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1 **Time-dependent deformation and failure of granite based on the virtual** 2 **crack incorporated numerical manifold method**

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9 **Abstract**

10 Micro-cracks are known to greatly affect the mechanical properties of granite and subcritical crack growth (SCG) is
11 considered to be the main mechanism of brittle creep in rocks, including granite. Here, we provide new uniaxial
12 compressive strength and creep experiments for Lanhélin granite, and a new multi-crack numerical model to explain the
13 experimental observations. We first thermally-stressed our granite samples to create thermal micro-cracks. Uniaxial
14 compressive strength experiments were then used to find the uniaxial compression strength of the thermally-cracked granite,
15 a pre-requisite for brittle creep experiments. We introduced a new model that combines SCG theory and the numerical
16 manifold method (NMM) to link the local damage caused by micro-crack propagation and the macroscopic creep
17 deformation observed in the granite samples. We also investigated the influence of virtual micro-crack length, confining
18 pressure, and differential stress on brittle creep behavior. According to our model, we can numerically simulate the **entire**
19 creep process, from the small deformation caused by micro-cracks to the large displacement **characteristic of** brittle creep.
20 The fact that **the** numerical simulations are in good agreement with experimental results shows that the NMM combined
21 with the SCG theory is a suitable method for modeling the creep behavior of rocks.

22 **Keywords:** Time-dependent deformation; Numerical manifold method; Subcritical crack growth; Multi-crack
23 propagation; Lanhélin granite

24

25 1. Introduction

26 The long-term stability of engineering rock mass structures have been of practical concern for many years[1].
27 Time-dependent deformation of rock has a significant influence on the stability of rock slopes and underground
28 structures such as mines and tunnels, as well as the long-term strength of the Earth's upper crust[2, 3]. Time-
29 dependent deformation under these conditions is known as brittle creep[4], which is the name applied to the
30 slow deformation of solids under loads below their short-term strength [5]. Creep strain and strain rates are very
31 sensitive to differential stress, confining pressure, and temperature. Indeed, even small changes in any of these
32 parameters will produce order of magnitude changes in creep strain rates[6, 7]. Many researchers have found
33 that micro-crack propagation in a rock mass is one of main parameters that influences the creep strain rate[8].
34 [The influence of slow crack growth \(subcritical crack growth \(SCG\)\) on fracture stress](#) was first examined [in](#)
35 [detail](#) by Charles[9]. One of the most important mechanisms of subcritical crack growth (known as static fatigue
36 or delayed failure) is stress corrosion[10]. During crack propagation, stress corrosion can explain a certain
37 proportion of the relationship between the stress intensity factor (K) and the crack velocity (V)[11]. SCG theory
38 in rock masses has been also used to explain the growth and development of joints, volcanic eruptions, and
39 underground excavations[12]. The double torsion experimental method was adopted to investigate SCG
40 theory[13, 14]. Nara and co-workers have provided a large number of SCG experiments to better understand
41 the influence of environment on crack velocities in rocks[15]. These authors also found that the physical
42 properties of different rocks also have an effect on SCG[16]. Because creep experiments take a lot of time,
43 numerical simulations, which can give good approximate results in a relatively short time[17], are of great
44 interest. Indeed, many researchers have used numerical simulations to study rock creep and SCG. For example,
45 Desai[18] used a finite-element (FE) program with the implementation of the disturbed-state concept (DSC)
46 creep model to study creep- and rate-dependent behavior of glacial tills to better understand the motion of
47 overlying glaciers. Brantut[19] used a micromechanical model to describe the brittle rock creep, based on the
48 sliding wing-crack model. Konietzky[20] used Fast Lagrangian Analysis of Continua (FLAC) to simulate time-
49 dependent crack growth of granite. These authors used the SCG theory to model a virtual crack within an
50 element to describe rock damage. Discontinuous deformation analysis (DDA), which analyzes the force-
51 displacement interactions of block systems, was proposed by Shi[21] and has also been used to investigate creep
52 problems[22]. Zheng[23] researched the contact force to improve the algorithm of DDA. Xu[24, 25] proposed

53 a thermomechanical time-dependent deformation model based on laboratory tests on Beishan granite under
54 constant elevated temperatures and found that this model can accurately describe brittle creep.

55 The numerical manifold method (NMM) is a new simulation method which provides a unified framework for
56 solving problems dealing with continuous media, discontinuous media, or both[26]. The independent response
57 variables when using NMM, such as displacements, can be continuous or discontinuous throughout the problem
58 domain[27]. The NMM has gained a wide attention and application in rock mechanics and engineering due to
59 its efficient treatment of problems involving continuous and discontinuous deformations in a unified way[28-
60 32]. An[33] introduced a new concept of “weak-discontinuous physical covers” to improve the advantage of
61 the partition of unity of NMM. NMM is suitable to solve crack propagation problems in simulations (Wu[34],
62 Yang[35] and Zhou[36]). Zheng[37] combined the moving least squares (MLS) interpolation with NMM to
63 analyze the unconfined flow in porous media, which can provide high accuracy and numerical stability.
64 Yang[38] proposed a general mass lumping scheme applicable to higher order elements, which improved the
65 efficiency of NMM to calculate the large deformation of distorted meshes. Using the time step–initial strain
66 method, the creep equation was coupled with the NMM to simulate the time-dependent deformation of rocks[39,
67 40]. Wu[41] also used the NMM to analyze viscoelastic material creep crack problems by incorporating a
68 generalized Kelvin-Voigt model into the NMM.

69 **2. Constitutive model**

70 *2.1 Crack Growth Criteria*

71 Before using the NMM to consider crack problems, the criteria that have an important influence on calculating
72 the crack initiation and propagation angle must be considered. In this paper, two different crack growth criteria
73 are introduced: the maximum circumferential stress criterion (MCSC)[42] and Mohr–Coulomb’s shear strength
74 criterion (MCSSC)[43]. For MCSC, the crack propagates in the direction of the maximum circumferential
75 stress. The stress field at the crack tip can be formulated in terms of stress intensity factors (SIFs):

$$76 \quad K_I \sin \theta_0 + K_{II} (-1 + 3 \cos \theta_0) = 0 \quad (1)$$

77 where K_I and K_{II} are respectively the stress intensity factors corresponding to mode I and mode II loading, and
 78 θ_0 is the kinking angle, which corresponds to the crack propagation angle. The direction of maximum
 79 circumferential stress is then:

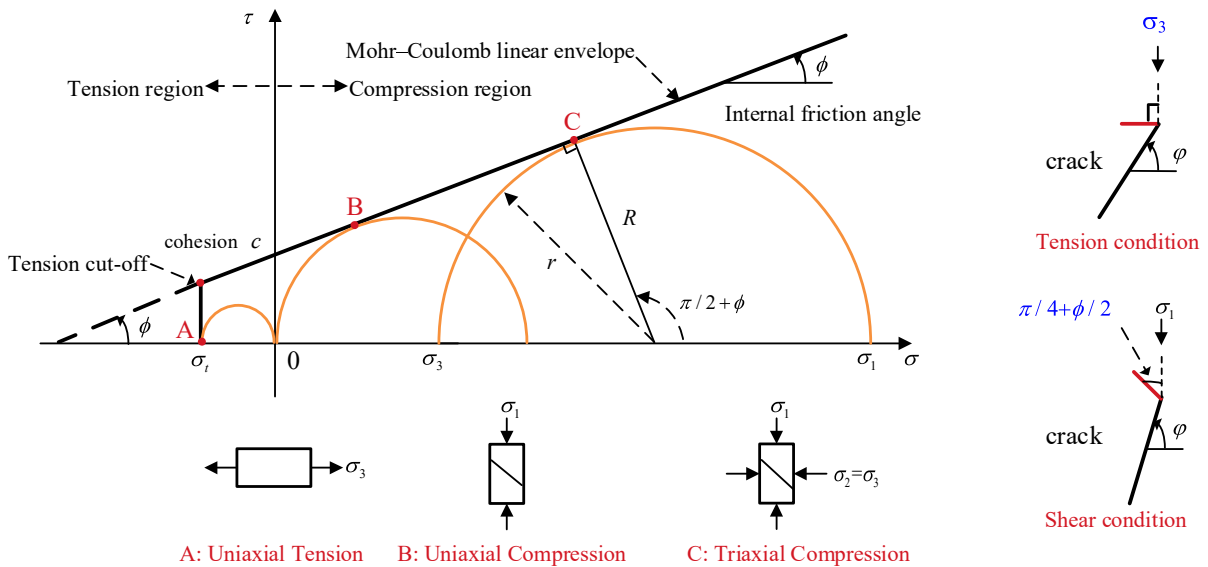
$$80 \quad \theta_0 = 2 \arctan \frac{1}{4(K_{II}/K_I)} \left\{ 1 \pm \left[1 + 8(K_{II}/K_I)^2 \right]^{1/2} \right\} \quad (2)$$

81 The MCSC can well predict the fracture direction in two mixed modes if there are several cracks in the
 82 sample. In nature, however, there are many cracks, joints, and holes in rock masses, and every flaw will
 83 influence the local stress field, which can greatly influence the direction of the maximum circumferential stress.
 84 We used the MCSSC to deal with this problem.

85 In classical engineering science, the SIFs are greatly sensitive to the local stress. Coulomb postulated that the
 86 crack initiation depends on the local stress relative to the strength of the material rather than on the SIFs[44]. In
 87 this framework, the material strength parameters are the cohesion and the angle of internal friction, as shown in
 88 Figure 1. The MCSSC can be used to calculate shear and tensile cracks. As shown in Figure 1, σ_1 and σ_3 are the
 89 maximum and minimum principle stresses, respectively, σ_t is the tensile stress, and R and r are expressed as:

$$90 \quad \begin{cases} R = c \cos \phi + \frac{\sigma_1 + \sigma_3}{2} \sin \phi \\ r = \frac{\sigma_1 + \sigma_3}{2} \end{cases} \quad (3)$$

91 where c and ϕ are cohesion and the internal friction angle, respectively.



92 **A: Uniaxial Tension B: Uniaxial Compression C: Triaxial Compression**
 93 **Figure 1** The Mohr-Coulomb criterion for the elements of the numerical model

94

95 When the crack satisfies the shear condition:

$$96 \quad \begin{cases} R=r \\ \sigma_3 > -\sigma_t \end{cases} \quad (4)$$

97 the crack initiation angle will be:

$$98 \quad \theta = \pi / 4 + \varphi / 2 \quad (5)$$

99 where φ is the crack initiation angle.

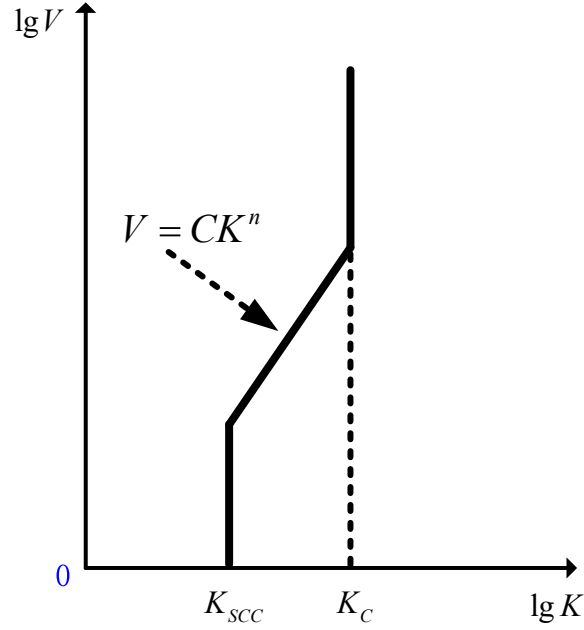
100 When the crack satisfies the tensile condition:

$$101 \quad \begin{cases} R > r \\ \sigma_3 < -\sigma_t \end{cases} \quad (6)$$

102 According to Wu[43], when the condition expressed in Eq. 6 is satisfied, the direction of crack propagation will
103 be perpendicular to the direction of the minimum principal stress, σ_3 . On the other hand, the crack initiation
104 angle will be in the direction of the maximum principal stress, σ_1 .

