A new bi-objective optimization model for bus priority network design

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ABSTRACT : *A widely adopted measure to alleviate urban traffic congestion is to give priority to public transport systems. Bus priority lanes enable buses to move quickly with less disruption, improve transmission reliability, and provide better scheduling-related performance. This article proposes a new bi-objective optimization model for bus priority network design. The objectives are to maximize the total benefits which are calculated by the total saving time by deploying bus priority lanes and the connectivity defined by a balanced connection between the selected terminal nodes subject to a given budget, simultaneously. At first, a novel bi-objective integer linear programming model is developed for resolving the problem. Then, an iterative ε-constraint method is proposed to obtain the Pareto frontier. Finally, a fuzzy-logic-based approach is used to suggest a preferred Pareto-optimal solution for decision makers. The results of a case study demonstrate that the proposed approach is able to produce a satisfactory and balanced bus priority network.*

KEYWORDS : *Transportation planning, bus priority lane, bi-objective optimization, iterative method.*

1 INTRODUCTION

With the rapid development of economies, the daily travel needs of people are increasing rapidly. Due to the rapid and continuous growth of car ownership, traffic congestion has become more and more serious. Traffic congestion and its associated problems, such as air and noise pollution, energy consumption and traffic accidents, have become major issues in many cities around the world. Much worldwide practical experience has proved that developing public transport systems is highly expected to solve the above problems.

However, due to the nature of low speed and uncontrollable time of buses, the bus service becomes less and less attractive. Therefore, it is necessary to design efficient and reliable public transport networks. Because road resources are limited, it is difficult to expand existing road sections. To achieve the above goal, deploying bus priority lanes to fully use existing road sections becomes an effective and wise alternative.

In recent years, numerous bus-priority studies have been proposed. Ceder (2004) describes the lessons learned from six case studies in Athens, Dublin, Munich, Turin, Vienna, and Zurich, and the benefits derived from the implementation of public transit priorities in these cities. Mesbah *et al.* (2008, 2010, 2011a, 2011b) first introduce a system-wide approach for designing priority lanes, and they propose a bi-level model combining priority lanes selection and traffic assignment.

In order to improve the efficiency of bus operation, more and more cities have begun to deploy bus priority lanes, but most of bus priority lanes in these cities are scattered and lack a systematic planning. Thus, traffic jams often occur at the origin and destination of a bus priority lane. In this case, it is usually interrupted when a bus arrives at the intersection, affecting the overall operation. Therefore, it is necessary to deploy a connected bus priority network instead of reserving isolated bus lanes. However, as far as we know, limited study has deliberated optimal bus priority network design.

Hadas *et al.* (2014) propose an optimization model for selecting bus lanes for constructing a connected bus priority network, which aims to maximize the total saving time while maintaining balanced origin and destination terminals subject to a given budget. The proposed model in Ceder *et al.* (2014) involves two stages in which the first stage is to enumerate a set of possible paths between any pair of terminal nodes and the second one is to formulate an integer program that is solved by solver CPLEX. We note that the existing model cannot guarantee that the obtained solution is an optimal solution, since the enumeration stage may miss optimal paths for an optimal solution. Moreover, the work (Ceder *et al.* 2014) essentially considers a single-objective optimization problem.

Different from Ceder *et al.* (2014), this paper develops a new single-stage bi-objective bus priority network optimization model, which always ensure that an optimal solution is obtained after it is exactly solved. For the bi-objective model, an iterative and fuzzy-logic approach based on ε-constraint is proposed to obtain Pareto frontier and provide a preferred Pareto-optimal solution for decision makers.

The remainder of this paper is organized as follows. Section II describes the considered problem, and proposes an optimal model for connected urban bus priority network. Section III introduces the concept of multi-objective optimization and an iterative and fuzzy-logic approach based on ε-constraint which is proposed to solve the proposed program. A case study based on an Israel mid-size city (Ceder *et al.* 2014) is conducted to verify the performance of the proposed model in Section IV. Section V concludes this paper.

