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Minimising the travel time on congested urban rail lines with a dynamic bi-modelling of trains and users

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Abstract

User assignment models in transit networks are relevant for the planning of new lines but less relevant for management. Other types of models are used for management, which is very efficient for the simulation of supply but less efficient for the simulation of users, who nevertheless influence train traffic on urban railway lines. This paper proposes to use a combined train and passenger simulation model on a railway line to propose line management solutions in order to minimise the passenger travel time. An application of this method on line 13 of the Paris subway network, shows how the relationship between frequency and dwell-time impact the railway traffic. Precisely, the current frequency of the line associated with a maximum dwell time of the 40s minimise the travel time. These optimal time conditions correspond to significant congestion in the line's train traffic.

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Keywords: Dynamic modeling; optimisation; transit line; transit assignment; line model; train regulation; train circulation

1. Introduction

Improving the modeling of users in transit systems is an important research field. There are many operational uses, particularly for estimating demand and time gains in scenarios for planning new transportation infrastructure. Models are often network scale, and the complexity and large size of the networks limits the accuracy of models, which have difficulty taking into account congestion and the temporal dynamics of the system (Hamdouch et al. 2014; Leurent et al. 2014).

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Railway line operators use supply planning software that enables them to optimise timetables and rolling stock dynamically on their line or network. Optimisation models are also an important research field. A number of these models focus on service optimisation based on service quality criteria and not only on train movements (Wang et al. 2017), others optimise passenger train timetables taking waiting times into account (Niu et al. 2015). Some researchs go further in optimising the timetable table by directly considering travel time or waiting time as an objective function (Sun et al., 2014; Wang et al., 2020; Zhang et al., 2018). However, if passenger assignment is well considered with capacity constraints, train movements do not directly depend on waiting time and thus on the number of passengers on the line.

On some highly congested urban railway lines, the maximum frequency that can be achieved depends as much on the number of passengers boarding and alighting at a station as on the minimum safety interval between trains. The number of trains in circulation and the travel times of trains will therefore also depend on the trajectories of the line's users. In order to improve the quality of service on these congested urban rail lines, optimisation of the timetable is not enough, so other solutions must be proposed. Some very strong ones, often chosen by operators and transit operating authorities, are the construction of alternative lines. Others are simpler but expensive, such as replacing rolling stock or counter-intuitive, such as slowing down the flow of trains at station terminals. In this research we propose to explore a first step of this kind of solutions: the maximal dwell time and his link with the frequency on this type of line.

The aim of this paper is twofold. The first is to propose a minimisation of the travel time calculated by simulations based on two variables that can be parameterised by the operator: frequency at a terminus station and maximum dwell time. The second is to better understand the influence of these two variables on the operation of the line and in particular on passenger travel times especially on a very congested line. The contributions are as follows: (i) the use of a simulation model to minimise travel time on a railway line, (ii) the determination of a maximum waiting time that corresponds to a minimum travel time and (iii) a better understanding of the link between frequency and waiting time with the operation of the line.

At the intersection of these research fields, we propose an optimisation of the transport supply on congested urban rail lines based on a coupled modeling between dynamic user assignment and train traffic (Poulhès, 2020; Poulhès et al. 2017). A simulation is used to estimate a real average travel time of users per origin and destination station. This is based on simulation variables that rarely appear as adjustment variables in the models. Some of these variables can be adjusted by the operator and thus serve to improve the quality of service on the line. This paper proposes to find the minimum of an objective function that evaluates quality of service indicators such as the average travel time. To do this, a mapping of the travel time space according to discretisation of the frequency values and dwell time max makes it possible to approach the minimum of the objective function. The analyse of the solutions' space provides a better understanding of the interactions between the key variables of the line dynamics and to consider quantifying the gains of operational solutions.

2. Method

The method is based on simulations of trains and passengers on a railway line. This model of train and passenger assignment on the line has been described in two previous papers. Poulhes et al (2017) describes the assignment of passengers on the line from an origin station to a destination station. Poulhes (2020) takes the passenger assignment and adds the assignment of interacting trains. The first sub-section will take up the essence of this model but we invite the reader to consult these two papers for more details. The following sub-sections describe the objective function and the characteristics of the line used to minimise the travel time of passengers.

