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COMPRATIVE STUDY OF TWO CLASSICAL OPTIMIZATION ALGORITHMS SOLVING ECONOMIC DISPATCH PROBLEM

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ABSTRACT: Solving the Economic Dispatch (ED) problem is an essential task in electrical power management systems. The aim of the ED is to minimize the fuel consumption costs of thermal power plants which could be carried out by finding the optimal generation power for each committed generating unit while providing the load demand and satisfying the operational constraints. This paper presents a comparative study of two classical optimization techniques for solving the conventional ED problem which are Gradient Search and Levenberg-Marquardt Optimization (LMO). The proposed techniques are used to reach the mentioned objective with consideration to real power line transmission loss. They are adapted to IEEE 9-bus 3-unit test system and simulated via MATLAB environment. The results reveal that LMO generates superior and efficient solutions.

KEYWORDS: System optimizations, operations search, energy, production management, economic dispatch, Levenberg-Marquardt optimization.

1 INTRODUCTION

Since the electrical power system of most of the countries mainly consists of thermal plants, the conventional ED is an important task for energy management which deals with power mismatch, fuel cost economy and transmission losses reduction.

The ED problem is solved traditionally using mathematical programming based on optimization techniques such as lambda Iteration method (Dewangan et al., 2015), gradient search (Wood and Wollenberg, 2012), Interior point method (Singhal et al., 2014), Newton's method (Dogra et al., 2014), and Dynamic Programming method (Liang and Glover, 1992).

Classical methods have some advantages such as their optimality that is mathematically proven (Xia and Elaiw, 2010), they can be applied to large scale problems (Bansal, 2005) and they have no problem-specific parameters to specify (Mahdi et al., 2018).

This paper intends to enhance the resolution of a traditional lossy economic active power dispatch via the two conventional optimization techniques; Gradient Search and Levenberg-Marquardt Optimization.

The rest of this paper is organized as follows; **Section 2** introduces the traditional ED mathematical formulation along with the Lagrange function. **Section 3** presents the Gradient Search technique applied for the proposed system. **Section 4** presents the Levenberg-Marquardt Algorithm. **Section 5** elaborates a comparison to discuss the convergence behavior between the Gradient method and LMO. **Section 6** highlights the main conclusion.

2 PROBLEM FORMULATION

The ED is defined as a static constrained mono-objective optimization problem that minimizes the total power cost while satisfying a specific power demand (including the transmission losses) and respecting the power limits of the generators. It is mathematically formulated as follows (Xia and Elaiw, 2010).

2.1 Cost objective function

The main objective of the conventional ED is to minimize the fuel cost to ensure optimal output of generated power while satisfying the operational constraints.

$$FT = \min \sum_{i=1}^{N_g} F_i(Pg_i) \quad (1)$$

where

FT : total fuel cost function

F_i : fuel cost function of the i^{th} generating unit

N_g : number of generating units

Pg_i : real power output of the i^{th} generating unit

$$F_i(Pg_i) = a_i Pg_i^2 + b_i Pg_i + c_i \quad (2)$$

a_i, b_i, c_i : cost coefficients of the i^{th} generating unit

2.2 Problem constraints

The objective cost function (1) is subject to the following equality and inequality constraints.

2.2.1 Power balance constraint

The total generation power must satisfy the demand and the transmission losses.

$$\sum_i^{Ng} Pg_i - Pd - Pl = 0 \quad (3)$$

Where

Pd : total power demand in MW

Pl : transmission lines network power losses in MW

The transmission lines network power loss Pl could be calculated by solving the Power Flow problem. The objective of a power flow study is to calculate the voltages (magnitude and angle) for a given load, generation, and network condition. Once voltages are known for all buses, line flows and losses can be calculated (Albadi, 2019).

The total real power losses can be calculated using the total net injected real power at all buses using the following relation (Ciornei and Kyriakides, 2013)

$$Pl = \sum_{k=1}^{Nl} g_k [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \quad (4)$$

Where

Nl : the number of the transmission lines

g_k : conductance of the k^{th} line that connects bus i to bus j

V_i : the voltage magnitudes at bus i .

δ_i : the voltage angles at bus i .

2.2.2 Real power operating limit constraint

The power generation of each unit should be between its minimum and maximum limits in order to guarantee a stable operation.

$$Pg_i^{\min} \leq Pg_i \leq Pg_i^{\max} \quad (5)$$

Where Pg_i^{\min} and Pg_i^{\max} are the minimal and maximal power limits of the i^{th} generating unit in MW.

