

# A dynamical model of toxic cyanobacteria growth in the case of lake eutrophication

Auguste Caen<sup>1</sup>

Jean-Denis Mathias<sup>1</sup>

Delphine Latour<sup>2</sup>

<sup>1</sup>IRSTEA, LISC

<sup>2</sup>UBP, LMGE

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# Introduction

## Cyanobacteria

- proliferation in the aquatic environment
- caused by eutrophication and high temperature



# Introduction

## Cyanobacteria

- seasonal blooms
  - ⇒ disrupts the ecology
  - ⇒ prohibits any economic exploitation



# Introduction

## Cyanobacteria

- toxicity  
⇒ health hazard

- *Anabaena*  
→ neurotoxins
- *Microcystis*  
→ neurotoxins and hepatotoxins
- *Aphanizomenon*  
→ neurotoxins
- etc.



# Introduction

## Cyanobacteria

Within the same species, two different populations :

- the **toxic population**
- the **non-toxic population**

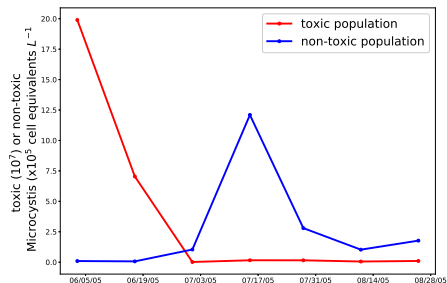


FIGURE – Toxic and non-toxic Microcystis in Lake Ronkoma, 2005 (data from *Davis et al., 2009*)



# Introduction

## Cyanobacteria

Within the same species, two different populations :

- the toxic population → blooms at the beginning and/or end of the summer
- the non-toxic population → blooms in the middle of the summer

⇒ **two different trajectories**

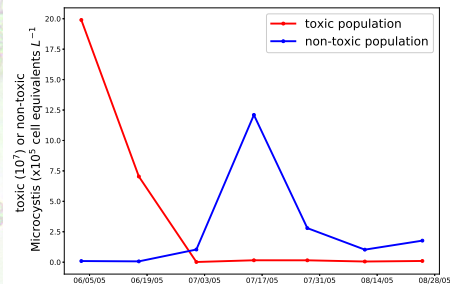


FIGURE – Toxic and non-toxic Microcystis in Lake Ronkoma, 2005 (data from *Davis et al., 2009*)



# Introduction

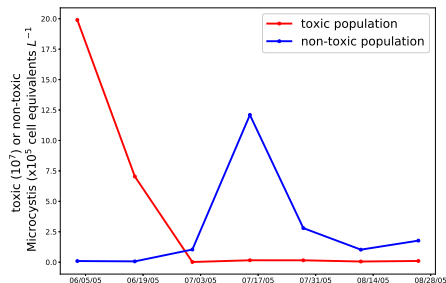
## Cyanobacteria

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**modelling toxic and non-toxic cyanobacteria populations : an economic, ecological and health issue**



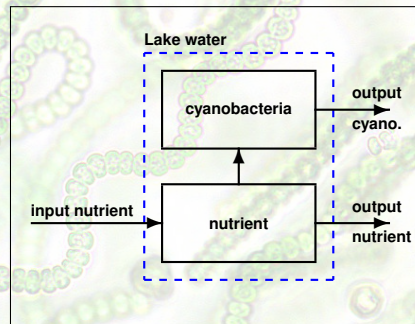
**FIGURE** – Toxic and non-toxic Microcystis in Lake Ronkoma, 2005 (data from *Davis et al., 2009*)



# Introduction

## Cyanobacteria

a current dynamic model of the bacteria population : the chemostat model.



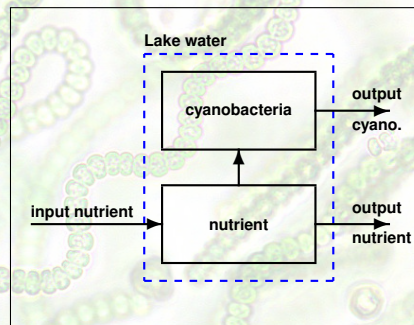


# Introduction

## Cyanobacteria

a current dynamic model of the bacteria population : the chemostat model.