2.1. The line model

This modelling considers train vehicles movements along the line as discrete events from block to block depending on the signalling and dwell time, which can vary with the number of passengers boarding and alighting.

The model assigns trains on the line from their terminus. Without incident or congestion, the signalling and the theoretical time per block govern the progress of trains. In this model, no incidents are considered, but the exchange of passengers in the station influences the travel time. This exchange time is calculated as a function of the number of user wishing to board and alight each train and the density of users on the platform and on board vehicles. If this time becomes too long, train traffic can be slowed down, reducing travel times and possibly increasing waiting time for the next trains. This increased waiting time may itself affect the number of passengers boarding and alighting for the next stations. The systemic interactions between train progress and passengers can be summarised in Figure 1.

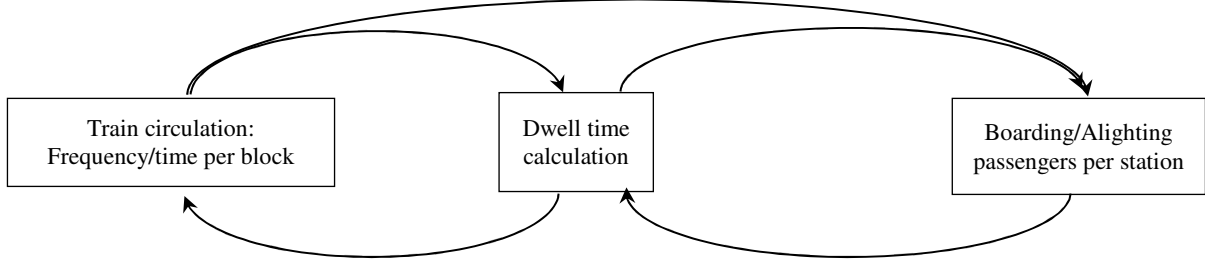


Figure 1: Relation scheme between trains and passengers

2.1.1. The train circulation

Train movements on the line are governed by a discrete event model that moves trains in chronological order from their terminus at the end of the block to the end of the block. Note $b_{i,i+1}$ the block between nodes i and $i + 1$. The travel time over this block is noted as $T_{par}(b_{i,i+1})$ and is found in the input data provided by the line operator. We therefore assume that there is no variability in travel time due to the human behaviour of the driver on a non-automatic line. We can therefore calculate the arrival time $H_{i+1}^-(k)$ of train k at node $i + 1$ as a function of the departure time of node i , $H_i^+(k)$:

$$H_{i+1}^-(k) = H_i^+(k) + T_{par}(b_{i,i+1}) \quad (1)$$

Train k then crosses node $i + 1$ which means leaving block $b_{i,i+1}$ and arriving at block $b_{i+1,i+2}$. If $b_{i,i+1}$ is a station block, the duration $D_{i,i+1}(k)$ is defined which corresponds to the stopping time at station $b_{i,i+1}$ of train k . If $b_{i,i+1}$ is a station block, then $D_{i,i+1}(k) = 0$ (no station stop). It is also necessary to take into account the traffic rules and determine how long the next block $b_{i+1,i+2}$ has been free. Note the previous train that passed through this block $\beta_{i+1,i+2}$. The instant $H_{i+2}^+(\beta_{i+1,i+2})$ of departure of the previous train from the block of interest has already been calculated because of the chronological order in which the events are taken into account. Thus, the instant from which train k can pass over block $b_{i+1,i+2}$ due to the running rules is $H_{i+2}^+(\beta_{i+1,i+2}) + T_{sec}(b_{i+1,i+2})$, with T_{sec} the safety time for another train to enter a block after the departure of the previous train. Finally we have the following formula determining the departure time of train k of $+1$:

$$H_{i+1}^+(k) = \max[H_{i+2}^+(\beta_{i+1,i+2}) + T_{sec}(b_{i+1,i+2}), H_{i+1}^-(k) + D_{i+1}(k)] \quad (2)$$

Thus passengers' travel times can be degraded by the exchange times of the train they have taken but also by the increase in the exchange times of the trains preceding that train.

