(30)</sup>. *Cats are nocturnal* <sup>(31)</sup>. *Lions are gregarious* <sup>(32)</sup>.

*Carnivores are hunters* <sup>(33)</sup>. *Herbivores are prey for hunters* <sup>(34)</sup>. *Hunters generally attack prey* <sup>(35)</sup>. *If the prey is larger than the hunter, the latter does not attack* <sup>(36)</sup>. *Ostriches are bigger than cats* <sup>(37)</sup> and *owls* <sup>(38)</sup>.

It might be possible to model this knowledge using a formalism other than  $L_c$ . The default predicate logic could probably do this, for example. We have not attempted this. Our aim is not to compare the capabilities of the formalisms. The objective of this article is to show that the syntax of propositional logic is sufficient to model certain knowledge that was, until now, deemed to escape it.

We propose to translate this knowledge into  $L_p$  by the following formulas.

01	$Bird \rightarrow Animal$	
02	$Feline \rightarrow Animal$	
03	$Bird \rightarrow \neg Feline$	
04	$Animal \rightarrow Diurnal$	(em)
05	$Diurnal \rightarrow \neg Nocturnal$	
06	$Bird \rightarrow Flying$	(em)
07	$Bird \rightarrow Insectivore$	(em)
08	$Bird \rightarrow Gregarious$	(em)
09	$Feline \rightarrow Carnivore$	
10	$Feline \rightarrow Solitary$	(em)
11	$Gregarious \rightarrow \neg Solitary$	
12	$Insectivore \rightarrow \neg Carnivore$	(em)
13	$Swallow \rightarrow Bird$	
14	$Sparrow \rightarrow Bird$	
15	$Ostrich \rightarrow Bird$	
16	$Owl \rightarrow Bird$	
17	$Swallow \rightarrow \neg Sparrow$	
18	$Swallow \rightarrow \neg Ostrich$	
19	$Swallow \rightarrow \neg Owl$	
20	$Sparrow \rightarrow \neg Ostrich$	
21	$Sparrow \rightarrow \neg Owl$	

22	$Ostrich \rightarrow \neg Owl$	
23	$Ostrich \rightarrow \neg Flying$	
24	$Owl \rightarrow Solitary$	
25	$Owl \rightarrow Nocturnal$	
26	$Owl \rightarrow Carnivore$	
27	$Owl \rightarrow Insectivore$	
28	$Cat \rightarrow Feline$	
29	$Lion \rightarrow Feline$	
30	$Cat \rightarrow \neg Lion$	
31	$Cat \rightarrow Nocturnal$	
32	$Lion \rightarrow Gregarious$	
33	$Carnivore \rightarrow Hunter$	
34	$Herbivore \rightarrow (Hunter \rightarrow Prey)$	(pk)
35	$Hunter \rightarrow (Prey \rightarrow Attack)$	(em) and (pk)
36	$Hunter \rightarrow (Prey \rightarrow (Larger \rightarrow \neg Attack))$	(pk)
37	$Ostrich \rightarrow (Cat \rightarrow Bigger)$	(pk)
38	$Ostrich \rightarrow (Owl \rightarrow Bigger)$	(pk)

These formulas take up the whole statement. We are not in  $L_p$  because of the additional information (em – epistemic modality) and (pk – predicate knowledge). To fit into a strictly propositional syntax, it is necessary to translate them, respecting the principles of transformations presented in the previous paragraphs. We use for this the following conversion algorithm:

For each formula  $f_i$   
 If  $f_i$  is of type  $pk$  (of the form  $a \rightarrow (c \rightarrow b)$ )  
   Creating the thoughts  $c_{i0,1}$ ,  $c_{i1,2}$ ,  $c_{i2,3}$  et  $c_{i3,3}$   
   Creating the formula  $c_{i0,1} \rightarrow (c \rightarrow b)$   
   Creating the formula  $c_{i1,2} \rightarrow (a \rightarrow c_{i1,1})$   
   Creating the formula  $c_{i2,3} \rightarrow c_{i1,2}$   
   Creating the formula  $c_{i3,3} \rightarrow \neg c_{i1,2}$   
 Else if  $f_i$  is of type  $em$   
   Creating the thoughts  $c_{i0,2}$ ,  $c_{i1,3}$  et  $c_{i2,3}$   
   Creating the formula  $c_{i0,2} \rightarrow f_i$   
   Creating the formula  $c_{i1,3} \rightarrow c_{i0,2}$   
   Creating the formula  $c_{i2,3} \rightarrow \neg c_{i0,2}$   
 Else  
   Creating the thought  $c_{i0,2}$   
   Creating the formula  $c_{i0,2} \rightarrow f_i$

This algorithm requires some comments:

- It takes as input a list of formulas and the associated information (em) or (pk). We are not aware of a method to automatically identify whether a piece of information is modal or epistemic. Mechanisms can facilitate learning (automatic identification and questioning of the “instructor” in case of apparent semantic inconsistency). We will come back to this subject in a future article, devoted to the functional and technical description of the algorithms we have developed.
- We restrict the expression of predicates to sentences of the type “ $f g h$ ”, which we model as  $f \rightarrow (h \rightarrow g)$ .  $f$  is the subject,  $g$  is the action verb and  $h$  is the complement. We have not developed the case in which  $f$ ,  $g$  or  $h$  would carry a second predicative level. In fact, the general algorithm for translating a formula of type (pk) is complex and we have not yet managed to formalize it completely. To present our thoughts to date would have made our presentation unreadable. Limiting our work to simple affirmative sentences seems to us to be a promising basis for further work.
- $Hunter \rightarrow (Prey \rightarrow Attack)$  (formula number 35) has the particularity of being a predicate (pk) on an

epistemic modality (*em*): it is false if the prey is large (formula number 36). Formula number 35 is translated as predicative knowledge. Indeed, predicate already verify, at rank 3, the *em* particularity of to be and not to be (see the proposals  $c_{352.3}$  and  $c_{353.3}$ ).

- The translation algorithm uses the modelling of the initial formulas. So, the set  $\{f, g, h\}$  does not translate like the set  $\{f \wedge g, h\}$ . In some cases, this may change the result of the semantic analysis. This remark introduces, in a more general way, the subject of learning. We will discuss it in a future publication.

After transformation, we obtain the following set.

