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# Computational details for : “Differential Dynamic Programming for Maximum a Posteriori Centroidal State Estimation of Legged Robots”

François Bailly<sup>a,b,\*</sup>, Thomas Flayols<sup>a</sup>, Justin Carpentier<sup>c</sup> and Philippe Souères<sup>a</sup>

This document complements the paper entitled “Differential Dynamic Programming for Maximum a Posteriori Centroidal State Estimation of Legged Robots” [1]. The purpose of this work was to estimate the centroidal dynamics of legged robots by formulating a maximum a posteriori problem and solving it thanks to differential dynamic programming (DDP). In the following, the computations of the partial derivatives of the unoptimized value function ( $Q_k$ ) are provided for the DDP algorithm.

## APPENDIX I

### PARTIAL DERIVATIVES OF $Q_k$

- $Q_{xk} = \nabla_{\mathbf{x}_k} l_k + \nabla_{\mathbf{x}_k} \mathcal{V}_{k+1}(f(\mathbf{x}_k, \boldsymbol{\omega}_k)),$   
 $\nabla_{\mathbf{x}_k} l_k = 2 \frac{\partial(g(\mathbf{x}_k) - \mathbf{y}_k)^T}{\partial \mathbf{x}_k} \Sigma_{\boldsymbol{\eta}_k}^{-1} (g(\mathbf{x}_k) - \mathbf{y}_k),$   
 $\frac{\partial g(\mathbf{x}_k)}{\partial \mathbf{x}_k} = \frac{C(\mathbf{x}_k)}{\partial \mathbf{x}_k} \mathbf{x}_k + C(\mathbf{x}_k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2c_k \times & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \hat{=} \tilde{C}(\mathbf{x}_k),$   
 $Q_{xk} = 2\tilde{C}(\mathbf{x}_k)^T \Sigma_{\boldsymbol{\eta}_k}^{-1} (g(\mathbf{x}_k) - \mathbf{y}_k) + A^T \mathcal{V}'_x.$
- $Q_{\omega k} = \nabla_{\boldsymbol{\omega}_k} l_k + \nabla_{\boldsymbol{\omega}_k} \mathcal{V}_{i+1}(f(\mathbf{x}_k, \boldsymbol{\omega}_k)),$   
 $\nabla_{\boldsymbol{\omega}_k} l_k = \frac{\partial \|\boldsymbol{\omega}_k\|_{\Sigma_{\boldsymbol{\omega}_k}^{-1}}^2}{\partial \boldsymbol{\omega}_k} = 2\Sigma_{\boldsymbol{\omega}_k}^{-1} \boldsymbol{\omega}_k,$   
 $Q_{\omega k} = 2\Sigma_{\boldsymbol{\omega}_k}^{-1} \boldsymbol{\omega}_k + B^T \mathcal{V}'_x.$
- $Q_{xxk} = \nabla_{\mathbf{x}_k}^2 l_k + \nabla_{\mathbf{x}_k}^2 \mathcal{V}_{i+1}(f(\mathbf{x}_k, \boldsymbol{\omega}_k)),$

<sup>a</sup> Laboratoire de Simulation et Modélisation du Mouvement, Faculté de Médecine, Université de Montréal, Laval, QC, Canada LAAS-CNRS, 7 Avenue du Colonel Roche, F-31400 Toulouse, France

<sup>b</sup> LAAS-CNRS, 7 Avenue du Colonel Roche, F-31400 Toulouse, France

<sup>c</sup> Inria, Département d’informatique de l’ENS, École normale supérieure, CNRS, PSL Research University, Paris, France

\*corresponding author: francois.bailly@umontreal.ca

where, the element of  $\nabla_{\mathbf{x}_k}^2 l_k$  at the  $i^{th}$  row and  $j^{th}$  column is denoted by:

$$[\nabla_{\mathbf{x}_k}^2 l_k]_{ij} = \sum_{m=1}^Y \sum_{n=1}^Y [\Sigma_{\boldsymbol{\eta}_k}^{-1}]_{mn} \left( \frac{\partial \tilde{C}_{im}^T}{\partial \mathbf{x}_j} [g(\mathbf{x}_k) - \mathbf{y}_k]_n + \tilde{C}_{im}^T \frac{\partial [g(\mathbf{x}_k)]_n}{\partial \mathbf{x}_j} \right),$$

$$[\nabla_{\mathbf{x}_k}^2 l_k]_{ij} = [\tilde{C}^T \Sigma_{\boldsymbol{\eta}_k}^{-1} \tilde{C}]_{ij}$$

$$+ \sum_{n=1}^Y \sum_{m=1}^Y \frac{\partial \tilde{C}_{km}^T}{\partial \mathbf{x}_l} [\Sigma_{\boldsymbol{\eta}_k}^{-1}]_{mn} [g(\mathbf{x}_k) - \mathbf{y}_k]_n,$$

$$[\nabla_{\mathbf{x}_k}^2 l_k]_{ij} = [\tilde{C}^T \Sigma_{\boldsymbol{\eta}_k}^{-1} \tilde{C}]_{ij} + \tilde{\mathbf{c}}_{ij}^T \Sigma_{\boldsymbol{\eta}_k}^{-1} (g(\mathbf{x}_k) - \mathbf{y}_k),$$

where,  $\tilde{\mathbf{c}}_{ij} \in \mathbb{R}^Y$  is the stacked vector of  $\frac{\partial \tilde{C}_{im}^T}{\partial \mathbf{x}_j}$  for  $m \in [1..Y]$ .

- $Q_{\omega \omega k} = \nabla_{\boldsymbol{\omega}_k}^2 l_k + \nabla_{\boldsymbol{\omega}_k}^2 \mathcal{V}_{i+1}(f(\mathbf{x}_k, \boldsymbol{\omega}_k)),$   
 $\nabla_{\boldsymbol{\omega}_k}^2 l_k = 2\Sigma_{\boldsymbol{\omega}_k}^{-1} \frac{\partial \boldsymbol{\omega}_k}{\partial \boldsymbol{\omega}_k} = 2\Sigma_{\boldsymbol{\omega}_k}^{-1}$   
 $Q_{\omega \omega k} = 2\Sigma_{\boldsymbol{\omega}_k}^{-1} + B^T \mathcal{V}_{xx} B$

## REFERENCES

- [1] F. Bailly, T. Flayols, J. Carpentier, and P. Souères, “Differential dynamic programming for maximum a posteriori centroidal state estimation of legged robots,” in *Internat. Conf. on Robotics and Automation*. IEEE, 2021.