2.1.2. The demand side

Users are considered with an aggregated form, a flow from an origin station to a destination station by considering a simulation step as fine as the data can inform. They are assigned to the next trains arriving at the station according to the residual capacity on board and their destination. If the residual capacity after passengers have alighted is not sufficient to accommodate all passengers wishing to board, queues are created with a FIFO principle. Passengers who have not been able to board a train are given priority for the next train and so on. We propose a very simplified formula of the classic bottleneck model presented in the article Poulhes et al. (2017). Let $y^+(k)$ the number of passengers who succeed to board in the train k , $h_{ij}^+(x)$ the arrival time on the platform of the x -th passengers from the station i to the station j . $q(t)$ the number of passengers arrived on the platform during the time step $[t; t - 1]$. $X_{ij}^-(q(t))$ the cumulative flow who board a train when $q(t)$ passengers are on the platform and

$X_{ij}^-(k)$ the total boarding flow when k leaves i from i to j since the beginning of the simulation. Thus the average waiting time is:

$$w_{ij}(k) = \frac{1}{y^+(k)} \sum_{t>0} \int_{X_{ij}^-(q(t-1))}^{X_{ij}^-(q(t))} h_{ij}^+(x) dx \quad (3)$$

2.1.3. Interaction

An interaction is then created between the forward movement of trains and the forward movement of passenger flows in the dwell time. This dwell time is a function of the number of passengers wishing to board y^+ , the number of passengers wishing to alight y_- , the density of passengers standing on board after alighting passengers d_t , the density of passengers on the platform upon arrival of the latter d_q and the number of doors of a train set N_p . The passenger exchange approach studied is sequential. Users first alight and then those wishing to board do so later. The formula for passenger exchange time without operational constraints used in this article is therefore in the following form (Poulhes, 2020):

$$D_i(k) = \alpha \left(\theta^+ \frac{y^+}{N_p} + \theta^- \frac{y_-}{N_p} \right), \theta^+ = \theta_0^+ e^{\alpha_q^+ d_q + \alpha_t^+ d_t}, \theta^- = \theta_0^- e^{\alpha_q^- d_q + \alpha_t^- d_t} \quad (4)$$

θ_0^+ (resp. θ_0^-) is the basic time it takes a passenger to board (or alight) the train if there is no congestion on the platform and on board. The parameters α_q^+ et α_t^+ (resp α_q^- and α_t^-) represent the influence of the density of passengers on the platform and on board on the boarding (resp. alighting) time of passengers. The multiplicative parameter α represents the influence on the exchange time of random and heterogeneous passenger behaviour as well as their distribution on the platforms.

The expected dwell time also depends on the minimum limit D_{ik}^0 and on the maximum limit $D_{ik}^{0,max}$ as well as the time taken for the doors to open and shut $t^{o+c}(k)$:

$$\tilde{D}_{i,\mu_k(i)}(k_z) = t^{o+c}(k) + \min \left(D_{ik}^{0,max}, \max(D_{ik}^0, D_{ik}) \right) \quad (5)$$

$D_{ik}^{0,max}$ is the maximum dwell time considered as an operational variable in this paper. However, stationary time could be upper this value if previous trains occupies the next blocks. The doors of the train then remain open during this added time.

2.2. The objective function

The aim is to find a set of parameters that minimises the total travel time for users of an urban railway line. The travel time per passenger on the line is given by the sum of the waiting time $w_i(j, k)$ and the sum of the train travel time $k \in K_i$ taken by block $t_{b_{k,k+1}}$ between the origin station $i \in N$ and the destination station $j \in N$. N is the set of the all stations of the line and K_i the set of all trains passing through station i during the simulation period. Note q_{ij} the passenger flow between the station i and the station j among the set of arriving passengers on the line Q during the simulation period. This gives the following result for all users of the line during the simulation period:

$$T = \min \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_i} \sum_{q_{ij}(k) \in Q} \sum_{b_{k,k+1} = b_{i,i+1}}^{b_{j-1,j}} t_{b_{k,k+1}} + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_i} \sum_{q_{ij} \in Q} w_i(j, k) \quad (6)$$

The input variables of the simulation model that we want to estimate in order to have the minimum total travel times for the users of the line are the timetable and the maximum exchange time. On a busy urban rail line, the timetable can be simplified in the search for optimisation by a frequency of trains departing from each terminus of the line. The maximum exchange time is the time that the operator defines not to be exceeded for the station stop time. This time is often regulated by an audible signal announcing the closing of the doors. This signal is often

triggered by the train driver who has no instructions from the line operator. This variation in station stopping time is an important control strategy for the operator in the event of an incident or congestion.