- the dynamics of a population coupled with the dynamics of a resource

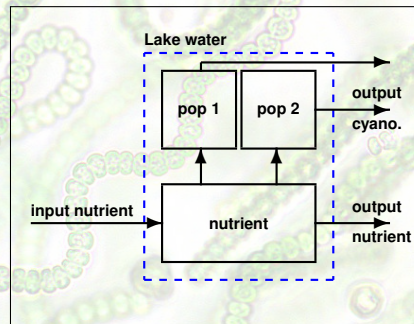


# Introduction

## Cyanobacteria

a current dynamic model of the bacteria population : the chemostat model.

- the dynamics of a population coupled with the dynamics of a resource
- Two populations

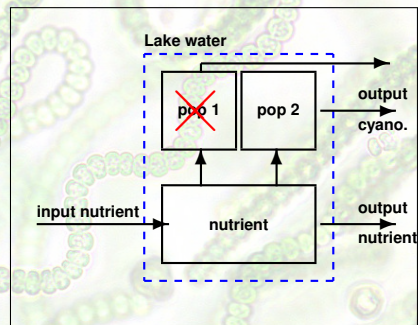


# Introduction

## Cyanobacteria

a current dynamic model of the bacteria population : the chemostat model.

- the dynamics of a population coupled with the dynamics of a resource
- Two populations  $\Rightarrow$  one survival population : **no coexistence** (*Sari and Mazenc, 2011*)

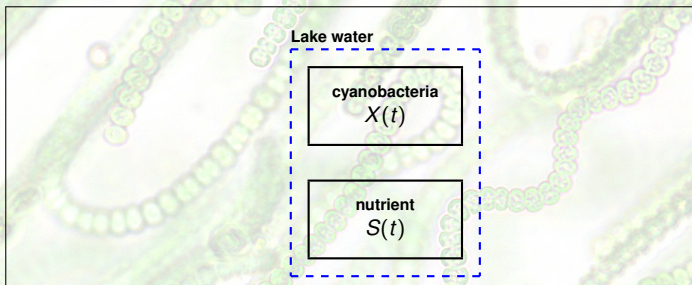


## Introduction

- A biological issue : What are the factors favouring one or the other population ?
- A modelling issue : How to model the coexistence ?



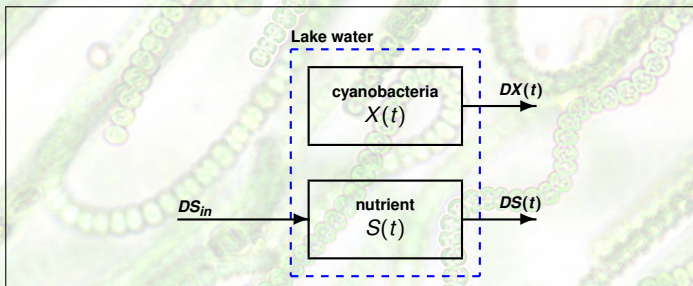
## A dynamical model



The chemostat model with temperature :

$$\frac{dX(t)}{dt}$$
$$\frac{dS(t)}{dt}$$

## A dynamical model

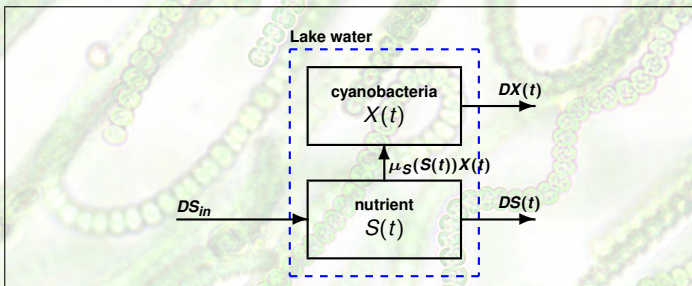


The chemostat model with temperature :

