

# Sub-proton range induced large-scale transitions in solar wind magnetic field

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2	magnetic field
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#### Abstract

We investigate the role of sub-proton dynamics in inducing large-scale transitions in the solar wind magnetic field by means of dynamical systems metrics based on instantaneous fractal dimensions. By looking at the corresponding multiscale features, we observe a break in the average attractor dimension occurring at the crossover between the inertial and the sub-proton regime. Our analysis suggests that large-scale transitions are induced by sub-proton dynamics through an inverse cascade mechanism driven by local correlations, while electron contributions (if any) are hidden by instrumental noise.

The solar wind has been shown to be a natural laboratory for plasma physics, covering a 45 wide range of scales and being characterized by a large variety of phenomena as turbulence, 46 intermittency, waves, instabilities, and so on [1]. A lot of attention has been directed towards 47 understanding the scale-invariant features and self-organization of both the MHD/inertial 48 and the kinetic/dissipative regimes [2–4]. Indeed, by searching for scaling-law behaviors and 49 looking at high-order statistics several insights have been provided on turbulence and inter-50 mittency [5] as well as on both the direct and inverse energy/enstrophy cascade mechanisms 51 [6, 7]. When exploring the multiscale variability of solar wind parameters, these approaches 52 are not able to investigate scale-to-scale effects, only providing a global view of the sys-53 tem over a specific range of scales. Moreover, with the solar wind being characterized by 54 nonlinearities, emergent phenomena, and cross-scale coupling, the natural framework to ob-55 tain a suitable description of scale-dependent features is via dynamical system theory [8, 9]. 56 Within this framework, Alberti *et al.* [10] recently introduced a novel formalism to deal with 57 the characterization of the multiscale nature of fluctuations by deriving suitable multiscale 58 measures of complexity when looking at the behavior of scale-dependent phase-space tra-59 jectories. The basic idea is to combine a time series decomposition method (like Empirical 60 Mode Decomposition [11]) and the concept of generalized fractal dimensions [12] to char-61 acterize how complexity varies among scales in a complex system, allowing a description of 62 scale-dependent underlying (multi)fractal features [10]. Let x(t) be a time series assumed to 63 be composed of dynamical patterns at a "collection" of scales, i.e.,  $x(t) = x_0 + \sum_{\tau} \delta_{\tau} x(t)$ , 64 with  $\delta_{\tau} x(t)$  being a fluctuation at a mean scale  $\tau$  and  $x_0$  the steady-state average value. 65

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<sup>66</sup> Then, for each scale  $\tau'$  we can define a natural measure  $\mu_{\tau'}$  such that

$$D_{q,\tau'} = \frac{1}{q-1} \lim_{\ell \to 0} \frac{1}{\log \ell} \log \int d\mu_{\tau'}(x) \mu_{\tau'} \left( B_{x,\tau'}(\ell) \right)^{q-1}, \tag{1}$$

with  $B_{x,\tau'}(\ell)$  being the hyperball of size  $\ell$  centered at the point x on the scale-dependent 68 phase-space of  $\sum_{\tau < \tau'} \delta_{\tau} x(t)$  [10]. This approach has been demonstrated to be very promising 69 for revealing different dynamical features and behaviors of both paradigmatic model systems 70 and real-world time series [10, 13]. A modification of this newly introduced formalism is 71 to replace the generalized fractal dimensions, allowing a global topological and geometric 72 view of the scale-dependent phase-space, with instantaneous measures of the actual degrees 73 of freedom of a system, namely the local dimension [14] and phase-space local persistence. 74 As shown by Caby et al. [15], these quantities can be formally related to generalized fractal 75 dimensions and the local stability of the system. In particular, the distribution of the local 76 dimensions is modulated by the multifractal properties of the system. 77

In this Letter, we extend the formalism introduced by Alberti *et al.* [10] to characterize 78 the scale-dependent phase-space topology of solar wind magnetic field fluctuations over a 79 wide range of scales, moving from the kinetic, through the sub-proton to the inertial scales. 80 We use measurements coming from the Fast Plasma Investigation (FPI) instrument on board 81 of the Magnetospheric Multiscale Mission (MMS) satellites. Our goal is to test whether this 82 formalism is able to highlight the nature of the sudden changes in the large-scale dynamics 83 of the solar wind by looking at the interplay between the sub-proton and the inertial range 84 in triggering those transitions. 85

<sup>86</sup> On November 24, 2017, the MMS orbit allowed to collect measurements in the pristine <sup>87</sup> solar wind, well outside the Earth's magnetosheath and the bow shock, for a long period (i.e., <sup>88</sup> a few times longer than the typical correlation scale) of approximately 1 hour from 01:10 to <sup>89</sup> 02:10 UT. Figure 1 (upper panel) displays an overview of the magnetic field measurements <sup>90</sup> collected by the FIELDS instrument suite [16] on board of MMS1 with a temporal resolution <sup>91</sup>  $\Delta t = 128$  sample/s [17].