$c_{010.2} \rightarrow (Bird \rightarrow Animal)$   
 $c_{020.2} \rightarrow (Feline \rightarrow Animal)$   
 $c_{030.2} \rightarrow (Bird \rightarrow \neg Feline)$   
 $c_{040.2} \rightarrow (Animal \rightarrow Diurnal)$   
 $c_{041.3} \rightarrow c_{040.2}$   
 $c_{042.3} \rightarrow \neg c_{040.2}$   
 $c_{050.2} \rightarrow (Diurnal \rightarrow \neg Nocturnal)$   
 $c_{060.2} \rightarrow (Bird \rightarrow Flying)$   
 $c_{061.3} \rightarrow c_{060.2}$   
 $c_{062.3} \rightarrow c_{060.2}$   
 $c_{070.2} \rightarrow (Bird \rightarrow Insectivore)$   
 $c_{071.3} \rightarrow c_{070.2}$   
 $c_{072.3} \rightarrow \neg c_{070.2}$   
 $c_{080.2} \rightarrow (Bird \rightarrow Gregarious)$   
 $c_{081.3} \rightarrow c_{080.2}$   
 $c_{082.3} \rightarrow \neg c_{080.2}$   
 $c_{090.2} \rightarrow (Feline \rightarrow Carnivore)$   
 $c_{100.2} \rightarrow (Feline \rightarrow Solitary)$   
 $c_{101.3} \rightarrow c_{100.2}$   
 $c_{102.3} \rightarrow \neg c_{100.2}$   
 $c_{110.2} \rightarrow (Gregarious \rightarrow \neg Solitary)$   
 $c_{120.2} \rightarrow (Insectivore \rightarrow \neg Carnivore)$   
 $c_{121.3} \rightarrow c_{120.2}$   
 $c_{122.3} \rightarrow \neg c_{120.2}$   
 $c_{130.2} \rightarrow (Swallow \rightarrow Bird)$   
 $c_{140.2} \rightarrow (Sparrow \rightarrow Bird)$   
 $c_{150.2} \rightarrow (Ostrich \rightarrow Bird)$   
 $c_{160.2} \rightarrow (Owl \rightarrow Bird)$   
 $c_{170.2} \rightarrow (Swallow \rightarrow \neg Sparrow)$   
 $c_{180.2} \rightarrow (Swallow \rightarrow \neg Ostrich)$   
 $c_{190.2} \rightarrow (Swallow \rightarrow \neg Owl)$   
 $c_{200.2} \rightarrow (Sparrow \rightarrow \neg Ostrich)$   
 $c_{210.2} \rightarrow (Sparrow \rightarrow \neg Owl)$   
 $c_{220.2} \rightarrow (Ostrich \rightarrow \neg Owl)$   
 $c_{230.2} \rightarrow (Ostrich \rightarrow \neg Flying)$   
 $c_{240.2} \rightarrow (Owl \rightarrow Solitary)$   
 $c_{250.2} \rightarrow (Owl \rightarrow Nocturnal)$   
 $c_{260.2} \rightarrow (Owl \rightarrow Carnivorous)$   
 $c_{270.2} \rightarrow (Owl \rightarrow Insectivore)$   
 $c_{280.2} \rightarrow (Cat \rightarrow Feline)$   
 $c_{290.2} \rightarrow (Lion \rightarrow Feline)$   
 $c_{300.2} \rightarrow (Cat \rightarrow \neg Lion)$   
 $c_{310.2} \rightarrow (Cat \rightarrow Nocturnal)$   
 $c_{320.2} \rightarrow (Lion \rightarrow Gregarious)$   
 $c_{330.2} \rightarrow (Carnivore \rightarrow Hunter)$   
 $c_{340.1} \rightarrow (Hunter \rightarrow Prey)$   
 $c_{341.2} \rightarrow (Herbivore \rightarrow c_{340.1})$   
 $c_{342.3} \rightarrow c_{341.2}$   
 $c_{343.3} \rightarrow \neg c_{341.2}$

$c_{350.1} \rightarrow (Prey \rightarrow Attack)$   
 $c_{351.2} \rightarrow (Hunter \rightarrow c_{350.1})$   
 $c_{352.3} \rightarrow c_{351.2}$   
 $c_{353.3} \rightarrow \neg c_{351.2}$   
 $c_{360.1} \rightarrow (Prey \rightarrow (Larger \rightarrow \neg Attack))$   
 $c_{361.2} \rightarrow (Hunter \rightarrow c_{360.1})$   
 $c_{362.3} \rightarrow c_{361.2}$   
 $c_{363.3} \rightarrow \neg c_{361.2}$   
 $c_{370.1} \rightarrow (Cat \rightarrow Fatter)$   
 $c_{371.2} \rightarrow (Ostrich \rightarrow c_{370.1})$   
 $c_{372.3} \rightarrow c_{371.2}$   
 $c_{373.3} \rightarrow \neg c_{371.2}$   
 $c_{380.1} \rightarrow (Owl \rightarrow Larger)$   
 $c_{381.2} \rightarrow (Ostrich \rightarrow c_{380.1})$   
 $c_{382.3} \rightarrow c_{381.2}$   
 $c_{383.3} \rightarrow \neg c_{381.2}$

We are now completely and strictly compatible with the syntax of propositional logic.

To obtain epistemic contexts is very tedious, so we have computerized the process to overcome this difficulty. To do this, we use a classical propositional logic solver and a classical combinatorial algorithm. These identify the epistemic contexts associated with the indicated stimulus.

