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# FRICITION AND ADHESION

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**Abstract** The studies carried out on adhesion by the group "Modeling in Contact Mechanics" at the LMA are reviewed in this paper and recent applications are presented. Based on the introduction of the adhesion intensity variable developed by M. Frémond, different forms of a model coupling adhesion to unilateral contact and friction have been developed. The formulations are given either under the form of implicit variational inequalities or the one of complementarity problems. Both quasi-static and dynamic formulations are considered.

The model is non smooth because we do not use any regularization for the unilateral conditions and for the friction, i.e. Signorini conditions and strict Coulomb law are written. In the thermodynamics analysis, the state and the complementarity laws are then written using sub-differentials and differential inclusions because of the non convexity and non differentiability of the potentials. For the dynamics, the formulation is given in term of differential measures in order to deal with the non continuity of the velocities that may occur in the solutions.

This work therefore owes much to the theories and the numerical scheme developed by J.-J. Moreau and M. Jean.

**Keywords:** Unilateral contact, friction, adhesion, non smooth dynamics

## Introduction

In order to describe the smooth transition from a completely adhesive contact to a usual unilateral contact (Signorini conditions) with Coulomb friction, a model based on interface damage has been first developed for quasi-static problems in (Raous et al, 1997) (Cangémi, 1997)

(Raous et al, 1999) (Raous, 1999). Using a dynamic formulation, the model was then extended to account for the brittle behaviour occurring when a crack interacts with fiber-matrix interfaces in composite materials in (Raous-Monerie, 2002) (Monerie, 2000). More recently, the model has been used to study metal/concrete interfaces in reinforced concrete in civil engineering (Karray et al, submitted), delamination of coated bodies (Raous et al, 2002), delamination of glued assembling in civil engineering (Raous et al, 2004), cohesive masonry (Jean et al, 2001) (Acary, 2001) and production of wear particles in bio-engineering (Baudriller, 2003).

The quasi-static formulation was extended to deal with hyperelasticity in (Bretelle et al, 2001). Mathematical results about the existence of the solutions were given in (Cocou-Rocca, 2000) without using any regularization on the contact conditions.

## 1. THE MODEL

The RCC model (Raous-Cangémi-Cocou) has been first given in (Raous et al, 1997)(Cangémi, 1997) and then extensively presented in (Raous et al, 1999). It has been extended to the present form including progressive friction with the term  $(1 - \beta)$  in (Monerie, 2000) (Raous-Monerie, 2002). Adhesion is characterized in this model by the internal variable  $\beta$ , introduced by Frémond (Frémond, 1987)(Frémond, 1988), which denotes the intensity of adhesion. The introduction of a damageable stiffness of the interface ensures a good continuity between the two models during the competition between friction and adhesion. The behaviour of the interface is described by the following relations, where (1) gives the unilateral contact with adhesion, (2) gives the Coulomb friction with adhesion and (3) gives the evolution of the adhesion intensity  $\beta$ . Initially, when the adhesion is complete, the interface is elastic as long as the energy threshold  $w$  is not reached. After that, damage of the interface occurs and consequently, on the one hand, the adhesion intensity  $\beta$  and the apparent stiffness  $\beta^2 C_N$  and  $\beta^2 C_T$  decrease, and on the other hand, friction begins to operate. When the adhesion is completely broken ( $\beta = 0$ ), we get the classical Signorini problem with Coulomb friction.

$$-R_N^r + \beta^2 C_N u_N \geq 0, \quad u_N \geq 0, \quad (-R_N^r + \beta^2 C_N u_N) \cdot u_N = 0, \quad (1)$$

$$\begin{aligned} R_T^r &= \beta^2 C_T u_T, \quad R_N^r = R_N, \\ \|R_T - R_T^r\| &\leq \mu(1 - \beta) |R_N - \beta^2 C_N u_N|, \end{aligned} \quad (2)$$

with

$$\begin{aligned} \text{if } \|R_T - R_T^r\| &< \mu(1 - \beta) |R_N - \beta^2 C_N u_N| \Rightarrow \dot{u}_T = 0, \\ \text{if } \|R_T - R_T^r\| &= \mu(1 - \beta) |R_N - \beta^2 C_N u_N| \Rightarrow \exists \lambda \geq 0, \dot{u}_T = \lambda(R_T - R_T^r), \end{aligned}$$

$$\begin{aligned} \dot{\beta} &= - \left[ \left( w - \beta (C_N u_N^2 + C_T \|u_T\|^2) - k\Delta\beta \right)^- / b \right]^{1/p} \text{ if } \beta \in [0, 1[, \\ \dot{\beta} &\leq - \left[ \left( w - \beta (C_N u_N^2 + C_T \|u_T\|^2) - k\Delta\beta \right)^- / b \right]^{1/p} \text{ if } \beta = 1. \end{aligned} \quad (3)$$

