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Semi-analytical modeling of the flow redistribution upstream from the mixing grids in a context of nuclear fuel assembly bow

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Abstract

Fuel assembly bow is a phenomenon that has drawn a significant attention over the last decades, due to its role in several issues related to nuclear core management. As a result, nuclear-based electricity production stakeholders support extensive research on this topic.

Recent studies agree that the various hydraulic forces affecting fuel assemblies over their whole height constitute a major cause of deformation. More specifically, the lateral force acting on a mixing grid strongly depends on the coolant main flow distribution between the grid itself and the surrounding water gaps. This flow redistribution varies according to the position of the assembly relatively to its neighbours. Often ignored, the modeling of the water gaps is in the limelight of this article. In order to design a simple yet accurate model of flow redistributions, we approximate the space between two successive grids (further called convergent-diffuser) as a 1D-channel. The latter is set up with suitable properties and its variable thickness reflects the deflection of the assemblies. A simple hydraulic network connecting the gap model to its adjacent grids (both represented by loss coefficients) is built to enable the flow redistribution. Successive enhancements are proposed to comply with the physical aspects neglected by the initial model.

The final version of our semi-analytical model achieved very close agreement with both CFD simulations at local scale and dedicated experimental results. The latter are obtained thanks to a mock-up made of 3D-printed grids, especially designed to validate our approach's trustworthiness. In the end, the computational efficiency of the proposed models is drawn upon to initiate a first sensitivity analysis. It aims at emphasizing the most influential inputs of the models, in order to set their values at best and obtain the most accurate description of the flow redistribution phenomena. The water gap model described in the present article is meant to be included within a larger, properly designed hydraulic network in future simulations at the scale of a full reactor core.

1 INTRODUCTION: CONTEXT AND PROBLEMATICS

The present article is intended to gain a better understanding of the underlying mechanisms behind the bow of Fuel Assemblies in the core of Pressurized Water Reactor. It represents a topic of great interest for both performance and safety of reactors of this family, with issues such as power tilt or drop times of reactivity control devices. It is also scientifically challenging as intricate multiphysical phenomena are involved. The proposed research especially focuses on forces coupling the fuel

45 assembly deflections to the water flow through the core (steady-state fluid-structure interaction). The
46 fluid-structure coupling would probably explain the deformed shape of some initial cores, occurring
47 early during the startup procedure. Starting from numerous works published over the last decade, a
48 special attention is given to the ins and outs of flow redistribution in the vicinity of mixing grids.
49 Basically, the fraction of the inlet flow going through the grids or through the water gaps between them
50 is highly dependant on the width of these gaps. The related hydromechanical forces still require to be
51 properly assessed along with uncertainty levels. The specific model built in this article will later be
52 inserted into a more macroscopic one, necessary for running the simulations of full cores or of a
53 selected row of fuel assemblies. To achieve this goal, a full multi-scale strategy is designed, with the
54 main objective of providing a computationally efficient model for hydraulic forces resulting from flow
55 redistribution. It could be used to fully characterize the fluid-structure coupling patterns originated
56 from such phenomena through generalized sensitivity analyses. Local scale 3D simulations are
57 provided to validate the elementary components of the model, which is then confronted to a new and
58 specifically designed experiment.

59 The paper is thus organized as follows. First, a review of the existing state-of-the-art is provided,
60 in order to fully establish the content and relevance of the proposed research. This is followed by the
61 description of the aforementioned model for hydromechanical forces accounting for local flow
62 redistribution, from general principles to an extended formulation presented as a generalized
63 hydraulic network. The next two sections are dedicated to the validation of the model through
64 comparison to both local scale simulations and experiment. The last section provides a sensitivity
65 analysis, as a first step to obtain insights on relevant phenomena to be accounted for when modeling
66 coupled hydromechanics for fuel assembly bow. It can also draw open perspectives for an extended
67 application of the introduced approach to configurations of growing complexity involving several fuel
68 assemblies.

69 **2 REVIEW OF FUEL ASSEMBLY BOW HYDROMECHANICS AND OF THE INFLUENCE** 70 **OF FORCES RELATED TO FLOW REDISTRIBUTION**

71 **2.1 Short state-of-the art for fuel assembly deformation and associated fluid-structure** 72 **interaction**

73 Deformation of fuel assemblies is an issue first reported in the 90's [Andersson, 2005]. Initial
74 concerns arose specifically regarding IRI (incomplete rod insertion) [Andersson, 2005][Gabrielsson,
75 2018]. Over the last decade, the phenomenon has drawn attention for its effects on several other
76 matters, including neutronics or core management policies [de Lambert, 2019][Gabrielsson, 2018].
77 The study of fuel assembly bow on an exhaustive basis is actually a very intricate task as many
78 technical fields in nuclear engineering are involved simultaneously [de Lambert, 2019]: neutronics,
79 thermal mechanics, and thermal hydraulics.

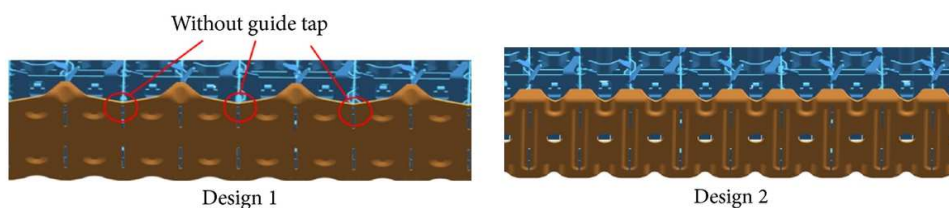
80 Three main effects are often invoked to explain fuel assembly bow: irradiation creep and growth
81 and stationary hydraulic forces [IAEA, 2010][Wanninger, 2018]. The first two mechanisms concern
82 the structural and material behavior of fuel assembly in operational conditions. They are out of the
83 scope of the present article and several studies involving them can be found in the literature (see for
84 instance [Wanninger, 2016]). Instead, the proposed work focuses on the hydraulic forces applied on
85 the assemblies. They result from macroscopic and local flow redistributions occurring within the core,
86 placing this research in the field of Fluid-Structure Interaction (FSI) for nuclear cores.

87 In this context, FSI can be seen either from a dynamical point of view, mainly in the context of
 88 seismic loadings [Ricciardi, 2009], or from a quasi-static point of view, by retaining the low-frequency
 89 component of the fluid velocity and structure deformation [Horvath, 2012]. In this paper, we focus on
 90 the latter only, as we are looking at the core deformation over one or several full operating cycles.
 91 Numerically, this problem is often treated with a partitioned two-way coupling: two solvers are used
 92 separately for each physics and, at each time step, an inner loop is performed consisting in solutions
 93 for fluid and structure problems alternatively, until convergence is reached [Benra, 2011]. Time
 94 dependence arises only due to creep processes in the fuel assemblies bow calculations and one seeks
 95 for a static fluid-structure interaction equilibrium at each creep time step. This kind of coupling
 96 induces higher computation time, and simpler models are thus necessary for both fluid and structure
 97 calculations. Studies have been pursued to develop low-cost models for the flow travelling through the
 98 nuclear core, including porous medium [Wanninger, 2018][Lascar, 2015], and hydraulic networks
 99 [Stabel, 2011]. Such models inevitably come with a series of hypotheses and assumptions over both the
 100 structural behavior and the fluid flow, needing to be confirmed or improved through dedicated studies
 101 and parametric analyses. Among those topics, the dependence of the lateral fluid forces acting on the
 102 fuel assemblies to the local flow redistributions is of primary importance. This article specifically
 103 addresses this issue through an advanced, yet computationally efficient, 1D-model of the flow pattern
 104 around and across spacer grids, validated upon both CFD and experimental data, and compatible with
 105 the requested sensitivity studies in terms of numerical cost.

106 2.2 Specific influence of the grids

107 Spacer grids located on fuel assemblies play an important role on the flow in the core. They
 108 increase the heat transfer rate from the fuel rods to the water by creating turbulent swirls and cross-
 109 flows downstream. Two important features have to be highlighted here.

110 First, the design of the grid outer straps (Fig. 1) significantly influences the flow behavior across
 111 the grid. Yan, in [Yan, 2014], performed calculations on a cluster of 4 quarters of assembly separated
 112 by a cross-shaped water gap, considering two types of grid straps used in the industry, respectively
 113 called ‘Design 1’ and ‘Design 2’ (Fig. 1). The study demonstrated that pressure drop and cross-flow
 114 upstream/downstream of the grids depend on the adopted design.



115
 116 *Figure 1 - Grids' outer straps as considered by Yan [Yan, 2014]*

117 Secondly, the flow in the upstream region of the grids is subject to redistributions, especially in
 118 the case ‘Design 2’ in [Yan, 2014]. Practically, the flow can either stay in an assembly (and then cross
 119 the grid itself) or go through the bypasses (or water gaps) between the outer straps of the grids. Studies
 120 showed that this redistribution towards the bypasses depends on their width (which can vary due to
 121 bow) and on the fluid velocity [Puragliesi, 2019][Xu, 2019]. In [Bieder, 2020], the author noticed that
 122 only some extensions of the long diagonals cross flows developing within the assembly, not hindered
 123 by the control rod guide tubes, can go through the water gap. According to Bieder, the water gap acts
 124 as a barrier to inter-assembly exchanges, it tends towards isolating the flow within one assembly from
 125 each other for a 2 mm nominal width.

126 These aspects play a significant role in the unfolding of fuel assembly bow in an actual nuclear
127 core. When an assembly is deflected, the widths of all the bypasses surrounding it are modified from
128 the bottom to the top. Indeed, all spacer grids can draw closer or away from their neighbors. The flow
129 distribution among the grids and the bypasses between them is then affected, depending on the grid
130 geometry and the bypasses width. This globally entails differences in the pressure exerted on the
131 opposite outer straps of one given grid, which thus yields a lateral hydraulic force of significant
132 intensity.

133 **2.3 Previous work introducing lateral hydraulics forces resulting from flow** 134 **redistribution through the concept of added stiffness**

135 In his experimental and modeling work, Ricciardi [Ricciardi, 2014] showed that the fuel
136 assembly's displacement during its deformation induced velocity fluctuations in the two bypasses
137 surrounding it. This difference of velocity brings about a pressure difference. The resulting force,
138 depending on the displacement, is thus interpreted in terms of flow-induced added stiffness.

139 Then, Ricciardi [Ricciardi, 2015] proposed to estimate this effect supposing that pressure drops
140 along the grid and the bypasses were equal. With the help of Bernoulli's principle and the Darcy-
141 Weisbach equation, the author is able to bring out a nonlinear force depending on the fuel assembly's
142 displacement. The simulations with this model reproduce important phenomena observed in the
143 experiments and Ricciardi concluded that bypasses must play a key role on the dynamics of the fuel
144 assembly. Yet, his model considers that the fluid's velocity is constant in the bypasses, and represents a
145 fuel assembly located between two rigid walls (with larger bypasses than in a PWR context).

146 The force induced by the fuel assembly's displacement is thus not completely understood today in
147 a PWR context, and recent studies neglect a fine modeling of grids' bypasses to save up simulation
148 time. For instance, Wanninger [Wanninger, 2018] with Ruiz Antón-Pacheco [Ruiz Antón-Pacheco,
149 2017] considered the fuel assembly deformations by modifying the porous medium parameters;
150 however, the pressure loss coefficient equals zero in the bypass, meaning that it does not offer any
151 resistance to the flow in the axial direction whatever its width.

152 **2.4 Research contribution introduced in the current paper**

153 Starting from the state-of-the-art provided above, the goal of the present paper is to contribute to
154 the understanding of the influence of the flow redistribution upstream from the grids on the hydraulic
155 forces coupled to structural motion in the framework of fuel assembly bow. Some insights can be
156 found in the work of Ricciardi who computed the added stiffness from Bernoulli's equation, but his
157 model deals roughly with the grid geometry and neglects important phenomena regarding this local
158 flow redistribution. A more sophisticated approach is therefore needed, based in the current article on
159 1D hydraulic networks applied to the configuration of interest, *i.e.* bundles of rods in the vicinity of
160 supporting grids. The first objective is to fully describe and understand the fluid flow behavior in water
161 gaps close to grids. The second objective is allow producing efficient simulations involving several
162 assemblies and redistributions at a larger scale with suitable flow profiles at the core inlet and outlet,
163 while ensuring that the relevant hydromechanical couplings are accurately reproduced.