2.3 Lagrange function

The above constrained optimization problem can be converted into an unconstrained problem using the Lagrange function (Wood and Wollenberg, 2012) which is given by:

$$L(Pg_i, \lambda) = FT(Pg_i) + \lambda(Pd - Pl - \sum_{i=1}^{Ng} Pg_i) \quad (6)$$

Where λ is the Lagrangian multiplier.

Necessary conditions for the optimization problem are:

$$\frac{\partial L}{\partial Pg_i} = \frac{d(F_i)}{dPg_i} = 0 \quad (7)$$

$$\frac{\partial L}{\partial \lambda} = Pd - Pl - \sum_{i=1}^{Ng} Pg_i = 0 \quad (8)$$

Where $\frac{d(F_i)}{dPg_i}$ is called the incremental cost, it represents

the increase of the supplementary cost of a generating unit.

3 GRADIENT SEARCH

The gradient method is based on the construction of a progression x_k that approaches to the minimum. Suppose to start with a random initial value x_0 , and establish the gradient progression using the next relation:

$$x_{k+1} = x_k + \eta f'(x_k) \quad (9)$$

Where:

η is a chosen positive constant value that helps to adjust the speed of the convergence of the algorithm.

k is the number of iterations.

For the ED problem $x = [Pg_i, \lambda]$ and $f'(x) = \nabla L(Pg_i, \lambda)$

then the problem can be solved by finding the solution of the Gradient vector which is expressed as follows:

$$\nabla L = \begin{bmatrix} \frac{\partial L}{\partial Pg_1} \\ \frac{\partial L}{\partial Pg_2} \\ \vdots \\ \frac{\partial L}{\partial Pg_{Ng}} \\ \frac{\partial L}{\partial \lambda} \end{bmatrix} \quad (10)$$

The initial values of the generated power Pg_i and the Lagrange multiplier λ are calculated with the use of the relations (7) and (8):

$$\lambda = \left(\sum_i^{Ng} \frac{1}{2c_i} \right)^{-1} (Pd + Pl + \sum_i^{Ng} \frac{b_i}{2c_i}) \quad (11)$$

$$Pg_i = \frac{1}{2c_i} \left[\left(\sum_i^{Ng} \frac{1}{2c_i} \right)^{-1} (Pd + Pl + \sum_i^{Ng} \frac{b_i}{2c_i}) - b_i \right] \quad (12)$$

3.1 Gradient Search Algorithm

Step 1: Read given data cost coefficients (a_i, b_i, c_i) , power demand Pd and power limits $(Pg_i^{\min}, Pg_i^{\max})$

Step 2: Calculate the power losses Pl

Step 3: Calculate the initial values of the Lagrangian multiplier λ^0 and the initial values of the active generated power Pg_i^0

Step 4: Check generation limits for each unit

Set $Pg_i = Pg_i^{\min}$ if $Pg_i \leq Pg_i^{\min}$

Set $Pg_i = Pg_i^{\max}$ if $Pg_i \geq Pg_i^{\max}$

Step 5: Calculate the power mismatch

$$\Delta Pg = \sum_{i=1}^{Ng} Pg_i - Pd - Pl \quad (13)$$

Step 7: If $\Delta Pg \leq \varepsilon$ (where ε is the convergence criteria, set to $\varepsilon = 0.01$), then stop calculation and move to Step 11 with the obtained optimal value. Otherwise, go to the next step.

Step 8: Calculate the gradient vector ∇L

Step 9: Calculate Pg_i^{k+1} and λ^{k+1} using (9)

$$Pg_i^{k+1} = Pg_i^k - \eta \nabla L \quad (14)$$

$$\lambda^{k+1} = \lambda^k - \eta \nabla L \quad (15)$$

Where $\eta = 10^{-4}$

Then update Pg_i and λ values.