2 PROBLEM DESCRIPTION AND FORMULATION

In this section, we first describe the studied problem formally. Then, a new single-stage bi-objective bus priority lane network optimization model is developed.

2.1 PROBLEM DESCRIPTION

Graph $G(N, A)$ is called an urban bus transportation network, where N is the set of nodes and A is the set of directed arcs. Give a set of tasks and terminal pairs, and each task corresponds to one terminal pair.

The purpose of the considered problem is to design a connected urban bus network of priority lanes. However, deploying priority lanes on existing road sections will lead to increasing traffic pressure in other lanes. Hence, maximizing the total benefits which are calculated by the total saving time by deploying bus priority lanes is one objective of the problem. The other objective is to maximize the network connectivity defined by a balanced connection between origin and destination nodes.

To better define and formulate the problem, we make the following assumptions : i) there is at least one path between any pair of terminal nodes; and ii) at most one lane in a road section is considered as a bus priority lane, regardless of adding one lane as a bus priority lane on an existing road section.

2.2 FORMULATION

Here we introduce the indices, parameters, and decision variables which throughout the article.

2.2.1 INDICES

i, j : index of nodes
k : index of terminal pairs

2.2.2 PARAMERTERS

N : set of nodes
A : set of arcs, $(i, j) \in A$
K : set of terminal pairs, $k \in K$
I : set of terminal nodes
B : the given budget of the construction of priority lanes
c_{ij} : construction cost of road section (i, j)
f_{ij} : total passengers' flow of all routes passing through road section (i, j)
τ_{ij} : travel time of buses on a priority lane on road section (i, j)
τ'_{ij} : travel time of buses on road section (i, j) when no lanes are reserved
v_{ij} : bus travel time saved by deploying a priority lane on road section (i, j), $v_{ij} = \tau'_{ij} - \tau_{ij}$
o_k : origin station of a terminal pair $k \in K$
d_k : destination station of a terminal pair $k \in K$

2.2.3 DECISION VARIABLES

z_{ij} : a binary variable equals to 1 if a priority lane is set on road section (i, j); 0 otherwise, where $(i, j) \in A$.
x_{ij}^k : a binary variable equals to 1 if a priority lane is set on road section (i, j), and a path of a terminal pair k passes through it; 0 otherwise, where $(i, j) \in A$, $k \in K$.
y_k : a binary variable equals to 1 if terminal pair k is selected, priority lanes are deployed on road sections which a path of a terminal pair k passes through, $y_k = 1$; 0 otherwise, where $k \in K$.

2.3 FORMULATION

$$P: \quad F_1 = \max \sum_{(i,j)\in A} z_{ij} v_{ij} f_{ij} \tag{1}$$

$$F_2 = \max \min_{i\in I} \min\Big(\sum_{k\in K|o_k=i} y_k, \sum_{k\in K|d_k=i} y_k\Big) \tag{2}$$

$$\text{s.t.} \quad \sum_{(i,j)\in A} z_{ij} c_{ij} \le B \tag{3}$$

$$\sum_{(i,j)\in A} x_{ij}^k = y_k, i = o_k, \forall k \in K \tag{4}$$

$$\sum_{(i,j)\in A} x_{ij}^k = y_k, j = d_k, \forall k \in K \tag{5}$$

$$x_{ij}^k \le y_k, \forall (i, j) \in A, \forall k \in K \tag{6}$$

$$x_{ij}^k \le z_{ij}, \forall (i, j) \in A, \forall k \in K \tag{7}$$

$$\sum_{k\in K} x_{ij}^k \ge z_{ij}, \forall (i, j) \in A \tag{8}$$

$$x_{ij}^k, y_k, z_{ij} \in \{0, 1\}, \forall (i, j) \in A, \forall k \in K \tag{9}$$

$$\sum_{(i,j)\in A} x_{ij}^k = \sum_{(j,i)\in A} x_{ji}^k, \quad \forall j \in N \backslash \{o_k, d_k\}, \forall k \in K \tag{10}$$