164 **3 STEP-BY-STEP CONSTRUCTION OF AN ADVANCED MODEL OF THE FLOW** 165 **REDISTRIBUTION**

166 **3.1 Model 0: the water gap seen as a simple channel flow**

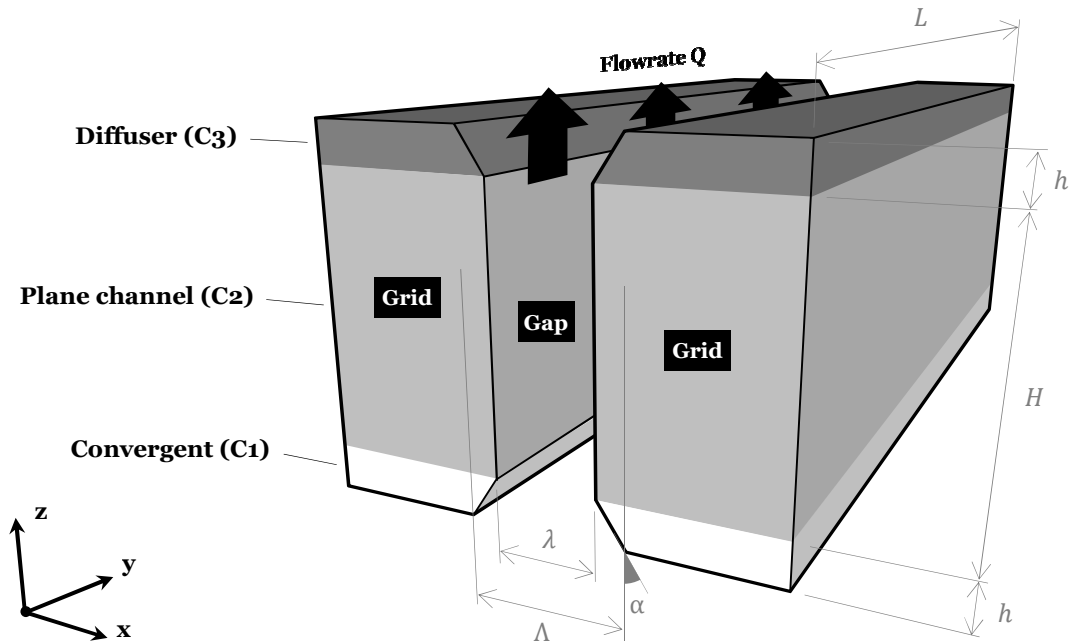
167 The global objective is to determine the pressure field on the outer straps of two adjacent mixing
 168 grids. A natural approach thus consists in modeling the flow inside the gap between them with the so-
 169 called *extended Bernoulli equation*:

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2 + \Delta P \quad (\text{Eq. 1})$$

170 where P stands for the pressure and V for the bulk velocity, subscripts A and B denote points
 171 located respectively upstream and downstream along the water gap, and $\Delta P > 0$ represents the
 172 irreversible pressure loss between A and B.

173 Eq. 1 is actually a 1D flow model of the bypass. As such, it requires to adopt a conceptualized
 174 representation of the bypass real geometry. First, the water gap is seen as a 3D-extruded plane channel
 175 flow, whose “walls” are flush with the last rows of the two adjacent tube bundles. Then the outer strap
 176 of each spacer grid is a small protrusion in the channel, akin to speed bumps on a straight road. The
 177 guide vanes (visible on Fig. 1) provide some smoothness to this obstacle, which we represented by a
 178 bevel at the inlet and outlet of the channel. The symmetry thus created between the inlet and the outlet
 179 makes our system more similar to Design 2 in Fig. 1. The design of the mixing vanes can vary to a
 180 certain extent, and the 3D extrusion does not account for the discontinuous presence of vanes, so that
 181 in the end the angle and curvature of the bevels are uncertain quantities.

182 This simplified geometry is represented in Fig. 2. It consists of three different parts: a convergent
 183 C1, a straight plane channel C2 and finally a diffuser C3. We will refer to this whole flow region as the
 184 convergent-diffuser (CD) thereafter. It approximates the part of the water gap located between two
 185 spacer grids at the same level.



186
 187 *Figure 2 - Geometry considered for the area between two mixing grids*

188 In this model, L is the grid depth (in the spanwise direction y). λ represents the channel width in
 189 C2, whereas Λ is the width of the water gap at the entrance of C1 as well as at the exit of C3. These two
 190 quantities are linked by Eq. 2 below:

$$\Lambda = \lambda + 2h \cdot \tan(\alpha) \quad (\text{Eq. 2})$$

191 where h is the height of both the convergent C1 and diffuser C3, H is the height of the middle part
 192 C2, α is the bevel angle.

193 The value of $\Delta\mathbb{P}$ in Eq. 1 can now be determined as the sum of pressure drops across C1, C2 and
 194 C3, for which empirical formulas are widely known [Idel’Cik, 1986]. The evaluation of these three
 195 pressure drops independently of one another is not straightforward, because H in actual grids is of the
 196 same order of magnitude as h (in other words, the velocity profile at C1 inlet may still be related to the
 197 one at C3 outlet). Even if individual empirical formulas cannot be summed *a priori* to account for the
 198 total pressure drop, CFD computations were carried out to check the validity of this approximation *a*
 199 *posteriori*, with quite conforming results (see section 4).

200 Pressure losses in hydraulic pipe systems are usually written in the following general form [Cross,
 201 1936] (Eq. 3):

$$\Delta\mathbb{P} = KQ^a \quad (\text{Eq. 3})$$

202 where K is the hydraulic resistance coefficient, Q is the volumetric flow rate, and a is an exponent
 203 depending on the used formula. The general formula presented above covers both frictional losses
 204 (*i.e.* energy dissipation per unit length due to the fluid rubbing against the duct wall) and local losses
 205 (*i.e.* energy dissipation due to sharp changes of the duct geometry).

206 Pressure drop across C1

207 In the convergent C1, we assume that the resistance is composed of both local and frictional parts.
 208 The total pressure loss across a “rectilinear converging bellmouth” is adapted from [Idel’cik, 1986]:

$$\Delta\mathbb{P}_1 = \frac{1}{2}\rho \left[\frac{\zeta_1}{(L\lambda)^2} \left(1 - \frac{\lambda}{\Lambda}\right) + \frac{1}{2L^2 \cos(\alpha)} \int_0^h \frac{f(\ell_1(z_1), Q)}{\ell_1(z_1)^3} dz_1 \right] Q^2 \quad (\text{Eq. 4})$$

209 where ζ_1 and f are two resistance coefficients for the local and frictional parts respectively and ρ is
 210 the fluid density.

211 The local gap width $\ell_1(z_1)$ is a linear map such that $\ell_1(0) = \Lambda$ and $\ell_1(h) = \lambda$. We considered that
 212 ζ_1 varied little enough on the range of commonly studied gap values $\lambda \in [0 \text{ mm}; 20 \text{ mm}]$ so that it
 213 could be treated as a constant. It is also common to assume that f is constant provided that the
 214 Reynolds number is high. Rather than this assumption, we retained a computational-friendly
 215 correlation from Haaland to assess f [Asker, 2014]). It yields the expression of the frictional part in
 216 Eq. 4 as the integration from $z_1 = 0$ to h of the Darcy-Weisbach equation applied to a pipe with a
 217 linearly varying hydraulic diameter (and thus linearly varying bulk velocity). One can check that taking
 218 f as a constant in Eq. 4 would have yielded the original Idel’Cik correlation [Rennels, 2012].

219 Pressure drop across C2

220 The middle part C2 is a plane channel of hydraulic diameter 2λ . The flow resistance is then only
 221 due to friction. The Darcy-Weisbach equation directly gives the streamwise evolution of pressure loss
 222 in C2:

$$\Delta\mathbb{P}_2 = \frac{\rho f H}{4\lambda} \left(\frac{Q}{L\lambda}\right)^2 \quad (\text{Eq. 5})$$

223 Pressure drop across C3

224 Similarly to part C1, this resistance in the diffuser C3 depends on a local and a frictional part. The
 225 frictional resistance is equal to that of Eq. 4 by symmetry. We obtain in Eq. 6 the streamwise evolution
 226 of the pressure loss across C3 in the same way as we did for C1:

$$\Delta P_3 = \frac{1}{2} \rho \left[\frac{\zeta_3}{(L\lambda)^2} \cdot \left(1 - \frac{\lambda}{\Lambda}\right)^2 + \frac{1}{2L^2 \cos(\alpha)} \left(\int_0^h \frac{f(\ell_1(z_1), Q) dz_1}{\ell_1(z_1)^3} \right) \right] Q^2 \quad (\text{Eq. 6})$$

227 Conclusion: total resistance coefficient of model o

228 We derive the total pressure loss coefficient of model o from the summation of Eq. 4 to 6:

$$K_{CD} = \frac{\Delta P}{Q^2} = \frac{\rho}{2(L\lambda)^2} \left[\zeta_1 \cdot \left(1 - \frac{\lambda}{\Lambda}\right) + \frac{fH}{2\lambda} + \zeta_3 \cdot \left(1 - \frac{\lambda}{\Lambda}\right)^2 + \frac{\lambda^2}{\cos(\alpha)} \left(\int_0^h \frac{f(\ell_1(z_1), Q) dz_1}{\ell_1(z_1)^3} \right) \right] \quad (\text{Eq. 7})$$

229 Used to estimate lateral hydraulic forces due to pressure differences on each side of a spacer grid,
 230 this initial model turned out to significantly overestimate the forces compared to the ones observed in
 231 the literature (see for instance [Wanninger, 2018] for a set of hydraulic forces calculated in the case of
 232 a row of fuel assemblies). This originates in the neglected, and unknown, fraction of the flow passing
 233 the grids' level through the rod bundle rather than through the water gap, lowering the actual effect of
 234 the convergent-diffuser.

235 **3.2 Model 1: distribution of the flow between the water gap and the mixing grids**

236 In the model o, the flow is bound to circulate in separate domains, with some kind of
 237 impermeable boundary between the rod bundles and the water gap. Water particles initially inside one
 238 fuel assembly will go through the mixing grid and remain inside the assembly downstream of the grid.
 239 Water particles initially in the water gap between assemblies will go through the bypass between grids
 240 and continue along the water gap.

241 In reality, when the gap width λ decreases, the flow in the bypass dries up. On the contrary, when
 242 the gap width increases enough, water rushes from the rod bundles and towards the gap. Model 1 is
 243 designed to account for these two phenomena in addition to the convergent-diffuser system of Model
 244 o.

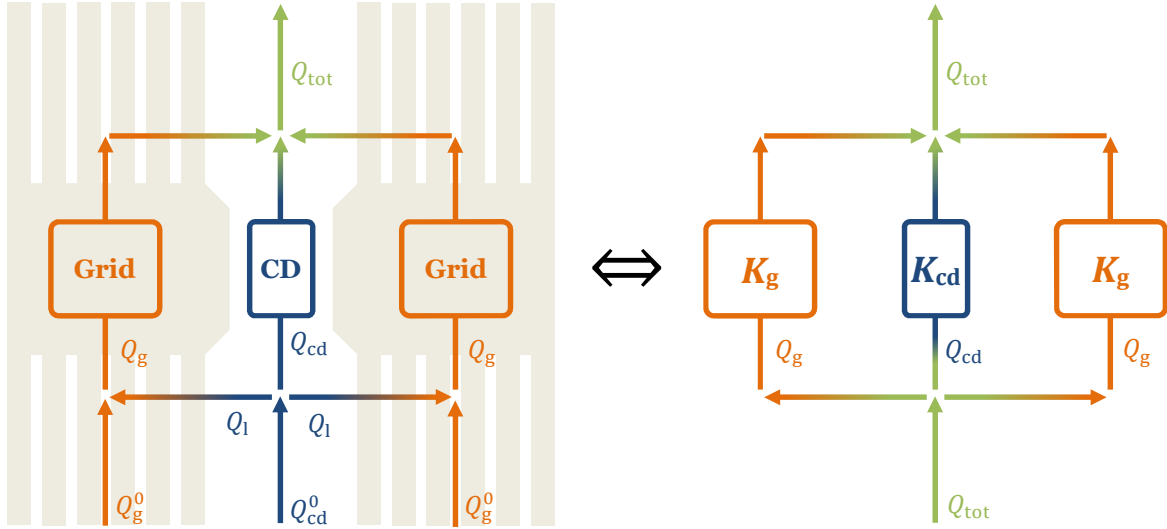
245 The grid is geometrically complex and modeling the behavior of the flow passing through it is a
 246 challenging task. Within this article, we stick to a simple expression of the irreversible pressure drop
 247 associated to the mixing grids in the following form (Eq. 8):

$$\Delta P_g = \frac{1}{2} \rho C_g V_g^2 = K_g Q_g^2 \quad (\text{Eq. 8})$$

248 where C_g is the coefficient of local resistance of the grid, V_g is the bulk velocity upstream of the
 249 half-grids, Q_g is the volumetric flow rate crossing one half-grid and K_g is the resistance given by Eq. 9:

$$K_g = \frac{1}{2} \rho \frac{C_g}{S_g^2} \quad (\text{Eq. 9})$$

250 with S_g being the flow cross-section upstream one half-grid.