2.3. Minimisation research

Minimisation must make it possible to give an indication of the optimum time that each train should remain at the platform at most, considering this time constant whatever the station and the train. This stopping time is associated with an optimal programmed frequency. In this article we use a simple approach by discretisation of the solutions space and not a classical optimization approach. Considering only two variables, this method provides an approximate value of the optimum but also a cartography of the values taken by the objective function for all the pairs of possible variables limited by minimum and maximum values. We can then better understand the influence of the frequency and the maximum dwell time on the total travel time of the passengers on the line.

We propose to illustrate this travel time minimisation method and to show its interest through the example of line 13 of the Parisian subway.

3. Use case

3.1. The line 13 in the Paris network

According to Google data, line 13 is the 5th busiest subway line in the world. According to users, it is one of the busiest and most uncomfortable (FNAUT). From south to north, the line serves 18 stations, including two national train stations in the Paris region (Montparnasse and St-Lazare) and major tourist centres. Then there are two branches to the north. The western branch serves 6 stations, the eastern branch serves 8 stations. Both serve major business centres.

On the morning peak from 7.30 am to 9.30 am, service frequency is scheduled at about 34 runs per hour on the main trunk. So, theoretically, a train shall leave from the Châtillon-Montrouge terminal station every 1 min 45. Every second train leaves for each of the two branches. The trains have 128 seats and considering a maximum of 5 persons/m² and a surface of 111.5m² the maximum capacity is near to 685 passengers. And the trains are all the same with 15 doors. This inputs data describing the supply, the timetables and the travel time for each block is provided by the line operator.

To mimic the morning peak, the demand matrix is built from a survey which provides information on station-to-station flows. This survey was done by the RATP, which operates the line. Then, AFC data per entry station is used to make demand per entry station dynamic with a step time of 5 minutes. In order to make congestion phenomena clear and to inflate the counting data may be too underestimated, the demand matrix is increased by a factor of 1.3. The total number of passengers using the line is then 77 367 during the simulation period.

Finally, the computing of the dwelling time comes from equation (4) and the parameters of table 1 are used. They were obtained from experiments on the Paris metro and calibrated on real-time data provided by the operator of line 13.

Table 1: Parameters to mimic the boarding/alighting of passengers.

Parameters	Values	Parameters	Values
Number of seats	128	On board density alighting	0.04
Capacity of trains	685	On board density boarding	0.05
Base alighting time	0.0117	On platform density alighting	0.02
Base boarding time	0.0167	On platform density boarding	-0.05

The model was coded in Matlab. Each iteration is a simulation of the assignment of trains and passengers between 7:30 and 9:30. The simulation time is 30s per iteration, and an acceptable convergence takes about thirty iterations.

3.2. Results

Firstly, to have a reference as the best average time that can be obtained on the line, we estimate the waiting time and the travel time without congestion with the model with infinite capacity trains and infinite number of doors. This way, the passenger waiting time average is 1.03 min and the travel time average is 11.82 min. Thus, the average of the total travel is 12.85 min. As a second element of comparison, a simulation is made with demand and supply conditions as provided by the operator. The maximum dwell time taken for the simulation is 50 seconds. The average waiting times is 1.46 min and the average travel time is 12.55 min. The resulting average total time is then 14.01 min. It can already be concluded that under these simulation hypotheses, passengers lose on average 1.16 min of time due to congestion on the line, i.e. 9% extra time. In order to highlight the effect of the frequency and the maximum dwell time on the total time spend on the line (waiting time + travel time on boarding) we propose a numerical analysis. We simulate our model by trying out a set of values. The tested frequency values are between 30 and 36 with a step of 0.1. And, the tested values of the maximum dwell time are between 0.5 and 1 minutes with a step of 0.01 minutes. The results are shown in figure 2. For each couple of nominal frequency and dwell time tested the total travel time is plotted and some isolines are drawn.

