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# Optimization of monochromatic primaries in RGB system: on the specific question of purples

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## Abstract

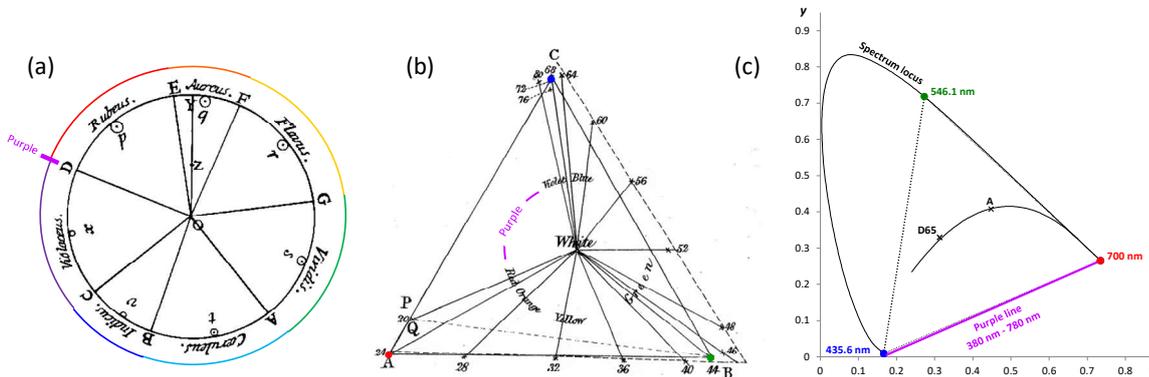
*In the CIE1931 xy chromaticity diagram, the set of visible colors is delimited by the spectrum locus representing the monochromatic radiations and by the purple line, set of non-spectral colors obtained by binary mixing between monochromatic radiations at the two extremities of the visible range. In contrast with the spectrum locus, which represents, for most wavelengths, bright and maximally saturated colors, the purple line rather represents colors at the perception limit, corresponding to spectral powers with very low luminous efficacy. The notion of "practical" purple lines is discussed. It comes down to the definition of the blue and red monochromatic primaries of RGB light emitting systems. The choice of monochromatic primaries allowing the widest gamut while maintaining a good luminous efficacy over its whole perimeter is still an open question: by choosing red and blue primaries further in the extremes of the visible spectrum, the gamut size is increased but the luminous efficacy of the primaries, thereby of all reproducible colors, is decreased. As optimal trade-offs, we suggest two RGB systems favoring either the gamut size, or the luminous efficacy.*

## 1. Introduction

Created in 1931 by the CIE, the XYZ color space is a 3D space representing all perceptible colors. Simultaneously was created the xy chromaticity diagram, a 2D representation resulting from a projective transformation of the XYZ color space discarding the luminance component. This latter is suitable for lighting systems for which lightness makes no sense. For example, by decreasing the power of a white source, one still obtains a source of white color not grey. Its well-known shape is delimited by the *spectrum locus*, line drawn by the chromaticity values attached to the visible monochromatic radiations, and by the *purple line*, straight line joining the chromaticity values attached to the two extremities of the visible spectrum. Owing to its ingenious mathematical construction, this diagram has the interesting property to represent the chromaticities of light stimuli obtained by additive mixing of two primary stimuli as a segment of straight line, whose extremities are the chromaticities of the two primary stimuli, a linearity inherited from the linearity of the XYZ color space in respect to additive color mixing. These mathematical properties probably explain the success, still well observable today, that this diagram has met for the color characterization of displays, video projectors, and any other RGB system based on additive color mixing of three primary red, green and blue lights. Even though their color gamut is actually a 3D volume, more precisely an hexahedron, in the XYZ color space, its projection (a triangle) in the xy chromaticity diagram is often preferred, probably because a 2D diagram is more convenient to display than a 3D volume, and also because the triangle in the chromaticity diagram exhibits well the color reproduction quality in terms of chroma for each hue.