$R_N$  and  $u_N$  are the algebraic values of the normal components of the contact force and those of the relative displacement between the two bodies (occupying domains  $\Omega^1$  and  $\Omega^2$ ) defined on the contact boundary  $\Gamma_C$ , and  $R_T$  and  $u_T$  are the tangential components of this contact force and those of this relative displacement. Subscript  $r$  denotes the reversible parts. The constitutive parameters of this interface law are as follows :  $C_N$  and  $C_T$  are the initial stiffnesses of the interface (full adhesion),  $\mu$  is the friction coefficient,  $w$  is the decohesion energy,  $p$  is a power coefficient ( $p = 1$  in what follows) and  $k = 0$  in what follows.

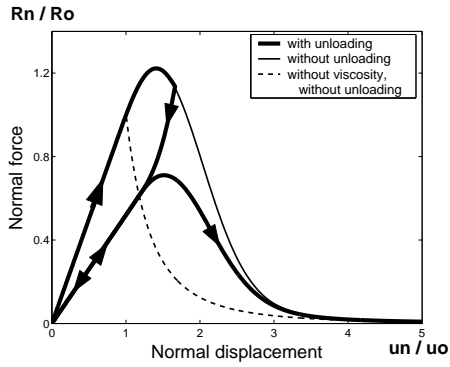


Figure 1.1. Normal behaviour of the interface

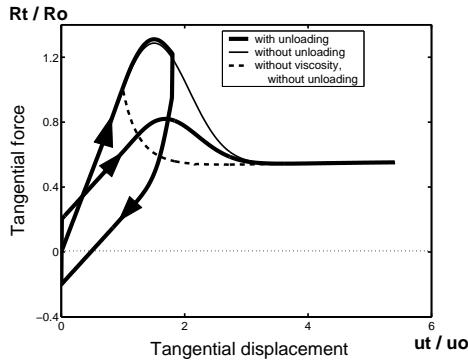


Figure 1.2. Tangential behaviour of the interface

Fig. 1.1 and Fig. 1.2 give the normal and tangential behaviour of the interface during loading and unloading ( $C_N = C_T = C$ ,  $u^0 = \sqrt{\omega/C}$  and  $R^0 = \sqrt{\omega C}$ ). It should be noted that the Signorini conditions are strictly imposed when compression occurs. References on other models can be found in (Raous, 1999) (Raous et al, 1999) and a comparison between some of them is made in (Monerie et al, 1998) (models developed by Tvergaard-Needleman, Girard-Feyel-Chaboche, Michel-Suquet, Allix-Ladevèze). Using penalization and augmented Lagrangian on a similar model to the RCC one, Talon-Curnier have solved the quasi-static problem using generalized Newton method (Talon-Curnier, 2003).

## 2. THE THERMODYNAMICS

In the framework of continuum thermodynamics, the contact zone is assumed to be a material surface and the local constitutive laws are obtained by choosing two specific forms of the free energy and the dissipation potential associated to the surface. The following thermodynamic variables are introduced : the relative displacements ( $u_N, u_T$ ) and the adhesion intensity  $\beta$  are chosen as the state variables, and the contact force  $R$  and a decohesion force  $G$ , as the associated thermodynamic forces. The thermodynamic analysis given for the RCC model in (Raous et al, 1999) has been extended to the present model in (Monerie, 2000) (Raous-Monerie, 2002) in order to obtain relations (1) to (3), where friction is progressively introduced in the form of the term  $(1-\beta)\mu$  into (2). Expressions (4) and (5) are adopted for the free energy  $\Psi(u_N, u_T, \beta)$  and the potential of dissipation  $\Phi(\dot{u}_T, \dot{\beta})$ . In (4), the indicator function  $I_{\tilde{K}}$  (where  $\tilde{K} = \{v / v \geq 0\}$ ) imposes the unilateral condition  $u_N \geq 0$  and the indicator function  $I_P$  (where  $P = \{\gamma / 0 \leq \gamma \leq 1\}$ ) imposes the condition  $\beta \in [0, 1]$ . In (5), the indicator function  $I_{C^-}(\dot{\beta})$  (where  $C^- = \{\gamma / \gamma \leq 0\}$ ) imposes that  $\dot{\beta} \leq 0$  : the adhesion can only decrease and cannot be regenerated (it is irreversible) in the present model.

