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A Plate Electrostrictive Finite Element – Part I: Modeling and Variational Formulations

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ABSTRACT: Piezoelectric plates have been widely used for the vibration reduction and noise control of structures. Due to power forces considerations, electrostrictive patches present a growing interest. It is thus the purpose of the present research to contribute to modeling aspects of thin structures integrating such actuators. Many three-dimensional finite element models have then been elaborated to simulate these structures behavior but no two-dimensional models have been presented up to now. The aim of this article is thus to set out the elaboration of a thin plate electrostrictive finite element for PMN-PT type ceramics used as actuators. This element is developed for dynamic purposes and thus takes into account phenomena induced by applying to the patch a cycling electric field. The finite element formulation is based on electromechanical constitutive equations derived in a previous paper, mechanical and electrical considerations and direct a priori plate assumptions. The electrostrictive finite element is here derived using techniques inspired from a piezoelectric finite element. This method has the particular property of reducing the initial electro-mechanical problem to a purely mechanical problem based on a modified elastic constitutive law. The electrical unknowns are then explicitly derived from the mechanical displacements. This method considerably simplifies the resolution of the problem since classical finite elements for laminated plates can be used to model the electrostrictive plate with a modified constitutive law. This paper is focused on the plate modeling and corresponding variational formulations.

INTRODUCTION

PIEZOELECTRIC plates have been widely used for the vibration reduction and noise control of structures. Due to power forces considerations, electrostrictive patches present a growing interest. It is thus the purpose of the present research to contribute to modeling aspects of thin structures integrating such actuators.

Several researches have been performed on electrostrictive ceramics for the last fifteen years with the view of characterizing their electromechanical behavior under varying electric fields. These studies particularly showed a great sensitivity of induced polarization to operating parameters: bias electric field and dynamic electric field magnitudes, dynamic electric field excitation frequency, surrounding temperature, and pre-stress (Uchino et al., 1980, 1981; Zhang et al., 1988, 1989; Pan et al., 1989; Namboodri and Rogers, 1992; Blackwood and Ealey, 1993; Namboodri, 1993; Rittenmyer, 1994; Brown et al., 1996; Sherrit et al., 1998; Kurutcharry, 2000).

We additionally proved in a paper devoted to the establishment of electrostrictive constitutive laws (see (Pablo and Petitjean, 2000a)), that an important heating of electrostrictive patches occurs under cycling electric field. We then underlined that the sensitivity of the electromechanical behavior of electrostrictive ceramics to operating parameters was, in fact, a sensitivity of this behavior to the ceramic own temperature. A modeling of the electrostrictive ceramic heating has been elaborated through an experimental curve fitting with respect to operating parameters and led to a complex law. We thus opted for using lookup tables so that to determine the ceramic temperature. Finally, isothermal constitutive laws for electrostrictive ceramics were established in a similar way as Hom and Shankar (1994). We remark here that recent works, such as those performed by Kurutcharry (2000), established modified constitutive laws in order to take into account hysteresis observable in ferroelectric cycles.

Results presented in the present paper has been obtained within the framework of a study aimed at performing active vibration control of plate structures using electrostrictive patches as actuators (Pablo, 2002).

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As, very little research has – up to now – been done with the view of using these materials in such applications (Fripp et al., 1994; Fripp, 1995; Fripp and Hagood, 1997), new studies were necessary.

The behavior simulation of the controlled structure coupled with electrostrictive ceramics is needed in order to be able to elaborate the controller. Such simulations will then be carried out through a Finite Element Method.

The aim of this article is thus to present the development of an electrostrictive finite element. The first part of this paper, here presented, is focused on the plate theory leading to finite element formulation which will be studied more precisely in the second part.

The electromechanical coupling in active vibration control applications using electro-actuators, implies the necessity of simultaneously solving electrical and mechanical equations which increases computation times. This could be a penalty for controller elaboration. The element here to be described is characterized by the faculty of reducing this electromechanical problem to a purely mechanical problem having a particular elastic constitutive law. Finally, we underline that the theory allowing such a simplification is only based on a priori assumptions, and can thus, just as well, be applied to continuous media as to discrete ones (as exposed hereafter).

This paper will first deal with an important discussion on the driving input to be used in active vibration control applications. The structure of this article in two main parts, each of one being focused on consequences of using a particular driving input on the plate theory, is then induced by this discussion. Additional parts will, afterwards, expose considerations on the electrostrictive plate finite element and application to the active vibration control.