Objective (1) maximizes the total benefits of deploying priority lanes. The total benefits are computed by the total saving time brought by bus priority lanes. Objective (2) is to balance the selected terminal nodes. This balance is maintained in this way that maximizing the minimal in-degrees and outdegrees of all terminal nodes among all feasible solutions. If a priority lanes set is unbalanced, it will produce an effect on the total reliability of the public traffic network and reduce the level of service. Constraint (3) restricts that the entire construction cost of the deploying bus priority network can not exceed the given budget B. Constraint (4) describes that if a terminal pair k is selected, priority lanes will be deployed on all road sections ending at origin station of a terminal pair $k \in K$. The same to constraint (4), constraint (5) describes that if a terminal pair k is selected, priority lanes will be deployed on all road sections ending at destination station of a terminal pair $k \in K$. Constraint (6) means that for a terminal pair k, if a priority lane on road section (i, j) is passed (i.e., $x_{ij}^k = 1$), then this pair of terminal nodes are considered to set a priority path (i.e.,$y_k = 1$). Constraint (7) implies that if a path of a terminal pair k passes through road section (i, j), then a priority lane should be reserved in this road section (i, j). Constraint (8) indicates that a road section (i, j) is set as a priority lane and must have at least a path of a terminal pair k passes through it. Constraint (9) implies x_{ij}^k, y_k and z_{ij} are binary decision variables. Constraint (10) is a flow balance constraint. It makes sure flow balance of the intermediate nodes between the origin and destination of each task.

It is not easy to directly deal with objective (2), hence we reformulate objective (2) by the following equivalent way. First, we define a new non-negative decision variable R. Then, objective (2) can be reformulated as :

$$F_2 = \max \quad R \tag{11}$$
$$R \leq \sum_{k \in K | o_k = i} y_k, \forall i \in I \tag{12}$$
$$R \leq \sum_{k \in K | d_k = i} y_k, \forall i \in I \tag{13}$$

Constraints (12) and (13) enforce the in-degree and out-degree bounds.

3 SOLUTION METHODS

In this section, the basic principle of multi-objective optimization is presented at first. Then, we propose an iterative and fuzzy-logic approach based on ε-constraint to solve the proposed bi-objective model.

3.1 THE BASIC PRINCIPLE OF MULTI-OBJECTIVE OPTIMIZATION

On the whole, a multi-objective optimization problem is composed of multiple objective functions and some related equations and inequality constraints. It can be generally described as follows.

$$\begin{aligned} & \min \quad F(X) = [f_1(x), f_2(x), ..., f_m(x)]^T \\ \text{s.t.} \quad & g_i(x) \leq 0, \qquad i = 1, 2, ..., I \\ & h_j(x) = 0, \qquad j = 1, 2, .., J \end{aligned}$$

where m indicates the number of objectives. The number of inequality constraints and equality constraints are denoted by I, J, respectively.

For the sake of analysis, we make the following equivalent substitution :

$$F_1' = -F_1$$
$$F_2' = -F_2$$

Then we reformulate the bi-objective model P as follows :

$$\min \quad F(\phi) = \{F_1'(\phi), F_2'(\phi)\} \quad \text{s.t.} \quad \phi \in \Phi$$

where ϕ and Φ are the solution vector formed by all decision variables and the solution space defined by (3)-(9) and (12)-(13). $F_1'(\phi)$ and $F_2'(\phi)$ imply the total benefits by deploying priority lanes and balanced connection between the selected terminal nodes, respectively. $\{F_1'(\phi), F_2'(\phi) | \phi \in \Phi\}$ defines the objective space. Then we give the definitions related to bi-objective optimization.

DEFINITION 1 A non-dominated (Pareto-optimal) solution $\phi^* \in \Phi$ satisfies that no solution $\phi \in \Phi$ exists make $F_1'(\phi) \leq F_1'(\phi^*)$ and $F_2'(\phi) \leq F_2'(\phi^*)$, where at least one of these inequalities is strict.

