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## PDE-Driven Spatiotemporal Disentanglement

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## Motivation

- Disentanglement improves interpretability and prediction
- Prior spatiotemporal disentanglement works are often complex and do seldom analysis of its meaning
- We aim at grounding spatiotemporal disentanglement on stronger foundations

## Separation of Variables as Disentanglement

- Consider the heat equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad u(0, t) = u(L, t) = 0, \quad u(x, 0) = f(x),$$

with separable solutions:

$$u(x, t) = \underbrace{\mu \sin\left(\frac{n\pi}{L}x\right)}_{\phi(x)} \underbrace{\exp\left(-\left(\frac{cn\pi}{L}\right)^2 t\right)}_{\psi(t)} = \xi(\phi(x) \times \psi(t))$$

- This method **separates factors of variations** and can be generalized and applied to numerous systems

## Separation of Variables for Observations

- **State**  $u_v: (x, t) \mapsto u_v(x, t)$ , observations  $v = (v_{t_0}, \dots, v_{t_1})$  corresponding to a **spatial measurement** of  $u$ :

$$v_t = \zeta(u_v(\cdot, t))$$

- Following the **functional separation of variables**, we seek  $\phi, \psi, U, \xi$  and  $\zeta$  such that:

$$z = \xi(\phi(x), \psi(t)), \quad u(x, t) = U(z), \quad v_t = \zeta(u(\cdot, t)),$$

with associated ODEs or PDEs on  $\phi, \psi$  and  $U$

- We abstract the unknown spatial coordinates:

$$v_t = (\zeta \circ U \circ \xi)(\phi(\cdot), \psi(t)) = D(\phi, \psi(t))$$

- In practice, we learn vectorial  $\phi \equiv S$  and  $\psi \equiv T_t$
- $S$  and  $T_{t_0}$  are inferred with encoders  $E_S$  and  $E_T$  from conditioning frames:

$$V_\tau(t_0) = (v_{t_0}, \dots, v_{t_0+\tau})$$

## Prediction Constraints

- $T \equiv \psi$  is driven by an ODE:  $\frac{\partial T_t}{\partial t} = f(T_t)$
- Forecasting and alignment losses:

$$\mathcal{L}_{\text{pred}} = \sum_t \|\hat{v}_t - v_t\|_2^2$$

$$\mathcal{L}_{\text{AE}} = \left\| D\left(S, E_T(V_\tau(t'))\right) - v_{t'} \right\|_2^2$$