The widest possible gamut is generally sought so that the maximum chroma can be achieved for each hue angle. For RGB systems, increasing the size of the triangular gamut can hardly be done otherwise than by choosing three monochromatic primaries. The choice of the spectral color green will strongly influence the system's ability to reproduce with a high chroma the colors of hue ranging from orange to cyan through greens, favouring either yellows or turquoises. The choice of the blue and red spectral colors will mainly affect the system's ability to reproduce blue, purple, red and other shades with high chroma. To increase the gamut coverage, one is tempted to take the red and blue primaries at the extremes of the visible spectrum, but as the luminous efficacy is very low at these extreme wavelengths, some colors, especially white, cannot be reproduced with sufficient luminous efficacy. This is an obvious technical and energy issue for displays or projectors. A trade-off has therefore to be found between gamut size and luminous efficacy. In this paper, we detail the solutions that have been proposed and try to better describe the criteria for achieving an optimal solution.

Enlarging the gamut is therefore equivalent to trying to achieve the highest chroma possible for each hue. It is worth considering the best way to quantify this color attribute. However, even with the component Y representing luminance added to the chromaticity diagram, CIE xyY color space is not a color appearance model, in contrast with CIELab or CIECAM02 spaces. The notions of hue, saturation and lightness are not directly accessible. To overcome this difficulty, the concepts of dominant wavelength (for hue) and excitation purity (for chroma) have been defined. The dominant wavelength is the wavelength indicated by the meeting point S between the spectrum locus and the line meeting the chromaticity points C and W representing respectively the considered color and a white reference color selected beforehand. The excitation purity is then the ratio of Euclidian distances CW/SW in the diagram. Spectral colors therefore have an excitation purity unity and a dominant wavelength equal to that of their radiation. These notions make less sense for colors close to the purple line, whose hue does not benefit from a spectral color to define them. By convention, a negative dominant wavelength is assigned corresponding to the complementary spectral color, i.e. on the same segment and opposite to the reference white. The excitation purity is calculated in the same way, taking the purple line as boundary with unit purity. This solution is not satisfactory since colors on the purple line are practically imperceptible because it results from the mixture of two red and blue lights at the limit of the locus spectrum, which are themselves almost imperceptible. This difference between purples and other hues runs counter to the main ideas that have accompanied the evolution of color representation systems since Newton, notably the idea of a continuous color circle, where all hues are treated similarly.



**Figure 1.** (a) Newton's chromatic disc [1]. The purples are on the OD segment that connects the two ends of the spectrum. (b) Maxwell's color triangle [3] with monochromatic primaries noted 24, 44 and 68. An angular sector is dedicated to purple between the two extreme wavelengths noted 20 and 80. (c) CIE  $xy$  chromaticity diagram. The purple line connects the extreme wavelengths of the visible: 380 nm and 780 nm. The monochromatic primaries of the CIE1931 RGB color space correspond to the wavelengths 700 nm, 546.1 nm and 435.8 nm.

We therefore provide a history of these ideas, starting in Section 2 with the place of non-spectral purple colors in color representation systems. We present in Section 3 the difficulty of determining the maximum chroma location for each hue, including purples. In Section 4, we discuss the choice of monochromatic primaries in existing RGB systems for which a practical purple line is defined as the line joining the blue and red monochromatic primaries. In the trade-off between gamut coverage and luminous efficacy, we highlight two new RGB systems with monochromatic primaries. In Section 5, the proposed RGB system favours a high luminous efficacy to obtain the reference white by ternary mixing between the three primaries. In Section 6, we present an alternative that allows for a wider color gamut and a constant and not zero luminous efficacy on the line connecting the blue and red primaries.

## 2. On purple in the History of color science

In 1665, the Great Plague epidemic in England forced the universities to close. Students had to confine themselves into their homes. Isaac Newton, then student at Cambridge, went back home to Woolsthorpe. History does not say whether Newton gained weight during his confinement, but opinions agree with the fact that he began to revolutionize physics at that time. He made his first studies on the decomposition of white light through the prism. He then demonstrated the composite nature of white light and the fact that the colors coming out of the prism are related with the elementary components of light. By desired analogy with the Dorian scale, he divided the spectrum into 7 intervals, 5 main colors (tones) to which he added orange and indigo (semitones). The names of the colors did not change between Newton's various writings except at one end of the spectrum where Newton first chose *purple* before opting for *violet*. In a few pages in *Opticks* published in 1704 [1], Newton sketched the beginnings of colorimetry. The idea was not the one of a physicist. It was more a matter of intuition that the colors of the spectrum form a continuum. In this way, he connected the two ends of the spectrum and created a color disc (Figure 1.a). The periphery was made of the spectral colors while white was at the centre. Newton proposed a geometric method for determining the perceived color of a mixture to which he associated what we would today call the dominant hue and saturation. On the diagram in Figure 1 is shown