105 2.2 Subcritical Crack Growth (SCG)

106 In classical fracture mechanics theory, the crack will not propagate if the SIFs at the crack tip are lower than
107 the fracture toughness K_c . However, some researchers also find that cracks still propagate when the SIFs are
108 lower than K_c , but at a certain lower velocity. This is called subcritical crack growth (SCG) theory[45]. Figure
109 2 shows a schematic stress intensity factor (K) against crack velocity (V) curve. K_{sc} is a critical value, which
110 is a material property. When $K < K_{sc}$, the crack cannot propagate. When $K_{sc} < K < K_c$, the crack propagates
111 as described by SCG theory (Figure 2). Nara and co-workers performed a lot of SCG experiments[46], which
112 show that, regardless of the environmental conditions, the subcritical crack velocity always increases as a
113 function of increasing stress intensity factor (Figure 2).



114

115 **Figure 2** Schematic diagram showing a typical stress intensity factor (K) against crack velocity (V) curve

116 The crack velocity is governed by Charles' equation:

$$117 \quad \begin{cases} V = CK^n \\ K = \sqrt{K_I^2 + K_{II}^2} \end{cases} \quad (7)$$

118 where C is a material constant, K is the total SIF, which consists of K_I and K_{II} , n is the stress corrosion index, V
 119 is the crack velocity, and Δt is the time step. The crack length L' at every step is $V\Delta t$.

120 The SIFs K_I and K_{II} are calculated as:

$$121 \quad \begin{cases} K_I = \sigma_n \sqrt{\pi a} \\ K_{II} = \tau_n \sqrt{\pi a} \end{cases} \quad (8)$$

122 When the crack is under the tension-shear condition, where σ_n and τ_n are the maximum tensile normal stress
 123 and maximum shear stress, respectively, a is the half crack length, and σ_n and τ_n are expressed as:

$$124 \quad \begin{cases} \sigma_n = \frac{1}{2} [(\sigma_1 + \sigma_3) + (\sigma_1 - \sigma_3) \cos 2\varphi] \\ \tau_n = \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\varphi \end{cases} \quad (9)$$

125 where φ is the original angle of the crack.

126 When the crack is under the compress-shear condition, where τ_{eff} is effective shear stress and μ is the
 127 friction coefficient, τ_{eff} and K_{II} are expressed as:

128
$$\tau_{\text{eff}} = |\tau_n| - \mu |\sigma_n|$$

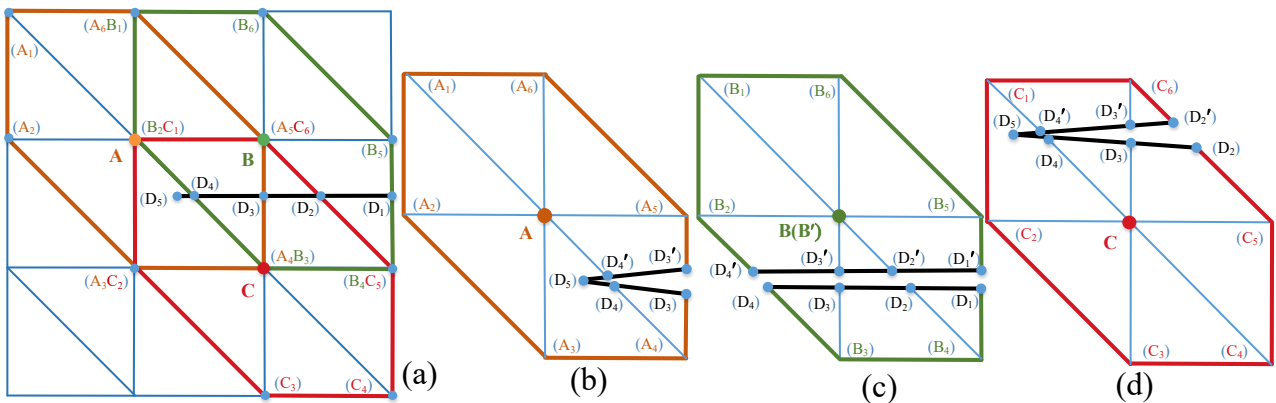
 129
$$K_{\text{II}} = |\tau_{\text{eff}}| \sqrt{\pi a}$$
 (10)

129 **3. Fundamentals of NMM**

130 *3.1 Dual cover system of NMM*

131 The NMM consists two of important components: the cover system and the block dynamics. The cover system
 132 is a dual cover system, which includes a mathematical cover (MC) of three-node triangle elements and a physical
 133 cover (PC) that includes the boundary, the material interface, and the crack. The block dynamics is used to solve
 134 the mechanical behavior of block systems under loading and block contact. The NMM program runs on the
 135 MATLAB[®] platform.

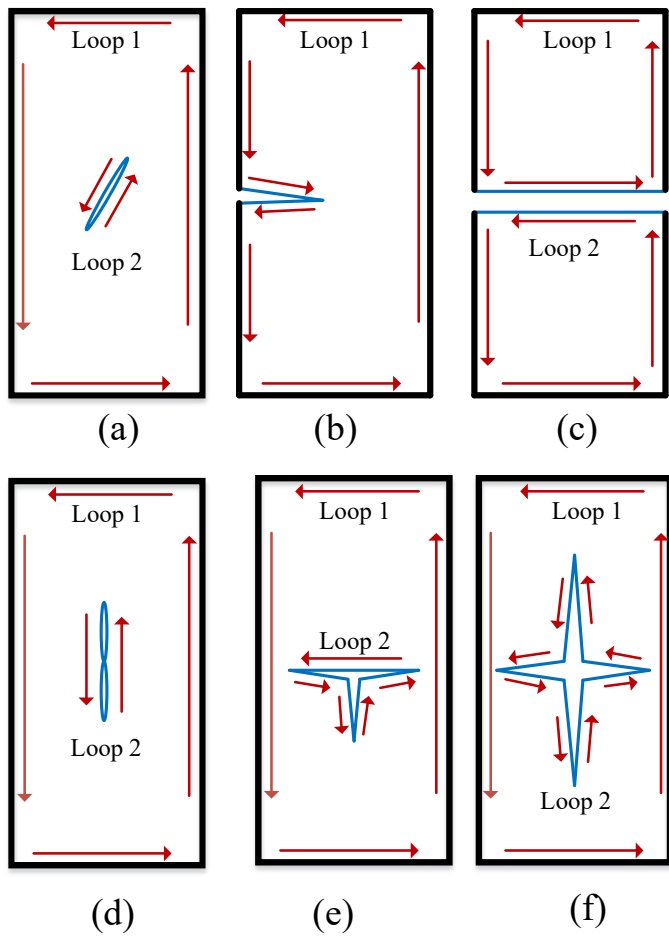
136 Figure 3 shows the dual cover system containing MC and PC. We can see in Figure 3(a) that there are three
 137 MCs that are coloured orange, green, and red, that every MC is a hexagon consisting of six triangles, and that
 138 the node of every triangle is at the centre of the MC. If there a crack that cuts through the triangle ABC (such
 139 as D₁-D₅ in Figure 3), the three MCs are changed. It is shown in Figures 3(b) and (d) that the crack does not cut
 140 the MC (A and C) into two parts. Instead, the hexagon and a part of the crack consist of a PC, which are A₁-A₂-
 141 A₃-A₄-D₃-D₄-D₅-D₄'-D₃'-A₅-A₆-A₁ and C₁-C₂-C₃-C₄-C₅-D₂-D₃-D₄-D₅-D₄'-D₃'-D₂'-C₆-C₁, respectively. In Figure
 142 3(c), the crack cuts the MC (B) into two parts (B and B') and so there are two PCs, which are B₁-B₂-D₄'-D₃'-
 143 D₂'-D₁'-B₅-B₆-B₁ and D₄-B₃-B₄-D₁-D₂-D₃-D₄, respectively.



144 **Figure 3** Dual cover system and crack cutting theory of NMM
 145

146 *3.2 Contact theory of NMM*

147 The loop contact is very important in NMM to identify the physical cover (PC), which includes the boundary,
 148 the material interface, and the crack. The loop consists of closed segments connected from the first to the last.
 149 In Figure 4, the black line is the boundary of model and the blue line is the crack. In NMM, the loop rotates
 150 counter-clockwise and a complete loop indicates a whole body (i.e. the crack boundary zone is empty in NMM),
 151 which is the same as making a hole in the model (Figure 4).



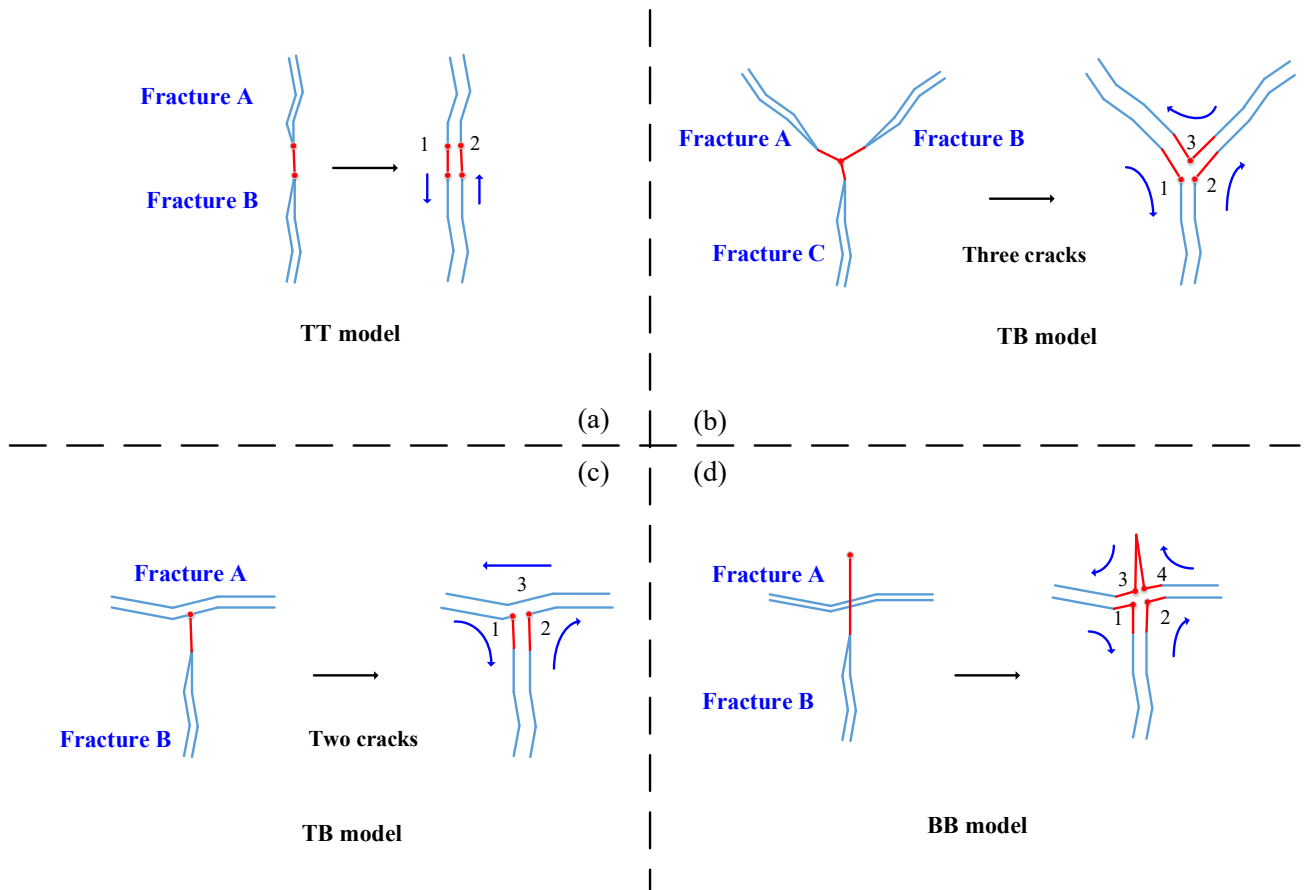
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153