<sup>92</sup> With the solar wind being usually considered as an example for nonlinear multiscale <sup>93</sup> dynamical systems, we diagnose its dynamical properties of the instantaneous (in time) <sup>94</sup> and local (in phase-space) states as represented by the three components of the magnetic <sup>95</sup> field. We use two dynamical systems metrics [18], the local dimension (d) and the in-<sup>96</sup> verse persistence ( $\theta$ ). The former is a measure of the active number of degrees of free-

dom, while the latter is a measure of phase-space persistence [19, 20]. Those instantaneous 97 metrics are obtained by sampling the recurrences of a state  $\zeta_B$  and observing that they 98 are distributed according to extreme value theory. Formally, let  $\zeta_B$  be a state of inter-99 est in phase-space and  $g(B(t), \zeta_B) = -\log [\operatorname{dist}(B(t), \zeta_B)]$  be the logarithmic return, where 100  $dist(B(t), \zeta_B)$  is the Euclidean distance between B(t) and  $\zeta_B$ . If we define exceedances as 101  $X(\zeta_B) = g(B(t), \zeta_B) - s(q, \zeta_B)$ , with  $s(q, \zeta_B)$  being an upper threshold corresponding to the 102 q-th quantile of  $g(B(t), \zeta_B)$ , then the Freitas-Freitas-Todd theorem modified by Lucarini 103 et al. [21] states that the cumulative distribution  $F(X,\zeta_B)$  converges to the exponential 104 member of the Generalized Pareto family 105

$$F(X,\zeta_B) \simeq \exp\left[-\theta(\zeta_B)\frac{X(\zeta_B)}{d^{-1}(\zeta_B)}\right],$$
(2)

where  $0 \le d < \infty$  is the local dimension and  $0 \le \theta \le 1$  is the inverse persistence of the state  $\zeta_B$  in units of  $\Delta t$  [22]. Figure 1 shows the behavior of the local dimension d (middle panel) and the inverse persistence  $\theta$  (lower panel). For all computations, we fix q = 0.98.

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FIG. 1. (From top to bottom) The magnetic field components in the GSE reference system collected by the flux-gate magnetometer (FGM) on board of MMS1 at a resolution of  $\Delta t = 128$  sample/s, the local dimension d, and the inverse persistence  $\theta$ . The blue, red, and yellow lines refer to  $B_x$ ,  $B_y$ , and  $B_z$ , respectively.

We observe a wide range of variability for the local dimension of  $1 \leq d \leq 9$  with an average value  $\langle d \rangle = 2.3 \pm 0.3$ , while the inverse persistence  $\theta$  is strictly confined to values

lower than 0.2 with  $\langle \theta \rangle = 0.07 \pm 0.02$ . This suggests that globally, the number of degrees of 112 freedom is less than the phase-space dimension (i.e.,  $\langle d \rangle < 3$ ). This means that the dynamics 113 is different from that of a stochastic process, for which d should exhibit small fluctuations 114 around 3 and  $\theta = 1$  [23]. A visual inspection suggests that larger  $d, \theta$  are found in close 115 correspondence with sudden changes in both  $B_x$  and  $B_z$ . The high number of degrees of 116 freedom (d > 3) associated with sudden variations in magnetic field components suggests 117 the existence of an unstable fixed point in the proximity of the transitions [14]. This is 118 also observed in fluid turbulence showing multi-stability [24] and hints at the existence of 119 an underlying strange stochastic attractor, i.e., the system lies in a subset of points of the 120 whole phase-space. Indeed, although the cascade mechanism involves a wide range of scales, 121 some of them seem to be less important than others and their description can arise from 122 stochastic theory [24]. However, only looking at the whole time series does not provide 123 information on the topology and the triggers of these transitions which depend on processes 124 occurring at different scales. 125