an example of mixture made of the seven colors according to the proportions indicated by the size of the small circles on the periphery of the disc. The centre of gravity is obtained in Z. The line OZ cuts the disc periphery in Y: the hue of the mixture is an orange color that tends slightly towards red. The OZ distance gives an indication of the saturation of the color: here, less than half compared to the corresponding spectral color (OY). Maximum saturation is logically achieved by the spectral colors at the periphery of the disc. It should be noted that the non-spectral colors, finally called purples, are not represented on the circumference of the circle. However, Newton did not ignore them completely and placed them on the OD segment as a result of mixing violet and red. In contrast to Newton's disc, current color classification systems such as Munsell or NCS grant purples a large angular sector of between 25 and 30 %.

In the 19th century, the English scientist Young suggests that our visual system should contain three types of particles, sensitive to the colors red, green or violet [2]. This trichromacy principle allowed to represent all colors perceptible in an equilateral triangle at the vertices of which are located the three primaries. A few decades later, Maxwell developed a "light box" that allows to visually equalize chromatic area by mixing spectral colors [3]. He chose three primaries labelled 24, 44 and 68, corresponding to wavelengths respectively around 631 nm, 529 nm and 457 nm (Figure 1.b). This shows that the spectral colors are slightly outside the equilateral triangle defined by the three primaries. Similar observation applies with non-spectral colors, even though Maxwell did not explicitly draw a "purple line" between the extreme wavelengths of his representation, labelled 20 and 80, corresponding to 664 nm and 435 nm. Naturally, Maxwell does not choose these ends of the spectrum as primaries because the perceived lights are too weak. The results obtained by visual equalization using these extreme wavelengths are not very satisfactory.

The CIERGB color space defined in 1931 from the work of Wright [4] and Guild [5] was finally a Maxwell triangle. After discussions, the CIE adopted the monochromatic primaries 700 nm, 546.1 nm and 435.8 nm. The two latter wavelengths are mercury emission lines. For red, 700 nm was chosen arbitrarily because there is no reference emission line in this range, but this was not a big issue since only L-cones are excited in this spectral domain and the perceived hue is stable.

The CIE defined the color matching functions that translate the amounts of red, green and blue lights needed to match with a monochromatic stimulus at any wavelength. In practice, each monochromatic light mixed with one primary (negative contribution) was visually equalized by mixing the other two primaries (positive contributions). The negative values generate computation issues at a time when computation was mainly done manually. The CIE thus proposed a linear transformation converting the RGB color space into the XYZ color space in which all tristimulus values are always positive. Consequently, allchromaticity values are also positive and contained within the triangle drawn by the chromaticity points attached to the virtual primary color stimuli X, Y and Z. In the  $xy$  chromaticity diagram, the set of perceptible colors is represented by a surface delimited by the *spectrum locus* for the spectral colors and by the purple line joining the coordinates of the monochromatic radiations of the two ends of the visible spectrum, 380 nm and 780 nm (Figure 1.c). Sometimes these standardized limits of the visible domain are even extended to 360 nm and 830 nm. In the chromaticity diagram, the two ends of the purple line correspond to monochromatic radiations between 360 and 380 nm for the blue end and between 700 and 830 nm for the red end. As with Maxwell, the purple line slightly differs from the line connecting the blue (435.6 nm) and red (700 nm) primaries of the CIE1931 RGB color space. Color annotations have been added to the original documents to make them easier to read.