Figure 4 Single-and multi-crack loop conditions of NMM

154 Figures 4(a), 4(b), and 4(c) are single crack loop models, which show different possible locations of cracks
 155 within the model. Figure 4(a) shows the condition when there is a crack inside the model. In this case, the crack
 156 itself can generate an independent loop. Indeed, the two loops in Figure 4(a) have no intersection point. Figure
 157 4(b) shows the condition when there is crack tip on the boundary of model. In this scenario, the crack will
 158 combine with the boundary to generate a single whole loop (Figure 4(b)). If the crack cuts the model into two
 159 parts, which is shown in Figure 4(c), the program will generate two independent loops. However, the sample
 160 will have to contain a lot of cracks to simulate a natural rock and, as the population of micro-cracks grow, they

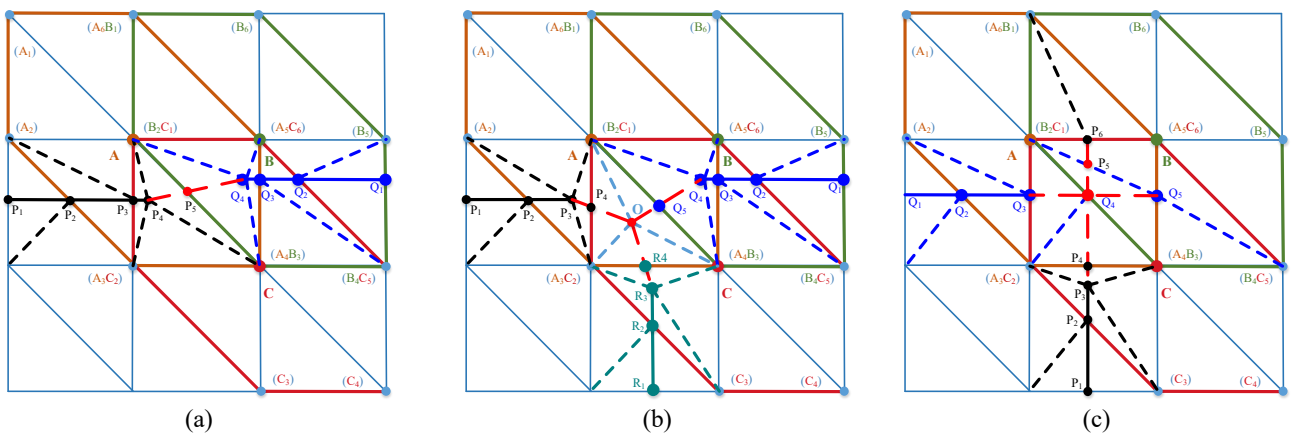
161 can coalescence and transform into a macro-crack. This transformation forms a large crack zone. It is therefore
 162 important to study the coalescence of cracks. There are three crack coalescence types during crack propagation,
 163 which are crack tip to crack tip (“tip and tip” crack model, **TT** model), crack tip to crack boundary (“tip and
 164 boundary” crack model, **TB** model) and crack boundary to crack boundary (“boundary and boundary” crack
 165 model, **BB** model) respectively. Figure 4(d) shows the **TT** model. In this case, the two cracks have a common
 166 crack tip and so they can generate a new larger crack with an independent loop, and the crack tip of the
 167 intersection will disappear (Figure 4(d)). If a crack propagates to another crack boundary, there will be a **TB**
 168 model, which is shown in Figure 4(e). In the **TB** model, the two cracks can generate a new crack with three
 169 crack tips. If one crack propagates across the other crack boundary, there will be a **BB** model. In the **BB** model,
 170 the two cracks can generate a new crack with four crack tips (Figure 4(f)).



171
 172 **Figure 5** Different types of loop intersection during crack propagation in NMM

173 Figure 5 shows the different types of loop intersection during crack propagation. The different loop
 174 intersection types correspond to the different number of intersection points: the **TT** model has two intersection
 175 points (Figure 5(a)), the **TB** model has three intersection points (Figure 5(b) and (c)), and the **BB** model has

176 four intersection points (Figure 5(d)). The **TB** model has two different types, the first type is when three crack
 177 tips propagate to the same intersection point (Figure 5(b)) and the second type is when a crack tip propagates to
 178 the boundary of another crack (Figure 5(c)). When three crack tips are propagating, the order of the crack
 179 propagation is that the first two cracks propagate according to the **TT** model and the third crack propagates
 180 according to the **TB** model. As a result, the three crack tips propagation type is still considered as a **TB** model.
 181 For a complete single loop connected from the first and last loop points, the loop segments at the intersection
 182 points cannot be knotted, and the loop point is a whole loop that should rotate counter-clockwise. Inside every
 183 crack loop should be empty.



184 (a) (b) (c)

185 **Figure 6** Form of element cutting during multi-crack intersection in NMM (Fig. 6 shows the propagating and
 186 cutting progress during multi-crack intersection in NMM. In Fig. 6(a), the black crack tip is P_4 and, when
 187 black crack propagates to the blue crack tip Q_4 , this is TT model. Following crack propagation, crack tips P_4
 188 and Q_4 disappear. The dotted line is the element segmentation progress. Fig. 6(b) is the TB model. There are
 189 three cracks in the TB model: cracks P_1P_3 , R_1R_4 , and Q_1Q_4 . These three cracks propagate to the same point O .
 190 Cracks P_1P_3 and Q_1Q_4 first propagate according to the TT model and, later, the crack R_1R_4 propagates. After
 191 the three cracks propagate, the three crack tips P_4 , R_4 , and Q_4 disappear. Fig. 6(c) is the BB model. In this
 192 model, the crack P_1P_4 propagates across the crack Q_1Q_5 , and the crack tips P_6 and Q_5 do not disappear.)

193 Figure 6 shows the form of element cutting by multi-crack intersection. Figure 6(a) shows the intersection of
 194 two crack tips, which is a **TT** model. We assume that crack P propagates, and that crack Q does not propagate.
 195 The new crack is P_4Q_4 . After the crack propagation event, the program generates four new elements, which are
 196 AP_4P_5 , CP_5P_4 , AP_5Q_4 , and CQ_4P_5 , respectively. The two cracks then connect to form a new crack and the
 197 intersection point of the new crack is Q_4 . In order to have the new loop, the program rearranges the four new

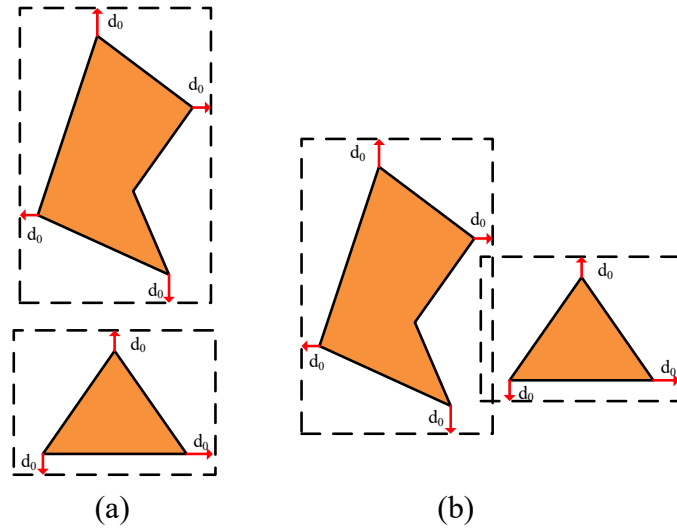
198 elements and the former elements cut by cracks P and Q. The program deals with multi crack intersection in the
199 same way. Figure 6(b) shows the three crack tips intersection: the three cracks propagate to the same point O,
200 which is the intersection point. Figure 6(c) shows the two cracks boundary intersection, and the intersection
201 point is Q₄.

202 Each element on the loop is likely to come into contact with other elements on the loop. As a result, both the
203 contact search and judgment are based on elements that are located on the loop. These elements located on the
204 loop are the contact elements. NMM uses the Mohr–Coulomb criterion and maximum tensile strength criterion
205 to handle the possibility of embedding, pulling, and friction during contact. The laws of contact theory of NMM
206 are as follows:

- 207 1. Penetration does not occur during contact.
- 208 2. The pulling stress is less than the tensile strength during contact.
- 209 3. The Mohr–Coulomb criterion is satisfied during contact.

210 These three laws are executed by applying or releasing contact springs (open–close iteration) to modify the
211 contact status and realize the no tension force and no penetration between two contacts[32, 47, 48].

212 The rectangular contact search method (RCSM) is used to determine whether there is a contact between two
213 potential CE. Figure 7 shows a diagram of the RCSM, where d_0 is the one step maximum displacement (safe
214 displacement). Before every time step of the numerical simulation, the program makes contact judgements and
215 calculates the contact condition of every element in the model. There are two contact conditions. If distance
216 between two contact elements is larger than d_0 , these two elements are non-contact elements (Figure 7(a)).
217 However, if distance between two contact elements is smaller than d_0 , these two elements are contact elements
218 (Figure 7 (b)).



219
220 **Figure 7** Diagram of rectangular contact search method of NMM: (a) non-contact element; (b) contact
221 element

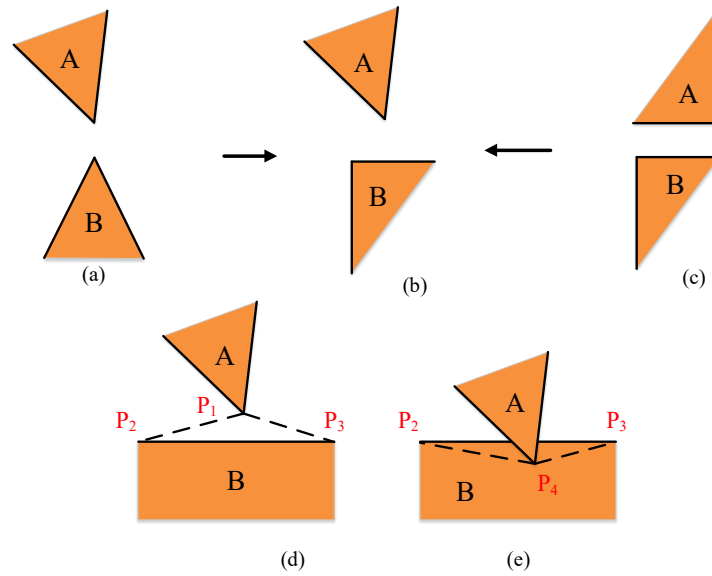
222 In NMM, there are three contact conditions, as shown in Figure 8: (a) angle to angle, (b) angle to edge, and
223 (c) edge to edge. “A” and “B” are two elements. Both angle to angle and edge to edge contacts can be converted
224 into angle to edge contacts. Therefore, it is the distance between the angle in one loop and edge of other loops
225 that influences the contacts judgment in the NMM. Contact distance can be obtained by calculating the distance
226 from angle to edge.