The algorithm of contextual semantic interpretation function is:

Stimulus input, in the form of a set of atomic propositions;

For each atomic proposition  $p$  in the stimulus  
 Deletion if necessary, of the previous formula  $c_{s,3} \rightarrow f$ ;

Creation of the formula  $c_{s,3} \rightarrow p$ ;

Extraction of the subset of formulas of rank 3;

**Saturation of this set of formulas** (we use a tool developed to take advantage of the  $c \rightarrow f$  form, but any available  $L_p$  saturation tool will do);

Identification of impossible contexts of rank 3 (i.e. the set of contexts  $C$  of rank 3 such that  $\vdash \neg C$ );

**Combinatorial production of maximum rank 3 contexts;**

For each of these maximum context

Extraction of the subset of formulas of rank 1 at 2;  
 Adding the maximum context;

**Saturation of this set of formulas;**

Identification of impossible contexts of rank 2 or 3;  
 Adding to the maximum rank 3 context the associated rank 2 credible context;

Memorising the result (it is an epistemic context);

Deletion of the formula carrying the stimulus;

For each pair  $(p_1, p_2)$  of the stimulus

Addition of the context composed of the conjunction of the 2 associated epistemic contexts;

**Production by saturation of all credible formulas;**

Deleting the pair  $(p_1, p_2)$ ;

This operation is then carried out for the triplets  $((p_1, p_2), p_3)$  resulting from the stimulus, then the quadruplets, etc., until all the possible combinations are obtained;

We present the algorithm in its raw form. Its complexity is carried by two functions: saturation and combinatorial production. It can be optimized, in particular by limiting the use of saturation functions, or by parallelizing the last steps (calculation of the productions on the peers, the triplets, etc.).

Note that the formulas obtained by  $((p_1, p_2), p_3)$  are identical to those obtained by  $((p_1, p_3), p_2)$ . The order in which the proposals are processed is not important.

Here are some examples obtained by applying contextual semantic interpretation function:

- if the stimulus is *Bird* or *Swallow*: birds (or swallows) are animals, diurnal, gregarious, insectivore, and fly,
- if the stimulus is *Ostrich*: ostriches are birds, insectivore, gregarious, diurnal, and do not fly,
- if the stimulus is *Owl*: owls are birds, carnivorous, solitary, nocturnal, and fly,
- if the stimulus is  $\{Cat, Sparrow\}$ : the sparrow will be attacked,
- if the stimulus is  $\{Cat, Owl, Sparrow, Ostrich\}$ : the sparrow is in a bad way, but the ostrich can go about its business.

To obtain the set of knowledges from this stimulus, we need to interpret the relevant models of  $\{Cat\}$ ,  $\{Owl\}$ ,  $\{Sparrow\}$  and  $\{Ostrich\}$  separately, then the pairs  $\{Cat, Owl\}$ ,  $\{Cat, Sparrow\}$ ,  $\{Cat, Ostrich\}$ ,  $\{Owl, Sparrow\}$ ,  $\{Owl, Ostrich\}$  and  $\{Sparrow, Ostrich\}$  then the triplets, etc.,

- and if the stimulus is  $\{Lion, Ostrich\}$ , the ostrich would have some reason to be worried.

To illustrate our work, we used the classical non-monotonic example of the bird Tweety. It would obviously have been possible to do this with any other theme that could be modelled with logical rules of  $L_c$  (and the complement information  $em$  and  $pk$ ).

## IX. OPENING TO COGNITIVE SCIENCES

One trap in this work is to bias the studies by unintentionally selecting inferences that lead to the desired conclusions. It is easy to succumb to this in  $L_p$ . In  $L_c$ , the presence of thoughts multiplies the risk of error. To counter this, we have developed a tool that reproduces the semantic interpretation function of  $L_c$ . It allowed us to verify the conclusions announced by the theory.

$L_c$  guarantees to the syntactic consistency of any knowledge set, which solves the first step of belief revision. We took advantage of this property to implement learning functions and allow the system to absorb any knowledge.

The tool is based on propositional logic algorithms. We use a classical propositional logic solver, which we have optimized to exploit the  $c \rightarrow f$  form of contextual formulas. But the time saving is not significant. For the example presented, the computation times are of the order of a second when the stimulus consists of only one proposition - and of a few minutes when it contains more than three propositions.