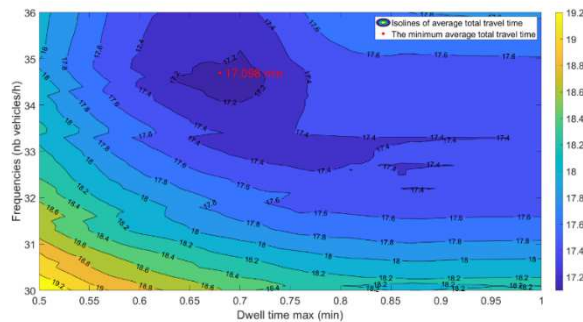


Figure 2: Average total travel time on the line 13 by frequencies and dwell time max.

Several interesting results are shown in Figure 2. The first is that the minimum time that can be obtained with these two variables is close to operational conditions. This is reassuring and shows that the operator seeks to minimise the travel time of the line's users. For instance, with a nominal frequency near to 33.3 a greater maximum dwell time allows to keep a total travel time less than 17.4min up to a limit of 0.97min. Note that, the lowest total travel time is 17.098min. It's obtained from a frequency of 34.7 train per hour and a maximum dwell time of 0.68 min. To discuss this result, it seems near to the nominal frequency programmed by the RATP which is 34 trains per hours. The model retrieves the same orders of magnitude. In regards the dwell time the interval of [0.65, 0.75] seems realistic too. In that case, the gap between the total travel time with and without congestion would be 4.25min.

The second concerns the average time variations in the value intervals for frequency and dwell time max. For example, the average time values that increase by 3% when removing a train per hour. Above 35 trains per hour, the signalling does not allow the trains to run in good conditions and the times increase. Even the dwell time control does not allow increasing the frequency beyond this threshold value of 35 trains. On the contrary, if the dwell time is not or cannot be controlled, the most efficient frequency for the line drops to 33 trains per hour. This value remains stable from 0.75 min of dwell time max considering our dwell time calculation model according to the congestion conditions of the line. An interesting result is the fact that there is a fairly high minimum dwell time max which proves that it is not necessarily important to want to leave the station as quickly as possible. The minimum value of the average dwell time on the line corresponds to the dwell time for passengers to get on and off the line without having to force the doors closed.

To explore more the results, the figures below dissociate the total travel time into the waiting times (Fig. 3.) and the travel time on board (Fig. 4.). On the first figure, the minimal waiting time (blue) is obtained with an high frequency, the red value correspond to the global minimal mean time. The waiting time without congestion is on average 1 min (on the central section on average a passenger would wait 53s for a frequency of 34 trains), with a 30% increase in demand, the waiting time is multiplied by 4 and the minimum is 3.41 min, i.e. the fact that on average a passenger allows at least one train to pass before being able to board a train, which reflects very

complicated transport conditions. A dwell time max that is too low does not leave enough time for all passengers to board, which mechanically increases the waiting time. Logically also, the higher the frequency, the lower the dwell time. But as can be seen on the second graph a high frequency slows down train traffic and therefore increases the travel time. Conversely, a low frequency allows good train movements but increases waiting time. Similarly, below the critical dwell time of 0.75 min, travel time decreases and traffic conditions are improved. On the other hand, few passengers are able to board, so waiting time is increased as can be seen in the previous graph and in total the average total time increases when the dwell time max decreases.