227 When we find the angle and edge of two contact elements, the program will calculate the distance between
228 the angle and the edge. Figures 8(d) and (e) show examples before and after the penetration of two elements,
229 respectively. P_1 is the angle before moving, P_4 is the angle after moving, and P_2P_3 is the penetration edge. The
230 equation for judging penetration is:

$$231 \quad \Delta = \begin{vmatrix} 1 & x_1 + u_1 & y_1 + v_1 \\ 1 & x_2 + u_2 & y_2 + v_2 \\ 1 & x_3 + u_3 & y_3 + v_3 \end{vmatrix} \quad (11)$$

232 where (x_i, y_i) and (u_i, v_i) are coordinate and displacement of P_i ($i=1,2,3$).

233 If $\Delta < 0$, the two elements have penetrated, and the program moves the penetration angle back to the outside
234 of the element along the minimum distance.



235

236

237

238

Figure 8 Three contact conditions and open-close iteration penetration judgement of NMM: (a) angle to angle, (b) angle to edge, (c) edge to edge, (d) before penetration of two elements, (e) after penetration of two elements

239

3.3 Crack initiation and coalescence of NMM

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Crack initiation and coalescence are a prerequisite for the macroscopic failure of rock. This is true for samples deformed at a constant strain or loading rate and those deformed under a constant stress (i.e. time-dependent brittle creep). During deformation, micro-cracks initiate and coalesce to form a macro-crack zone. It is therefore important to study the connection between micro-crack initiation, propagation, coalescence, and time-dependent deformation.

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Compared to XFEM, GFEM, and DEM, NMM is the most advanced to simulate crack initiation, propagation, and coalescence. The dual cover system and the loop concept in NMM allow for crack identification and crack propagation simulation without incorporating additional unknowns to the related nodes through enrichment functions.

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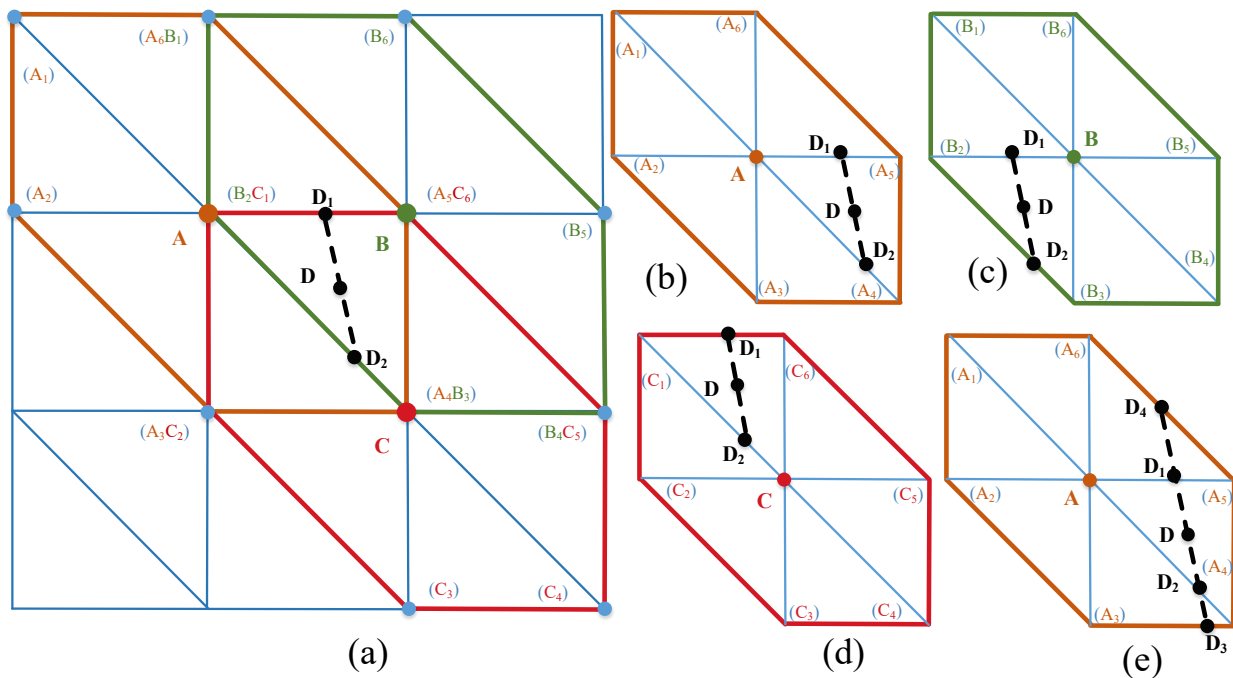
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Figure 3 introduces the dual cover system and crack cutting theory of NMM, which are the basic theories to realize crack initiation. There is a difference between crack initiation and propagation in the model. When cracks propagate, the crack tip can be located on either the boundary, node, or in the element, and there is no limit to the length of the crack propagation at each step. When cracks initiate, the crack has to traverse the boundary of three MCs at the same time where it is located. Figure 9 shows the crack initiation theory of NMM. In Figure

254 9(a), a crack “ D_1D_2 ” initiates in the triangle ABC, which is in contact with three MCs that are coloured orange,
 255 green, and red. The locations of crack “ D_1D_2 ” in every MC are shown in Figures 9(b)-(d). In Figures 9(c) and
 256 (d), one of the crack tips is located on the boundary of MC, the crack “ D_1D_2 ” can form in these two MCs.
 257 However, in Figure 9(b), both crack tips are located in the MC and so the crack “ D_1D_2 ” cannot generate in this
 258 MC. In order to make the crack “ D_1D_2 ” generate in this MC, we have to extend one of the crack tips to the
 259 boundary of MC and make sure the extended length is smaller than the other crack tip. For example, in Figure
 260 9(e), the two crack tips D_1 and D_2 extend to D_4 and D_3 , respectively. D_2D_3 is smaller than D_1D_4 and so the final
 261 initiated crack is $D_1D_2D_3$. When the crack initiates, it will transform into a real crack, cut the element, and
 262 propagate according to the MCSSC theory.

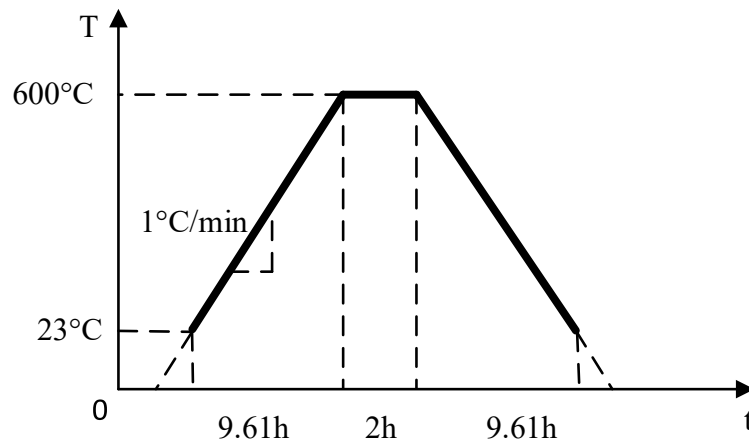


263
 264 **Figure 9** Crack initiation theory of NMM (Fig. 9 shows the crack initiation and element cutting progress. If
 265 crack D_1D_2 initiates at triangle element ABC, the crack center D is also the center of the triangle element. Every
 266 triangle element in NMM has three nodes, and every node is the center of a hexagon physical cover. Figs. 9(b),
 267 (c), and (d) are three hexagon physical covers. In NMM theory, if one crack wants to initiate successfully, the
 268 crack tip has to propagate to the boundary of three hexagons at the same time. In Figs. 9(c) and (d), the crack
 269 tip D_2 is at the boundary of hexagon B, and the crack tip D_1 is at the boundary of hexagon C. In Fig. 9(b), these
 270 two crack tips are inside hexagon A. Therefore, the crack tip propagates in two directions, yielding the shortest
 271 new crack D_1D_3 in Fig. 9(e).)

272 **4. Experimental materials, methods, and results**

273 *4.1 Sample preparation*

274 The material used in this research is Lanhélin granite, which is a coarse-grained, blue–grey granodiorite from
275 Brittany, France. It has an average crystal size of 2 mm[49, 50]. Lanhélin granite has important research value
276 in studies of rock deformation and fracture[49-54]. We prepared nine cylindrical rock samples that were 20 mm
277 in diameter and nominally 40 mm in length. We then thermally-stressed the nine rock samples in a furnace to a
278 maximum temperature of 600 °C so that our samples contained thermal micro-cracks. The samples were
279 thermally-stressed in an electric box furnace at room pressure. The furnace was programmed to heat at 1 °C/min
280 to a target temperature of 600 °C. The samples then held at the target temperature for two hours before being
281 cooled at 1 °C/min back to room temperature. Figure 10 shows the thermal-stressing procedure. The prepared
282 samples were then separated into two groups. The first group is the uniaxial compressive strength group, which
283 has five samples (U-1, U-2, U-3, U-4, and U-5). The second group is the creep deformation group, which has
284 four samples (C-1, C-2, C-3, and C-4).

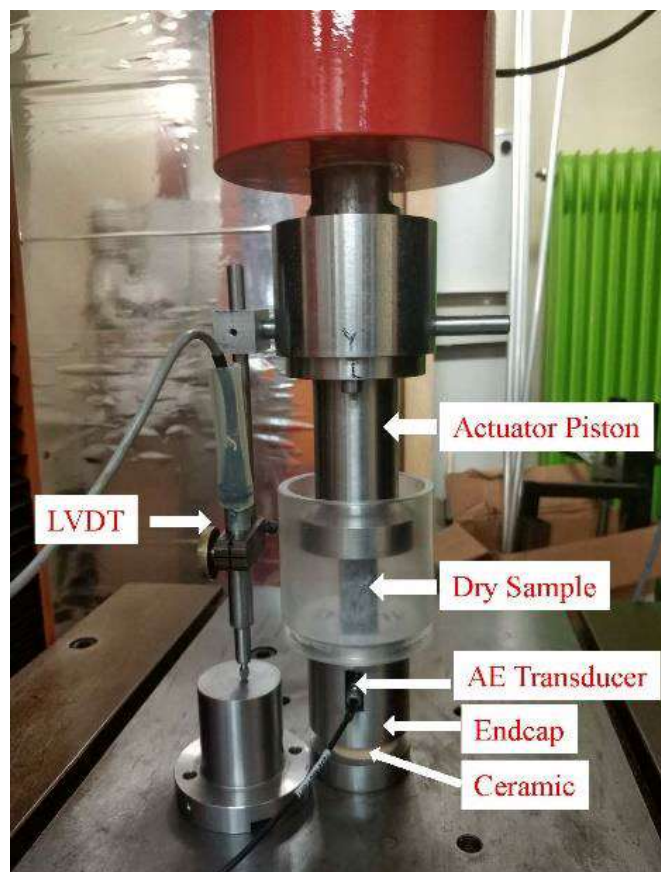


285

286 **Figure 10** Heating curve of Lanhélin granite samples

287 Following thermal-stressing, we determined the connected porosities of the nine samples. Connected porosity
288 was determined using the bulk sample volume (measured using digital callipers) and the connected (skeletal)
289 volume measured by an AccuPyc II 1340 helium pycnometer. Following thermal-stressing to 600 °C, the
290 connected porosity of the thermally-stressed samples of Lanhélin granite was about 2%, in agreement with
291 previously published porosity data for thermally-stressed granites[55].