## Disentanglement Constraints

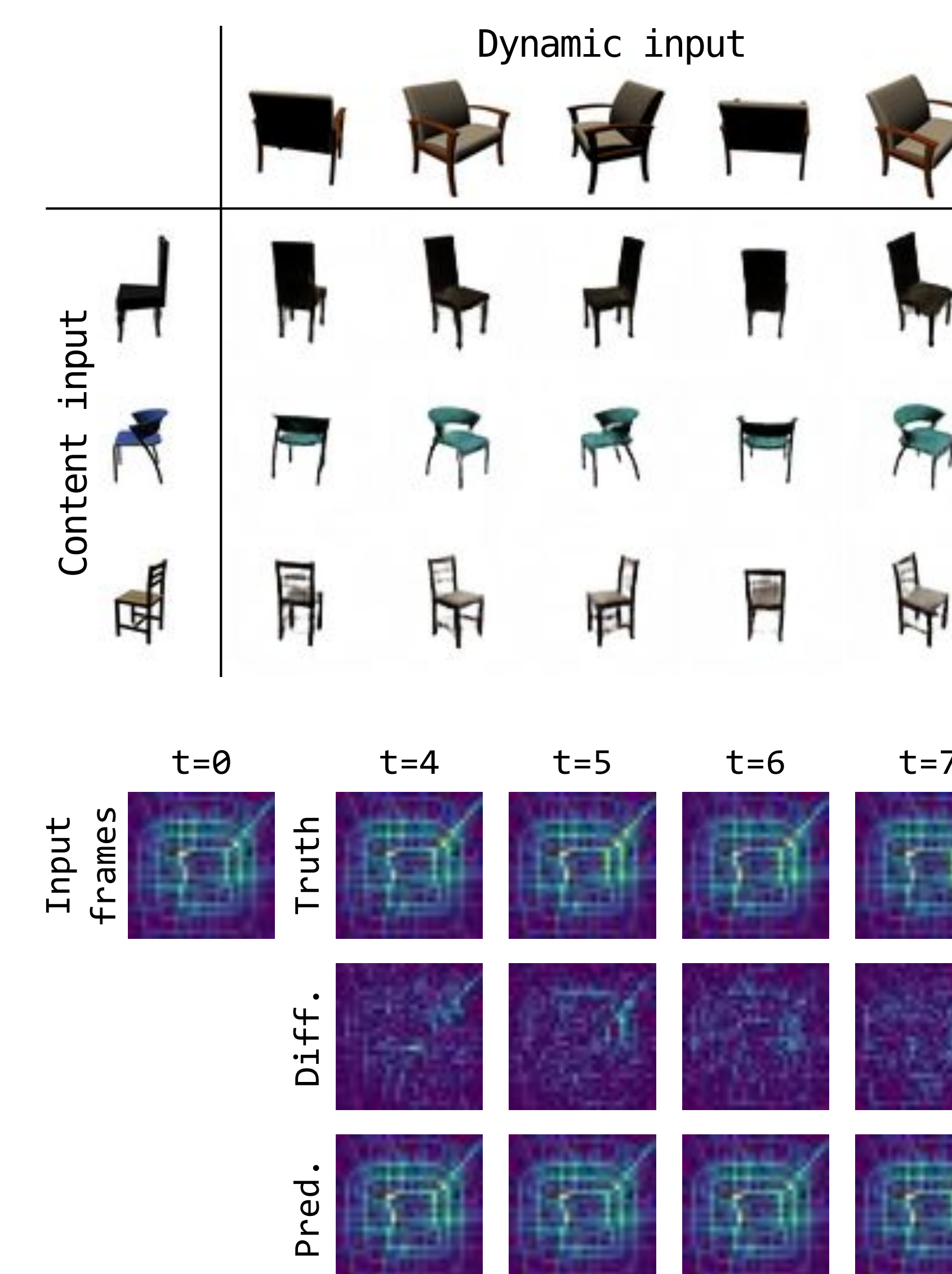
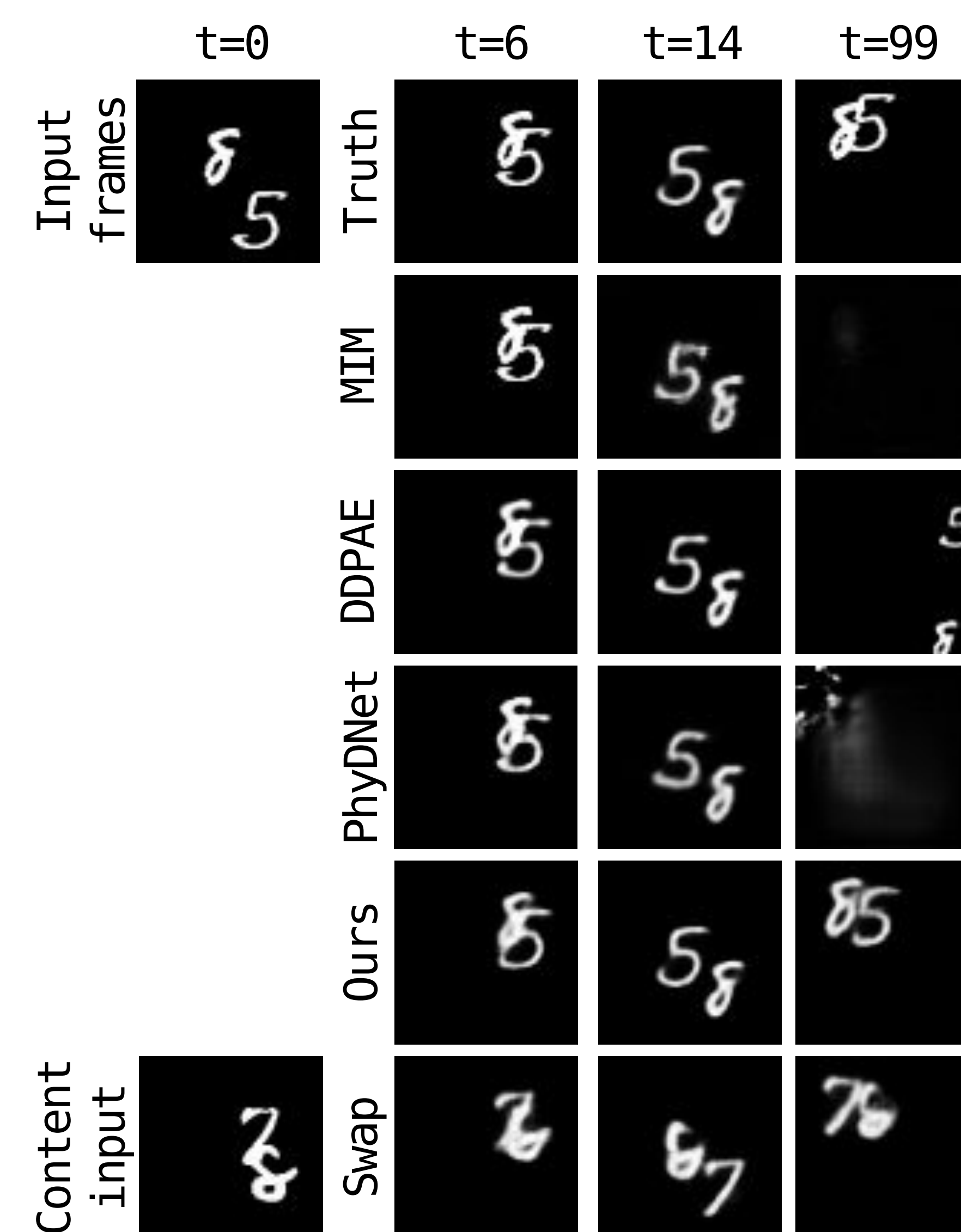
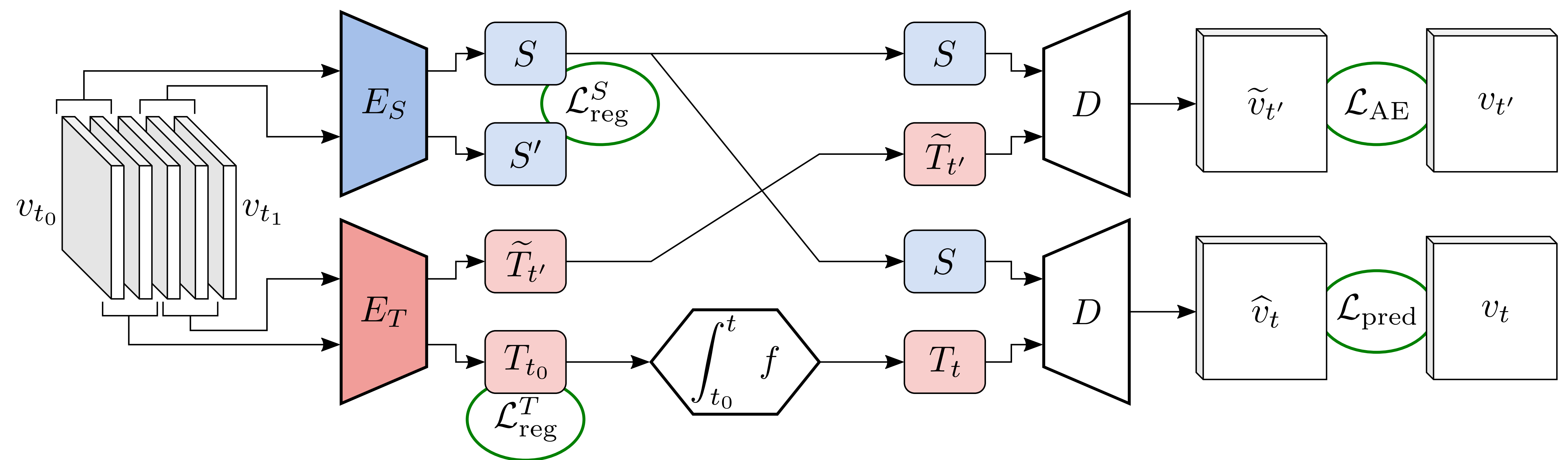
- From a strict  $S$  invariance constraint to a weaker one to take into account variations of observable content:

$$\frac{\partial E_S(V_\tau(t))}{\partial t} = 0$$

$$\Rightarrow \mathcal{L}_{\text{reg}}^S = \left\| E_S(V_\tau(t_0)) - E_S(V_\tau(t_1 - \tau)) \right\|_2^2$$

- Disentanglement loss:

$$\mathcal{L}_{\text{reg}}^T = \|T_{t_0}\|_2^2 = \|E_T(V_\tau(t_0))\|_2^2$$



Models	SST	Mov.	MNIST	TaxiBJ
	Pred.	Pred.	Swap	Pred.
	MSE	SSIM		MSE
PhyDNet	1.27	0.3878	0.2839	41.9
DDPAE	—	0.6446	0.6378	—
MIM	0.91	0.6529	—	42.9
Ours	<b>0.86</b>	<b>0.7990</b>	<b>0.7713</b>	<b>39.5</b>
Ours (no $S$ )	0.95	0.6707	—	43.7