The CIE  $xy$  chromaticity diagram retains Newton's distinction in *Opticks* between violet as the end of the visible spectrum and purple as a non-spectral color. In common parlance and in other color spaces, violet and purple are two shades of the same hue. The famous Tyrian purple that dressed the Roman emperors is close to red. The word "violet", probably derived from name of the flower, is more recent and designates colors closer to blue. In the 17th Century, François d'Aguilon was the first scientist to propose a color representation system based on mixtures [6]. The main hues were the red, blue and yellow triad, which would leave its mark Western culture for more than three centuries and is still a standard in art education today. Purple (*purpureus*) is presented as the result of the mixture between blue (*caeruleus*) and red (*rubeus*). Goethe choose same colors but arranged them into a chromatic circle [7]. Once again, *porpora* is placed between *blu* and *rosso*. There is a strong linguistic bias between French and English languages. French uses the word *violet* to designate a color obtained by mixing blue and red. This is the case, for example, in Chevreul's chromatic circle [8]. In Anglo-Saxon countries, the word *purple* is more common. It is, for example, one of the five main hues in Munsell's color space [9]. In the 19th century, advances in chemistry allowed the production of synthetic dyes based on aniline, which bore different trade names such as mauve, fuchsia or magenta. Marketed in 1859 in reference to the Battle of Magenta, which had taken place one year earlier, the color magenta was to impose itself as the primary of the ideal subtractive synthesis with cyan and yellow. For a symmetrical presentation, it is also the secondary colors in red, green, blue additive synthesis (and *vice versa*). Magenta has replaced violet and purple!

### 3. Maximum color chroma in the CIE1931 $xy$ chromaticity diagram

As mentioned in the introduction, the  $xy$  chromaticity diagram is not suitable for quantifying precisely the chroma of a color.

Nevertheless, *excitation purity* is a notion that comes close to it. It can be expressed as:

$$p_e = \frac{y - y_w}{y_b - y_w} \quad (1)$$

where  $y$ ,  $y_b$  and  $y_w$  are the CIE 1931 chromaticity coordinates  $y$  for respectively the color, the monochromatic or boundary color, and the reference white. The notions of excitation purity and dominant wavelengths are indeed relative to a chosen white light. This white light is generally chosen from among the illuminants defined by the CIE as equal-energy illuminant E ( $x_e, y_e$ ) = (1/3, 1/3), 6504 K daylight illuminant D65 ( $x_{D65}, y_{D65}$ ) = (0.31271, 0.32902), or 2856 K black body illuminant A ( $x_A, y_A$ ) = (0.44758, 0.40745).

Less widely used today, the CIE also defined *colorimetric purity* with a slightly different normalization:

$$p_c = \frac{y_b}{y} p_e = \frac{y_b}{y} \cdot \frac{y - y_w}{y_b - y_w} \quad (2)$$

It should be noticed that  $x$  coordinates may substitute the  $y$  coordinates in equations (1) and (2) when  $y_b$  and  $y_w$  are very close.

Both excitation purity and colorimetric purity vary between 0 for the reference white and 1 for the limits of the perceptible colors. Thus, by convention, the non-spectral colors of the purple line have a purity 1. This choice does not really make sense because the colors of the purple line are almost imperceptible.

This issue is perfectly stated in a paper by Pridmore [10]. To remedy this issue, he suggested calculating the luminous efficacy for obtaining the reference white by a binary mixture between a monochromatic radiation and its spectral complementary, which is located at the intersection between the spectrum locus and the segment joining the coordinates of the radiation and those of the selected white. The luminous efficacy maxima are obtained at the visible ends for  $442 \pm 1$  nm and  $613 \pm 1$  nm depending on whether illuminant D65 or illuminant A for white light is chosen. The line connecting these two wavelengths of the spectrum locus is substantially offset from the purple line. Pridmore proposed to keep the definitions of excitation and colorimetric purities given by the equations (1) and (2). He simply replaced the purple line with this new line (442 nm – 613 nm) as the unit and maximum purity border.

Another approach is to consider non-fluorescent surface colors and determine the limits of the color solid. For this purpose, Schrödinger [11] defined optimal colors, whose spectra only take on values 0 and 1 with a maximum of two transitions between these two values. MacAdam [12] [13] defined the boundary of the optimal color solid in the CIE $xY$  color space. For non-spectral colors, this boundary meets the purple line only for very low  $Y$  luminance values. In 1980, Pointer collected the colors of a big set of non-fluorescent surfaces [14]. Whereas MacAdam's limits set theoretical boundaries for optimal surface colors, Pointer's gamut is an attempt to establish the empirical limits of real surface colors. Although approximated, Pointer's gamut gives an idea of the colors that a color reproduction system should be able to render. **The most saturated purples in the Pointer gamut are noticeably away from the purple line in the  $xy$  chromaticity diagram.** It is worth noting that both MacAdam limits and Pointer's gamut are relative to the chosen illuminant and the relative luminance  $Y$ .