To complete this analysis, we use Multivariate Empirical Mode Decomposition (MEMD) [25] to evaluate the scale-dependent fluctuations of magnetic field measurements. By defining the multivariate signal  $\mathbf{B}_{\mu}(t) = [B_x(t), B_y(t), B_z(t)]^T$ , MEMD acts on its multivariate instantaneous properties to decompose it into a finite number of multivariate oscillating patterns  $\mathbf{C}_{\mu,k}(t)$ , called Multivariate Intrinsic Mode Functions (MIMFs), and a monotonic trend  $\mathbf{R}_{\mu}(t)$  as

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$$\mathbf{B}_{\mu}(t) = \sum_{k=1}^{N} \mathbf{C}_{\mu,k}(t) + \mathbf{R}_{\mu}(t).$$
(3)

The core of the MEMD is the so-called *sifting process* that allows to derive the MIMFs in an adaptive way by exploiting the instantaneous local properties of a signal in terms of local extrema interpolation [see Ref. 25, for more details]. Each  $C_{\mu,k}(t)$  is a multivariate pattern representative of a peculiar dynamical feature that evolves on a typical mean scale

$$\tau_k = \frac{1}{T} \int_0^T t' \langle \mathbf{C}_{\mu,k}(t') \rangle_\mu dt', \tag{4}$$

with T being the total length of the signal and  $\langle \cdots \rangle_{\mu}$  denoting an ensemble average over the  $\mu$ -dimensional space [13]. Moreover, the spectral features of the multivariate signal  $\mathbf{B}_{\mu}(t)$ can also be easily investigated by introducing an estimator of the power spectral density (PSD) as

<sup>142</sup> 
$$S(\tau) = \left( \mathbb{E} \left[ \mathbf{C}_{\mu,k}^2(t) \right] - \mathbb{E} \left[ \mathbf{C}_{\mu,k}(t) \right]^2 \right) \cdot \tau_k, \tag{5}$$

where  $\mathbb{E}$  denotes the expectation value. Thus, MEMD is particularly suitable for deriving scale-dependent patterns embedded in magnetic field data, providing the starting point for a multiscale characterization of the different dynamical regimes. The multivariate signal  $\mathbf{B}_{\mu}(t)$ is now interpreted as a collection of scale-dependent fluctuations belonging to different dynamical regimes (noise, kinetic, sub-proton, inertial, ...) that can be used to investigate how they contribute to the collective properties of the whole measurements. Indeed, following Alberti *et al.* [10], we can describe the dynamics at scales  $\tau' < \tau$  as

$$\mathbf{B}_{\mu}^{\tau}(t) = \sum_{k|\tau_{h} < \tau} \mathbf{C}_{\mu,k}(t) \tag{6}$$

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<sup>151</sup> such that we can define a scale-dependent local dimension  $d_k$  and inverse persistence  $\theta_k$  by <sup>152</sup> diagnosing the dynamical properties of  $\mathbf{B}^{\tau}_{\mu}(t)$ . To do this, we compute both  $d_k$  and  $\theta_k$  for <sup>153</sup> reconstructions of the first k MIMFs as in Eq. (3) until  $k \to N$  for which  $(d_k, \theta_k) \to (d, \theta)$ . <sup>154</sup> Figure 2 shows the distributions of both the local dimension and the inverse persistence as <sup>155</sup> a function of the different scales  $\tau$ .



FIG. 2. Probability distribution functions  $\rho(\cdot)$  of the local dimension d (left panel) and the inverse persistence  $\theta$  as a function of the scale  $\tau$  (colored lines).

<sup>156</sup> We observe a decreasing d and a decreasing  $\theta$  as  $\tau$  increases, suggesting that the whole <sup>157</sup> phase-space properties resemble those of a low-dimensional dynamical system. This is <sup>158</sup> again reminiscent of the fluid-dynamical behavior as observed in the turbulent von Kar-<sup>159</sup> man swirling flow, which despite very high Reynolds number, has been characterized by a <sup>160</sup> low-dimensional stochastic attractor [24]. Conversely, larger d and larger  $\theta$  characterize the <sup>161</sup> short-term variability of the solar wind magnetic field, providing evidence for an underlying <sup>162</sup> higher-dimensional structure. This seems to suggest a dynamical transition in the underly<sup>163</sup> ing structure of the phase-space when moving from short- to long-term dynamics, i.e., when <sup>164</sup> passing from sub-proton to inertial scales. To better underline these features we evaluate <sup>165</sup> the average values of d and  $\theta$  at the different scales  $\tau$  and compared with the behavior of <sup>166</sup> the PSD  $S(\tau)$  (see Eq. 5) as reported in Figure 3.