$$\Psi(u_N, u_T, \beta) = 1/2 \beta^2 C_N u_N^2 + 1/2 \beta^2 C_T \|u_T\|^2 - w \beta + I_{\tilde{K}}(u_N) + I_P(\beta) \quad (4)$$

$$\Phi(\dot{u}_T, \dot{\beta}) = \mu(1-\beta) |R_N - \beta^2 C_N u_N| \|\dot{u}_T\| + b/(p+1) |\dot{\beta}|^{p+1} + I_{C^-}(\dot{\beta}) \quad (5)$$

$\Psi$  has a part which is convex but not differentiable and another part which is differentiable but not convex with respect to the pair  $(u, \beta)$ .  $\Phi$  is convex but has a part which is not differentiable. The state laws and the complementarity laws are then written as follows in order to obtain the contact behaviour laws given in section 1 (Raous et al, 1999).

$$\begin{cases} R_T^r = \frac{\partial \Psi^d}{\partial [u_T]} & R_N^r - \frac{\partial \Psi^d}{\partial [u_N]} \in \partial I_{\tilde{K}}([u_N]) \\ -G_\beta - \frac{\partial \Psi^d}{\partial \beta} \in \partial I_{[0,1]}(\beta) \end{cases} \quad (6)$$

$$R_N = R_N^r \quad (R_T^r, G_\beta) \in \partial \Phi([u_T], \dot{\beta}) \quad (7)$$

### 3. THE QUASI-STATIC FORMULATION

The formulation and the approximation of quasi-static frictional problems given in (Cocou et al, 1996) have been extended to adhesion problems in (Raous et al, 1997) (Raous et al, 1999) (Raous, 1999). The problem can be here set as the coupling between two variational inequalities (one of which is implicit) and a differential equation.

**Problem ( $P_1$ ) :** Find  $(\tilde{u}, \beta) \in W^{1,2}(0, T; V) \times W^{1,2}(0, T; H)$  such that  $\tilde{u}(0) = \tilde{u}_0 \in K$ ,  $\beta(0) = \beta_0 \in H \cap ]0, 1[$  and for  $\forall t \in [0, T]$ ,  $\tilde{u}(t) \in K$ , and

$$\begin{aligned} \forall v \in V \quad & a(\tilde{u}, v - \dot{\tilde{u}}) + j(\beta, u_N, v_T) - j(\beta, u_N, \dot{u}_T) + \\ & \int_{\Gamma_C} \beta^2 C_T u_T \cdot (v_T - \dot{u}_T) ds \geq (\tilde{F}, v - \dot{\tilde{u}}) - \langle R_N, v_N - \dot{u}_N \rangle, \end{aligned} \quad (8)$$

$$-\langle R_N, z - u_N \rangle + \int_{\Gamma_C} \beta^2 C_N u_N \cdot (z - u_N) ds \geq 0 \quad \forall z \in K, \quad (9)$$

$$\dot{\beta} = -1/b \left[ w - (C_N u_N^2 + C_T \|u_T\|^2) \beta \right]^- \quad \text{a.e. on } \Gamma_C, \quad (10)$$

where :

- $\tilde{u} = (u^1, u^2)$  where  $u^1$  and  $u^2$  define the displacements in  $\Omega^1$  and  $\Omega^2$ ,
- $V = (V^1, V^2)$ ,  $V^\alpha = \left\{ v^\alpha \in [H^1(\Omega^\alpha)]^3; v^\alpha = 0 \text{ a.e. on } \Gamma_U^\alpha \right\}$ ,  $\alpha = 1, 2$ ,
- $H = L^\infty(\Gamma_C)$ ,
- $K = \{v = (v^1, v^2) \in V^1 \times V^2; v_N \geq 0 \text{ a.e. on } \Gamma_c\}$ , where  $\Gamma_c$  is the contact boundary between the two solids  $\Omega_1$  and  $\Omega_2$ ,
- $a(\cdot, \cdot)$  is the bilinear form associated to the elasticity mapping,
- $j(\beta, u_N, v_T) = \int_{\Gamma_C} \mu(1 - \beta) |R_N - \beta^2 C_N u_N| \|v_T\| ds$ ,
- $\tilde{F} = (F^1, F^2)$  are the given force densities applied to solid 1 and to solid 2 respectively.