#### 4. RGB monochromatic primaries

In his analysis, Pridmore “forgot” that he achieved a third maximum luminous efficacy in the green part of the spectrum, at 531.5 nm [10]. He actually defined a system of three monochromatic RGB primaries with the line connecting the blue and red primaries constituting a “practical” purple line. There is an infinite number of solutions to choose a triplet of wavelengths in the visible range. One condition is that the reference illuminant(s) must be reproducible by additive mixing. Therefore, the  $xy$  coordinates of this (these) illuminant(s) must be included into the triangle at the vertices of which the coordinates of the primaries are located. Other considerations come into play.

In his article [3], Maxwell chooses the three primaries (631 nm, 529 nm, 457 nm) “because they are well separated from each other on the scale, and because the colour of the spectrum at these points does not appear to the eye to vary very rapidly”. For the CIE1931 RGB color space (Fig. 1.c), the choice of mercury lines for the blue (435.8 nm) and green (546.1 nm) primaries was made because they are standards in metrology. The practical realisation of monochromatic primaries is also to be taken into consideration. Therefore, laser sources seem to be the most appropriate solution even if the spectral purity of the LED emissions would make it possible to approach the chosen configurations.

More recently, RGB color spaces with monochromatic primaries have been developed to quantify wide gamut devices. To estimate the extension of a color gamut, it is usual to give the percentage of coverage that this gamut represents in the  $xy$  chromaticity diagram compared to all perceptible colors. Counter-intuitively, the use of the perceptually non-uniform  $xy$  diagram allows a satisfying estimate of the size of color gamuts [15]. The Adobe Wide Gamut RGB color space (700 nm, 525 nm, 450 nm) developed by Adobe System has a very wide gamut covering 71.5 % of all chromaticities. It should be noted that the widest gamut (73.8 % coverage) is obtained with the triplet (700 nm, 518 nm, 431 nm). Here, we compare it with the RGB system (780 nm, 518 nm, 380 nm) which includes the purple line and has a coverage rate of 73.6%.

REC2020 is the standard for Ultra High Definition Television (UHD-TV). The primaries (630 nm, 532 nm, 467 nm) have been selected in order to cover Pointer’s gamut (99.9 % in the  $xy$  chromaticity diagram). The 100% optimum would be obtained with a green primary at 527 nm but more difficult to obtain in practice with LEDs or laser technologies. Although it is fortuitous, the primaries in the REC. 2020 standard are quite close to the ones proposed by Maxwell in the 19th Century.

Another approach is to consider human vision at the retinal level. The cones of types L, M and S have spectral sensitivity curves presenting a peak at 569 nm, 541 nm and 448 nm, respectively. These curves largely overlap each other, which explains why it is not possible to choose the wavelengths of the maxima as primaries. Thornton noted that whatever monochromatic RGB primaries are chosen, the three color matching functions preferentially peak at three wavelengths called Prime Colors [16]: 603 nm, 538 nm and 446 nm, with little dispersion on the wavelengths displayed in the literature. Worthey shows in an original way the existence of these Prime Colors [17] by using the Cohen works on the “locus of unit monochromats” [18], 3D representation of the “orthonormal opponent color functions”. Worthey explicitly defined a line of practical purples joining the blue and red Prime Colors. Pridmore’s primaries, although based on a different approach, are close to the ones of Prime Colors.

Table 1 shows the properties of the main monochromatic RGB systems. For some of them, their triangular representation in the  $xy$  chromaticity diagram is given in Figures 2.a and 2.b. Many practical purple lines can be obtained according to red and blue primaries selected by joining their representation in the diagram with a straight line.

In the following two Sections, we propose two monochromatic RGB systems using the REC2020 standard as a reference. In Section 5, we optimize the luminous efficacy for the reference white by maintaining the gamut coverage value of the RGB REC2020 system. In Section 6, we keep the blue primary of the REC2020 standard and we get a wider gamut by having a practical purple line with constant luminous efficacy.