$$\frac{dX(t)}{dt} = -DX(t)$$

$$\frac{dS(t)}{dt} = D(S_{in} - S(t))$$

## A dynamical model

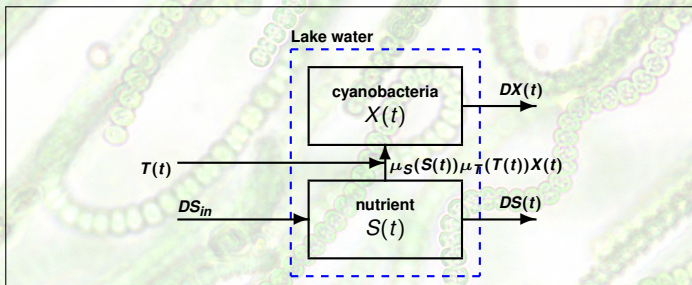


The chemostat model with temperature :

$$\frac{dX(t)}{dt} = \mu_S(S(t))X(t) - DX(t)$$

$$\frac{dS(t)}{dt} = D(S_{in} - S(t)) - \mu_S(S(t))X(t)$$

## A dynamical model



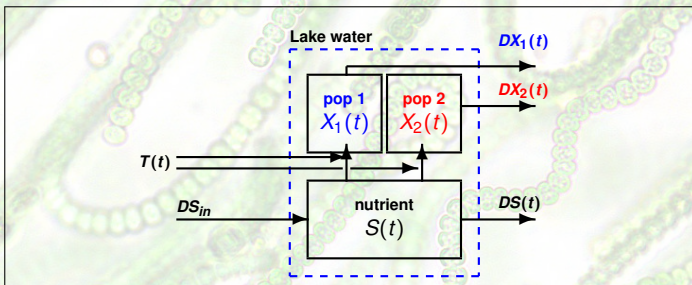
The chemostat model with temperature :

$$\frac{dX(t)}{dt} = \mu_T(T(t))\mu_S(S(t))X(t) - DX(t)$$

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## A dynamical model



The chemostat model with temperature :

$$\frac{dX_1(t)}{dt} = \mu_T^1(T(t))\mu_S^1(S(t))X_1(t) - DX_1(t)$$

$$\frac{dX_2(t)}{dt} = \mu_T^2(T(t))\mu_S^2(S(t))X_2(t) - DX_2(t)$$

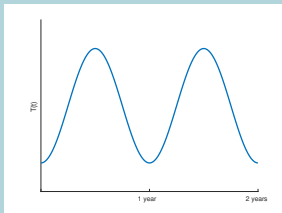
$$\frac{dS(t)}{dt} = D(S_{in} - S(t)) - \mu_T^1(T(t))\mu_S^1(S(t))X_1(t) - \mu_T^2(T(t))\mu_S^2(S(t))X_2(t)$$



# The temperature

The temperature :

A periodic function



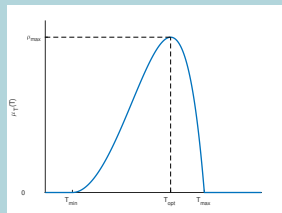
$\mu_T^i$  Grimaud et al. 2015 :

only one parameter :  $T_{opt}^i$

$$T_{min}^i = f_{T_{min}}(T_{opt}^i)$$

$$T_{max}^i = f_{T_{max}}(T_{opt}^i)$$

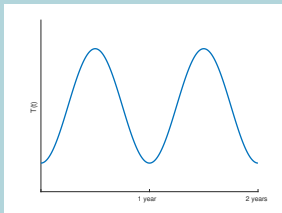
$$\mu_{max}^i = f_{\mu_{max}}(T_{opt}^i)$$



# The temperature

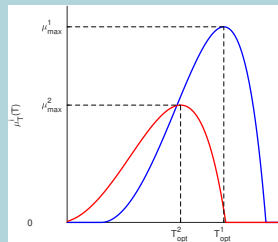
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$\mu_T^i$  Grimaud et al. 2015 :

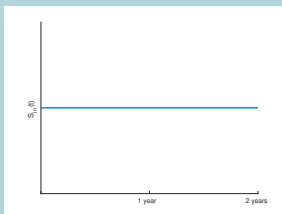
$$T_{opt}^1 \neq T_{opt}^2$$



# The nutrient

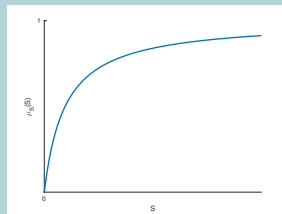
The nutrient :