DEFINITION 2 If $\phi^* \in \Phi$ is a non-dominated solution, $(F_1'(\phi^*), F_2'(\phi^*))$ is called a non-dominated (Pareto) point in objective space. The Pareto frontier is composed of all non-dominated points.

In the existing literature, there are two main methods for obtaining the Pareto frontier : the weighted-sum method and the ε-constraint method. Since the ε-constraint method avoids the difficulty caused by scaling objective functions and setting objective weights. Hence in this paper we adapt the ε-constraint-based method to solve the proposed problem next.

3.2 AN ITERATIVE AND FUZZY-LOGIC APPROACH BASED ON ε-CONSTRAINT

In this section, we propose an iterative and fuzzy-logic approach based on ε-constraint to solve the above program. This method is composed of two steps. At first, the initial bi-objective problem P is transferred into a single-objective problem by ε-constraint method. The ε-constraint method is to reserve the main objective while setting the other objective as an ε-constraint. Then, resolving a series of ε-constraint problems through updating ε's value in an appropriate way to obtain Pareto frontier. The second step is to select a preferred Pareto-optimal solution by employing a fuzzy-logic-based approach.

We first need to determine which objective is main objective. As the matter of fact, both objectives can be considered since the Pareto frontiers are the same no matter what the main function is. But the computational complexity of two versions are quite different. Based on our consideration, we choose F_1 as the main objective function. Then P is transformed into the single-objective ε-constraint problem $P(\varepsilon_j)$ as follows.

$$F_1' = \min - \sum_{(i,j)\in A} z_{ij} v_{ij} f_{ij} \tag{14}$$

s.t. (3)-(9), (12), (13)

$$F_2' = -R \leq \varepsilon_j \tag{15}$$

where $\varepsilon_j = F_2'(j-1) - \Delta$ in the j-th iteration, in winch $F_2'(j-1)$ is opposite number of decision variable R of the $(j-1)$-th iteration and Δ indicates the step length.

Identifying the value of ε is the key component of solving model $P(\varepsilon_j)$, and it further depends on the interval of ε. In order to determine this interval, we take the following four single-objective models to obtain the Ideal point and the Nadir point.

$$\begin{aligned}
&F_1'^{I} = \min \quad F_1' \\
&\quad \text{s.t. } (3)\text{-}(9), (12), (13) \\
&F_2'^{I} = \min \quad F_2' \\
&\quad \text{s.t. } (3)\text{-}(9), (12), (13) \\
&F_1'^{N} = \min \quad F_1' \\
&\quad \text{s.t. } F_2' = F_2'^{I}, (3)\text{-}(9), (12), (13) \\
&F_2'^{N} = \min \quad F_2' \\
&\quad \text{s.t. } F_1' = F_1'^{I}, (3)\text{-}(9), (12), (13)
\end{aligned}$$

Therefore, the obtained Ideal and Nadir points are $\left(F_1'^{I}, F_2'^{I}\right)$ and $\left(F_1'^{N}, F_2'^{N}\right)$, respectively. And the interval of ε is determined by $\left[F_2'^{I}, F_2'^{N}\right]$. For the first iteration, ε_0 is set as $F_2'^{N}$. For the $(j-1)$ iteration $(j > 1)$, ε_j is defined by

$$\varepsilon_j = F_2'\left(\varepsilon_{j-1}\right) - \Delta \tag{16}$$

where $F_2'(\varepsilon_{j-1})$ denotes $R(\varepsilon_{j-1})$ being the solution obtained by solving $P(\varepsilon_{j-1})$, and Δ is the step length which is set as the minimum unit value of F_2' (Wu *et al.* 2015). For the studied problem, the minimum unit is set as 1. The Pareto frontier of the studied bi-objective model can be derived from solving all $P(\varepsilon_j)$ with a step length which indicates 1. Note that Δ is set as 0 in the first iteration.