#### 5. High luminous efficacy RGB gamut

We take up the idea of Pridmore, which retains the primaries that enable the highest luminous efficacy for the reference white to be achieved. But in contrast with Pridmore who considers a binary mixture between each primary and its complementary, we consider the ternary mixture between the three primaries. We call  $c_R$ ,  $c_G$  and  $c_B$  the relative concentrations in R, G and B primaries yielding the reference white color. The luminous efficacy of a monochromatic radiation being  $K_m V(\lambda)$  with  $K_m = 683 \text{ lm/W}$ , the luminous efficacy  $\eta_w$  of the white color is given by:

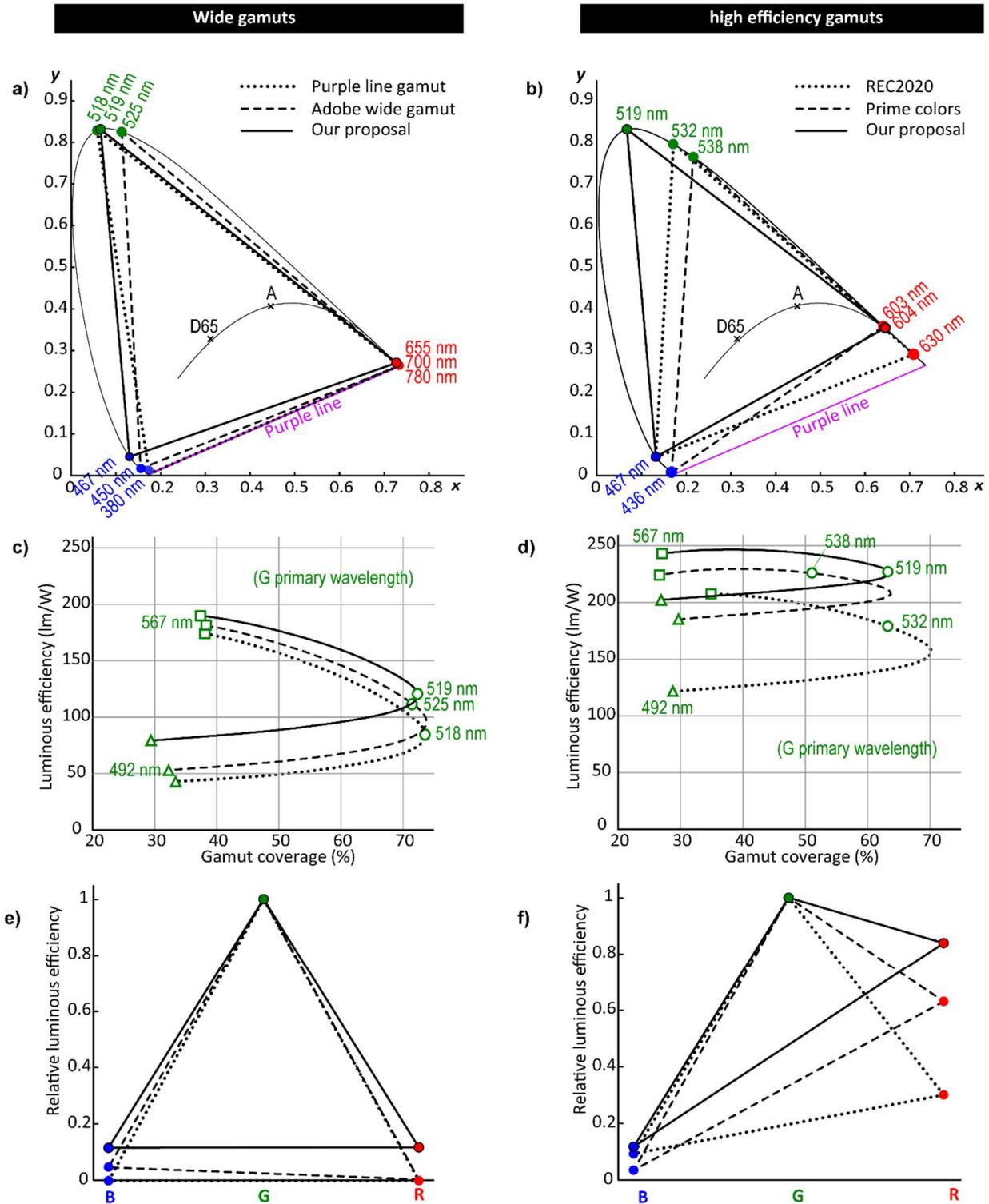
$$\eta_w = K_m [c_R V(\lambda_R) + c_G V(\lambda_G) + c_B V(\lambda_B)] \quad (3)$$

where  $c_R + c_G + c_B = 1$ .