251
252

Figure 3 - Hydraulic network representing the possible pathways at the grids level (model 1)

253 Having developed adequate resistances for the convergent-diffuser (Eq. 7) and the grid (Eq. 9), we
 254 can set up a simple hydraulic network (Fig. 3) representing the system of interest. It consists of two
 255 half-grids separated by a convergent-diffuser (meant for the water gap). Both half-grids are supplied
 256 with the same upstream flow rate noted Q_g^0 (symmetric system), while Q_{cd}^0 is the upstream flow rate
 257 feeding into the convergent-diffuser. The leaking flow rate between the grids and the convergent-
 258 diffuser is named Q_l . The effective flow rates crossing the grids and the convergent-diffuser are
 259 respectively named Q_g and Q_{cd} . The total outgoing flow rate leaving the system is named Q_{tot} . Applying
 260 the so-called *Kirchhoff hydraulic equations* [Cross, 1936], *i.e.* the network's mass conservation and
 261 energy, we obtain the following system of equations (Eq. 10):

$$\begin{cases} \Delta P_{cd}(Q_{cd}) - \Delta P_g(Q_g) = 0 \\ Q_g^0 + Q_l = Q_g \\ Q_{cd}^0 = 2Q_l + Q_{cd} \end{cases} \quad (\text{Eq. 10})$$

262 By defining $Q_{tot} = 2Q_g^0 + Q_{cd}^0$, the system can be changed into Eq. 11:

$$\begin{cases} K_{cd}Q_{cd}^2 - K_gQ_g^2 = 0 \\ 2Q_g + Q_{cd} = Q_{tot} \end{cases} \quad (\text{Eq. 11})$$

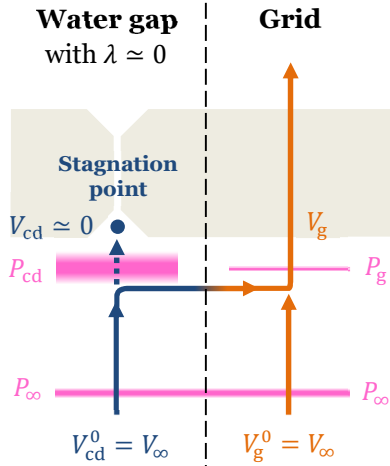
263 Basically, Eq. 11 shows that the splitting of Q_{tot} into Q_{cd} and Q_g does not depend on individual inlet
 264 boundary conditions Q_g^0 and Q_{cd}^0 but on their sum only. In other words, the geometry upstream of the
 265 grids level has no influence on the redistribution computed in Model 1 (see the equivalent network in
 266 Fig. 3). Practically, when modifying Q_{cd}^0 and Q_g^0 while keeping Q_{tot} constant, the leaking flow rate Q_l
 267 will adapt so that in the end Q_{cd} and Q_g remain the same (this is the meaning of the removed equation
 268 between Eq. 10 and Eq. 11).

269 One can notice that the final system depends on two functions $\Delta P_g(Q_g)$ and $\Delta P_{cd}(Q_{cd})$. It is non-
 270 linear in terms of unknowns Q_g and Q_{cd} , raised to the power of a in the energy equation ($a = 2$ in our
 271 case). While an advanced algorithm could be necessary for larger non-linear systems (see for instance
 272 [Wood, 1972][Wood, 1981] or [Nielsen, 1989]), this small one can be solved easily using any root-
 273 finding algorithm. It is however shown in section 4 that the predictions of this model deteriorate when
 274 considering low values of λ . Indeed, when the CD's thickness tends towards zero, CFD simulations

275 show that a stagnation point appears, increasing the pressure drop in a way this model cannot
 276 reproduce, yielding the need for a second improving step in the proposed modeling strategy.

277 **3.3 Model 2: stagnation point effect for thin water gaps**

278 When the CD gets very narrow ($\lambda \approx 0$), it acts as an opaque obstacle to the fluid, thus producing a
 279 stagnation point: the flow flees towards the grids, the CD upstream velocity decreases down to almost
 280 zero and the kinetic energy turns into a pressure peak at the inlet of the CD (Fig. 4).



281
 282

Figure 4 - Stagnation point setting up upstream from the convergent-diffuser

283 Therefore, we note that when λ is small enough, there is a conversion of the kinetic energy linked
 284 to V_{cd} into a kinetic energy linked to V_g . This observation introduces a ‘branch-coupling’ term in the
 285 energy equation.

286 Let us suppose that far from the obstacles, velocity V_∞ and pressure P_∞ are homogeneous (Fig. 4).
 287 As explained above, in a stagnation point context, upstream grid pressure P_g and CD pressure P_{cd} are
 288 not equal anymore. Applying the Bernoulli equation along two distinct axial streamlines across the
 289 grid and across the convergent-diffuser, we obtain Eq. 12:

$$\begin{cases} P_\infty + \frac{1}{2}\rho V_\infty^2 = P_g + \frac{1}{2}\rho V_g^2 \\ P_\infty + \frac{1}{2}\rho V_\infty^2 = P_{cd} + \frac{1}{2}\rho V_{cd}^2 \end{cases} \quad (\text{Eq. 12})$$

290 which leads to:

$$\Delta P_c = P_{cd} - P_g = \frac{1}{2}\rho(V_g^2 - V_{cd}^2) \quad (\text{Eq. 13})$$

291 A ‘branch-coupling’ term named ΔP_c appears, linking both grid and CD dynamic pressures. This
 292 additional term can be added to the equation energy (Eq. 11) in order to take into account that the
 293 pressure drops in grid and CD are no longer equal but are now shifted from each other by a value of
 294 ΔP_c :

$$\Delta P_{cd} - \Delta P_g = \Delta P_c \quad (\text{Eq. 14})$$

295 Introducing the flow rates Q_g and Q_{cd} , and the cross-section S_{cd} of the CD upstream of the grids,
 296 Eq. 14 becomes:

$$K_{cd}Q_{cd}^2 - K_gQ_g^2 = \frac{1}{2}\rho\left(\frac{Q_g^2}{S_g^2} - \frac{Q_{cd}^2}{S_{cd}^2}\right) \quad (\text{Eq. 15})$$

297 We can then define the modified resistance coefficients as below:

$$\tilde{K}_{cd}(\lambda) = K_{cd}(\lambda) + \frac{\rho}{2S_{cd}^2(\lambda)} \quad (\text{Eq. 16})$$

$$\tilde{K}_g = K_g + \frac{\rho}{2S_g^2} \quad (\text{Eq. 17})$$

298 We thus obtain the final equation system for Model 2:

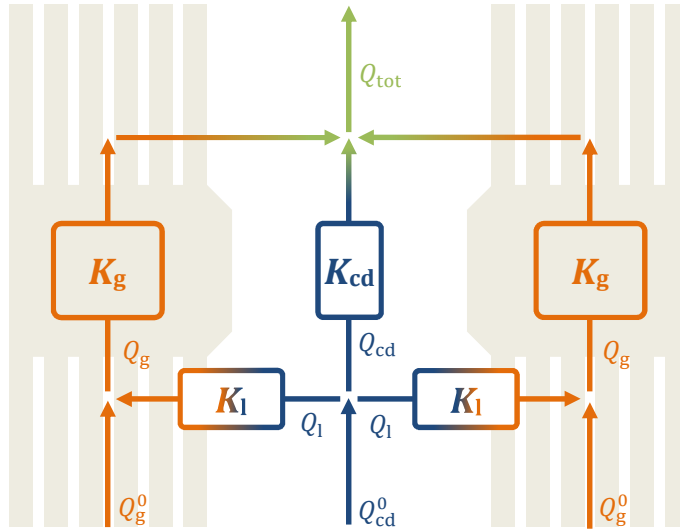
$$\begin{cases} \tilde{K}_{cd}Q_{cd}^2 - \tilde{K}_gQ_g^2 = 0 \\ 2Q_g + Q_{cd} = Q_{tot} \end{cases} \quad (\text{Eq. 18})$$

299 One can remark that Eq. 18 (Model 2) is formally identical to Eq. 11 (Model 1), except that both
 300 resistance coefficients are increased by distinct constant offsets, which entails that Eq. 11 and Eq. 18
 301 will yield different solutions. This means that the flow rate redistribution is actually modified when the
 302 stagnation point is taken into account (the added pressure drop is not the only outcome).

303 3.4 Model 3: lateral resistance due to the rods

304 In the previous developments, the lateral obstruction due to the fuel rods presence was neglected
 305 and the fluid could switch among the grid and CD channels without any resistance. This means that in
 306 Models 1 and 2, the geometry upstream of the grids level has no actual influence on the flow
 307 redistribution.

308 In reality, an additional lateral resistance causes the total axial pressure drop to slightly increase.
 309 CFD simulations show in the next section that the bypass pressure drop computed by Model 2 is for
 310 instance underestimated by approximately 10% in the case of a 1mm-thin bypass without fuel rods.
 311 Practically, the next evolution of the model, *i.e.* Model 3, includes the resistance K_1 for the leaking flow
 312 rate Q_1 coming from the bypass and going to the grid through several rows of fuel rods (see Fig. 5).

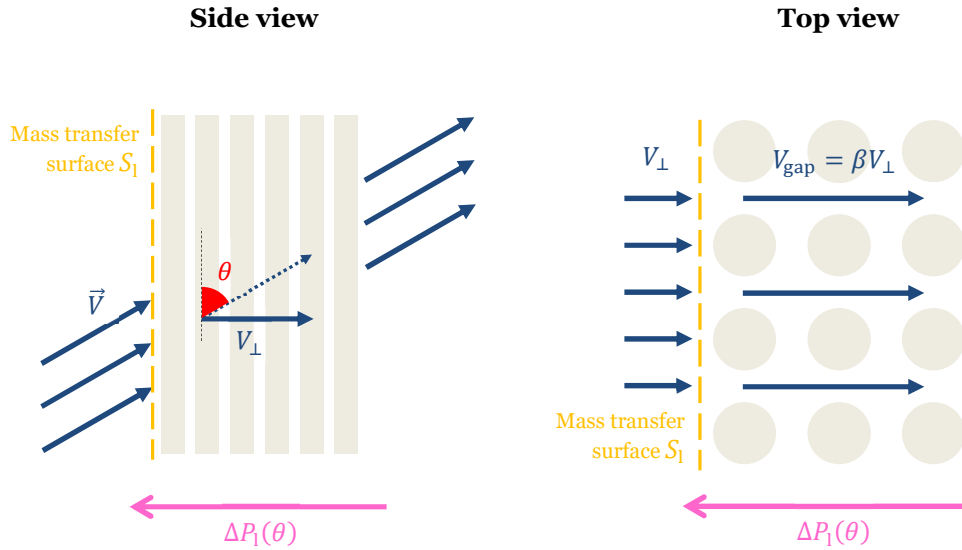


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 314

Figure 5 – Additional lateral resistance due to cross-flows through the rod bundle (model 3)

315 Quite intuitively, the coefficient K_1 depends on the 3D incidence angle of the leaking flow rate in a
 316 very complicated way. To provide a first approximation of this contribution, a 2D representation is
 317 chosen (see Fig. 6), so that some results are available in the literature. In his work, Peybernès
 318 [Peybernès, 2005][Bieder, 2015b] came up with a correlation for the lateral pressure drop across an
 319 inclined rod bundle, based on the Eole experimental results. The same correlation has also been used

320 recently in [Wanninger, 2018] in a more global effort to estimate the hydraulic forces on PWR fuel
 321 assemblies.



322
 323

Figure 6 – General layout of an inclined flow passing through a rod bundle

324 The correlation states that the mean pressure drop per row is:

$$\Delta P_{\text{row}}(V_{\text{gap}}, \theta) = \frac{1}{2} \rho K_{\perp} \xi(\theta) V_{\text{gap}}^2 \quad (\text{Eq. 19})$$

325 where V_{gap} stands for the fluid velocity in the gaps between the rods, K_{\perp} is the pressure drop
 326 coefficient for purely transverse flows ($\theta = 90^\circ$) and $\xi(\theta)$ is a correction factor stemming from the Eole
 327 installation operated at various incidence angles θ , given by Eq. 20 below:

$$\xi(\theta) = \left(\frac{\sin(\theta)}{\cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)} \right)^{1.7} \quad (\text{Eq. 20})$$

328 After some computations, the resulting lateral pressure drop across N rows of fuel rods is then
 329 given by:

$$\Delta P_1(\theta) = \frac{\rho N K_{\perp} \xi(\theta) \beta^2}{2 S_1^2 \sin^2(\theta)} Q_1^2 = K_1(\theta) Q_1^2 \quad (\text{Eq. 21})$$

330 where β is the geometrical ratio of the gap velocity V_{gap} to the pitch velocity V_{\perp} and S_1 the lateral
 331 rectangular surface over which the fluid transfer Q_1 takes place.