Finally, a preferred Pareto-optimal solution can be selected for decision makers by the fuzzy-logic-based method which is proposed by Esmaili *et al.* (2009) among the Pareto points obtained above. The fuzzy-logic-based method can recommend the optimal solution according to the preference of the decision makers. For the studied problem, linear membership functions $\delta\{F_1(j)\}$ represents the optimality degree of the total benefits of the j-th Pareto point in the Pareto frontier F are defined as follows :

$$\delta\{F_1(j)\} = \begin{cases} 1, & if F_1(j) \leq {F_1}^{I} \\ \dfrac{{F_1}^{N} - F_1(j)}{{F_1}^{N} - {F_1}^{I}}, & if {F_1}^{I} < F_1(j) < {F_1}^{N}, j \in F \\ 0, & if F_1(j) \geq {F_1}^{N} \end{cases} \tag{17}$$

where $F_1(j)$ denote the objective value of F_1 the j-th Pareto point. Both ${F_1}^{N}$ and ${F_1}^{I}$ are the upper and lower bounds of F_1', respectively.

The membership functions $\delta\{F_1(j)\}$ can be similarly defined. The definition of the total membership degree δ_j which indicates the optimality degree of the j-th Pareto solution is as follows :

$$\delta_j = \frac{w_1 \delta\left(F_1\right) + w_2 \delta\left(F_2\right)}{w_1 + w_2} \tag{18}$$

where w_1 and w_2 are the weights for the total passenger travel time and the balance function, respectively, and $\sum_{i=1}^{2} w_i$ in this model. Without loss of generality, decision makers select the Pareto point of maximum membership as the Pareto optimal solution.

4 CASE STUDY

We evaluate the performance of the proposed method in this section by testing the benchmark example in Petah-Tiqwa, Israel (Ceder *et al.* 2014). The test is done on a PC with Intel Core i5 CPU, 2.2 GHz, and 8GB RAM. The bi-objective program is exactly solved by using C++ embedded with commercial solver CPLEX 12.8 in Visual Studio 2019.

In Israel, Petah-Tiqwa is the fifth largest city which is located in Israel's largest metropolitan area (Gush-Dan). Urban public traffic network which presented in figure 1 is provided by one bus company for this city. Private and commercial vehicles share the same road resources. Each arc is a segment that can be used as part of a possible priority lane. All circled nodes are a set of possible origins and destinations for bus lanes. This instance has 44 arcs and 34 nodes, including 15 terminals. Transport tasks may occur between any two terminals. The OD pairs are known in advance.

Since the related data is not availiable in Ceder *et. al* (2014), we randomly generate the data according to the information provided in Ceder *et. al* (2014). After solving the bi-objective model, we can obtain 6 different non-dominated points. To visually and clearly show the trade-off between the objectives, figure 2 draws the Pareto frontier of

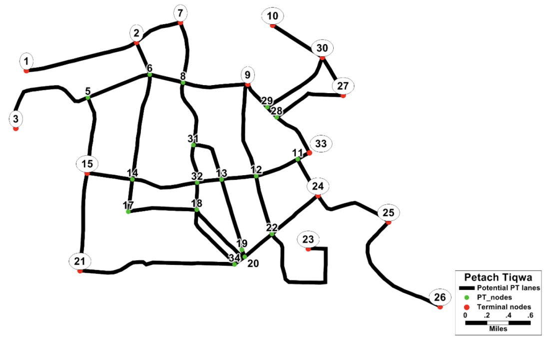

FIGURE 1 – Urban public traffic network in Petah-Tiqwa

the benchmark instance with setting B = 60,000,000. Figure 3 illustrates the obtained optimal bus priority network.