By using an incremental approximation, it has been established in (Raous et al, 1997) (Raous et al, 1999) that the numerical solutions can be obtained by adapting the methods that we have developed for dealing with

classical frictional unilateral contacts (Raous, 1999). The solutions have mainly been obtained as follows :

- either by taking a fixed point on the sliding threshold and solving a sequence of minimization problems with the choice between a projected Gauss-Seidel method (accelerated by relaxation or Aitken processes) and a projected conjugate gradient method (Raous-Barbarin, 1992),
- or by taking a complementarity formulation, using a mathematical programming method (Lemke's method).

With the adhesion model, these solvers are coupled with the numerical integration of the differential equation on  $\beta$  by using  $\theta$ -methods ( $\theta = 1$  is often chosen). Implementation of the algorithms has been conducted in our finite element code GYPTIS90 (Latil-Raous, 1991).

#### 4. THE DYNAMICS

Depending on the characteristics of the interface, especially when  $b$  tends towards zero (no viscosity for the evolution of the intensity of adhesion), brittle behaviour can be obtained and the inertia effects have to be taken into account. A 3D dynamic formulation has been developed (Raous-Monerie, 2002) (Monerie-Raous, 1999) (Monerie-Acary, 2001). Because of the non smooth character of these interface laws, the dynamics is written in terms of differential measures as follows (where  $q$  denotes the displacement and  $r$  the contact force) :

$$M(q, t)d\dot{q} = F(q, \dot{q}, t)dt - r d\nu, \quad (11)$$

where  $d\dot{q}$  is a differential measure associated with  $\dot{q}(t)$  :

$$\int_{t_1}^{t_2} d\dot{q} = q(t_2^+) - q(t_1^-) \quad \forall t_2 > t_1, \quad (12)$$

and hence :

$$\int_{t_1}^{t_2} M(q, t)d\dot{q} = \int_{t_1}^{t_2} F(q, \dot{q}, t)dt - \int_{]t_1, t_2]} r d\nu, \quad (13)$$

$$q(t_2) = q(t_1) + \int_{t_1}^{t_2} \dot{q}(\tau)d\tau. \quad (14)$$

The Non Smooth Contact Dynamics method developed by (Moreau, 1998) (Moreau, 1994) (Moreau, 1999) (Jean, 1999) has been extended to the treatment of the RCC model. A solver for dealing with adhesion has been implemented in the finite element code LMGC (Jean, 1999) (Monerie-Acary, 2001). Another solver based on complementarity formulation (Lemke's method) dealing with non smooth dynamics problems has been implemented in the finite element code SIMEM3 at the LMA (Vola et al, 1998).



## 5. APPLICATIONS

### ■ Delamination benchmarks

Various benchmarks for simulating delamination have been developed in the framework of a joint project with the Laboratoire Central des Ponts et Chaussées (Raous et al, 2004) focusing on adhesion and gluing in civil engineering.