**Table 1. Comparison of RGB systems<sup>a</sup>**

RGB system	RGB primaries (nm)			Gamut coverage (%)	Luminous efficacy of D65 or A white (lm/W)		Relative primary luminous efficacy (%)	
	$\lambda_R$	$\lambda_G$	$\lambda_B$		$\eta_{D65}$	$\eta_A$	$V(\lambda_R)/V(\lambda_G)$	$V(\lambda_B)/V(\lambda_G)$
Maxwell [3]	<b>631</b>	<b>529</b>	<b>457</b>	65.9	166	205	<b>30.0</b>	<b>6.2</b>
CIERGB	<b>700</b>	<b>546</b>	<b>436</b>	56.2	151	158	<b>0.4</b>	<b>1.8</b>
Pridmore primaries [10]	<b>613</b>	<b>532</b>	<b>442</b>	59.7	197	275	<b>52.6</b>	<b>2.9</b>
Purple line gamut	<b>780</b>	<b>518</b>	<b>380</b>	73.6	85	90	<b>0.0</b>	<b>0.0</b>
Adobe wide gamut	<b>700</b>	<b>525</b>	<b>450</b>	71.5	112	115	<b>0.5</b>	<b>4.8</b>
Wide gamut	<b>655</b>	<b>519</b>	<b>467</b>	72.4	121	130	<b>11.8</b>	<b>11.6</b>
REC2020	<b>630</b>	<b>532</b>	<b>467</b>	63.3	179	217	<b>29.9</b>	<b>9.1</b>
Thornton prime colors [16]	<b>603</b>	<b>538</b>	<b>446</b>	51.1	226	326	<b>63.0</b>	<b>3.3</b>
High efficacy	<b>604</b>	<b>519</b>	<b>467</b>	63.3	227	316	<b>83.9</b>	<b>11.6</b>

<sup>a</sup>The systems that we propose are in bold. The corresponding practical purple lines connect the red and blue primaries with a straight line.



**Figure 2.** Comparison between our proposed RGB systems and two existing wide gamut systems (column on the left) and two existing high efficacy, gamuts (column on the right). **a-b)** Gamut representation in the CIE  $xy$  chromaticity diagram. **c-d)** Parametric curve representing the gamut coverage and the luminous efficacy  $\eta_{D65}$  of a D65 white obtained by additive mixing of the three monochromatic primaries where R and B are fixed (indicated in the chromaticity diagram) and G is varied from 492 nm (triangles) to 567 nm (squares) through the G primary indicated in the chromaticity diagram (circles). **e-f)** Relative luminous efficacy of the monochromatic RGB primaries with respect to the one of the green primary.

For all RGB systems, the luminous efficacies  $\eta_{D65}$  and  $\eta_A$  for white corresponding to illuminant D65 and illuminant A respectively, are given in Table 1. There is a trade-off to be found between high luminous efficacy  $\eta_w$  and wide gamut coverage. Many choices are possible. Here, we impose a minimum coverage, and it seemed relevant to us to choose the one of the REC2020 standard, i.e., 63.3%. We then consider all sets of three monochromatic primaries, each of them being an integral number in nm, and keep the set for which the luminous efficacy  $\eta_{D65}$  is the highest while achieving this coverage. We obtain the primaries  $\lambda_R = 604$  nm,  $\lambda_B = 519$  nm, and  $\lambda_G = 407$  nm, for which the luminous efficacy  $\eta_{D65} = 227$  lm/W is to be compared with the value  $\eta_{D65} = 179$  lm/W for the RGB REC2020 standard.

Figures 2.c and 2.d show the luminous efficacy  $\eta_{D65}$  as a function of the gamut coverage where the blue and red primaries are fixed. The green primary wavelength varies from 492 nm (triangle dots) to 567 nm (square dots) and each circle corresponds to the green primary of the corresponding RGB system. The gamut coverage goes through a maximum for  $\lambda_G = 518 \pm 1$  nm for all RGB systems. It can be seen from Figure 2.d that the choice of a practical purple line between  $\lambda_B = 467$  nm and  $\lambda_R = 604$  nm optimises the luminous efficacy criterion.