DRIVING INPUT DISCUSSION

Current active vibration control applications integrating electromechanical actuators such as piezoelectric or electrostrictive ceramics, mainly use voltage as the driving input. These actuators then work as follows: an electric potential difference (V) is applied between the lower and upper faces of the patches which induces a transverse electric field (E) in the ceramic, which, in turn, generates strains (S).

When a voltage driving input is used for piezoelectric actuators, strains are linear functions of the applied electric field (when actuators are working in the electric field range defined by the manufacturer), but, when used for electrostrictive ceramics the strains are no longer linear functions of this field.

This remark is, for example, observable in Figure 1(a), where electrostrictive strains are plotted as functions of the electric field for a 500 V/mm bias electric field magnitude, a 500 V/mm dynamic electric field

magnitude, a 200 Hz excitation frequency and a 20°C surrounding temperature. Indeed, one can observe in this figure that strains are quadratic functions of electric field for low magnitudes but saturate from about 400 V/mm. We moreover remark that this electromechanical cycle underscores non negligible hysteresis.

If we now plot these previous electrostrictive strains as functions of polarization (or electric displacement) for similar operating parameters (Figure 1(b)), a quadratic dependence of strains on polarization is observable, but neither saturation nor hysteresis can be strongly noticed.

We underline here, that, observing experimental curves obtained for various operating parameters, we can note an hysteresis in polarization–strain curves, but this hysteresis is small compared to the one observable in electric field–strain curves.

From these observations it is obvious that saturation and hysteresis non-linearities in the $S(E)$ curves are mainly induced by the ferroelectric cycle (electric field–polarization curves). This conclusion can clearly be verified in Figure 2 where the ferroelectric cycle of the electrostrictive ceramic obtained for similar operating parameter used to obtain curves of Figure 1 is plotted. Indeed, this ferroelectric cycle is characterized by a

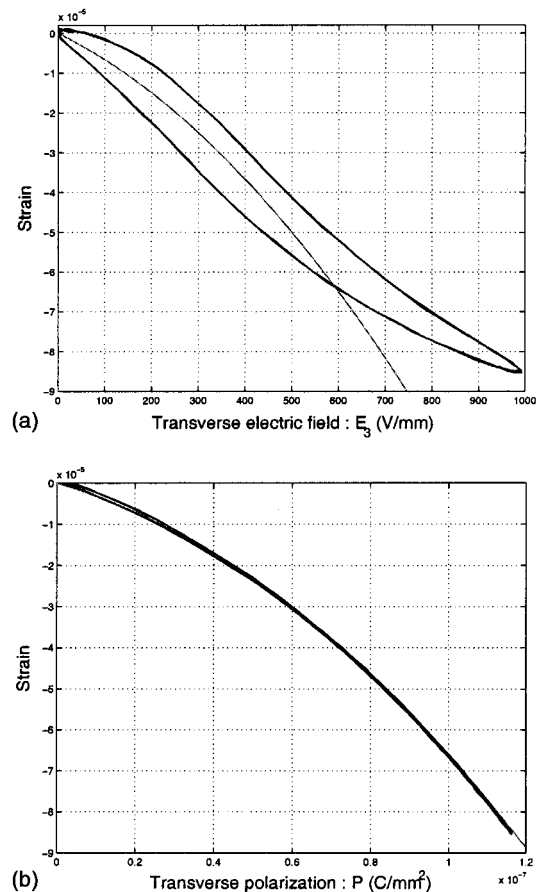


Figure 1. Electrostrictive strain as a function of electric field and polarization. (a) electrostrictive strains functions of applied electric field; (b) electrostrictive strains functions of polarization.

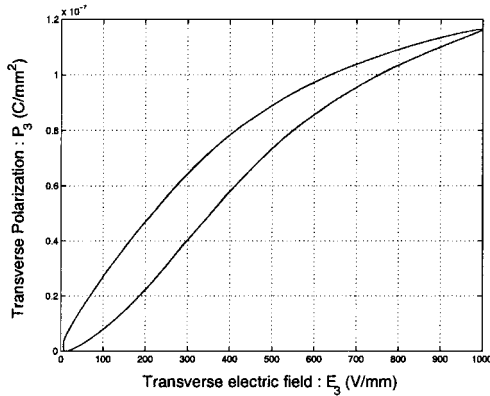


Figure 2. 0.9PMN-0.1PT ferroelectric cycle.

significant hysteresis, and a polarization saturation for electric field magnitudes greater than 400 V/mm.

As the electromechanical behavior of electrostrictive ceramics seems to be less non-linear when strains are plotted as a function of polarization rather than electric field, being able to impose polarization on electrostrictive patches would thus considerably simplify the use of these ceramics as actuators. This would be done by using current instead of voltage as the driving input of these actuators.