332 Writing $S_1 = L h_1$ with L the total width of a fuel assembly (see Fig. 2) and h_1 the corresponding
 333 height, we eventually give the following expression for K_1 :

$$K_1(\theta) = \frac{\rho N K_{\perp} \xi(\theta) \beta^2}{2 L^2 h_1^2 \sin^2(\theta)} \quad (\text{Eq. 22})$$

334 In our context of multi-1D modeling, the incidence angle θ can be approximated from the ratio of
 335 the lateral velocity to the axial velocity in the upstream branch (*i.e.* grid or CD), which corresponds to
 336 the following expressions:

$$\theta = \begin{cases} \tan^{-1} \left(\frac{S_g |Q_1|}{S_1 |Q_g|} \right) & \text{if } Q_1 < 0 \\ \tan^{-1} \left(\frac{S_{cd} |Q_1|}{S_1 |Q_{cd}|} \right) & \text{if } Q_1 > 0 \end{cases} \quad (\text{Eq. 23})$$

337 Using the newly added resistance to lateral flow, system (Eq. 18) becomes:

$$\begin{cases} \tilde{K}_{cd} Q_{cd}^2 - \tilde{K}_g Q_g^2 - K_1 Q_1^2 = 0 \\ 2Q_g + Q_{cd} = Q_{tot} \\ Q_g^0 + Q_1 = Q_g \end{cases} \quad (\text{Eq. 24})$$

338 Unlike in previous models, the inlet flow rate Q_g^0 (or Q_{cd}^0) now plays an important role in the
 339 redistribution of Q_{tot} between Q_g and Q_{cd} , due to the presence of Q_1 in the first equation of system (Eq.
 340 24). This means that in Model 3, the geometry of the fuel assemblies upstream of the grids does have
 341 an influence on the redistribution. In practice, this is shown by the appearance of several new
 342 geometrical parameters, notably β , h_1 and N . The value of β can be easily calculated given the pitch and
 343 diameter of fuel rods. However, h_1 and N characterize the dimensions of the region where cross-flows
 344 take place between the water gap and the grids. As such, those two parameters are complex to estimate
 345 and are likely to depend on the bypass thickness λ .

346 4 COMPARISON WITH LOCAL CFD SIMULATIONS

347 4.1 Presentation of the CFD models

348 CFD simulations are carried out in order to validate our semi-analytical model. The local-scale
 349 solutions provided by two different programs, namely TrioCFD [Angeli, 2015] and Code_Saturne
 350 [Archambeau, 2004], are cross compared to produce reliable reference data.

351 The geometry considered for the calculations is shown on Fig. 7. It consists in two assemblies
 352 separated by a central bypass of adjustable thickness λ (1, 2, 3, 5, 10 and 20 mm). Two rods are
 353 represented along the y-axis while there are four and a half rods per assembly along the x-axis. The
 354 convergent-diffuser area is modelled as shown in Fig. 2 (i.e. 3D extruded), but the nearer fuel rods go
 355 through parts C1 and C3. In this way, a notch is set up around the fuel rod positions. Thus, the outer
 356 straps geometry created comes closer to ‘Design 2’ (i.e. to a real grid design). The tube bundle regions
 357 use a rather fine mesh and the intricate geometry of the grids is not reproduced. Instead, we use a
 358 porous media approach with a coarser mesh and calibrate the source term in order to reach exactly the
 359 desired value of K_g . For the same reasons, some volume inside the grids close to the convergent-
 360 diffuser is not meshed (see again Fig. 7), since it would bring unwanted recirculations in the flow, thus
 361 increasing artificially the value of K_g .

362 The mesh is unstructured, consisting in 3D tetrahedral elements with two prismatic layers applied
 363 on surfaces with a no-slip boundary condition (fuel rods and outward faces of the grids, see Fig. 7). As
 364 TrioCFD does not allow prismatic volumes, a specific version of the mesh was created where the
 365 prisms near the walls are split into five tetrahedrons.

366 A mesh convergence study has been realized for the case $\lambda = 5$ mm. For both codes, four
 367 calculations have been run with various grid refinements whose y^+ averaged on wall surfaces always
 368 laid in the [30; 100] range required by the wall function. Volumetric flow rates in the water gap and
 369 pressure drop across the grid exhibited deviations of less than 6% between all the meshes. We selected

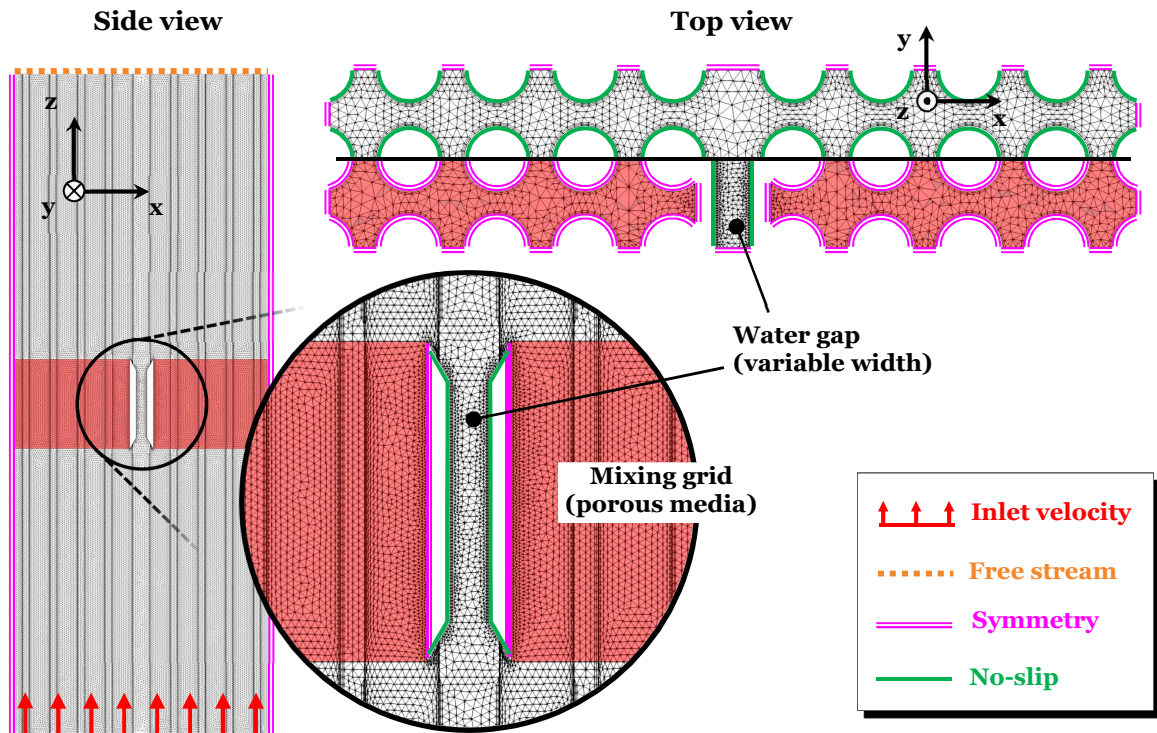
370 the second thinnest mesh considering it was a fair trade-off between accuracy and performance. It
 371 contains approximately 5.10^6 cells.

372 Symmetry boundary conditions are applied on each lateral side of the fluid domain (see Fig. 7).
 373 Inside the grids, a symmetry condition is applied on all the lateral surfaces (which are mainly fuel
 374 rods) in order to disable friction that would otherwise increase the pressure drop artificially (K_g is
 375 defined explicitly as a source term, see above). Outside of the grids, a no-slip condition is applied on
 376 the fuel rods and on the convergent-diffuser walls (Fig. 7). An axial flow velocity of 6 m/s is imposed
 377 uniformly on the bottom inlet surface (the Reynolds number based on bulk velocity and rod diameter
 378 is 10^5). Finally, a constant pressure is set on the top outlet surface (see Fig. 7).

379 Turbulence is handled by a standard k- ϵ RANS model accompanied by a wall function (1-scale
 380 model - log law for Code_Saturne and Richardson for TrioCFD). We aim at reaching a steady state.
 381 This turbulence model could have difficulty computing accurately the transverse flows. However
 382 studies based on large eddy simulations (LES) and two experiments, AGATE [Bieder, 2014][Bieder,
 383 2015a] and MATHYS_H [Bieder, 2012], have shown that the k- ϵ model can reproduce the coolant
 384 behavior near the mixing grid where the flow is mainly advection-controlled. More recently, Bieder
 385 also pointed out that conclusions pulled from LES and k- ϵ calculations on an inter assembly setup
 386 were similar [Bieder, 2020]. Dedicated LES calculations are considered as prospects in the future, but
 387 at this time those results encouraged us to use the k- ϵ model for our CFD simulations.

388 Water density is 700 kg/m^3 , close to the operating value in a PWR.

389



390
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Figure 7 – Description of the mesh ($\lambda = 5 \text{ mm}$ here) and boundary conditions used in the CFD simulations

392 **4.2 Pressure drop across the grids**

393 The first physical value of interest is the pressure drop across the convergent-diffuser. Two
 394 different locations were probed, respectively called ‘middle’ and ‘bevel’ (Fig. 8). The pressure drops
 395 obtained for the different CD thicknesses are plotted on Fig. 9.

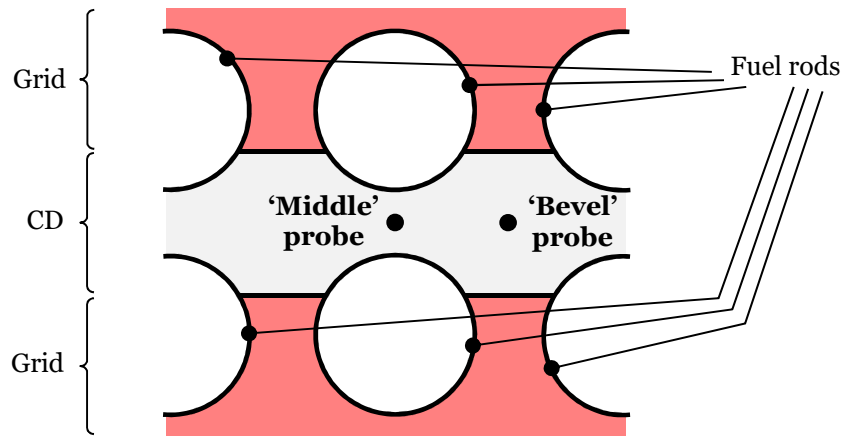


Figure 8 - Position of the probes of pressure drop

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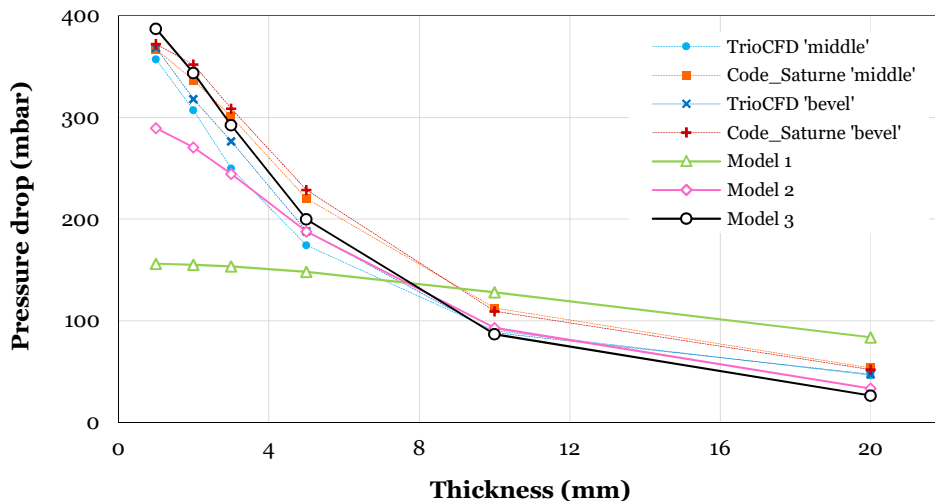


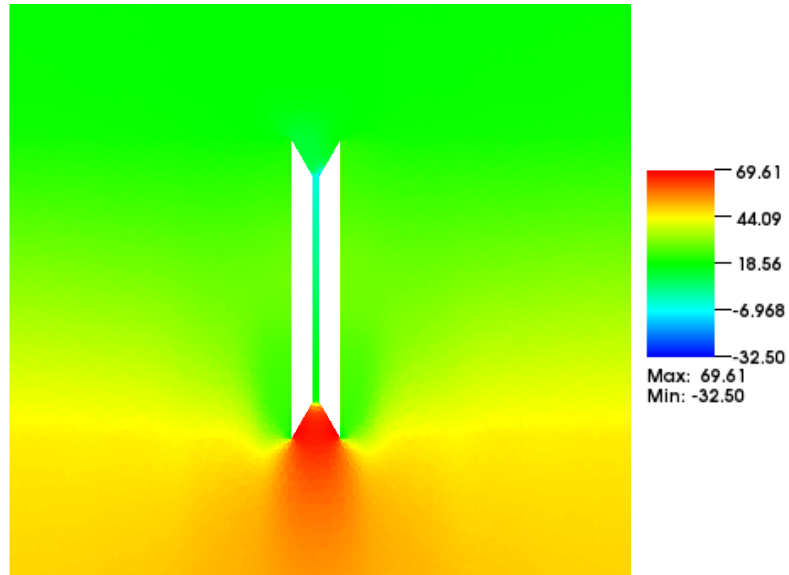
Figure 9 - Pressure drop as a function of the water gap thickness

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400 Both CFD codes found that the pressure drop is almost the same for the ‘middle’ probe and for the
 401 ‘bevel’ probe. This means that the pressure hardly varies in the spanwise direction.