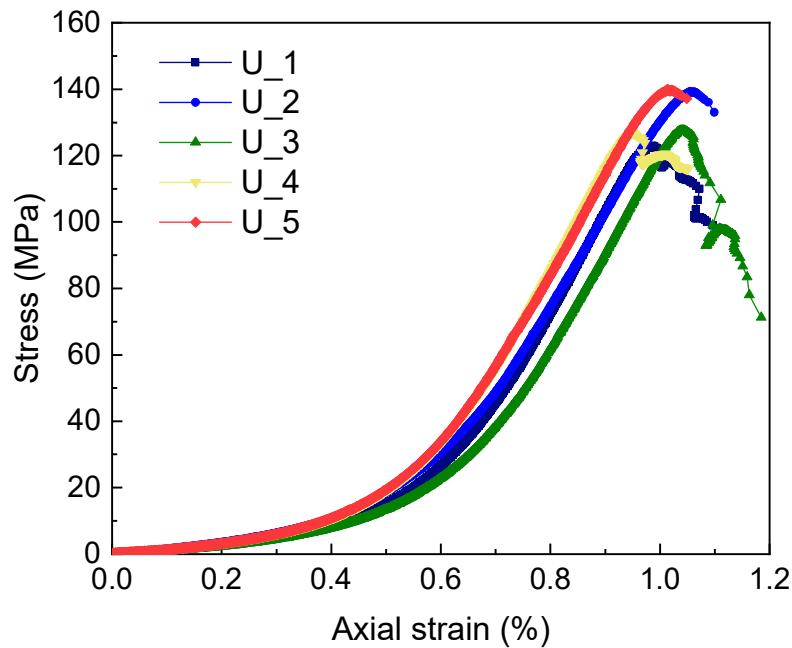
292 Uniaxial compressive strength (UCS) tests and uniaxial creep experiments were performed on dry samples
293 of thermally-stressed Lanhélin granite under ambient laboratory conditions (pressure and temperature) using a
294 uniaxial load frame (Figure 11). Axial displacement and axial load were measured by a linear variable
295 differential transducer (LVDT) and a load cell, respectively. Axial displacement and axial load were converted
296 to axial strain and axial stress using the sample dimensions. During the UCS tests, we also recorded the output
297 of acoustic emission (AE) energy using a USB AE Node (from Physical Acoustics). The threshold for an AE
298 hit was set at 30 dB in the accompanying program, AEwin. The UCS tests were performed at a constant strain
299 rate of $1 \times 10^{-5} \text{ s}^{-1}$ until macroscopic sample failure. For the creep tests, the samples were first loaded at a
300 constant strain rate of $1 \times 10^{-5} \text{ s}^{-1}$ to a pre-ascribed axial stress (a high percentage of the short-term failure stress
301 determined from the UCS tests). The samples were then left to deform under a constant stress until macroscopic
302 sample failure.



303
304 **Figure 11** Uniaxial compression apparatus used for the uniaxial compression strength tests and creep
305 experiments presented in this study

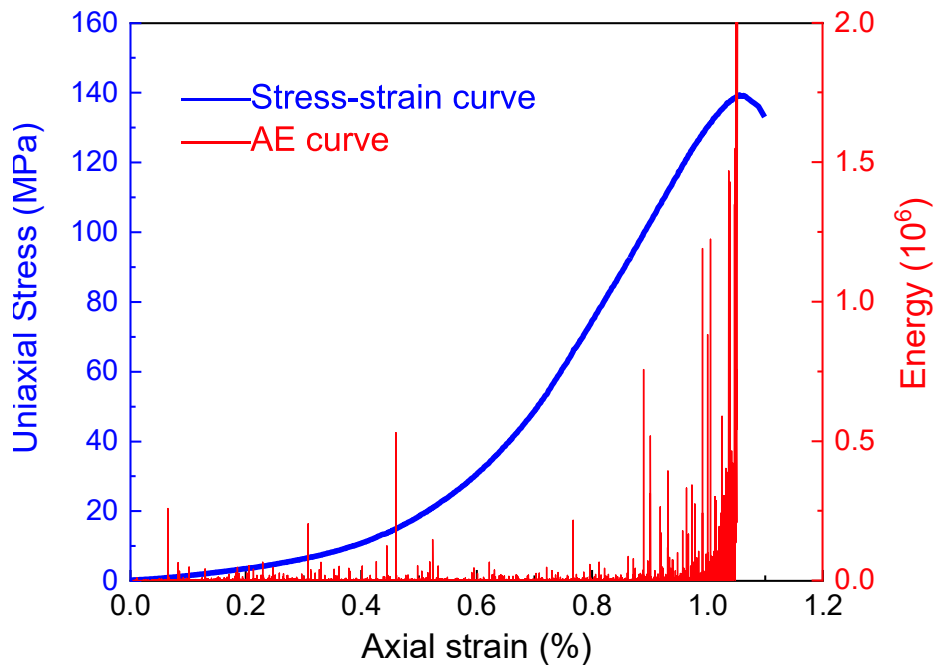
306 *4.2 Uniaxial compressive strength experiments*

307 The stress-strain curves for the five UCS tests performed on samples of thermally-stressed Lanhélin granite
308 are shown in Figure 12. These stress-strain curves are typical for crystalline rock deforming under uniaxial
309 compression[56, 57]. The UCS values of the five samples are 123.0, 139.2, 127.6, 120.6, and 139.9 MPa,
310 respectively, and the mean UCS is 130 MPa (σ_p). The mean UCS was used to determine the constant stresses
311 used in the uniaxial creep experiments. We can also calculate the Young's modulus (the slope of the stress-
312 strain curve in the elastic region), which are 26.2, 24.7, 25.5, 26.9, and 16.4 GPa for the five samples,
313 respectively. The mean Young's modulus is 23.9 GPa.



314
315
316 **Figure 12** Stress-strain curves for the five uniaxial compressive strength (UCS) tests performed on samples
317 of thermally-stressed Lanhélin granite

318 In Figure 13 we show the stress-strain curve for sample U-2 alongside the output of AE energy (the area
319 under the received waveform, in arbitrary units). Figure 13 shows that the AE activity increases during the
320 approach to macroscopic sample failure, and that the AE activity was greatest at the point of failure (marked by
321 the stress drop in the mechanical data).



322

323 **Figure 13** Stress-strain curve for a uniaxial compressive strength (UCS) test performed on a sample of
 324 thermally-stressed Lanhélin granite (sample U-2) alongside the output of acoustic emission (AE) energy

325 *4.3 Uniaxial creep experiments*

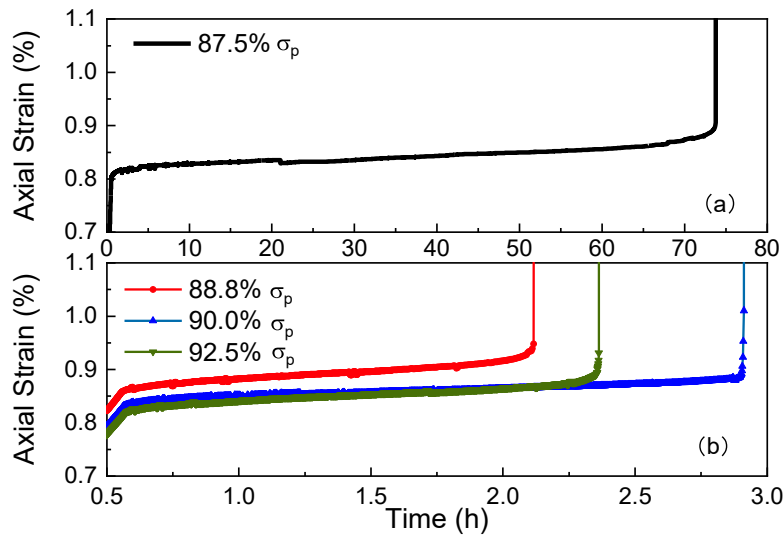
326 Conventional creep experiments were performed on thermally-stressed samples of Lanhélin granite at
 327 constant uniaxial stresses of 113.75, 115.5, 117, and 120.25 MPa, corresponding to 87.5, 88.8, 90, and 92.5%
 328 of σ_p , respectively. Table 1 shows a summary of these uniaxial creep tests. The minimum creep strain rates of
 329 the four samples were determined to be 1.9×10^{-9} , 7.4×10^{-8} , 4.2×10^{-8} , and $5.8 \times 10^{-8} \text{ s}^{-1}$, respectively. The
 330 time-to-failure of the four samples were 73.79, 2.12, 2.91, and 2.36 h, respectively. These data show that, as the
 331 constant axial stress was increased, the minimum creep strain rate and the time-to-failure increased and
 332 decreased, respectively (Table 1).

333 **Table 1** Summary of the conventional uniaxial creep tests performed on thermally-stressed Lanhélin granite

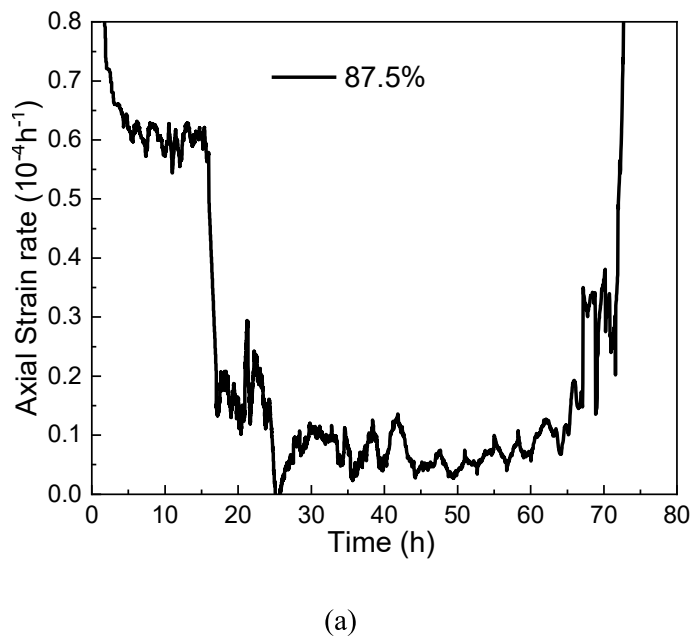
Sample number	Constant stress/ MPa	Percentage of σ_p /%	Minimum Creep Strain Rate/ s^{-1}	Time to failure/h
C-1	113.75	87.5	1.9×10^{-9}	73.79
C-2	115.5	88.8	7.4×10^{-8}	2.12
C-3	117	90	4.2×10^{-8}	2.91
C-4	120.25	92.5	5.8×10^{-8}	2.36

334

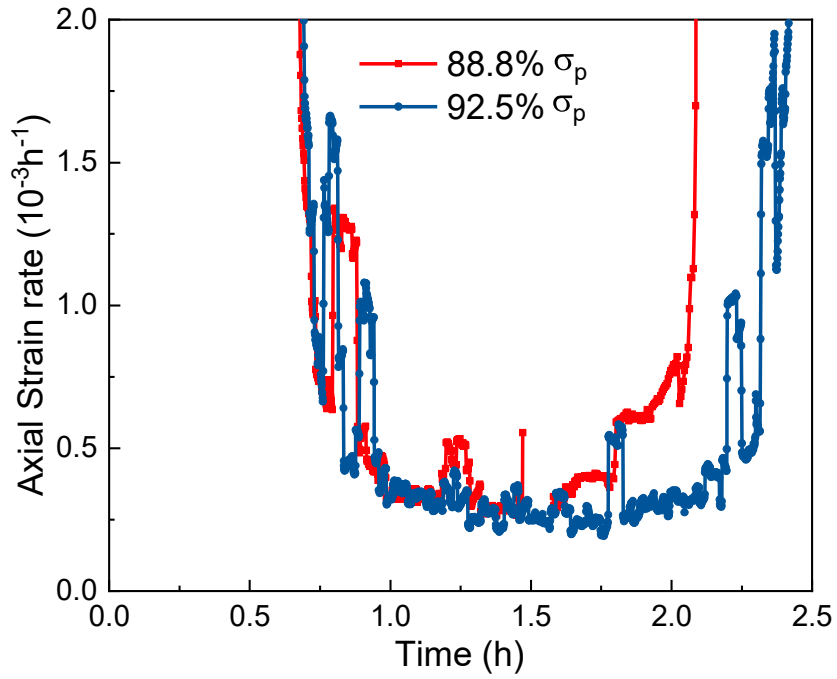
335 The four uniaxial creep (i.e. strain as a function of time) curves (for Lanhélin granite performed at constant
 336 stresses corresponding to 87.5, 88.8, 90, and 92.5% of σ_p) are shown in Figure 14. The strain rate as a function
 337 of time for these four experiments is shown in Figure 15. These curves all exhibit the decelerating followed by
 338 accelerating creep behavior observed in previous laboratory creep experiments[6].