Step 10: Repeat the procedure from **Step 4**

Step 11: Calculate the cost function with the optimal values of Pg_i and λ

The Gradient Search algorithm is illustrated in the following flowchart:

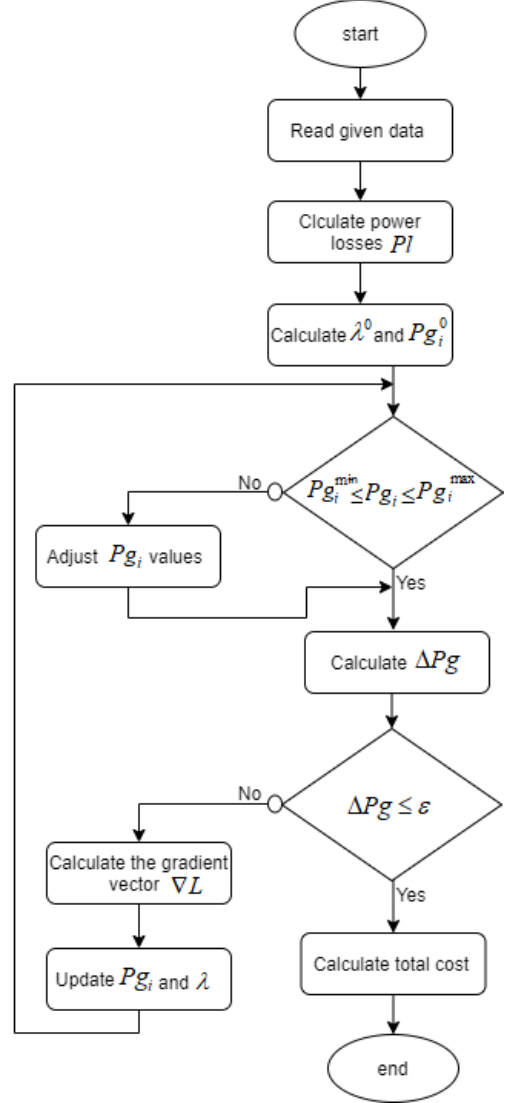


Figure 1: Flowchart of gradient search for the ED

4 LEVENBERG-MARQUARDT OPTIMIZATION

Levenberg-Marquardt Optimization (Marquardt, 1963) is a combination of the directions of the algorithms Gauss-Newton and Gradient descent. It is capable of finding a solution even if it started far from the minimum (Hagan and Menhaj, 1994). LMO is faster than several numerical techniques such as Lambda iteration method, interior point method (Daniel et al., 2018).

Suppose that we have a function that we aim to minimize $V(x)$ with the respect of the parameters vector x where Newton's update for this vector is as follows (Dogra et al., 2014):

$$x_{k+1} = x_k - [\nabla^2 V(x)]^{-1} \nabla V(x) \quad (16)$$

Where $\nabla^2 V(x)$ is the Hessian matrix and $\nabla V(x)$ is the gradient.

If we assume that the function $V(x)$ is a sum of squares function:

$$V(x) = \sum_{i=1}^n e_i^2(x) \quad (17)$$

$e(x)$ is the difference between the target and the network output then $\nabla^2 V(x)$ and $\nabla V(x)$ can be formulated to:

$$\nabla V(x) = J^T(x)e(x) \quad (18)$$

$$\nabla^2 V(x) = J^T J + S(x) \quad (19)$$

Where $J(x)$ is the Jacobian matrix:

$$J(x) = \begin{bmatrix} \frac{\partial e_1(x)}{\partial x_1} & \frac{\partial e_2(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \\ \frac{\partial e_2(x)}{\partial x_1} & \frac{\partial e_2(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \frac{\partial e_n(x)}{\partial x_1} & \frac{\partial e_n(x)}{\partial x_2} & \dots & \frac{\partial e_n(x)}{\partial x_n} \end{bmatrix} \quad (20)$$

And

$$S(x) = \sum_{i=1}^n e_i(x) \nabla^2 e_i(x) \quad (21)$$

The gauss-newton method assumed that $S(x) \approx 0$, so the update equation becomes:

$$x_{k+1} = x_k - [J^T(x)J(x)]^{-1} J^T(x)e(x) \quad (22)$$

The Levenberg-Marquardt modification to the Gauss-Newton method is:

$$x_{k+1} = x_k - [J^T(x)J(x) + \mu_k I]^{-1} J^T(x)e(x) \quad (23)$$

$$x_{k+1} = x_k - [H(x) + \mu_k]^{-1} \nabla V(x) \quad (24)$$

The Important characteristic of this algorithm is μ_k , generally it's set to ($\mu_k = 0.01$) as a starting point then it is multiplied by 10 whenever a step results an increased $V(x)$. Otherwise, if a step decreases $V(x)$, μ_k is divided by 10.