402 Compared to CFD, Model 1 shows poor results below $\lambda = 6$ mm. As mentioned in section 3, this
 403 discrepancy is related to the stagnation point at the entry of the CD (clearly visible in Fig. 10 showing
 404 TrioCFD results in mid-plane for $\lambda = 1$ mm) but not taken into account in the model. Model 2 logically
 405 performs better, with a relatively good agreement with both CFD reference results, even though a 25%
 406 deviation could be underlined for the smallest values of λ . Finally, Model 3 yields excellent results for
 407 all values of λ between 1 and 20 mm, with the internal parameters given in section 6.3.

408 It is worth putting emphasis on the CPU cost associated to one data point of Fig. 10, reaching
 409 several days in the case of CFD versus a split second in the case of the semi-analytical Model 3 (for
 410 basically the same results).



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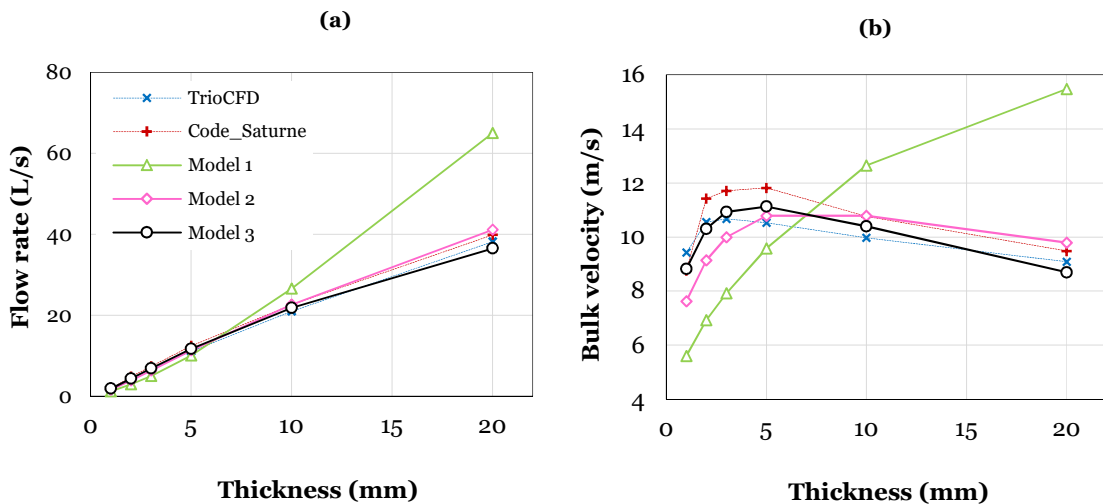
Figure 10 - Pressure peak underneath the CD for $\lambda = 1$ mm

413 **4.3 Flow rate and mean axial velocity**

414 In this section, flow rate and velocity in the water gap obtained with CFD are compared to those
415 given with the proposed semi-analytical models (see Fig. 11). Since the mesh for CFD simulations only
416 contains two rows of rods along the y-axis (see Fig. 7), the flow rate must be rescaled to match the grid
417 length L used in the models (see Fig. 2).

418 Fig 11a shows that Model 1 fails to reproduce the correct slope for the flow rate, but Models 2 and 3
419 give quantitatively accurate results. Model 3 stands out for its ability to stick to CFD curves even for
420 the smallest values of λ (below 3 mm).

421 The bulk velocity in the bypass is plotted on Fig. 11b. When λ increases starting from very small
422 values, a maximum is reached for $\lambda \approx 3$ mm and the bulk velocity decreases thereafter. Model 1 does
423 not reproduce this phenomenon as the bulk velocity keeps increasing with the bypass thickness. The
424 modification brought in Model 2 (branch coupling term for capturing stagnation points) is the key to
425 capturing the physics correctly. In this case, again Model 3 allows to further tune the solution for small
426 values of λ to get very close to the reference curves.

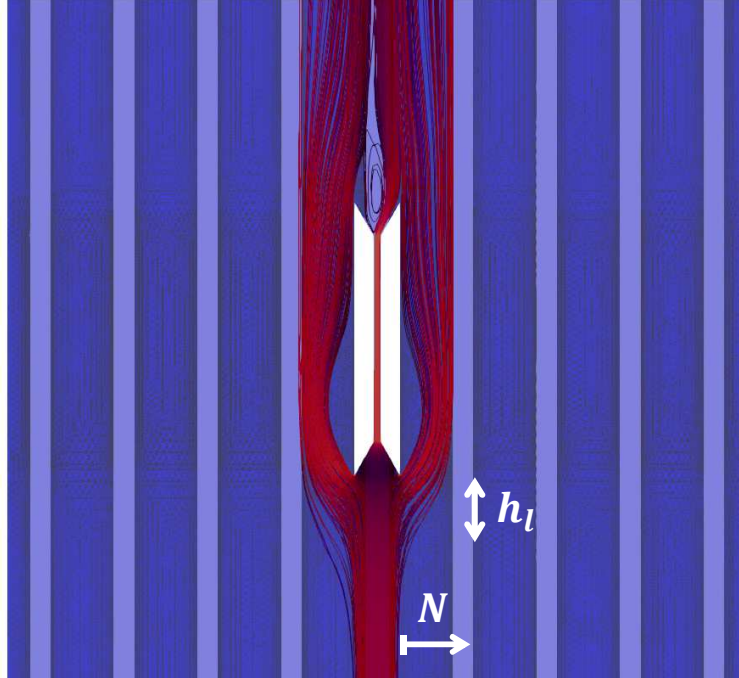


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Figure 11 - Flow rate (a) and axial mean velocity (b) as a function of the water gap thickness

429 **4.4 Advanced calibration for parameters of Model 3**

430 As mentioned in Section 3, Model 3 (*i.e.* integrating a resistance to the leaking flow Q_l) requires
 431 two empirical parameters to be set, namely the number of fuel rod rows N and the height h_l involved in
 432 the leak flow.



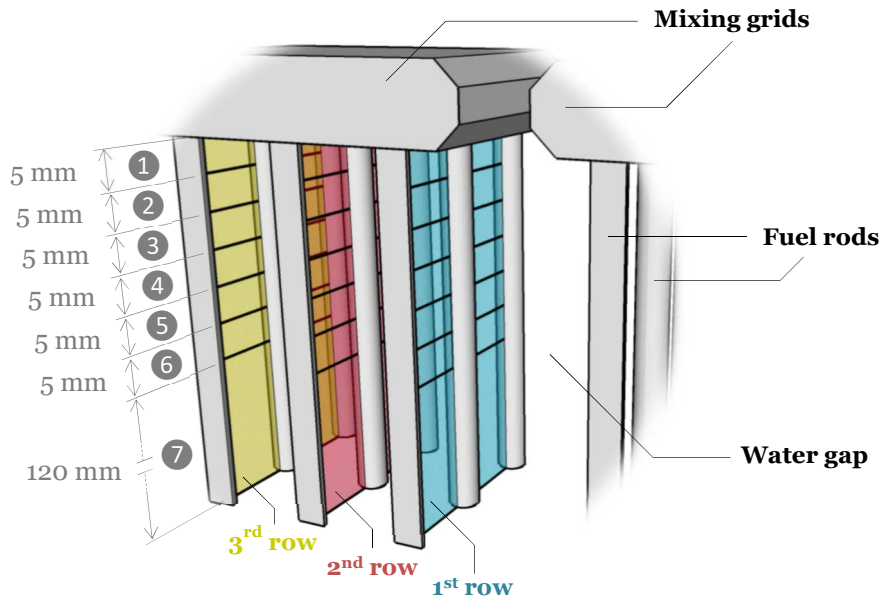
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Figure 12 - Streamlines in the water gap for $\lambda = 1$ mm

435 First qualitative insights regarding these parameters are given by the streamlines in the vicinity
 436 of the grid for the flow coming initially from the water gap inlet boundary. An example is provided in
 437 Fig. 12 for the streamlines obtained from the Code_Saturne simulation with a gap width $\lambda = 1$ mm.
 438 For this configuration, the redistribution (from the bypass to the grids) is initiated at a very small
 439 distance h_l upstream of the obstacle. Furthermore, the fluid enters only the first row of the rod bundle
 440 inside the grids, yielding $N = 1$ in this particular case.

441 A more generic and quantitative approach is to calculate the lateral flow rate for successive rows
 442 upstream of the grids. The flow rates are calculated the same way as Yan [Yan, 2014]: clip planes are
 443 created parallel to the water gap for the first three rows and each of them is cut into several sub-
 444 surfaces along the bundle direction to estimate the evolution of the computed flow rates (see Fig. 13 for
 445 the details). The first sub-surface is located between 0 and 5 mm upstream from the grids, and so on
 446 for the next 5 up to 30 mm upstream, with a 5 mm height each. The seventh and last sub-surface
 447 represents the lateral flow rate through the remaining geometry (between 30 mm and the 150 mm in
 448 the upstream direction).

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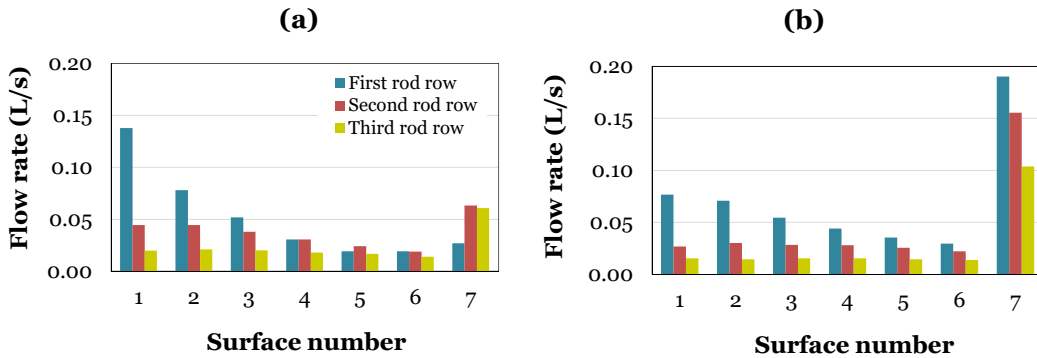


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Figure 13 – Clip planes used for post-processing of local lateral flow rates and subdivision into sub-surfaces

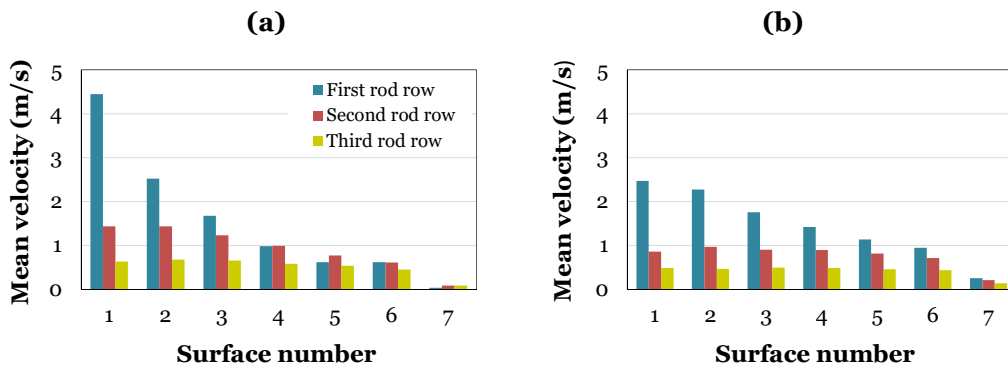
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This post-processing has been applied to the simulations performed with TrioCFD, for the two extremum values of CD's thicknesses (*i.e.* $\lambda = 1$ and $\lambda = 20$ mm) and the first three rows of fuel rods.