339
 340 **Figure 14** Uniaxial creep curves for thermally-stressed Lanhélin granite performed at different constant
 341 uniaxial stresses: (a) the creep curve for experiment C_1 at 87.5% of σ_p . (b) The creep curves for experiments
 342 C-2, C-3, and C-4 at 88.8%, 90%, and 92.5% of σ_p



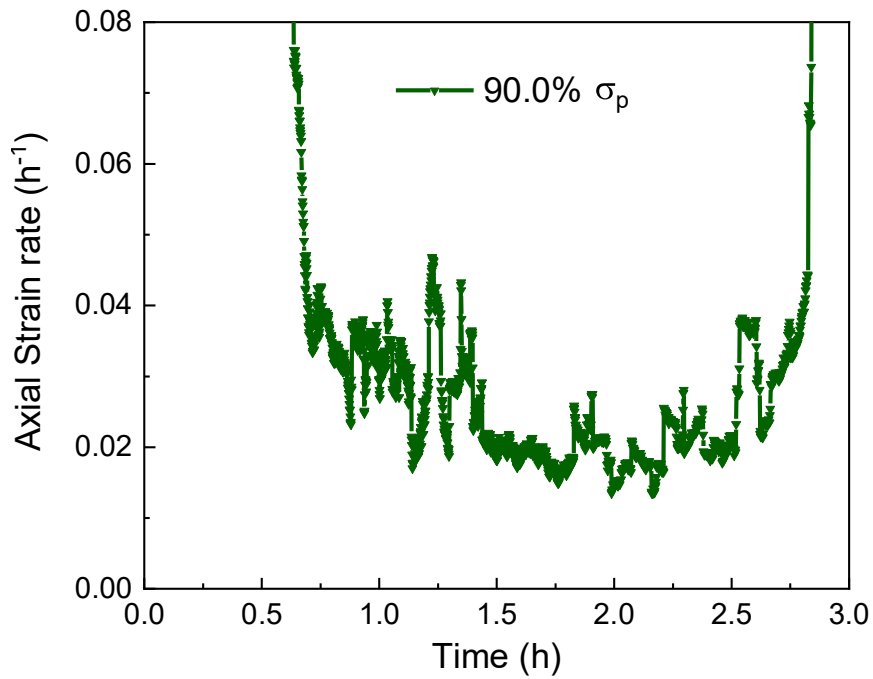
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 344



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(b)



347

348

(c)

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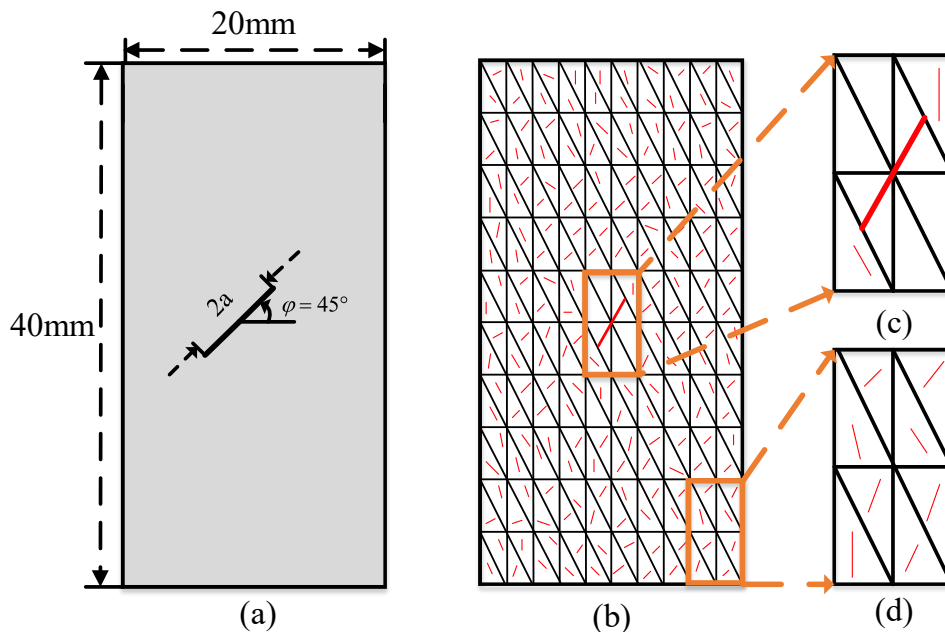
352

Figure 15 Axial strain rate as a function of time for four uniaxial creep experiments performed on thermally-stressed Lanhélin granite (the experiments shown in Figure 14), (a) the axial strain rate curve for experiment C_1, (b) the axial strain rate curves for experiments C_2 and C_4, (c) the axial strain rate curve for experiment C_3

353 **5. Numerical simulations**

354 *5.1 Multi-crack NMM model*

355 A pre-cracked granite specimen model of 20 mm in diameter and 40 mm in length is shown in Figure 16(a).
356 The real crack half-length is 4 mm, and the crack angle φ is 45° . The meshed specimen that contains multi-
357 cracks is shown in Figure 16(b). The three-node triangular elements in Figure 16(b) consist of mathematical
358 covers, and the solid black line in Figure 16(a) represents the physical boundary and discontinuities in the
359 specimen. The **thick and thin red solid** lines in Figures 16(c) and (d) represent real and virtual cracks,
360 respectively[20]. The model consists of 1108 elements, which means that this model has more than 1000 virtual
361 cracks. Every crack in the model has two crack tips, and the SIF of every crack tip can be calculated at every
362 simulation step. The initial virtual crack half-length is 0.042 mm. The real crack is a part of the physical cover,
363 and it can therefore cut an element and open and slide. There are no virtual cracks in the area of the real crack,
364 because these elements are damaged. Virtual cracks are only present in the three-node triangular elements, and
365 will not cut the elements. The angle and length of the virtual cracks is arbitrary, and their lengths are equal.



366

367 **Figure 16 NMM model of size and multi-crack distributions**

368 When the SIFs at the crack tips are lower than the fracture toughness K_{Ic} , both the real and virtual cracks will
369 propagate according to SCG theory. The elastic modulus of an element in which the virtual and real crack tips
370 are located will be damaged according to Equation (12):

$$\begin{cases} E' = D * E \\ D = (1 - L'/L)^n \\ L' = V * \Delta t \end{cases} \quad (12)$$

where E is the last step elastic modulus, E' is the damaged elastic modulus, D is the damage factor, L is the original crack length, L' is the crack propagation length, and n is the damage index.

Because the real crack length is larger than that of a virtual crack, the SIFs of the real crack will be first equal to Kc and then propagate according to the MCSSC. The virtual crack is very small and, because the crack length at each step is very small, crack growth will occur along the original direction and will take a long time. Before the virtual crack can change into a real crack, the main role of the virtual crack is to reduce the elastic modulus. The virtual cracks will not cut the elements. Once the virtual crack length propagates to a pre-defined value (related to model size and mesh density), it will be converted into a real crack according to the principles of NMM. During crack propagation, the boundary and node of the element cannot influence the crack direction. The crack can grow across the element boundary, and crack tip can be located on either the boundary, node, or within the element, which results in more freedom when simulating crack propagation.

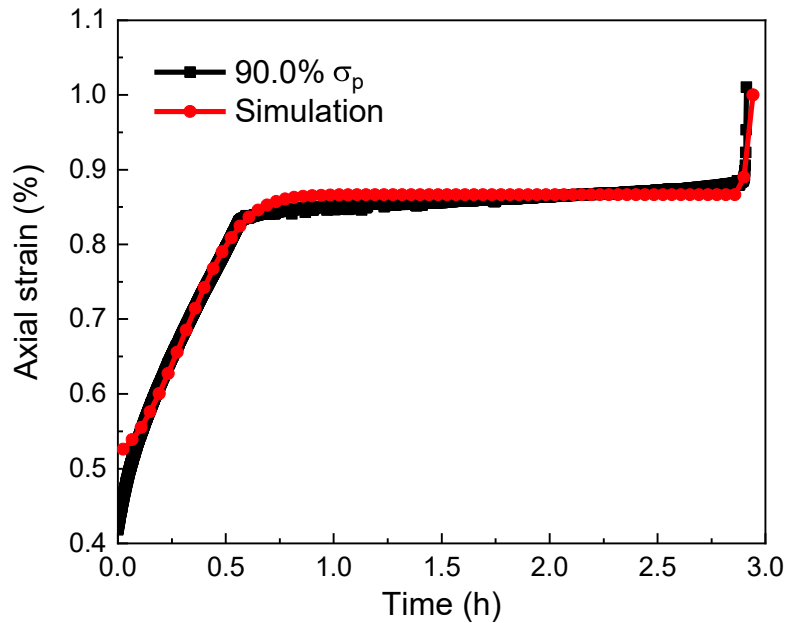
5.2 Simulation results

The improved NMM program was then used to simulate the time-dependent deformation of Lanhélin granite under constant uniaxial stress. The macroscopic mechanical properties of rock are shown in Table 2. An axial stress of 117 MPa was applied to the numerical specimen. The bottom of specimen was fixed.

Table 2 Physico-mechanical parameters of the numerical model

Items	Value
Elastic modulus: E(GPa)	21
Poisson's ratio: ν	0.25
Fracture growth constant: C	1×10^{-20}
Stress corrosion index: n	20.3
Time step: Δt (h)	0.04

388



389 **Figure 17** Comparison between experimental (black curve) and numerical (red curve) creep strain curve of
 390 Lanhélin granite
 391
 392

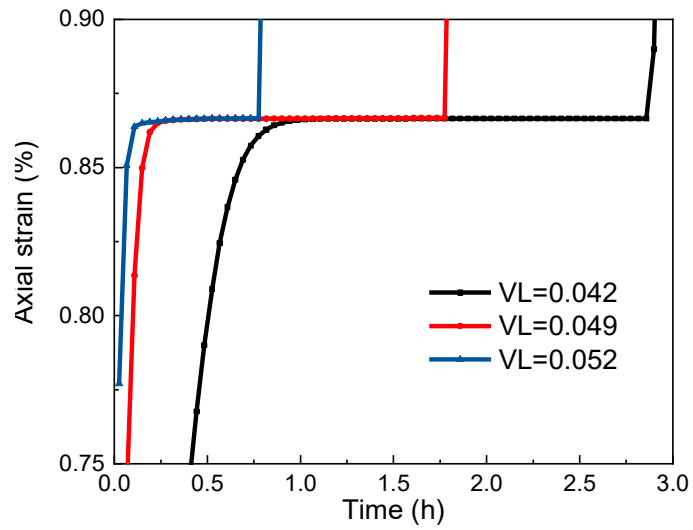
393 The classical creep simulation methods are the empirical method and the rheological component method,
 394 which use differential equations to realize the different stages of rock creep. In this research, when the SIFs of
 395 crack tips are smaller than K_{Ic} , both the virtual and real cracks propagate according to the SCG theory. During
 396 crack growth, when the crack is under the tension-shear condition, the program calculates the maximum tensile
 397 normal stress and maximum shear stress according Equation (9), when the crack is under the compress-shear
 398 condition, the program calculates the effective shear stress, after these judgements, the program then calculates
 399 the SIFs according to Equation (8), and gets the crack velocity according to Equation (7). The elastic modulus
 400 is decreased according to Equation (12) at the same time. When the SIFs are smaller than K_{Ic} , the axial strain
 401 decreases as an increasing function of time (i.e. the decelerating creep phase). When the SIFs are larger than
 402 K_{Ic} , the real crack propagates according to the MCSSC.