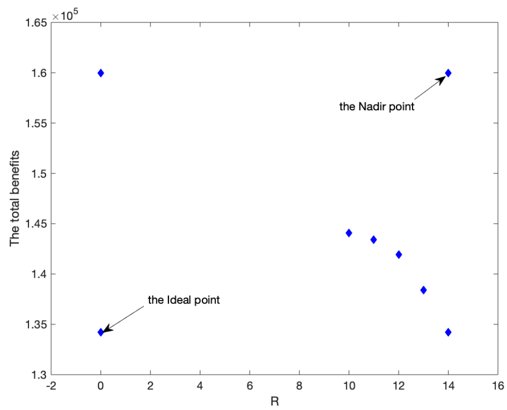

FIGURE 2 – The Pareto frontier of the case study

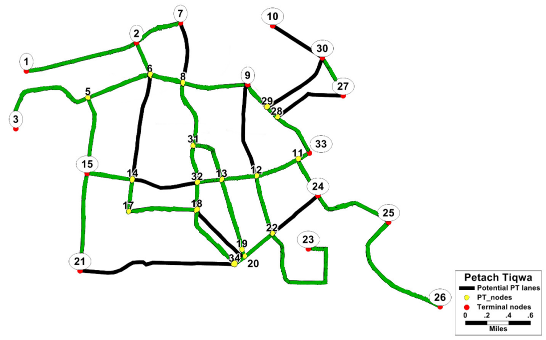

FIGURE 3 – Selected network of priority lanes

Table 1 present all Pareto points for the benchmark instance with setting B = 60,000,000. For the sake of recommending the most preferred solution for decision makers, we assume that three different preference combinations on the objectives (i.e., $w_1 > w_2$, $w_1 = w_2$, and $w_1 < w_2$). If $w_1 = 0.9$ implying $w_2 = 0.1$, which indicates that decision makers expect to provide a high service level for passengers. Consequently, the Pareto point $(134190, 14)$ is advised. On the contrary, if $w_1 = 0.1$ implying $w_2 = 0.9$, then the Pareto point $(159945, 0)$ is recommended, with which the F_1 is greatly increased but a much smaller F_2 is obtained. In addition, if decision makers exhibit equal preference, i.e., $w_1 = w_2 = 0.5$, then the Pareto point $(134190, 14)$ and $(159945, 0)$ are selected, with which neither too high F_1 nor too big F_2 is achieved.

5 CONCLUSION

This study proposes a novel model for a bus priority network design problem. The model aims to maximize the total benefits by deploying bus priority lanes, at the same time, maintain the balance by maximizing the minimal in-degrees and out-degrees of all terminal nodes among all feasible solutions. To solve the proposed bi-objective model, an iterative and fuzzy-logic approach based on ε-constraint is proposed. Computational results of a case study confirms the effectiveness and correctness of the proposed model.

In the future, our research directions will include but not be limited to : i) the effect on carbon emission reduction caused by bus priority networks should be considered. With the improvement of bus service level, more and more passengers will travel by bus instead of private cars, thus reducing carbon emissions. ii) due to the NP-hard characteristic of the problem, the proposed model is solved by solver CPLEX, and the solution time will increase with the growth of the problem size. Therefore, we focus on developing efficient algorithms (Montemanni *et al.* 2005, Che *et al.* 2015, Ghosh *et al.* 2019, Wu *et al.* 2019, Mishra *et al.* 2018), to solve large-scale problems in acceptable time.

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j	F_1	F_2	$\delta\{F_1(j)\}$	$\delta\{F_2(j)\}$	$\delta_j(w_1 = 0.9)$	$\delta_j(w_1 = 0.5)$	$\delta_j(w_1 = 0.1)$
1	159945	0	0	1.000	0.100	0.500	**0.900**
2	144063	10	0.617	0.286	0.584	0.451	0.319
3	143437	11	0.641	0.214	0.598	0.428	0.257
4	141950	12	0.699	0.143	0.643	0.421	0.198
5	138385	13	0.837	0.017	0.761	0.454	0.148
6	134190	14	1.000	0.000	**0.900**	0.500	0.100

Tableau 1 – Computational results for the case study

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