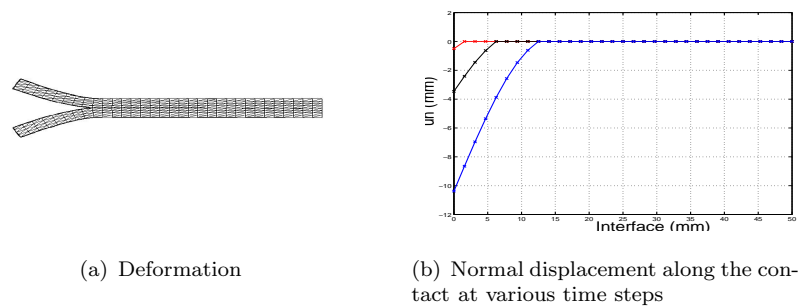


Figure 1.3. Delamination of two layers submitted to vertical traction

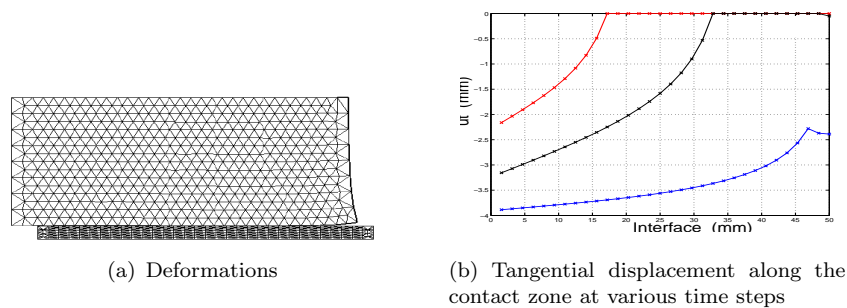


Figure 1.4. Shear delamination of a block submitted to horizontal loading

### ■ Micro-indentation of a single fiber in a composite material

This model was first developed in order to describe the behaviour of the fiber-matrix interface of composite materials (Cangémi, 1997) (Raous et al, 1999). The parameters of the model were identified on micro-indentation experiments performed at the ONERA on single fibers. Physical and mechanical considerations are taken into account in the identification procedure : for example, the

values of the initial interface stiffness  $C_N, C_T$  are taken to be in the range corresponding to the elastic properties of the oxides located at the interfaces. The viscosity parameter  $b$  is particularly difficult to identify : experiments with different loading velocities are required for this purpose. The validity of the model was then confirmed by taking various kinds of loadings (especially cyclic loadings) and comparing experimental results with those obtained in the numerical simulation with the same interface parameters. Details of these studies can be found in (Cangémi, 1997) (Raous et al, 1999) and later in (Monerie, 2000).

■ **Interaction between cracks and fiber/matrix interfaces in composite materials**

Again in collaboration with ONERA, during the thesis (Monerie, 2000), the RCC model has been used to investigate the different ways in which cracks propagate through a composite material and how they depend on the interface properties. The adhesion model has been used to account both the crack propagation (decohesive crack with no viscosity, i.e.  $b = 0$ ) and the interface behaviour. Crack bridging, crack trapping and fiber breaking can be observed depending on the interface characteristics (Raous-Monerie, 2002) (Monerie, 2000). On Fig. 1.5 adhesion is broken in the black zones.

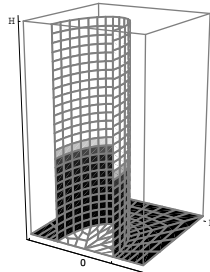


Figure 1.5. Interaction of a crack and a fiber/matrix interface

■ **Metal/concrete interfaces in reinforced concrete**

In civil engineering, we are now testing the RCC model for the adhesive contact between steel and concrete in reinforced concrete. Pull-out tests of a steel shaft are being simulated in the framework of a joint project between LMA and ENIT in Tunisia (Karray et al, submitted). In that case, a variable friction coefficient (depending on the sliding displacement) has been used in order to take into account the wear of the surface which occurs during the sliding

which seems to be quite significant with concrete. Another version of the way used to introduce friction has been used :  $(1 - \beta^2)$  instead of  $(1 - \beta)$ . Results with  $C_T = C_N = 16\text{N/mm}^3$ ,  $\mu$  varying from 0.45 to 0.3 and  $f(\beta) = (1 - \beta^2)$  are given in Fig. 1.6.

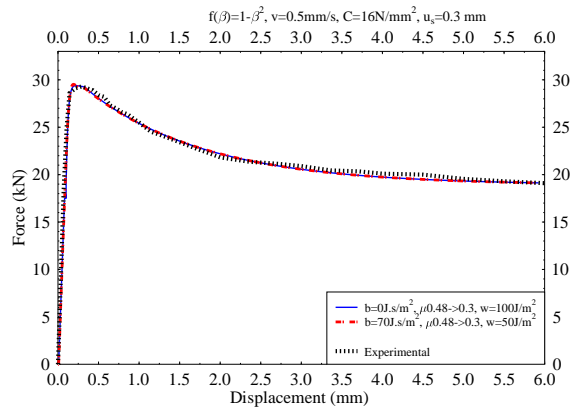


Figure 1.6. Simulation of a pull-out test on reinforced concrete

#### ■ Delamination of a coated body

A simplified approach to the delamination of a coated body has been conducted to simulate the indentation of a thin Chrome layer adhering to a metal body (Raous et al, 2002).