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Figure 14 - Lateral flow rates calculated for a water gap width of (a) 1 mm and (b) 20mm



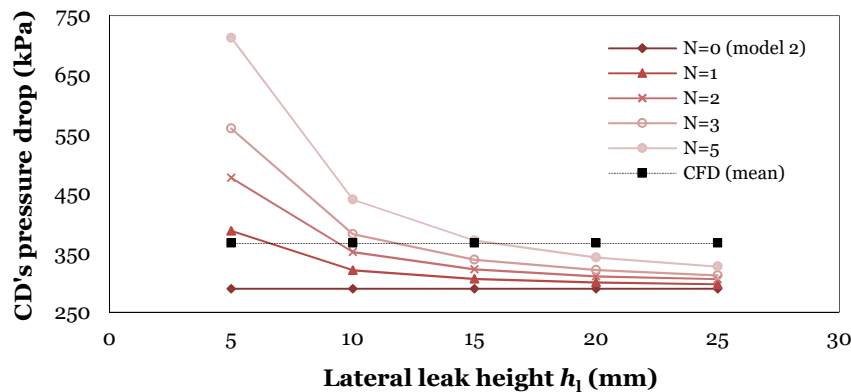
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Figure 15 - Lateral average velocity for for a water gap width of (a) 1 mm (b) 20 mm

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Fig. 14 and 15 show the resulting flow rates and lateral average velocities respectively for both considered values of the water gap width. Absolute values are displayed for the sake of clarity in the

460 comparisons, even though the fluid goes from the assembly to the bypass for the larger values of the
 461 water gap and in the opposite direction otherwise. The redistribution in the 1 mm-case (Fig. 14a and
 462 Fig. 15a) is strong near the grid (in agreement with what was observed on the streamlines of Fig. 12),
 463 and for the first row of fuel rods. Actually, 60% of the total lateral flow rate occurs through surfaces 1
 464 and 2 (*i.e.* within the first centimeter below the grid) and for the first rod row only. The lateral average
 465 velocity is also quite important within the first millimeters (almost the same order of magnitude as the
 466 axial velocity), but drops quickly under 1 m/s after only 1,5 cm. The 20 mm-case (Fig. 14b and Fig. 15b)
 467 shows smaller flow rates and they are more spread on all sub-surfaces. Only 30% of the total lateral
 468 flow rate is located within sub-surfaces 1 and 2 for the first row and the lateral velocity in the vicinity of
 469 the grid is again significantly larger for the first row of rods.



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Figure 16 - Pressure drop as a function of leak length (for several row numbers) when $\lambda = 1$ mm

472 To close this section, it is noticeable that parameters h_1 and N cannot be chosen independently.
 473 Fig. 16 plots the pressure drops in the CD determined by Model 3 for many different combinations of
 474 leak height h_1 and number of rows N , as well as the reference value computed by CFD: several couples
 475 of values provide a good match, for instance ($h_1 = 5$ mm ; $N = 1$), ($h_1 = 11$ mm ; $N = 3$) or ($h_1 =$
 476 15 mm ; $N = 5$). The first one is probably the most relevant for small widths λ , while the second one
 477 best describes the configurations where λ is large. However, it is still possible to keep constant values
 478 for h_1 and N for all values of λ . The sensitivity analysis in Section 6 will help to assess the influence of
 479 these choices on the results produced by the semi-analytical models.

480 5 COMPARISON WITH DEDICATED EXPERIMENTAL RESULTS

481 5.1 Experimental mock-up

482 In order to check that the previous semi-analytical models provide physically relevant results (in
 483 terms of pressure drop and flow rate distribution between the grids and the CD), a specific
 484 experimental setup was designed, named DIVA+G. The detailed description of the tests goes beyond
 485 the scope of the current article and will be subject to its own publication process. Two half grids were
 486 3Dprinted in plastic material. They consist of a thick plate filled with numerous holes arranged in a
 487 triangular pattern, allowing a controlled pressure drop while maintaining flow conditions as
 488 homogeneous as possible. The space between the half-grids uses the same simplified geometry for the
 489 water gap as the theoretical model described in Section 3 (Fig. 2). Several shims of various thicknesses
 490 allow modifying the distance between the half-grids (*i.e.* the water gap). All these elements are placed
 491 in the test section of an hydraulic loop (Fig. 17).

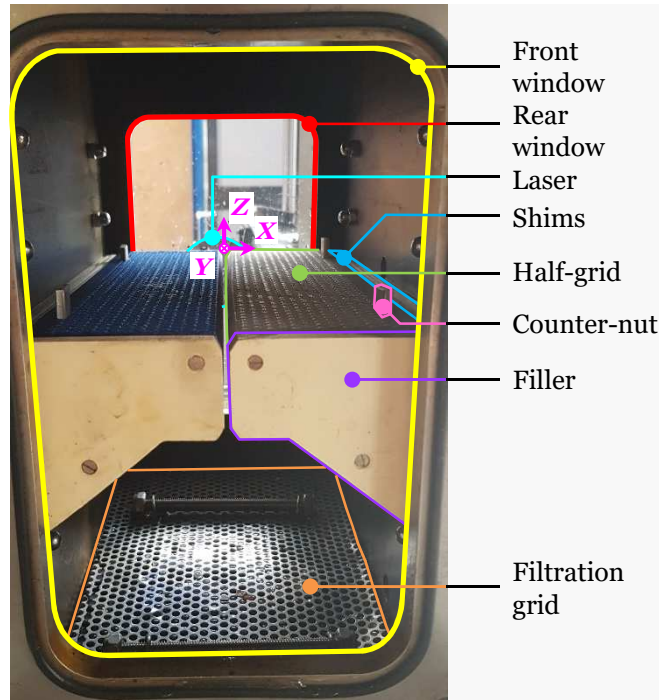


Figure 17 – The DIVA+G mock-up

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494 Two differential pressure probes are located on a side wall at the base of one of the grids, above
 495 and below it. The local mean axial velocity is measured thanks to Laser Doppler Velocimetry. Between
 496 60 and 340 locations are probed, according to the water gap width. This allows determining accurately
 497 the flow rate through the gap, and therefore the flow rate within the grids by difference with the total
 498 flow rate.

499 5.2 Parameters of models 1 and 2 for DIVA+G

500 In DIVA+G, metallic shims are successively added aside to put grids closer. As a result, the ratio
 501 between flow cross-sections inside the grids and at the inlet decreases as the water gap goes thinner.
 502 Using the correlation in [Idel’Cik, 1986] for thickened grids, we are able to match the pressure drop
 503 measurement when the gap is closed ($\lambda = 0$). By extrapolation, it is possible to express the coefficient
 504 of local resistance C_g (defined in Eq. 8) with the following relation (Eq. 25):

$$C_g(\lambda) = 4.23 - 0.06\lambda \quad (\text{Eq. 25})$$

505 Here λ stands for the average width of the water gap, in millimeters. Likewise, the flow cross-
 506 section upstream of one half-grid S_g can be evaluated thanks to the following equation (Eq. 26):

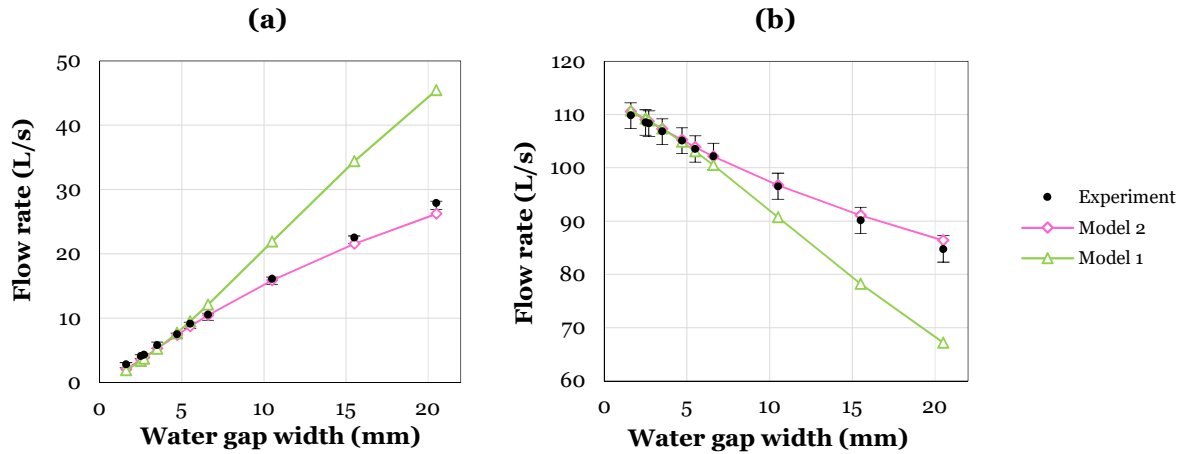
$$S_g(\lambda) = \frac{1}{2}(S_0 - L\lambda) \quad (\text{Eq. 26})$$

507 where S_0 is the mock-up’s cross-section (see Fig. 17). As mentioned above in the article, the
 508 pressure loss associated to crossing the grids is thus given by (Eq. 27):

$$\Delta P_g = K_g(\lambda)Q_g^2 = \frac{1}{2}\rho C_g(\lambda)\frac{Q_g^2}{S_g(\lambda)^2} \quad (\text{Eq. 27})$$

509 Those functions are used to compute the solution for both Model 1 and Model 2. Model 3 is
 510 obviously out-of-scope in this section since no rods are included in the DIVA+G mock-up.

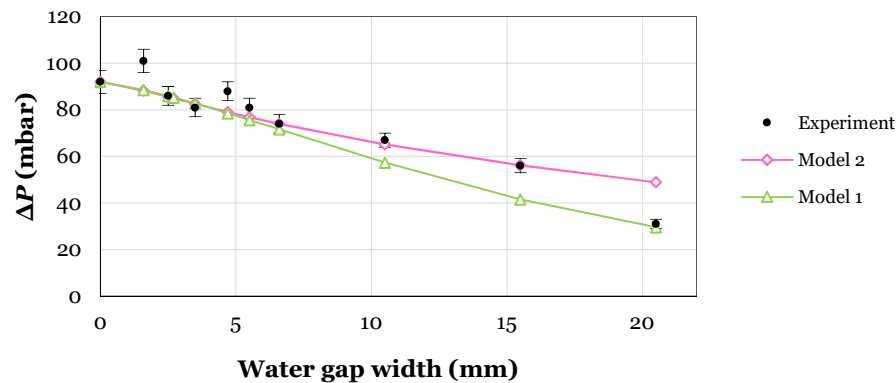
511 5.3 Comparison to DIVA+G results



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Figure 18 – Models vs DIVA+G: flow rates in (a) the water gap and (b) the grids (uncertainty ranges are plotted over experimental points)

515 Fig. 18 represents the volumetric flow rate in the CD (a) and in the grids (b) as a function of λ .
516 Model 2 is in very good agreement with experiments, whereas Model 1 shows significant discrepancies
517 for larger gaps, confirming the importance of the stagnation point modeling to account for the
518 heterogeneity of the pressure field.



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Figure 19 - Models vs DIVA+G: pressure drop, absolute uncertainties are indicated over measurements' points

521 Finally, Fig. 19 presents the pressure drop through the grids. In DIVA+G, this value is obtained
522 through the two pressure sensors placed on both sides of the right half-grid (see Fig. 17). Experimental
523 measurements are here subjected to additional uncertainties probably due to non uniform operating
524 conditions in the loop over the complete test sequence stretching over one week approximately (see for
525 instance the points for values of λ of 1.6 mm, 4.5 mm and 20 mm). However, both Models 1 and 2
526 accurately reproduce the decrease of the pressure drop through the grid with the opening of the water
527 gap.

528 As a conclusion, the very good match between Model 2 and DIVA+G experimental data (in terms
529 of pressure drop and flow rates) is remarkable, all the more so as this version of the semi-analytical
530 model needs no empirical parameter at all. It shows that the physics of the redistribution upstream of
531 the grids is correctly understood and well captured by our semi-analytical models.

532 6 SENSITIVITY ANALYSIS OF THE MODELS

533 This last section provides some early work illustrating the potential of the proposed semi-
 534 analytical approach in terms of advanced sensitivity analysis, made possible by the balance between
 535 accuracy for the phenomena of interest and computational efficiency. Two case studies are considered:
 536 first, the sensitivity of Model 2 results to variations of its internal parameters together with some of the
 537 DIVA+G experimental inputs, and second, a deeper dive into the inner workings of the ΔP_c and ΔP_l
 538 corrective terms resulting from the stagnation point modeling, through compared sensitivity analyses
 539 of Model 2 and Model 3.

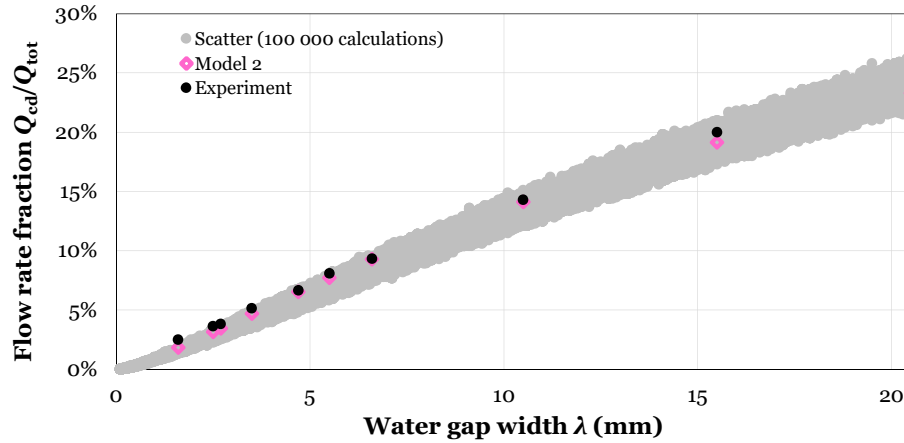
540 6.1 Model 2 (applied to the DIVA+G experiment)

541 In order to analyze the sensitivity of Model 2 and the experiment's parameters, 100 000
 542 calculations were performed thanks to CEA's Uranie uncertainty platform [Gaudier, 2010] with
 543 different values of input parameters (see Section 3) to estimate the Sobol Indexes [Jacques,
 544 2011][Iooss, 2011]. The sampling is operated by a Latin hypercube method [McKay, 1979]. The
 545 different inputs are drawn from uniform distributions whose bounds are detailed in Table 1. In Section
 546 5, it was explained why the grid coefficient C_g of the DIVA+G mock-up was a linear function of the
 547 water gap width λ . However, in order to make the sensitivity analysis simpler [Iooss, 2011], we act as if
 548 these two variables were uncorrelated.