FIG. 3. Average values  $\langle d \rangle$  (left panel, red asterisks) and  $\langle \theta \rangle$  (right panel, red asterisks) at the different scales  $\tau$  as compared with the behavior of the PSD  $S(\tau)$  (blue circles). The dashed and dashed-dotted lines indicate the FGM instrumental noise limit ( $f \sim 5$  Hz) to the inverse of the proton-cyclotron frequency ( $f_{c_p} = 1/\tau_{c_p} = eB/m_p$ ), respectively. The dashed blue line highlights the Kolmogorov-like spectral scaling  $\sim \tau^{5/3}$ .

The behavior of  $S(\tau)$  evidences the existence of three different dynamical regimes: the 167 instrumental noise range for  $\tau$  < 0.2 s, the sub-proton range for 0.2 s <  $\tau$  < 1.3 s, and 168 the inertial range for  $\tau > 1.3$  s. These findings are well in agreement with the results of 169 Bandyopadhyay et al. [17]. By looking at the behavior of  $\langle d \rangle$  and  $\langle \theta \rangle$  at the different scales 170  $\tau$  we clearly observe a scale-dependent behavior of  $\langle d \rangle$  that resembles that highlighted by 171  $S(\tau)$ . The instrumental noise regime is characterized by  $\langle d \rangle = 3$  and  $\langle \theta \rangle \approx 1$  suggesting, as 172 expected, a purely stochastic origin for the short-term variability of magnetic field fluctua-173 tions (i.e.,  $\tau < 0.2$  s). From a dynamical system point of view, this means that ergodicity 174 characterizes the phase-space, i.e., there exists a reference trajectory of a "typical" point 175 that can be used for deducing the average behavior of the system. 176

The sub-proton regime  $(\tau_{noise} < \tau < \tau_{c_p})$  is instead characterized by an increasing  $\langle d \rangle$  as  $\tau$  increases, reaching a maximum value  $\langle d \rangle_{max} \approx 3.2$  for  $\tau \sim \tau_{c_p}$  (i.e., larger than the topological dimension of the system), together with a nonlinear decreasing  $\langle \theta \rangle$  with  $\tau$ , reaching an inflection point for  $\tau \sim \tau_{c_p}$ . These features have not been reported before in the literature

and can be interpreted as an increase in the average number of degrees of freedom at sub-181 proton scales due to the nonlinear energy cascade effects moving energy towards these scales 182 [5]. Moreover, the behavior of  $\langle d \rangle$  and  $\langle \theta \rangle$  at  $\tau \sim \tau_{c_p}$  is clearly the reflection of a dynami-183 cal transition occurring at the boundary between the sub-proton and the inertial regimes. 184 Indeed, going towards larger scales (i.e., approaching the inertial range), we observe a de-185 creasing  $\langle d \rangle$  and  $\langle \theta \rangle$  with  $\tau$ , reflecting a reduced-order nature of large-scale magnetic field 186 fluctuations with an active number of degree of freedom that is lower than the topological 187 dimension of the system ( $\langle d \rangle < 3$ ). This points towards the possibility to describe the dy-188 namics across the inertial range as a low-dimensional dynamical system [9]. Furthermore, 189 the reduced values of  $\langle \theta \rangle$  suggest a long residence time of the system in the dynamical states 190 corresponding to the inertial range, indicating that they can be interpreted as a collection 191 of marginally stable fixed points of the dynamics with sub-proton scale induced transitions. 192 Taking together the above results, we can firmly state that (i) the inertial range dynamics can 193 be easily described as a reduced-order dynamical system, and (ii) the increasing dimension-194 ality of the sub-proton regime is a reflection of small-scale turbulence-induced magnetic field 195 fluctuations due to a dynamical component that is external to the sub-proton physics, i.e., 196 dynamical information is introduced from processes occurring through the inertial regime 197 and reflecting a direct energy cascade mechanism. 198

<sup>199</sup> To further support the claim that a larger number of degrees of freedom in magnetic field <sup>200</sup> fluctuations is only due to processes occurring at sub-proton scales while global properties <sup>201</sup> are mainly related to inertial processes, we compare the behavior of the  $d - \theta$  plane at two <sup>202</sup> different scales  $\tau_{c_p}$  and  $\tau_I$ , with  $\tau_I$  being the largest scale belonging to the inertial range (i.e., <sup>203</sup>  $\tau_I \sim 10^3$  s) in dependence on the ratio between magnetic field fluctuations at that scale and <sup>204</sup> their standard deviation as reported in Fig. 4.