These experimental observations are not brand-new ones. Indeed, Comstock (1976) already discovered in 1976 that piezoceramics exhibited less hysteresis when they were current driven than when they are voltage driven. Moreover, some papers have already presented methods allowing such a current driving for piezoelectric (Lindner and Chandrasekaran, 1999a,b) and electrostrictive (Znovar et al., 1996; Znovar and Lindner, 1997, 1998) stacked actuators.

Znovar and Lindner (1998) particularly developed small power amplifiers driving nine stacked actuators of total nominal capacitance around 45 μF .

In our active vibration control applications, $20 \times 30 \times 0.4 \text{ mm}^3$ electrostrictive patches are used instead of stacked actuators. These patches are especially characterized by a nominal capacitance of about 150 nF. This very low capacitance compared to the previous one implies the need of using high voltage and low current magnitudes – up to 400 V and less than 300 mA – compared to those given by Znovar and Lindner (1998) (voltage magnitude about 30 V and peak to peak current around 3 A). For these reasons, a new electronic technology was needed with the view of using current driven electrostrictive patch actuators.

As we underlined the necessity of using current as input of electrostrictive actuators, the plate theory presented in the following will take into account this assumption. Nevertheless, a voltage driven plate theory will also be established as to highlight difficulties introduced in a classical voltage based theory.

PROBLEM STATEMENT

The aim of the present study is to perform active vibration control of plate structures using electrostrictive actuators. The problem here to be solved is thus a fully coupled electromechanical problem.

Moreover, previous researches proved that the behavior of electrostrictive ceramics is non-linear with respect to electric field. This non-linearity is then taken into account through the constitutive laws. The problem to be solved is thus a materially non-linear one. We underline here that we moreover assume small strains and rotations, and that we will thus not take into account geometrical non-linearities.

The design of the controller implies the need of simulating the behavior of the coupled structure through, for example, a Finite Element Method.

Many piezoelectric finite elements have been developed for static and dynamic studies. Among this important literature we can, for example, quote Gaudenzi and Bathe (1995), Suleman and Venkayya (1995), Rahmoune et al. (1998), Bisegna and Caruso (2000).

On the other hand, very few electrostrictive finite elements have been elaborated up to now. Yet, three-dimensional ones have been developed with the view of simulating phase transition in non-linear ferroelectric materials (Ghandi and Hagood, 1996, 1997). Moreover, the simulation of electrostrictive stacks behavior also required the creation of two-dimensional bar finite elements (Hom and Shankar, 1996). Finally, Dubus et al. developed two two-dimensional electrostrictive finite elements (an 8 nodes quadrangular element and a 6 nodes triangular one) based on a three-dimensional variational formulation as a function of mechanical and electrical variables (Dubus et al., 1996, 1999; Debus et al., 1998).

As we wish to simulate the behavior of plate structures equipped with electrostrictive patches, the use of plate finite elements seems to be more suitable than three-dimensional ones. No electrostrictive plate finite element with a priori plate assumptions has – to our knowledge – been developed up to now.

The present section exposes the theory leading to such an elaboration. The underlying major ideas of the theory have been given in an earlier work carried out by the second author which was devoted to linear piezoelectric plates (see Osmont, 1996). This theory was moreover proved to be valid for linear Kirchhoff–Love plates through an asymptotic theory (Rahmoune et al., 1996, 1998).

This section is in particular focused on establishing the basic equations of the plate theory. Given the coupling of mechanics and electrics in electrostrictive ceramics, sets of equations able to model mechanical and electrical behaviors are needed to correctly set the problem. Constitutive laws established in a previous

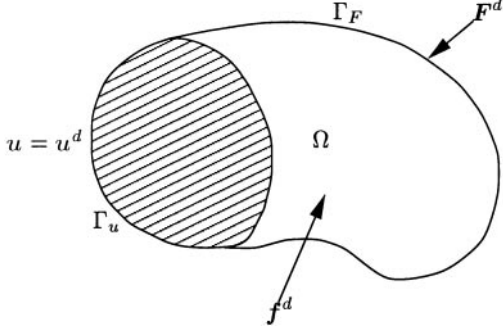


Figure 3. Classical mechanical problem.

paper and Kirchhoff–Love a priori plates assumptions will moreover be reminded.