403 A comparison between experimental and numerical creep curves is shown in Figure 17. The experiment is a
 404 creep test performed on Lanhélin granite under uniaxial compression and room temperature, and the numerical
 405 model was performed under the same conditions. The experimental results clearly show the two stages of creep
 406 curve, decelerating and accelerating creep, and the numerical results are in good agreement with the experiment.
 407 In Figure 17, the initial axial strain is about 0.5%. In the first 0.6 h, the initial crack length is very small and, the

408 cracks are growing at a slow rate. As a result, the strain increases slowly. From 0.6 to 2.9 h, the axial strain
409 remains at almost a constant value. After 3 h, the axial strain increases quickly, the cracks propagate quickly
410 and connect together to generate a macro-crack zone that, eventually, results in macroscopic sample failure.

411 Figure 18 shows the influence of virtual crack length (VL) on the time-dependent deformation of granite. As
412 shown in Figure 18, the creep strain increases with increasing VL. The larger the VL, the faster the specimen
413 goes through the decelerating creep stage and the sooner the specimen reaches the accelerating creep stage.
414 Before the simulation, the virtual crack angles in every element were fixed. **During the time-dependent
415 deformation progress, the axial stress is a constant value and so the SIFs of crack tips will not increase according
416 to fracture mechanics. However, during the subcritical crack growth progress, crack tips will propagate
417 according to Eq. 7 (Charles' equation), and the crack speed and SIFs are positively correlated. The crack speed
418 and the virtual crack length (VL) are also positively correlated.** We also note that increasing VL cannot change
419 the axial strain when the strain rates are constant values. The VL increase as the crack velocity increased
420 according to Equation (12), and crack velocity increase as the SIFs increased according to Equation (7). So, VL
421 and SIF are positively correlated: the **longer** the VL, the larger the SIF. Further, with an increase of SIF, the
422 velocity of SCG also increases. So, the propagation of micro-cracks can result in the failure of the specimen.

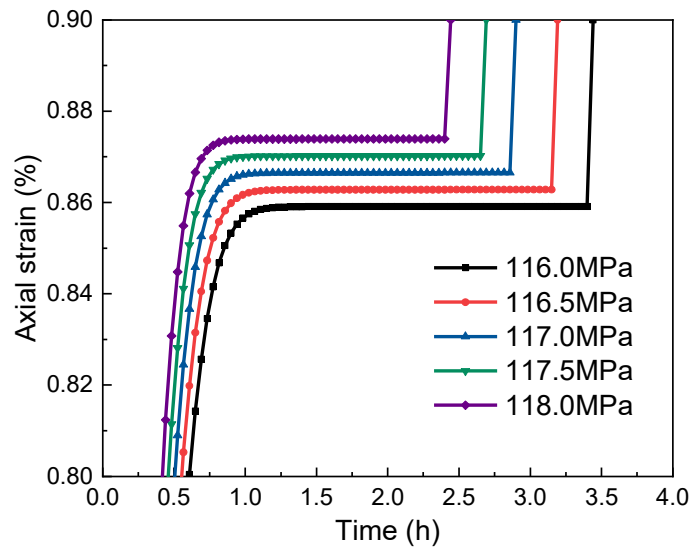
423 Figure 19 shows the influence of axial stress on the axial creep strain and strain rate. Five axial stress levels
424 were chosen to simulate the creep deformation, which are 116, 116.5, 117, 117.5, and 118 MPa (in Figure 19(a)).
425 The confining pressure was set to 0 MPa for these simulations. As the axial stress level was increased, the axial
426 strain also increased (Figure 19(a)). Figure 19(a) also shows that the time-to-failure decreased as the axial stress
427 level was increased. Figure 19(b) shows the evolution of axial strain rate as a function of time for the different
428 axial stress levels. We find that the axial strain rates are very sensitive to axial stress levels: the larger the axial
429 stress, the sooner the axial strain rate arrives at the minimum value, the sooner the sample enters the accelerating
430 creep phase, and the sooner the sample fails macroscopically (Figure 19(b)).



431

432

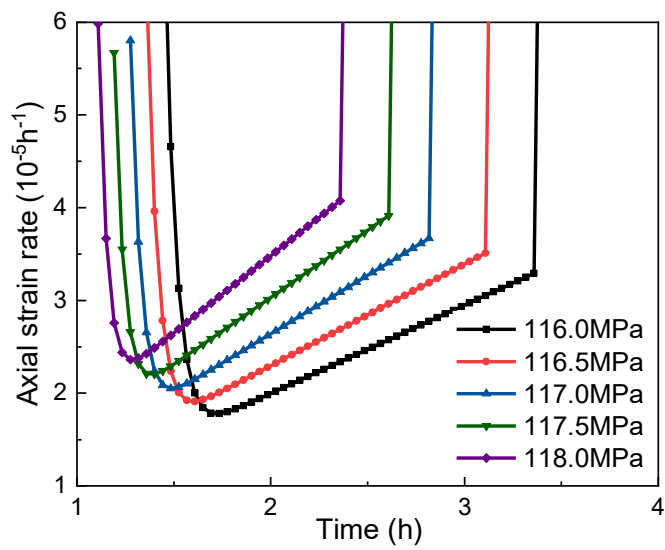
Figure 18 Influence of virtual crack length (VL) on time-dependent deformation



433

434

(a)



435

436

(b)

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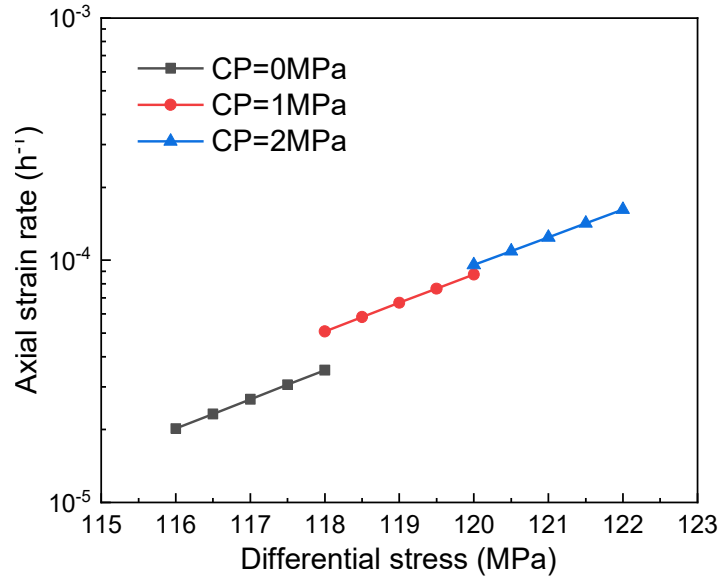
Figure 19 Influence of axial stress on time-dependent deformation. (a) Axial strain as a function of time

438

(creep curves) and (b) axial strain rate as a function of time for modeled samples at a confining pressure of 0

439

MPa

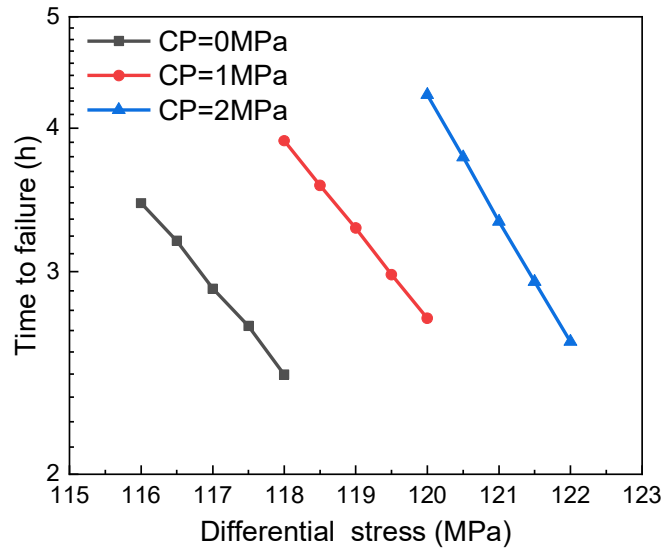


440

Differential stress (MPa)

441

(a)



442

Differential stress (MPa)

443

(b)

444

Figure 20 Influence of differential stress (DS) and confining pressure (CP) on time-dependent deformation.

445

(a) Axial strain rate as a function of differential stress and (b) time-to-failure as a function of differential stress

446

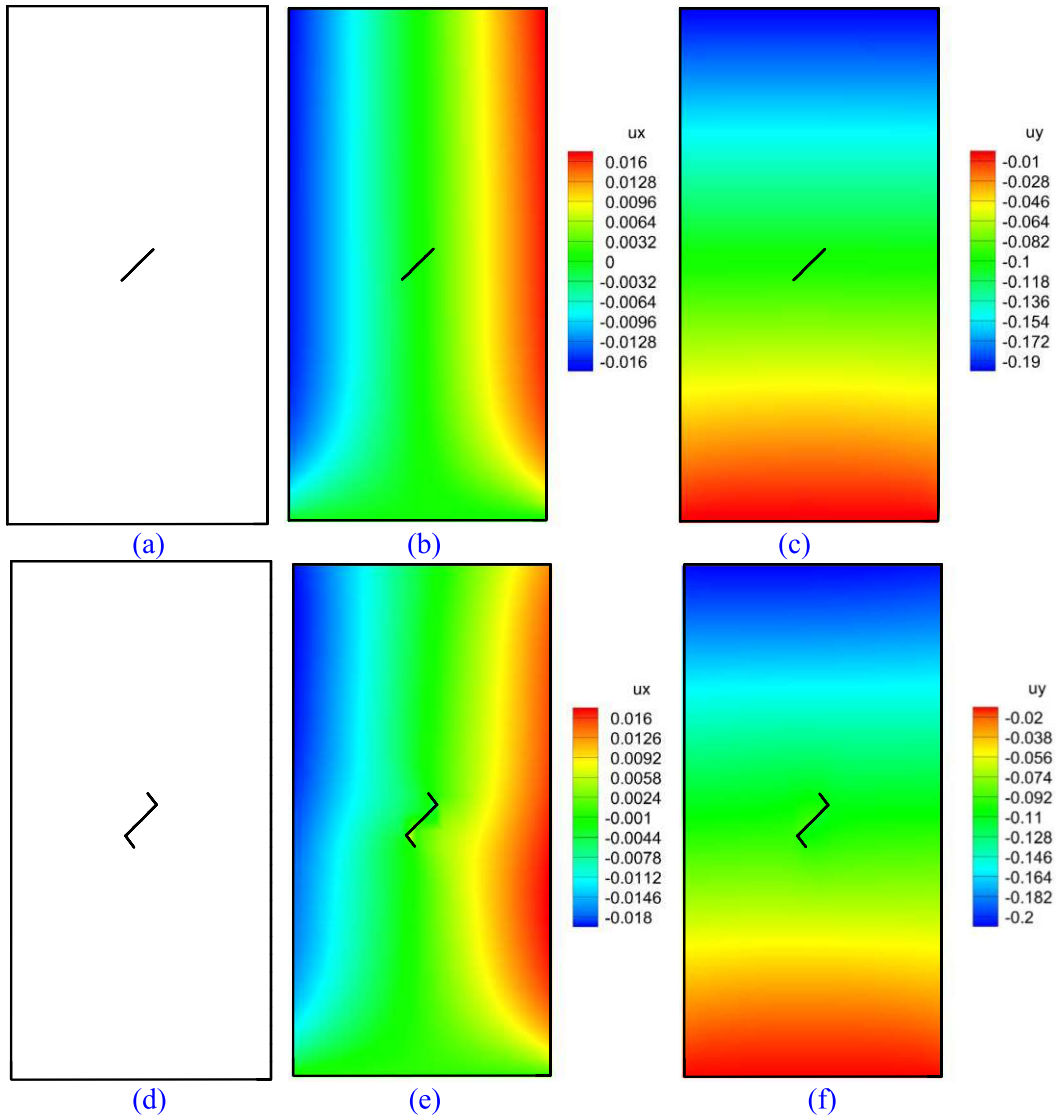
for models performed at confining pressures of 0, 1, and 2 MPa