Input	Minimum	Maximum	Comments
λ	0.1 mm	20.5 mm	
H, h, α	-10%	+10%	(Λ/λ) thus varies up to $\pm 20\%$ for $\lambda = 1$ mm and $\pm 5\%$ for $\lambda = 20$ mm (this ratio plays a major role on the flow redistribution, see Eq. 7)
ζ_1	0.1	0.6	+20% upper margin w.r.t. the sudden contraction ($\zeta_1 = 0.5$ when $\alpha = 90^\circ$)
ζ_3	1	1.3	$\zeta_3 = 1$ is for a sudden expansion ($\alpha = 90^\circ$), $\zeta_3 = 1.15$ the nominal value
C_g	3.00	4.23	C_g depends on λ (see section 3)
Q_{tot}	-10%	+10%	
ρ	997 kg.m ⁻³	1000 kg.m ⁻³	Temperature variation between 5°C and 25°C
μ	8×10^{-4} Pa.s	1.5×10^{-4} Pa.s	Temperature variation between 5°C and 25°C
ε	1 μ m	100 μ m	$\varepsilon = 1$ μ m is typical steel, $\varepsilon = 100$ μ m is an upper bound for 3D-printed PETG (ε is the wall rugosity, appearing in Haaland formula for f [Asker, 2014])
$S_0 = L \times W$	-1 mm \times -1 mm	+1 mm \times +1mm	

549 *Table 1 - Inputs for sensitivity analysis of model 2 in the context of DIVA+G experiments*

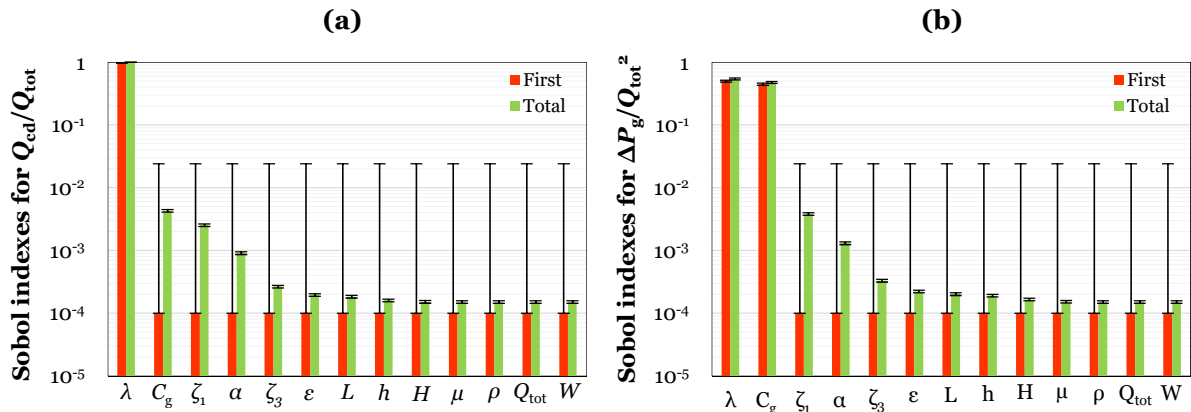
550 The chosen outputs of interest are the volumetric flow rate fraction going through the water gap,
 551 *i.e.* Q_{cd}/Q_{tot} , as well as the pressure drop divided by the square of total flow rate, *i.e.* $\Delta P_g/Q_{tot}^2$. Such
 552 nondimensionalization prevents any undesirable variation of Q_{cd} and Δp_g which would be imputed
 553 solely to an increase or decrease of the total flow rate Q_{tot} going into the system.



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Figure 20 – Flow rate in the CD computed with perturbed input parameters

557 Fig. 20 represents the nondimensional flow rate in the CD, for both experimental, initial Model 2
558 calculation (with nominal values), and the 100 000 calculations results. It highlights that whatever the
559 draw, the maximal relative deviation towards experimentation is around 20%. Keeping in mind that
560 every single possible input variable has been considered and that they have been drawn uniformly in
561 the whole range of admissible values, this is a very satisfactory result. First, it shows that a rational
562 change in the models' parameters will keep the solution (in terms of flow rate) in an acceptable range.
563 Second, the distribution of the scattered results around the experimental values shows some
564 robustness regarding the measurement errors. Finally, the shape of the scatter plot suggests that the
565 flow rate is mostly a function of the thickness.



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Figure 21 – Sobol indexes and 95% confidence intervals for Q_{cd}/Q_{tot} (a) and $\Delta P_g/Q_{tot}^2$ (b) obtained with 100 000 calculations with Model 2 in the context of DIVA+G experiments

569 To explicitly quantify the influence of the modeling parameters on the variation of Q_{cd}/Q_{tot} , first
570 and total Sobol ([Jacques, 2011] or [Jacques, 2006] in English) are computed and displayed in Fig.
571 21a. The first index associated to the thickness λ is indeed around 1. It means as anticipated in Fig. 20
572 that the output can be considered in first approximation as a function of λ only, neglecting some small
573 interactions with other inputs. Confidence intervals of the other inputs for the first index are too large
574 to be discussed. However, according to the total indexes (whose confidence intervals are small enough
575 to allow interpretation), one could see that C_g , ζ_1 and α are the next three more influential inputs on
576 the prescribed ranges, after the thickness λ . There is no way to tell if those parameters step in alone or
577 through their interactions, but their total effect stands out from the others. More specifically, caution

578 must be exercised with regards to C_g , as it was considered independent of λ to compute the Sobol
 579 indexes, whereas analyses from DIVA+G experimental campaign clearly concluded that these two
 580 parameters were strongly related (as such they should be considered as a same group of inputs
 581 [Jacques, 2011]). Further correlation analyses could be performed to highlight this topic, but this goes
 582 beyond the scope of the current paper.

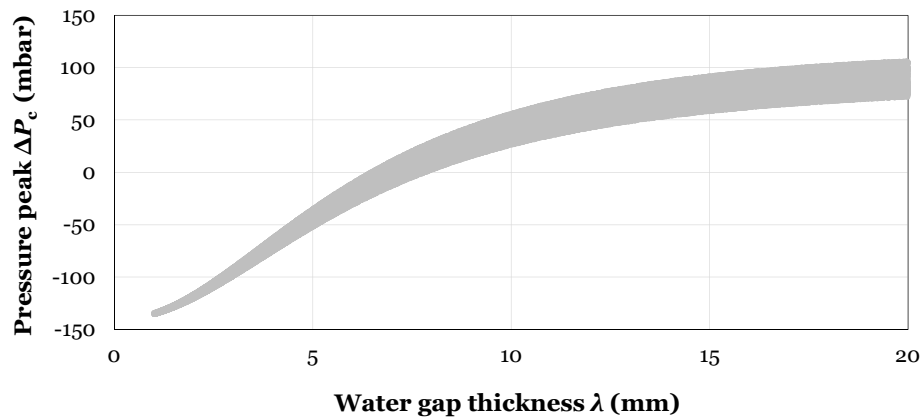
583 Fig. 21b indicates the Sobol indexes calculated for the output $\Delta P_g/Q_{tot}^2$. The remarks pointed out
 584 above are still relevant, and the four more influential inputs are the same. The difference is that the
 585 effect of C_g is much more important in this case (see the total and first indexes) because the pressure
 586 drop is directly proportional to the loss coefficient.

587 **6.2 Model 3 (and comparison to the simpler Model 2)**

588 The common distinctive feature of Model 2 and Model 3 is held in the ΔP_c corrective term, and
 589 Model 3 then stands out from Model 2 for the additional parameters introduced by the lateral
 590 resistance K_l . It is interesting to analyze the sensitivity of $\Delta P_c = P_{cd} - P_g$ to different values of new
 591 inputs associated with K_l and to compare with Model 2's conclusions discussed above. Building on the
 592 results of Fig. 21, every parameter whose influence seemed negligible are now considered as constants.
 593 Simulations are otherwise analog to the ones performed in section 5. For Model 3, the inputs of the
 594 sensitivity analysis are listed in Table 2.

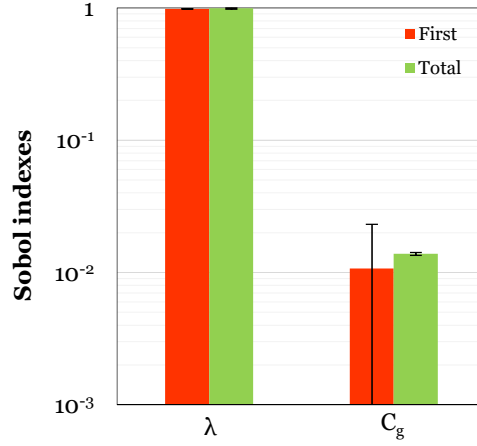
Input	Minimum	Maximum	Comments
$\lambda^{(*)}$	1 mm	20 mm	
$C_g^{(*)}$	0.9	1.3	
μ	10^{-5} Pa.s	10^{-4} Pa.s	Used to calculate the local Reynolds number in K_l (Eq. 22)
N	1	3	See Fig. 12
h_l	1 mm	100 mm	Linear probability density function (instead of uniform) in order to promote low values of h_l (see Fig. 12)

595 Table 2 - Inputs for sensitivity analysis of Model 2 (* only) and 3 in the context of real PWR fuel assemblies



596 Figure 22 – Pressure peak term ΔP_c obtained by Model 2 when varying λ and C_g only
 597

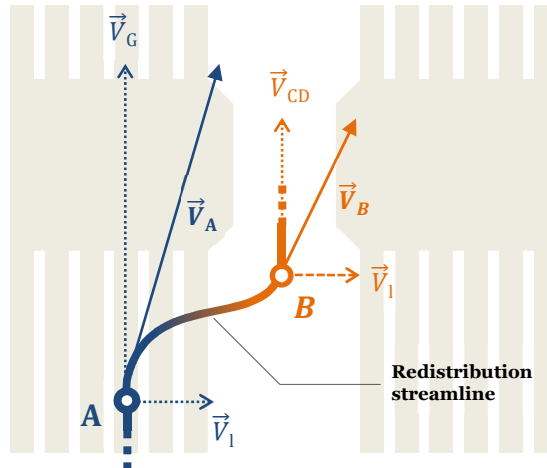
598 Fig. 22 depicts the variance of ΔP_c obtained with Model 2 when varying λ and C_g only. One can
 599 notice that ΔP_c is either positive or negative according to the flow rate value in each branch. Thus, zero
 600 values obtained for λ around 7 mm correspond to equal velocities through the grid and the water gap
 601 (as shown by Eq. 13), which means a uniform flow. The dispersion induced by C_g clearly increases as
 602 the thickness λ grows larger. The Sobol indexes associated to this sensitivity analysis are given in Fig.
 603 23.



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Figure 23 – Sobol indexes and 95% confidence intervals for ΔP_c obtained with 100 000 calculations with Model 2 in the context of real PWR fuel assemblies

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Figure 24 - Streamline upstream from the CD with fuel rods

610 In the case of Model 3, the quantity of interest is $\Delta P_{\text{tot}} = \Delta P_c + \Delta P_1$. Indeed, two effects are now are
611 involved in the upstream pressure drop. This can be shown by applying the generalized Bernoulli
612 principle on a streamline following the flow redistribution (Fig. 24) for instance from the grid (point A,
613 velocity $\vec{V}_A = \vec{V}_g + \vec{V}_1$ and pressure $P_A = P_g$) to the water gap (point B, velocity $\vec{V}_B = \vec{V}_{cd} + \vec{V}_1$ and pressure
614 $P_B = P_{cd}$). The sign convention for ΔP_1 comes from Eq. 24:

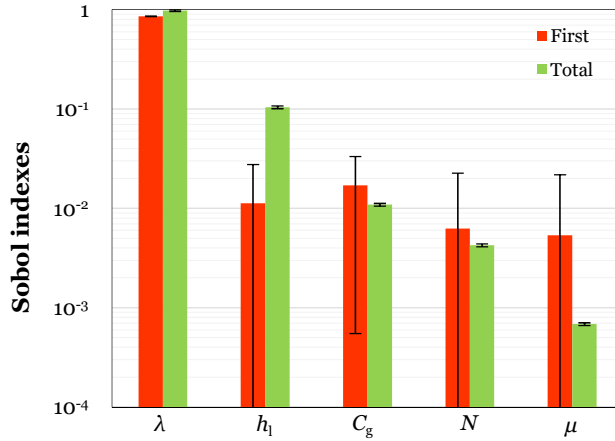
$$P_A + \frac{1}{2} \rho V_A^2 = P_B + \frac{1}{2} \rho V_B^2 - \Delta P_1 \quad (\text{Eq. 28})$$

615 which leads to the following formula for the total pressure drop:

$$\Delta P_{AB} = P_{cd} - P_g = \frac{1}{2} \rho (V_g^2 - V_{cd}^2) + \Delta P_1 = \Delta P_{\text{tot}} \quad (\text{Eq. 29})$$

616 (to be compared to Eq. 13 which gives $P_{cd} - P_g = \Delta P_c$ in the case of Model 2).