Remark: From now on, we will respectively note down \mathbf{T} , \mathbf{S} , u , \dot{u} , \ddot{u} , \mathbf{f}^d and \mathbf{F}^d the Cauchy stresses, strains, mechanical displacements, velocity, acceleration, imposed body forces and imposed surface forces, and respectively, \mathbf{D} , \mathbf{E} , V , q^d and Q^d the electric displacement, electric field, electric potential, imposed body charges and imposed surface charges.

Mechanical Equations

In this paragraph, we will recall classical mechanical equations and a modification that previous papers proposed for dielectric media.

We consider in the following a body Ω delimited by its surface boundary $\partial\Omega$. We suppose that this body is subjected to a body force \mathbf{f}^d , a surface traction \mathbf{F}^d on a part $\partial\Omega_F = \Gamma_F$ of its boundary, and an imposed mechanical displacement u^d on the complementary part of the boundary $\partial\Omega_u = \Gamma_u = \partial\Omega - \Gamma_F$, as presented in Figure 3.

CLASSICAL MECHANICAL GOVERNING LAWS

The three-dimensional equations of a classical elastodynamical problem are (with the usual initial Cauchy conditions) (Fung, 1965):

Dynamic Equations:

$$\mathbf{T}_{ij,j} + \mathbf{f}_i^d = \rho \ddot{u}_i \quad \text{in } \Omega. \quad (1)$$

Displacement boundary conditions:

$$u_i = u_i^d \quad \text{on } \Gamma_u, \quad (2)$$

Stress boundary conditions:

$$\mathbf{T}_{ij}, n_j = \mathbf{F}_i^d \quad \text{on } \Gamma_F, \quad (3)$$

Strain–displacement relationship:

$$\mathbf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (4)$$

Constitutive law:

$$\mathbf{T}_{ij} = \mathbf{T}_{ij}(\mathbf{S}). \quad (5)$$

ELECTROMECHANICS FOR CONTINUOUS DIELECTRIC MEDIA

Toupin (1956, 1963) and Eringen (1963) showed that in dielectrics (such as electrostrictive ceramics) subjected to an electric field, mechanical governing equations had to be slightly modified such as taking into account the influence of this electric field on the mechanical behavior. Hom and Shankar (1996) recently adapt this theory for small deformations and rotations, for static problems.

If we now moreover take into account elastic dynamical phenomena, the elastodynamical problem of dielectric continuous media subjected to an electric field can be set down through the following governing equations:

Dynamic equations:

$$\mathbf{T}_{ij,j} + m_{ij,j} + \mathbf{f}_i^d = \rho \ddot{u}_i \quad \text{in } \Omega,$$

Displacement boundary conditions:

$$u_i = u_i^d \quad \text{on } \Gamma_u,$$

Stress boundary conditions:

$$[\mathbf{T}_{ij} + m_{ij}], n_j = -\mathbf{F}_i^d \quad \text{on } \Gamma_F,$$

Strain–displacement relationship:

$$\mathbf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

Maxwell tensor definition:

$$m = \varepsilon_0 \left(\mathbf{E}_i \mathbf{E}_j - \frac{1}{2} \mathbf{E}_k \mathbf{E}_k \delta_{ij} \right),$$

Constitutive laws:

$$\mathbf{T}_{ij} = \mathbf{T}_{ij}(\mathbf{S}, \mathbf{E}) \quad \text{and} \quad \mathbf{D}_k = \mathbf{D}_k(\mathbf{S}, \mathbf{E}).$$

Nevertheless, as it will be shown in the following sections, in-plane electric field can, in our applications, be neglected compared to imposed transverse one. From this a priori assumption, the Maxwell electrostatic stress tensor (m) is then a diagonal matrix composed by terms of absolute value given by $0.5\varepsilon_0 \mathbf{E}_3^2$. If we now compare this value to the Cauchy stresses calculated from the electric part of the constitutive laws presented

hereafter (second term of the right hand side of Equation (12)), one can deduce that Maxwell electrostatic stresses are negligible compared to Cauchy stresses (the ratio between these two stresses is of the order of 10^{-6}).

Given these observations, the Maxwell electrostatic stress tensor will not be taken into account in our problem formulation, and the mechanical behavior will thus be given by mechanical governing Equations (1)–(5).

Electrical Equations

In the active vibration control applications we are interested in, time varying electric fields are imposed to the electrostrictive actuators in order to reduce the vibrations of the host structures. A formulation of the present electromechanical problem should thus use electromagnetic theories.

Nevertheless, in dielectric materials such as piezoelectric and electrostrictive ceramics, the electromagnetic waves are characterized by a much greater propagation speed – about that of light: $3 \cdot 10^8 \text{ m s}^{-1}$ – than elastic waves. Time scales used in structural mechanics then imply electric fields to be established at any time t . The electric field dynamics thus does not play any part in this study. Electrostatics will therefore be used from now on.