A constant input



$\mu_S^i$  (The Monod model) :

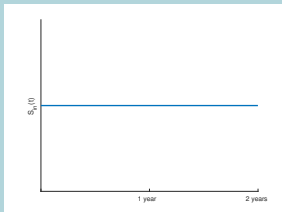
only one parameter :  $K_S^i$



# The nutrient

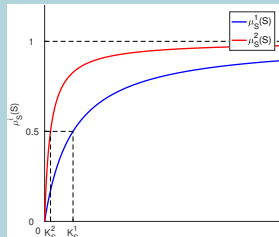
The nutrient :

A constant input



$\mu_S^i$  (The Monod model) :

$$K_S^1 \neq K_S^2$$



## A dynamical model

2 populations :

$$\frac{dX_1(t)}{dt} = \mu_T^1(T(t))\mu_S^1(S(t))X_1(t) - DX_1(t)$$

$$\frac{dX_2(t)}{dt} = \mu_T^2(T(t))\mu_S^2(S(t))X_2(t) - DX_2(t)$$

$$\frac{dS(t)}{dt} = D(S_{in} - S(t)) - \mu_T^1(T(t))\mu_S^1(S(t))X_1(t) - \mu_T^2(T(t))\mu_S^2(S(t))X_2(t)$$

$$\frac{dT(t)}{dt} = \Delta_T \cos(\omega t)$$

With four parameters :  $K_S^1$   $K_S^2$   $T_{opt}^1$   $T_{opt}^2$



## Issue

**What are the coexistence conditions ?**

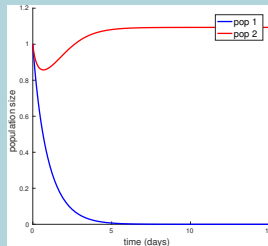


## The temperature impact

If the temperature is constant :

One survival population  $\Rightarrow$  **no coexistence**

*Sari & Mazenc 2010*



$$K_S^1 > K_S^2 \Rightarrow X_1 \rightarrow 0$$



## The limited nutrient impact

If the nutrient is no-limited :

$$\begin{aligned} \begin{pmatrix} \frac{dX_1(t)}{dt} \\ \frac{dX_2(t)}{dt} \end{pmatrix} &= \begin{pmatrix} \mu_T^1(T(t)) - D & 0 \\ 0 & \mu_T^2(T(t)) - D \end{pmatrix} \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \\ &= A(t) \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \end{aligned}$$

⇒ the system admits the following solution :

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \exp\left(\int_0^t A(\tau) d\tau\right) \begin{pmatrix} X_1(0) \\ X_2(0) \end{pmatrix}$$



## The limited nutrient impact

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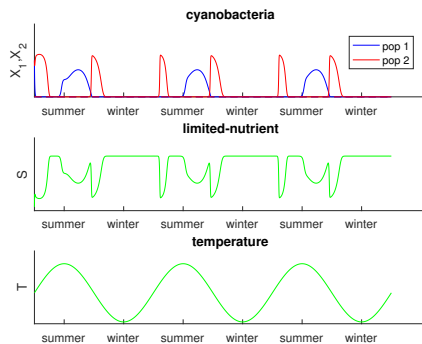
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⇒ **no coexistence** in the general case

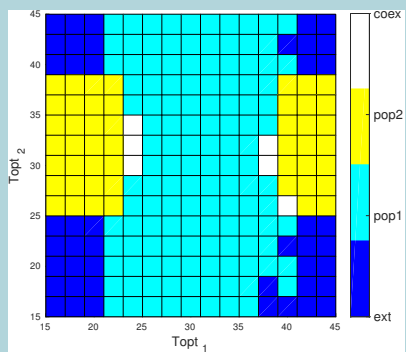
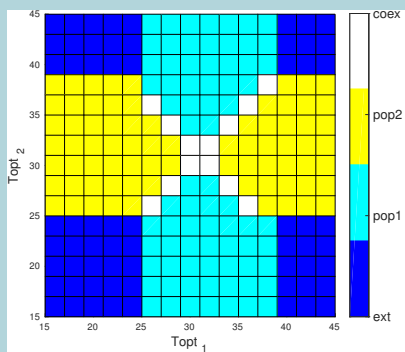