We observe that larger values of d are associated with larger fluctuations at sub-proton 205 scales, thus suggesting a key role of the organization of sub-proton scales in regulating the 206 active number of degrees of freedom of the system. We also show that a very wide range 207 of  $\theta$  values is observed at sub-proton scales, thus reflecting an unstable (from a dynamical 208 system point of view) nature of fluctuations within this regime. Conversely, when inertial 209 scales are reached we evidence a reduced range of  $\theta$ , confined below 0.2, together with a 210 clearly narrower range of d with respect to the sub-proton regime. This suggests that the 211 MHD dynamics is dominated by marginally stable fixed points conferring a low-dimensional 212



FIG. 4.  $d - \theta$  plane at two different scales  $\tau_{c_p}$  (left) and  $\tau_I$  (right) in dependence on the ratio between magnetic field fluctuations at that scale and their standard deviation (colors). White and black asterisks correspond to the mean values of d and  $\theta$  at the two different scales.

<sup>213</sup> nature to the system.

To quantitatively demonstrate these observations, we evaluated the mutual informa-214 tion [26] between  $d(\theta)$  and  $\mathbf{B}^{\tau}_{\mu}(t)$  (Eq. 6) as shown in Fig. 5. A statistically significant 215 dependency is clearly found between d and magnetic field fluctuations below the proton-216 cyclotron scale  $\tau_{c_p}$  as well as between  $\theta$  and the inertial range dynamics ( $\tau \gtrsim \tau_{c_p}$ ). This 217 quantitatively shows that the overall dynamics of solar wind magnetic field variability re-218 flects those observed at inertial scales, being characterized by a low-dimensional dynamics 219 around marginally stable fixed points, while a larger number of degrees of freedom in con-220 junction with an unstable nature of the phase-space topology reflecting an externally forced 221 dynamics is associated with the sub-proton range dynamics. The kinetic and electron scales 222 contributions (if any) to the dynamics remain difficult to evaluate because of the prominent 223 role of the noise overriding the kinetic and electron scales contributions at high frequency. 224

Our results provide the first quantitative evidence of large-scale turbulence-induced fluc-225 tuations, thus being a quantitative proof of the Richardson cascade effects on the sub-proton 226 regime from the inertial one. We found that the number of degrees of freedom is larger than 227 the topological dimension of the system as represented by the magnetic field (D = 3) at 228 sub-proton scales ( $\tau > \tau_{c_p}$ ), while a low-dimensional phase-space is found at MHD scales 229  $(\tau < \tau_{c_p})$ . On one hand, this suggests the existence of an externally induced dynamics at 230 sub-proton scales that can be related to the inertial range direct cascade mechanism; on 231 the other hand, this also implies that there exists a degree of correlation between mag-232 netic field components that tends to reduce the effective number of degrees of freedom at 233



FIG. 5. Mutual information between d and  $\mathbf{B}^{\tau}_{\mu}(t)$  (red) and between  $\theta$  and  $\mathbf{B}^{\tau}_{\mu}(t)$  (green). The dashed and dashed-dotted lines correspond to the FGM instrumental noise cutoff ( $f \sim 5$  Hz) and the inverse of the proton-cyclotron frequency ( $f_{c_p} = 1/\tau_{c_p} = eB/m_p$ ), respectively. The gray shaded area denotes the 95% significance level.

MHD scales. The latter result points towards a 2D nature of magnetic field fluctuations 234 at MHD scales with an inverse enstrophy cascade typically arising in near two-dimensional 235 incompressible decaying turbulence. Taken together, we firmly demonstrated that sudden 236 variations observed in magnetic field measurements are associated with unstable fixed points 237 characterizing the dynamics at sub-proton scales. These large-scale intermittent-like vari-238 ations cannot be definitely associated with coherent intermittent events belonging to the 239 MHD domain, but seem to be related to an underlying stochastic strange attractor [27], in 240 close analogy with the results obtained by Raphaldini et al. [28, 29] for MHD and Faranda 241 et al. [24] for fluid turbulence. Indeed, we also demonstrated the existence of a different 242 fixed point nature across the different scales, moving from an unstable point approached 243 at sub-proton scales to marginally stable fixed points at inertial scales. Thus, the overall 244 dynamics of solar wind magnetic field fluctuations consists of multi-stable and multiscale 245 fixed points, opening a novel way to describe the solar wind via stochastic low-dimensional 246 models featuring a large number of degrees of freedom [24, 27]. 247

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