Notice that when μ is large the algorithm becomes steepest Gradient descent (with step Up), while for small μ the algorithm becomes Gauss-Newton. The Mar-

quardt-Levenberg algorithm can be considered a trust region modification to Gauss-Newton (Battiti, 1992). For the ED problem $x = [Pg_i, \lambda]$ and $V(x) = L(Pg_i, \lambda)$

4.1 Levenberg-Marquardt Algorithm

Step 1: Read given data cost coefficients (a_i, b_i, c_i), power demand P_d and power limits (Pg_i^{\min}, Pg_i^{\max})

Step 2: Calculate the power losses Pl

Step 3: Set the initial values of the Lagrangian multiplier λ^0 , the active generated power Pg_i^0 the Levenberg-Marquardt characteristic μ^0

Step 4: Calculate the Langrange function $L(Pg_i, \lambda)$ for each unit

Step 5: Calculate the Jacobian matrix

$$J(Pg_i, \lambda) = \begin{bmatrix} \frac{\partial L_1}{\partial Pg_1} & \frac{\partial L_1}{\partial Pg_2} & \dots & \frac{\partial L_1}{\partial Pg_{Ng}} & \frac{\partial L_1}{\partial \lambda} \\ \frac{\partial L_2}{\partial Pg_1} & \frac{\partial L_2}{\partial Pg_2} & \dots & \frac{\partial L_2}{\partial Pg_{Ng}} & \frac{\partial L_2}{\partial \lambda} \\ \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \dots & \cdot & \cdot \\ \frac{\partial L_{Ng}}{\partial Pg_1} & \frac{\partial L_{Ng}}{\partial Pg_2} & \dots & \frac{\partial L_{Ng}}{\partial Pg_{Ng}} & \frac{\partial L_{Ng}}{\partial \lambda} \end{bmatrix} \quad (25)$$

Step 6: Calculate the Hessian Matrix

$$H(Pg_i) = J^T(Pg_i)J(Pg_i) \quad (26)$$

Step7: Update the power generation Pg_i and λ following the equation (20)

$$\begin{bmatrix} Pg_{i(k+1)} \\ \lambda_{(k+1)} \end{bmatrix} = \begin{bmatrix} Pg_{i(k)} \\ \lambda_{(k)} \end{bmatrix} - \Delta \begin{bmatrix} Pg_i \\ \lambda \end{bmatrix} \quad (27)$$

Step8: Calculate the new value of $L_{k+1}(Pg_i, \lambda)$ for each generating unit

Step9: Update of the characteristic μ

If $L_{k+1} \geq L_k$ set $\mu = \mu * 10$

else if $L_{k+1} < L_k$ set $\mu = \frac{\mu}{10}$

Step10: Check generation limits for each unit

Set $Pg_i = Pg_i^{\min}$ if $Pg_i \leq Pg_i^{\min}$

Set $Pg_i = Pg_i^{\max}$ if $Pg_i \geq Pg_i^{\max}$

Step 11: Calculate the power mismatch ΔPg using (13).

Step 12: If $\Delta Pg \leq \varepsilon$ (where ε is the convergence criteria, set to $\varepsilon = 0.01$), then stop the calculation and move to Step 14. Otherwise go to the next step.

Step13: Repeat the procedure from **Step 4**
Step14: Calculate the cost function with the optimal values of P_{g_i} and λ .

The LMO algorithm is illustrated in the following flowchart:

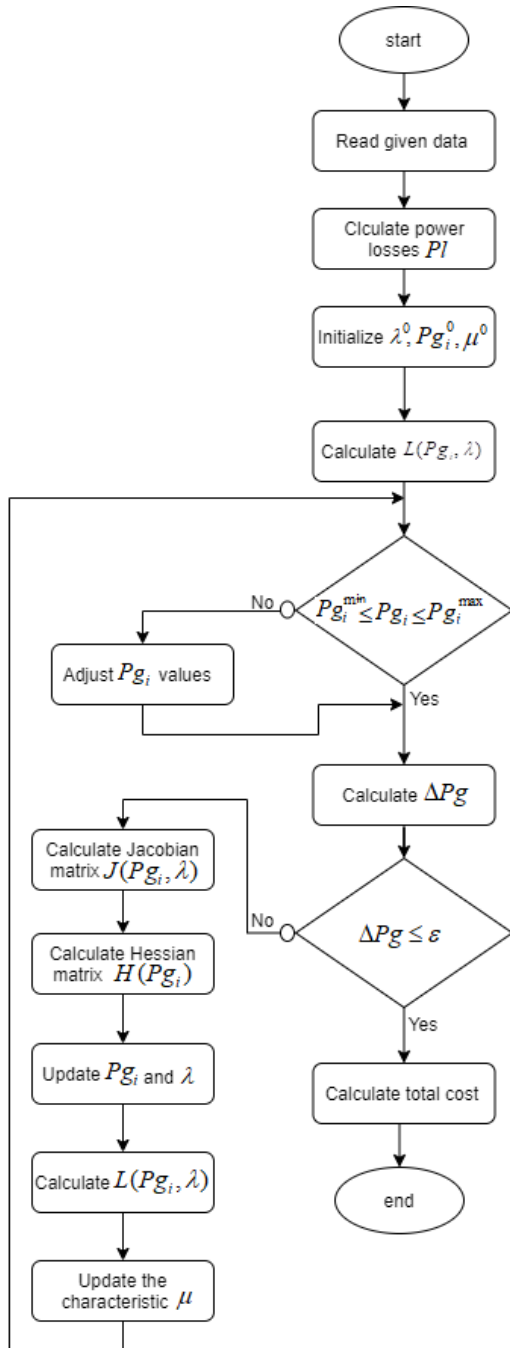


Figure 2: Flowchart of LMO for the ED

5 RESULTS AND DISCUSSIONS

In this part we present the results of applying the Gradient Search and the LMO separately for the static ED using a quadratic cost function.

The network considered is the IEEE.9 Bus system called WSCC 3 machines (also known as P.M Anderson 9 Bus). It consists of 9 buses, 3 generators, 3 power trans-

formers, 6 lines and 3 loads. The load demand to be met by the 3 units is 315MW.

As a first step, the power flow analysis problem was solved using the generators and transmission lines data which is taken from (Anderson and Bose, 1983). Results show that the total active power loss is 4.6410MW, which makes the total active power load set at 319.6410MW.

The cost coefficients and generators limits data are taken from MATPOWER4.1.

Unit	a	b	c	$P_{g_i}^{\min}$	$P_{g_i}^{\max}$
1	0.11	5	150	10	250
2	0.085	1.2	600	10	300
3	0.1225	1	335	10	270

Table 1: Network generators data

The simulations results using gradient search and LMO separately are illustrated in Table 2:

	Gradient Search	LMO
P_{g_1} [MW]	88.0918	88.0192
P_{g_2} [MW]	136.0010	136.2601
P_{g_3} [MW]	95.5521	95.3642
λ_{opt}	24.2057	73.0997
Iterations number	493	3
Total Cost [\$]	5.3284e+03	5.3282e+03

Table 2: results of ED using Gradient search and LMO

We note that LMO provides optimal power generation while it is faster (fewer iterations) and less costly than the Gradient search. However, the Gradient search could give optimal results but it requires more computational time.

As mentioned above the two optimizations techniques were applied for a specific instance of active power load which is 315MW.

In order to test the performance of the proposed method we changed the load at different buses randomly in wide range. For each load the power flow was ran and the power losses was calculated. Some of the simulations results are represented in Table 3.

$P_d = 130$ [MW]		
	Gradient Search	LMO
P_{g_1} [MW]	31.3098	31.2377
P_{g_2} [MW]	62.5172	62.7785
P_{g_3} [MW]	44.5649	44.3768
λ_{opt}	11.8082	23.6875
Iterations number	1061	2
Total Cost [\$]	2.0445e+03	2.0444e+03

$Pd = 400[\text{MW}]$		
	Gradient Search	LMO
Pg_1 [MW]	114.8892	114.8155
Pg_2 [MW]	170.6799	170.9377
Pg_3 [MW]	119.6150	119.4261
λ_{opt}	30.0836	121.0960
Iterations number	400	4
Total Cost [\$]	7.6645e+03	7.6642e+03
$Pd = 700[\text{MW}]$		
	Gradient Search	LMO
Pg_1 [MW]	213.4636	213.3900
Pg_2 [MW]	229.8868	298.5047
Pg_3 [MW]	208.1306	207.9420
λ_{opt}	51.7401	207.9672
Iterations number	231	4
Total Cost [\$]	2.0598e+03	2.0597e+03

Table 3: Test performance of Gradient search and LMO

6 CONCLUSION

We have presented in this paper a comparison between two classical optimization techniques for solving the conventional ED with convex cost function. The proposed techniques were applied to the IEEE.9 bus system with consideration of the transmission lines power loss. LMO has been shown to be more effective than Gradient Search in terms of precision and computation time.

Since the Gradient Search is incapable of solving the non-convex ED problem (with Valve Point Effects consideration), the LMO can be applied for such a problem and be compared to other classical and heuristic techniques.

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