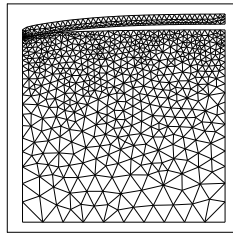


Figure 1.7. Indentation of a coated surface

#### ■ Cohesive masonry

The RCC model has been used in (Jean et al, 2001) (Acary, 2001) to simulate the behaviour of a cohesive dome. In Fig. 1.8, the deformation of a dome let on pillars and submitted to gravity is shown.

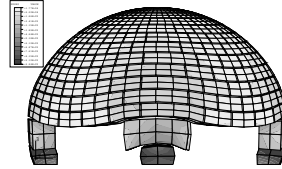


Figure 1.8. Behaviour of a cohesive dome

#### ■ Wear in biomechanics

In order to simulate the wear occurring in biomaterials and especially the production of wear particles, H. Baudriller (Baudriller, 2003) has used the RCC model in a volumic sense. The adhesive model is used on all the interfaces between the finite elements. Fig 1.9 gives an example of production of fragments.

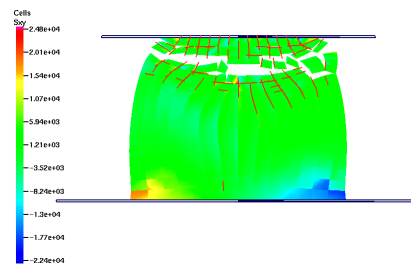


Figure 1.9. Fragmentation of a block

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## References

- Acary, V. (2001). Contribution la modélisation mécanique et numérique des édifices maçonnés . *Thesis*, Jean, M., supervisor, Université de la Méditerranée, Marseille.
- Baudriller, H. (2003). Approche micro-mécanique et modélisation de l'usure : application aux biomatériaux des prothèses totales de hanche. *Thesis*, Chabrand, P., supervisor, Univ. de la Méditerranée, Marseille.
- Alart, P., and Curnier, A. (1991). A generalised Newton method for contact problems with friction. *J. Mech. Théor. Appliq.*, (7): 67–82.

- Bretelle, A.-S., Cocou, M., and Monerie, Y. (2001). Unilateral contact with adhesion and friction between two hyperelastic bodies. *Int. J. Eng. Sci.*, (39): 2015–2032.
- Cangémi, L. (1997). Frottement et adhérence : modèle, traitement numérique et application à l’interface fibre/matrice, *Thesis*, Raous, M., supervisor, Université de la Méditerranée, Marseille.
- Cocou, M., Pratt, E., and Raous, M. (1996). Formulation and approximation of quasistatic frictional contact. *Int. J. Eng. Sci.*, (34)7: 783–798.
- Cocou, M., Cangémi, L., and Raous, M. (1997). Approximation results for a class of quasistatic contact problem including adhesion and friction. In Argoul, A., Frémond, M. and Nguyen, Q.S., editors, *Variation of domains and free-boundary problems in solids mechanics*, Solid Mechanics and its Applications, Number 66, Kluwer, 211–218.
- Cocou, M., and Rocca, R. (2000). Existence results for unilateral quasistatic contact problems with friction and adhesion. *Math. Model. Num. Anal.*, (34)5: 981–1001.
- Frémond, M. (1987). Adhérence des solides. *J. Méc. Thé. Appl.* (6)3, 383–407.
- Frémond, M. (1988) Contact with adhesion. In Moreau, J.-J. and Panagiotopoulos, P.D., editors, *Non Smooth Mechanics and Applications*, CISM Courses and lectures, Number 302, Springer, 177–221.
- Chaboche, J.-L., Feyel, F., and Monerie, Y. (2001). Interface debonding model : a viscous regularization with a limited rate dependency. *Int. J. Solids Structures*, (38)18: 3127–3160.
- Jean, M. (1999). The Non Smooth Contact Dynamics method. *Computer Meth. Appl. Mech. and Engrg.*, (177)3-4: 235–257.
- Jean, M., Acary, V. and Monerie, Y. (2001). Non-smooth contact dynamics approach of cohesive materials. *Phil. Trans. R. Soc. Lond. A*, (359): 2497–2518.
- Karray, M. A., Barbarin, S. and Raous, M. (submitted). Liaison béton-acier par un modèle d’interface couplant adhésion et frottement.
- Latil, J.-C. and Raous, M. (1991). Module GYPTIS 1.0 - Contact unilatéral avec frottement en mécanique des structures. *Note interne LMA*, Number 132, CNRS.
- Monerie, Y., Raous, M., Leroy, F.-H., Sudre, O., Feyel, F., and Chaboche, J.-L. (1998). Comparaison de lois d’interface fibre/matrice sur la base d’un modèle uni-axial d’expérience de micro-indentation. In Lamon, J. and Baptiste D., *XI J. Nat. Composites*, AMAC publisher, 565–574.
- Monerie, Y., and Raous, M. (2000). A model coupling adhesion to friction for the interaction between a crack and a fibre-matrix interface. *Zeitschrift für Angewandte Mathematik und Mechanik*, (80): 205–209.