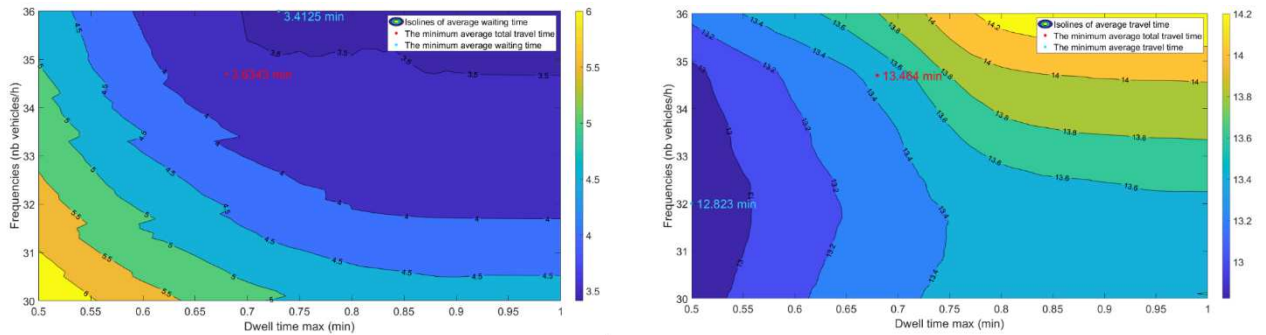


Figure 3: (a) Average waiting time on the line 13 by frequencies and dwell time, (b) Average travel time on the line 13 by frequencies and dwell time

Figure 4 shows the passengers boarding (green) and alighting (orange) between 8:00 and 9:00 along the line with the optimised parameters. The frequencies (black) and waiting time (blue) are also shown. The first station, called Châtillon, is already very busy. At Montparnasse station, the maximum waiting time has been exceeded for the first time. The increase in length of stay is already taking place at Plaisance and Pernety stations due to congestion on the Montparnasse line. The reason for exceeding the maximum dwell time at Montparnasse is an over-occupation of the tracks by other trains downstream of the line. Indeed, Saint-Lazare station is the busiest and trains are congested before this station. The slowdown due to passenger dwelling has repercussions upstream. However, at Saint-Lazard, the maximum dwell time is exceeded, indicating greater congestion downstream. The Place de Clichy station seems to be the last to be in a critical situation. Guy Mocquet and Porte de Clichy are not the busiest stations, but the exchange time seems to be important in relation to the relatively low total number of passengers. In future developments, it will be interesting to accelerate the dwell time only in this station to see the effect on the crowded upper station: place de Clichy, Saint-Lazare, Montparnasse. The variation in frequency on the line is another indicator that shows the congestion of trains on the line and the slowdown. One of the results of this research is therefore the fact that the optimal time solution for passengers is a solution where trains are congested and therefore slowed down. The reduction in waiting time then compensates for the increase in travel time by the large number of trains arriving at each station on the line.

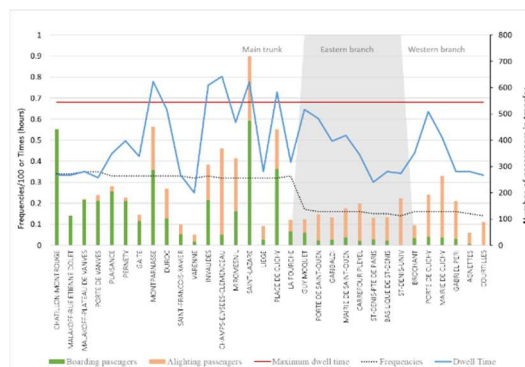


Figure 4: Frequencies and dwell time along the line 13 of Paris according to the optimised parameters

4. Conclusion

This research uses an original simulation model of train traffic that dynamically affects passengers by taking into account congestion problems on the line. The dwell time appears to be the central element of the good functioning of the line. By studying more precisely the travel time on the line and the compensations between a decrease in waiting time but an increase in travel time by increasing the frequency, this study brings several advances for the understanding of a congested urban railway line but also helps operators in their choices of line regulation and timetables.

The operator's frequency is the optimal frequency. A decrease in frequency is not beneficial for travel times even if maximum dwell times are respected. Conversely, too high a frequency reduces travel time. The dwell time is considered by operators as the key to the operation of a line. However, the study on line 13 of the Paris metro shows that a too low value of this maximum is not beneficial for passenger journey times. Conversely, it does not seem necessary to prevent passengers from boarding in order to keep the lowest travel time.

Several suites are being considered. The gap between the estimated time and perceived time may be important. We have considered a travel time and a travel time as having the same arduousness, but comfort on board and the perception of waiting time can considerably degrade the perceived time, which is also important to analyse. This research focuses on the optimal functioning of a line but under normal traffic conditions and passenger behaviour. One of the main outcomes is to test the impact of an incident or delay on the minimum time and the frequency and dwell time max values that minimise it. Finally, an optimisation algorithm would allow to add variables such as the interior layout of the rolling stock, in the minimization of time.

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