## Coexistence :

A numerical result :



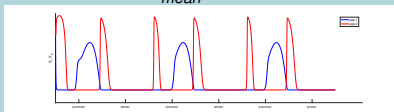
## Impact of the model parameters :

If  $K_S^1 < K_S^2$ If  $K_S^1 = K_S^2$ 

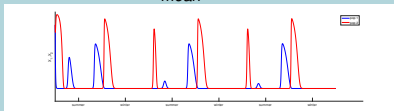
# Impact of the mean temperature :

$T_{mean}$  increasing

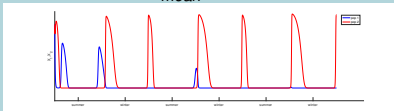
$T_{mean} = 17$



$T_{mean} = 20$

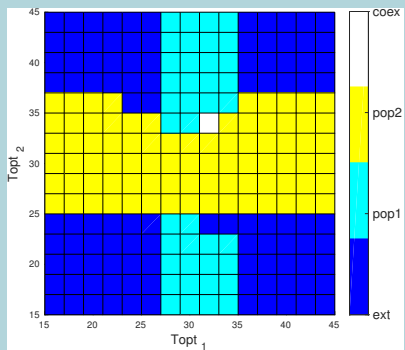


$T_{mean} = 23$

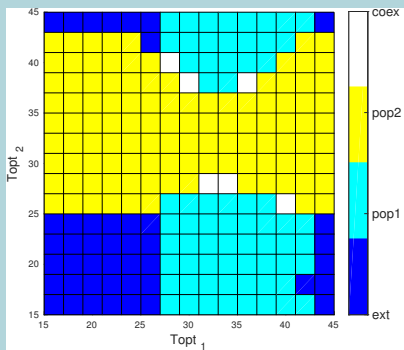


## Impact of the mean temperature :

If  $T_{mean} = 15C$  and  $K_S^1 > K_S^2$



If  $T_{mean} = 19C$  and  $K_S^1 > K_S^2$



## Coexistence :

### Conclusion

The mixed effects of limited resource and the periodical temperature allow coexistence for different parameters values.

### Properties of coexistence :

- If  $T_{opt}^1 < T_{opt}^2$  : the bloom of the population  $X_2$  is between two blooms of the population  $X_1$
- The increase in temperature leads to an initial coexistence



## Davis et al. 2009

Data from the Lake Agawam, summer 2005 :

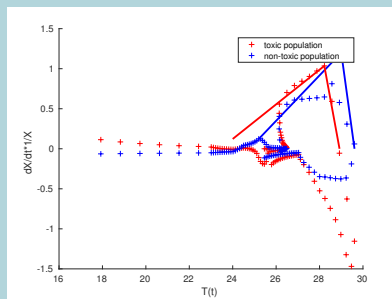
- toxic and non-toxic cyanobacteria populations
- temperature





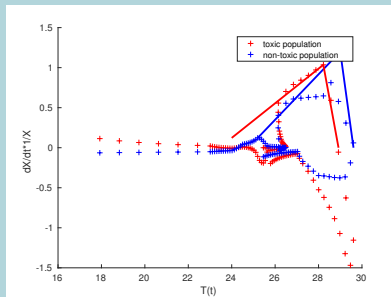
*Davis et al. 2009*

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## Davis et al. 2009

Data from the Lake Agawam, summer 2005 :



Qualitative comments :

- $T_{opt}^{tox} < T_{opt}^{notox}$

⇒ according to the model :

- the toxic population → blooms at the beginning and/or end of the summer
- the non-toxic population → blooms in the middle of summer



## Take Home Message :

- The mixed effects of limited resource and the periodical temperature allow coexistence
- an increase in temperature disrupts the balance of toxic and non-toxic populations



## To be continued

- A mathematical proof of the coexistence
- Adapt the cyanobacteria model to lake case :

$$\frac{dP(t)}{dt} = P_{input}(t) - (s + h)P(t) + r \frac{P(t)^q}{m^q + P(t)^q}$$

- An experimental study to confirm your results



**thanks for your attention**