## 6. Wide RGB gamut

Another way of representing an RGB system is to express the luminous efficacy of the primaries relative to that of the green primary, which is the highest. Table 1 lists for each RGB system the relative luminous efficacies in % for the blue  $V(\lambda_B)/V(\lambda_G)$  and red  $V(\lambda_R)/V(\lambda_G)$  primaries. The triangle representation in Figures 2.e and 2.f shows the relative luminous efficacy obtained by binary mixing between two primaries, in particular between blue and red for the purples.

In the case of wide gamuts (Fig. 2.e), this representation confirms that the purple line ( $\lambda_B = 380$  nm,  $\lambda_R = 700$  nm) corresponds to aluminous efficacy zero. With a relative luminous efficacy of less than 4.8% on the line joining  $\lambda_B = 450$  nm and  $\lambda_R = 700$  nm, Adobe's wide gamut system is not more interesting. RGB systems with higher luminous efficacy (Fig. 2.f) correct this defect. The system we have proposed in Section 5 presents the highest luminous efficacies for the blue  $\lambda_B = 467$  nm and red  $\lambda_R = 604$  nm primaries corresponding to respectively 11.6% and 83.9% relative to that of the green primary. Nevertheless, for the three RGB systems shown in Fig. 2.f, there is an imbalance between the red primary and the much less efficient blue primary. We propose another RGB system by keeping the blue primary of the REC2020 standard,  $\lambda_B = 467$  nm. It is indeed the blue primary with the longest wavelength and therefore the highest luminous efficacy among the presented RGB systems (Table 1). We thus choose a red primary  $\lambda_R$  with same luminous efficacy, satisfying  $V(\lambda_R) = V(\lambda_B = 467 \text{ nm})$ , and we found  $\lambda_R = 655$  nm. Finally, we determine the green primary  $\lambda_G = 519$  nm that maximizes the gamut coverage (Fig. 2.c). This RGB system has a coverage of 72.4%, higher than the one of the REC2020 standard (63.3%), and even higher than the one of Adobe wide gamut system (71.5%). Notice that with significantly different primaries, the luminous efficacy ( $\eta_{D65} \approx 120$  lm/W) is very close to the one of the Adobe wide gamut system (Table 1). The advantage of this RGB system is that the line connecting the blue and red primaries has a constant and significant luminous efficacy, corresponding approximately to 11.7% of the one of the green primary.

## 7. Conclusion

The purple line in the chromaticity diagram connects the two ends of the spectrum locus. This is a borderline in the human color gamut, but it also corresponds to colors that are almost imperceptible due to a very weak luminous efficacy. It is therefore interesting for color practitioners to define another, more practical purple line. This one is defined by linking the blue and red monochromatic primaries of an RGB system. Various criteria can be used, while a trade-off is generally to be found between a wide coverage rate and high luminous efficacy for the white generated by ternary mixing between the primaries. We thus propose two RGB systems:

- $\lambda_R = 604$  nm,  $\lambda_G = 519$  nm,  $\lambda_B = 467$  nm. For similar coverage as the REC 2020 RGB system, the luminous efficacy  $\eta_{D65}$  of the white color corresponding to the illuminant D65 is more than 25% higher.
- $\lambda_R = 655$  nm,  $\lambda_G = 519$  nm,  $\lambda_B = 467$  nm. By changing only the red primary in the previous system, a significantly higher coverage is obtained. The practical purple line has a constant and not negligible luminous efficacy (more than 11% of the one of the green primary).

However, the practicality of an RGB system with monochromatic primaries should not obscure the fact that there are colors outside the RGB triangle, in particular faintly perceptible purples. These practical considerations are therefore not suitable for artists working with color lights and exploring these limits of perception such as James Turrell, Olafur Eliasson or Adrien Lucca.

The next step would be to visualize iso-chroma curves in the CIE  $xy$  chromaticity diagram. Pridmore suggested redefining excitation and colorimetric purities in relation to the new practical purple line. This approach is pragmatic. However, it does not allow quantifying this purity of excitation for all colors between the line connecting the blue and red primaries and the 380 nm – 780 nm purple line. Above all, we must remind that the chromaticity diagram is not the appropriate space for chroma characterization. There are alternatives to the excitation purity, which are more in line with the perception of color [19]. Chroma could be evaluated in a selected color appearance model like CIECAM02 or  $J_{az}b_z$  [20]. It would then be possible to preserve the representation in the widely used  $xy$  chromaticity diagram and to discriminate the location of the most saturated colors from that of the perceptible colors, especially near the purple line.

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