617

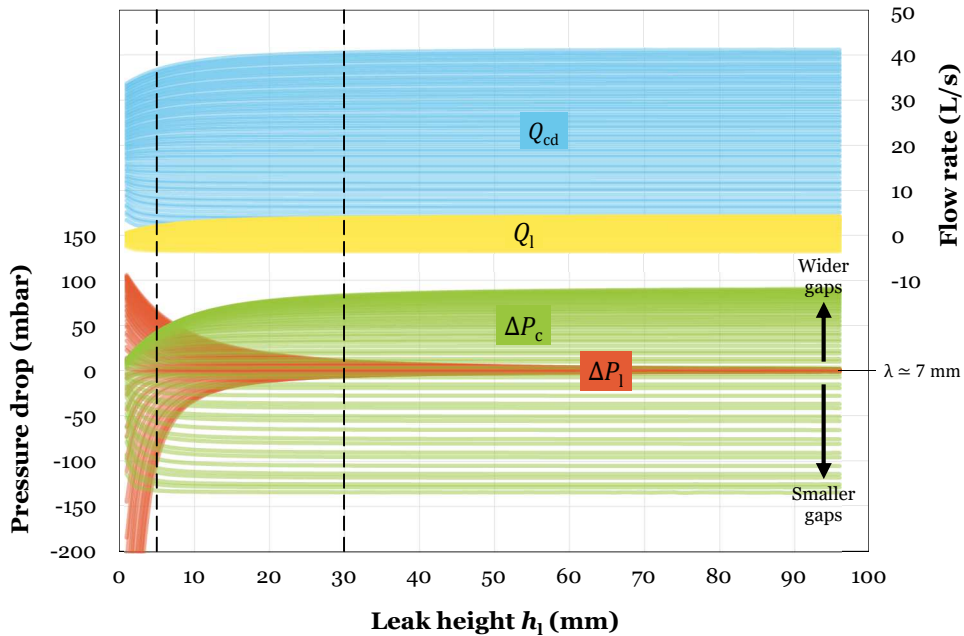


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Figure 25 - Sobol indexes and 95% confidence intervals for ΔP_{tot} obtained with 100 000 calculations with Model 3 in the context of real PWR fuel assemblies

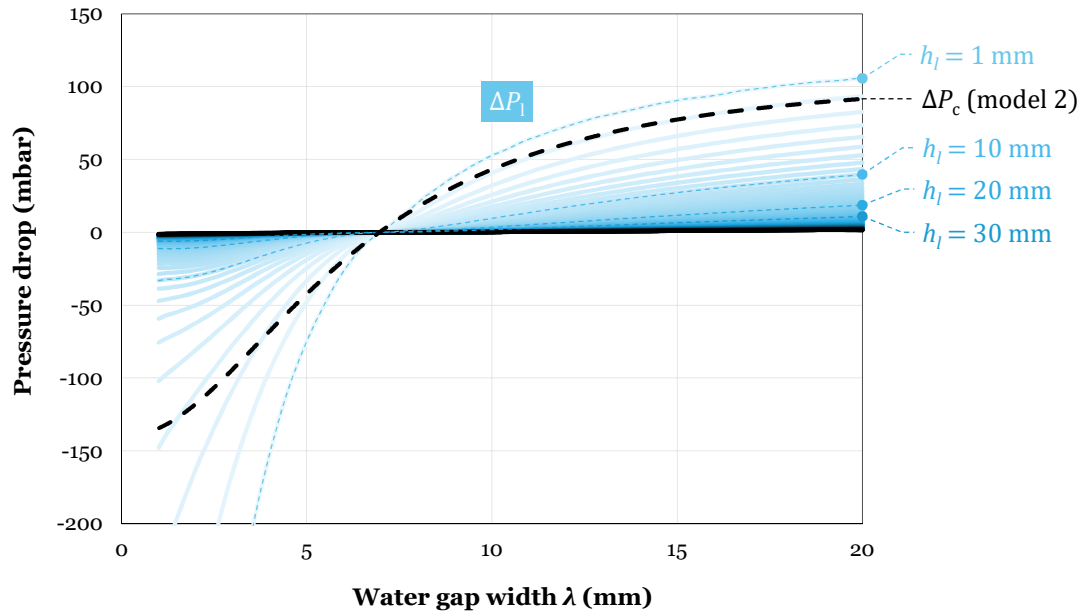
621 Fig. 25 highlights Sobol indexes obtained for ΔP_{tot} . Once again, λ has by far the highest effect
622 among all inputs. But in this new situation, the sum of all first indexes is not equal to 1, meaning that
623 correlations exist between inputs. From h_1 indexes, it can be inferred that this input is unimportant
624 alone but has a significant impact through interactions, most probably with λ .

625 In order to grasp a better understanding of the roles of λ and h_1 , the same calculations have been
626 performed considering all the other inputs as constants with nominal values. Quantities of interest
627 such as ΔP_c and ΔP_{tot} can therefore be visualized with smooth curves (instead of fuzzy scatter plots like
628 in Fig. 22 for instance). Fig. 26 shows the evolution of Model 3 outputs along with h_1 (for different
629 values of λ) whereas Fig. 27 and 28 show their evolution along with λ (for different values of h_1).



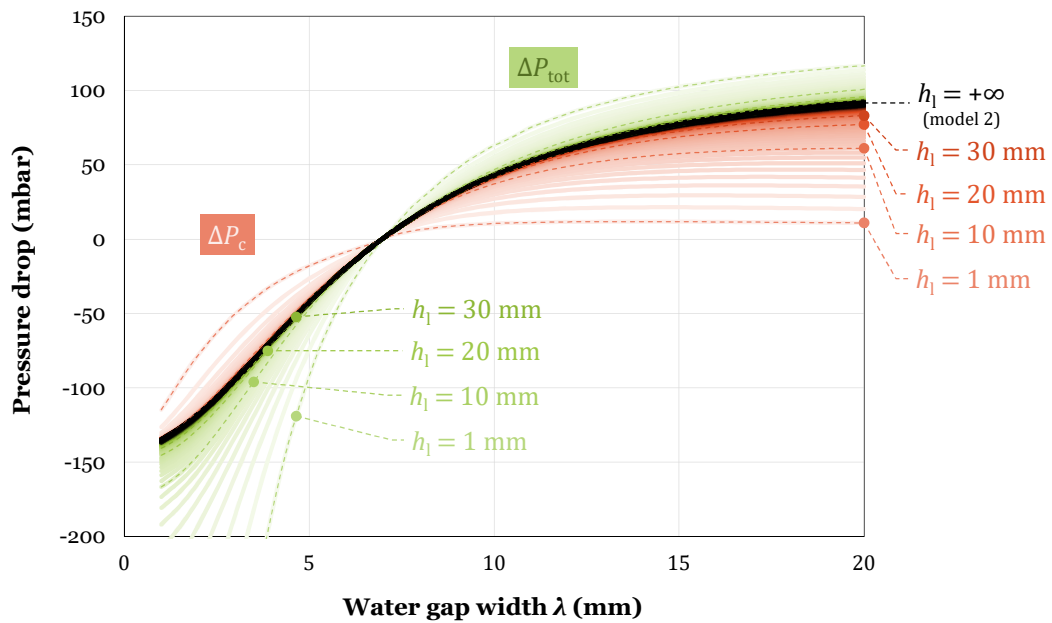
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Figure 26 - Evolution of the flow rates and pressure drops along with h_1 for different values of λ



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Figure 27 - Evolution of the lateral pressure drop along with λ for different values of h_1



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Figure 28 - Evolution of the upstream pressure drop along with λ for different values of h_1

636 One first observation is the presence of a singular point when both λ and h_1 approach zero, for
 637 which the upstream pressure drop ΔP_{tot} associated to the redistribution becomes potentially infinite.
 638 Actually, the singular term is rather ΔP_1 . It makes sense because that singular point correspond to a
 639 situation where almost all the incoming flow would be forced towards the grids ($\lambda \approx 0$) by going
 640 through an extremely thin horizontal slit ($h_1 \approx 0$). Hence, care must be taken so as not to choose too
 641 small a value for the h_1 parameter. We suggest to keep h_1 above 5 mm at the very least.

642 On the contrary, when h_1 grows larger, Fig. 26 and 27 show that ΔP_1 approach zero whatever the
 643 value of λ . In other words, Model 3 asymptotically tends towards Model 2 when h_1 becomes infinite.

644 Actually, it is possible to observe on Fig. 26 that the evolutions of flow rates and pressure drops
645 become somewhat unaffected by increasing h_1 beyond 3 cm.

646 The *loci* where pressure drop curves of Fig. 27 and 28 cross the horizontal axis is almost a single
647 point near $\lambda \simeq 7$ mm. When the water gap has this width, the axial flow profile is flat ($V_{cd} = V_g$, with no
648 cross-flow). Since it hardly depends on h_1 at all, even Model 2 or a poorly calibrated Model 3 would be
649 able to predict this equilibrium value accurately.

650 For intermediate values of h_1 (in the “reasonable range” between 5 mm and 3 cm), one can wonder
651 how the redistribution term of Model 2, namely ΔP_c , blends in the one of Model 3, namely $\Delta P_{tot} =$
652 $\Delta P_c + \Delta P_1$. For h_1 between 5 and 30 mm, Fig. 27 shows that the additional term ΔP_1 increases (in
653 absolute value) when h_1 gets smaller, staying below the asymptotic value of ΔP_c curve for Model 2 even
654 so. Conversely, Fig. 28 shows that ΔP_c decreases (in absolute value) along with h_1 , but not enough to
655 compensate for the additional upstream pressure drop brought by ΔP_1 . Everything boils down in the end
656 to the total pressure drop ΔP_{tot} being slightly increased (in absolute value) when the length h_1 becomes
657 smaller. One worthy observation is that the flow rate Q_{cd} in the water gap shows little dependence on
658 h_1 . These results are the very purpose of Model 3 enhancements versus Model 2, aimed at tweaking the
659 pressure axial profile without jeopardizing the good prediction of the flow rate redistribution, which
660 was already validated for Model 2 by comparison to DIVA+G experimental data.

661 CONCLUSION AND OPEN PROSPECTS

662 In this article, a series of different 1D hydraulic models of growing complexity were introduced to
663 reproduce the flow redistribution upstream from the grids for two fuel assemblies separated by a water
664 gap. All models are based on hydraulic networks, starting from the simple association of two pathways:
665 one for the bypass, and one for the grids. The first model does not take into account the pressure peak
666 which appears when the bypass is being closed, yielding a second model containing a modified energy
667 equation to consider the effects of the identified stagnation point. Finally, a third model adds the
668 lateral hydraulic resistance of the fuel rods, to further increase the accuracy of the network for small
669 water gaps.

670 Although Model 1 turns out to be far too simplistic, both Model 2 and Model 3 are well-suited for
671 reproducing the CFD results obtained with TrioCFD and Code_Saturne, with excellent agreement
672 achieved by Model 3 even in the case of very narrow bypasses. Models 1 and 2 were also compared to a
673 dedicated experiment called DIVA+G consisting in two 3D-printed porous grids facing each other,
674 between which is located a bypass of variable thickness. Model 3 was not relevant in this case as there
675 is no rod bundle in the mock-up. Model 2 accurately predicted flow rates and pressure losses measured
676 in DIVA+G, validating the global strategy proposed to account for the flow redistribution upstream
677 from the grids. As regards the improvements brought in Model 3 to account for the lateral obstruction
678 due to the fuel rods, sensitivity analyses suggest that a particular attention should be paid to the choice
679 of the h_1 parameter, due to its significant influence on the pressure drop. Relying on CFD simulations
680 is recommended to assess its value, with some insights given in the present paper. Dedicated LES
681 calculations are considered in the future to consolidate the results of the proposed k- ϵ validation cases.

682 The next step of this work consists in building a model for the lateral forces acting on the fuel
683 assembly resulting from the computed flow rates and pressures. This will be followed by the
684 construction of extended models involving hydraulic networks and fluid-structure interaction, first to
685 represent a 2D row of assemblies separated by water gaps, and second to test a 3D arrangement of
686 assemblies close to a full PWR core. These prospects are supported by the capability of the proposed

687 approach to carry out extended sensitivity analyses thanks to the computational efficiency of the semi-
688 analytical expression of the hydraulic networks. The final objective lies in identifying relevant patterns
689 related to flow redistribution with fluid-structure interaction, so as to draw some guidelines to improve
690 existing models involving classical porous approximations.

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