447

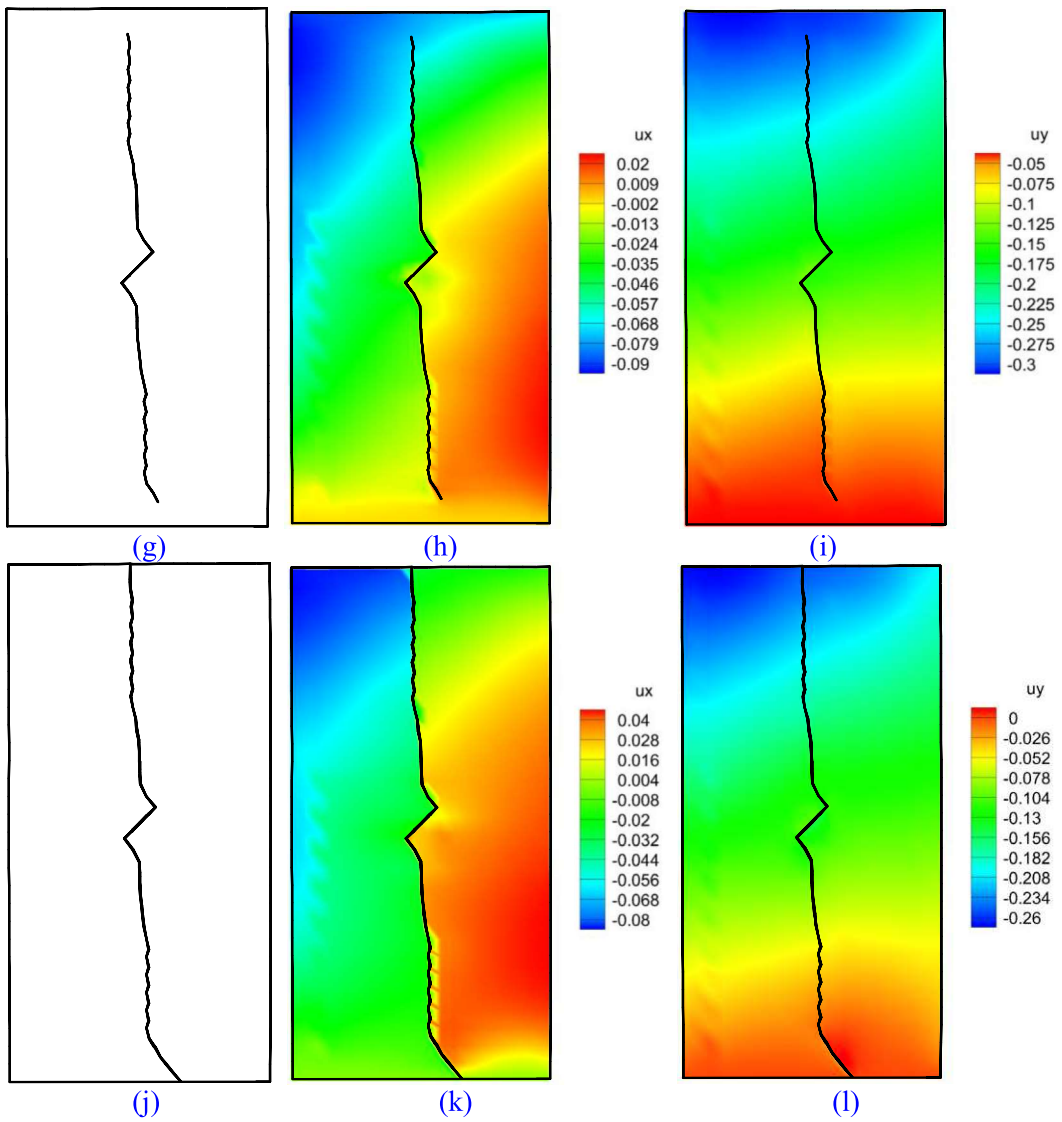
448 The influence of differential stress (DS) and confining pressure (CP) on the time-to-failure and creep strain
449 rate is illustrated in Figures 20(a) and (b), respectively. The data show that DS and CP has a profound influence
450 on times-to-failure and creep strain rates. In Figures 20(a) and (b), when the CP is a constant value, the axial
451 strain rate increases as a function of increasing DS: the larger the DS, the faster the axial strain rate. The time-
452 to-failure also decreases with increasing DS. When the CP is increased, the axial strain rate and time-to-failure
453 curves are shifted to higher values of DS. When we consider a constant value of DS (118 MPa, for example),
454 the axial strain rates are lower for the model performed at the highest CP, and the times-to-failure were
455 increased. These observations are similar to the experimental results of Heap[3] and Brantut[6]. These
456 simulation results also illustrate that a small change in differential stress and confining pressure can lead to
457 significant changes in axial strain rate and failure time.

458 According to the SCG theory, when K is smaller than K_c , both the virtual and real cracks will propagate at a
459 very low velocity. It is therefore not easy to observe the crack propagation process. On the other hand, if K is
460 larger than K_c , the real crack will propagate according to the MCSSC. According to Bobet[58, 59], since the
461 stresses adjacent to the flaw tip are much higher than the strength of the material, the crack tip will likely
462 propagate. In other zones of the material, it is not as easy to initiate cracking. Stress concentration effects at the
463 crack tips can also inhibit the initiation of surrounding cracks. The area where crack initiation is most likely is
464 the area containing the crack initial tip and crack boundary. Figure 21 shows the real crack propagation path
465 under a constant stress. Figures 21(a), (d), (g), and (j) show the initial real crack propagation, crack initiation,
466 and coalescence. As SCG progresses, the real crack propagates according to the MCSSC, because σ_1 is larger
467 than σ_3 . As a result, the crack extends in the direction of σ_1 . Because of NMM theory, the crack can propagate
468 by cutting through the elements, which is different from FEM and DEM in which the crack can only propagate
469 along the boundary of the elements. Crack tips in this simulation are not restricted to the node or boundary, but
470 can also exist within the element. After the crack propagates, the crack face is opened too. The program can
471 calculate the contact state at every node of the crack by adding and subtracting the spring to make sure the crack
472 surface is not embedded according to contact theory. Figures 21(b), (c), (e), (f), (h), (i), (k), and (l) show the
473 displacement of the specimen in the X- and Y-direction under a constant stress. At the beginning of simulation,
474 the X-direction displacement is symmetric from left to right, and the Y-direction displacement is symmetric

475 from top to bottom. The crack propagation can greatly influence the displacement and facilitate macroscopic
476 failure. The maximum displacement moves gradually toward the crack during crack propagation. Figure 22
477 shows a comparison between the failure mode of a specimen with a 45° crack from one of the NMM numerical
478 simulations and a sample deformed under a constant uniaxial stress in the laboratory (the experiment was
479 performed by Qin[60]). Figure 22 shows that the failure mode observed during the NMM simulation is very
480 similar to that observed in the laboratory.



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Figure 21 Crack propagation path and displacement in the X- and Y-direction under time-dependent deformation when K is larger than K_c

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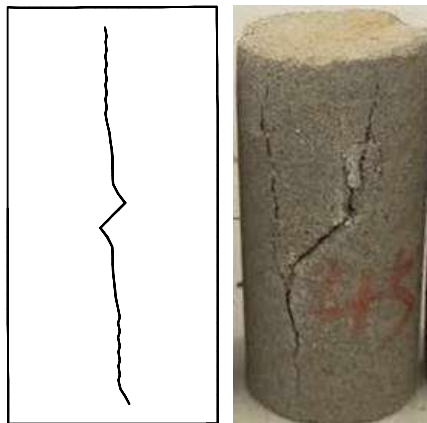


Figure 22 Comparison between the failure mode of a specimen with a 45° crack from one of the NMM numerical simulations and a sample deformed under a constant uniaxial stress in the laboratory (the experiment was performed by Qin[60])

496 6. Concluding remarks

497 In the present paper, the time-dependent deformation of granite is modeled by combining SCG and NMM on
498 the MATLAB® platform. This combined method replaces the traditional creep component model method. In
499 the model, we introduced virtual cracks into the NMM to realize crack initiation. Before the virtual crack can
500 change into a real cack, the virtual crack must be located at every element of the NMM, propagate in a line, and
501 reduce the elastic modulus of the elements. No opening or sliding occurs during virtual crack propagation: only
502 real cracks can open and slide. Because crack coalescence is very complicated during crack propagation, we
503 introduce three coalescence models to deal with this problem, which are the “tip and tip”, “tip and boundary”,
504 and “boundary and boundary” crack models. In order to reflect the influence of crack propagation on the creep
505 deformation of Lanhélin granite, we use Charles’ equation to calculate the crack velocity. When stress intensity
506 factor K is smaller than the fracture toughness K_c , the crack growth length is linked to the local damage (using
507 an exponential material softening law) to reduce the strength of the granite. When K is larger than K_c , the cracks
508 propagate according to the MCSSC. Parameter sensitivity analysis for the improved NMM program was
509 performed and validated against experimental data. The influence of crack length, confining pressure, and
510 differential stress were then investigated. The results show that the microstructure of granite has a great
511 influence on time-dependent deformation. We found that the virtual crack length (VL), differential stress (DS),
512 and confining pressure (CP) all play a major role in governing elastic deformation. With an increase of VL, the
513 specimen can reach the accelerating creep stage quickly, because the VL can greatly influence the SIF at the
514 crack tips. Therefore, the larger the VL, the higher the crack velocity. CP increases rock strength and therefore
515 decreases the creep strain rate and increases time-to-failure at a constant stress. Even a small change of DS can
516 significantly increase and decrease the creep strain rate and time-to-failure, respectively. In addition, we find
517 that the cracks propagate along the direction of the maximum principle stress. When the SIF of the crack tip is
518 larger than K_c , the real cracks start to initiate and propagate. The crack can also influence the specimen
519 displacement greatly during propagation. The maximum displacement also moves towards the crack during
520 crack propagation, which means that the crack zones are more likely to be broken. The results of the numerical
521 simulations are in good agreement with the experimental results and therefore show that the improved NMM
522 program is suitable for modeling the time-dependent creep deformation of rocks. Last but not least, the improved
523 NMM program also simulates large displacements during creep deformation. The cracks in the specimen can

524 open and slide without the influence of the element node or boundary, therefore reducing the reliance on the
525 grid, which makes it possible for the program to more accurately simulate crack propagation during creep
526 deformation.

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532 [comments of two reviewers helped improve this paper.](#)

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