This is not acceptable in the context of an AI. The reasoning (and subsequent decision making) of a human being is obviously much faster. We do not take several seconds to decide to run away from a hungry lion (in fact, this may not be the most appropriate response to this situation, but that's another story).

We use a personal computer from the supermarket, which does not have the capacity to parallelize tasks, for example. However, we do not believe that the solution lies solely in the use of force.

To solve this problem, we propose to use the technical properties of  $L_c$ . They are well suited to model human memory [12]. Indeed, the guarantee of syntactic consistency allows a safe distinction between long-term and short-term memory. This makes it possible to integrate cognitive thresholds from cognitive science research into the algorithms (see [9]):

- the minimal change in beliefs between  $t_n$  and  $t_{n+1}$ ,
- the evolution criterion, which favors the use of the most recent or "primitive" thoughts,
- the technical incapacity threshold, which limits the number of simultaneous thoughts that can be used simultaneously,
- the semantic thresholds: Jacques Pitrat's work [14] shows that a human is not able to reason on more than four levels of meta-knowledge,
- the threshold of proportionate reasoning: when confronted with a stimulus, the objective is not to perform the best theoretical analysis, but to reach a level of analysis sufficient to cause a reaction.

We see a three-level architecture emerging:

- a long-term memory, in which knowledge is stored as it arrives,
- a short-term memory, fed by a function that selects knowledge from the long-term memory using cognitive thresholds,
- and a working memory, which semantically interprets the content of the short-term memory.

This is a very simple first approach, but it already gives some results that we find interesting in the context of human reasoning - for example, deducing, because we have just passed an airport, that the object in the distance in the sky is a plane.

We are currently enriching the algorithms to integrate these notions. The aim is to be able to study its behavior in the face of practical use cases.

## X. CONCLUSION

Aristotle's three laws (the law of contradiction, the law of excluded middle, and the principle of identity) describe an amazingly simple syntax. Our aim in this paper is to show that they conceal a high degree of semantic complexity in their use, and a capacity for expression which has perhaps not yet been fully explored.

The guiding idea of our work is that new information does not syntactically modify old knowledge: it changes our semantic interpretation of it. So syntactic production is monotonic, which solves some technical problems. It is backed up by an adequate and complete syntactic interpretation function, which is also monotonic.

This assumption has two immediate consequences:

- the formal language must guarantee the syntactic consistency of the knowledge base for any new information,
- the non-monotonic property of reasoning is not captured by the syntactic interpretation function, which must be adequacy and completeness with the syntax production function. So, there is necessary a

semantic interpretation function. This is done via an analysis of the models of the knowledge set.

In this perspective, the contextual logic  $L_c$  strictly respects monotonic syntactic rules of propositional logic. However, by modelling reflexivity of reasoning in syntax and extending semantics by a recursive function, we associate to  $L_p$  a non-monotonic interpretation function. It adopts a fallibilist and perspectivist behavior which allows it to exploit incoherent, modal epistemic, temporal, and predicative knowledge.

We are careful not to conclude that propositional syntax is sufficient to model all reasoning.

An exhaustive study would be necessary, for example, based on the typology proposed by [2]. It will probably reveal other cases than those we have dealt with.

Pending the results of this forthcoming study, and as of now, we hope that by showing the capacity of propositional syntax to model modal and predicative information, we open up new avenues of research, which reexamine some of the difficulties (see [4]) encountered in symbolic Artificial Intelligence.

We have developed a tool to verify the conclusions announced by the theory. The algorithms are continuously enriched to integrate, and verify by use cases, the theoretical evolutions.

We have added to the classical logic solver a layer integrating notions from the cognitive sciences. This dimension seems to us to be essential if we wish to model human reasoning. It is also the solution we prefer to solve problems generated by algorithmic complexities.

In parallel, we analyze the subject of natural language, revisiting the semantic interpretation function of  $L_c$  through the theories of Jean Piaget [13].

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