- Monerie, Y. (2000) Fissuration des matériaux composites : rôle de l'interface fibre-matrice, *Thesis*, Raous, M., supervisor, Université de la Méditerranée, Marseille.
- Monerie, Y., and Acary, V. (2001). Formulation dynamique d'un modèle de zone cohésive tridimensionnel couplant endommagement et frottement. *Rev. Eur. Elém. Finis*, (10): 489–504.
- Moreau J.-J. (1994). Some numerical methods in multi-body dynamics: application to granular media. *Eur. J. of Mechanics*, (A13): 93–114.
- Moreau, J.-J. (1988). *Unilateral contact and dry friction in finite freedom dynamics*. In Moreau, J.-J. and Panagiotopoulos, P.D., editors, *Non Smooth Mechanics and Applications*, CISM Courses and lectures, Number 302, Springer, 1–82.
- Moreau, J.-J. (1999). Some basics of unilateral dynamics. In Pfeiffer, F. and Glocker, P.H., editors, *Unilateral Multibody Contacts*, Solid Mechanics and its applications, Number 72, Kluwer, 1–14.
- Raous, M. and Barbarin, S. (1992). Preconditioned conjugate gradient method for a unilateral problem with friction. In Curnier, A., Editor, *Contact Mechanics*, Presses Polytechn. et Univ. Romandes, 423–432.
- Raous, M., Cangémi, L., and Cocou, M. (1997). Un modèle couplant adhérence et frottement pour le contact unilatéral entre deux solides déformables. *C. R. Acad. Sci. Paris, Série II b* (329): 503–509.
- Raous, M. (1999). Quasistatic Signorini problem with Coulomb friction and coupling to adhesion. In Wriggers, P. and Panagiotopoulos, P.D., editors, *New developments in contact problems*, CISM Courses and lectures, Number 383, Springer, 101–178.
- Raous, M., Cangémi, L., and Cocou, M. (1999). Consistent model coupling adhesion, friction and unilateral contact. *Comp. Meth. in Appl. Mech. and Engrg.*, (177)3-4: 383–399.
- Raous, M. and Monerie, Y. (2002). Unilateral contact, friction and adhesion : 3D cracks in composite materials. In Martins, J.A.C. and Monteiro Marques M.D.P., eds, *Contact mechanics*, Kluwer, 333–346.
- Raous, M., Monerie, Y. and Rosu, I. (2002). Dynamic formulation in contact mechanics : flutter instabilities and adhesion problems. *EuroMech 434 Contact mechanics of coated bodies*, Moscow, May 2002.
- Raous, M., Belloeil, V. and Rosu, I. (2004). Modélisation de l'adhésion par collage, *LCPC contract report*, LMA, 85 pages.
- Talon, C. and Curnier, A. (2003) A model of adhesion coupled to contact and friction. *European Journal of Mechanics A/Solids*, (22): 545–565.
- Vola, D., Pratt, E., Jean, M., and Raous, M. (1998). Consistent time discretization for dynamical frictional contact problems and complementarity techniques. *Rev. Eur. Eléments Finis* (7): 149–162.