Let us now consider a body Ω delimited by its surface boundary $\partial\Omega$. We suppose that this body is subjected to a body charge density q^d , a surface charge density Q^d on a part $\partial\Omega_Q = \Gamma_Q$ of its boundary, and an electric potential V^d on the complementary part of its boundary $\partial\Omega_V = \Gamma_V = \partial\Omega - \Gamma_Q$, as presented in Figure 4).

Electrostatics laws for dielectric materials are then (Landau and Lifchitz, 1969):

Gauss law:

$$\mathbf{D}_{i,i} - q^d = 0 \quad \text{in } \Omega. \quad (6)$$

Electric potential boundary conditions:

$$V = V^d \quad \text{on } \Gamma_V, \quad (7)$$

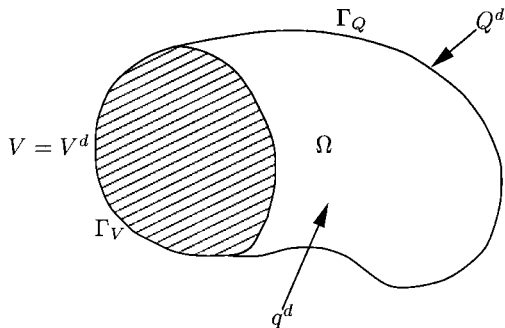


Figure 4. Classical electrical problem.

Electric displacement boundary conditions:

$$\mathbf{D}_i n_i = -Q^d \quad \text{on } \Gamma_Q, \quad (8)$$

Electric field–electric potential relationship:

$$\mathbf{E}_i = -V_{,i}. \quad (9)$$

Constitutive Laws

We underlined in Pablo and Petitjean (2000a) that electrostrictive ceramics are characterized by a non-linear behavior which is moreover fully electro-mechanically coupled. This strong coupling can then be mathematically formulated through the material constitutive laws.

This paragraph recalls the constitutive laws which have been analysed in Pablo and Petitjean (2000a) for electrostrictive patches subjected to a time varying electric field.

CURRENT DRIVEN ACTUATORS

If we are interested in current driven patches, one has to use the strain (\mathbf{S}) and the electric displacement (\mathbf{D}) as the mechanical and electrical independent variable, to establish the electrostrictive constitutive laws.

These current driven constitutive laws, obtained in Pablo et al. (2001b), then respectively express the mechanical and electrical intensive variables (\mathbf{T} and \mathbf{E}) as functions of mechanical and electrical extensive variables (\mathbf{S} and \mathbf{D}). One can then obtain:

$$\mathbf{T}_{ij} = \mathbb{C}_{ijkl}^D (\mathbf{S}_{kl} - \mathbb{Q}_{klmn} \mathbf{D}_m \mathbf{D}_n), \quad (10)$$

$$\mathbf{E}_m = (\chi_{mm}^*)^{-1} \mathbf{D}_m^s \operatorname{arctanh}\left(\frac{\mathbf{D}_m}{\mathbf{D}_m^s}\right) - 2\mathbb{C}_{ijkl}^D \mathbb{Q}_{klmn} \mathbf{D}_s (\mathbf{S}_{ij} - \mathbb{Q}_{ijpq} \mathbf{D}_p \mathbf{D}_q), \quad (11)$$

where χ^* and \mathbf{D}^s are respectively a pseudo-susceptibility obtained by the slope of the ferroelectric cycle around a nil polarization and a saturation constant which have an electric displacement dimension and which characterize the saturation of the ferroelectric cycle for high electric fields.

VOLTAGE DRIVEN ACTUATORS

If we are now interested in voltage driven patches, one has to use the strain (\mathbf{S}) and the electric field (\mathbf{E}) as the mechanical and electrical independent variables, to establish the electrostrictive constitutive laws.

Proceeding in a similar way as used to derive Equations (10) and (11), the voltage driven constitutive laws then respectively express the intensive mechanical

variable (\mathbf{T}) and the extensive electrical variable (\mathbf{D}) as functions of the extensive mechanical variable (\mathbf{S}) and the intensive electrical variable (\mathbf{E}). One can then write:

$$\mathbf{T}_{ij} = \mathbb{C}_{ijkl}^E \mathbf{S}_{kl} - \mathbb{C}_{ijkl}^E \mathbb{Q}_{klmn} \chi_{nm}^* \chi_{nm}^* \mathbf{E}_m^S \mathbf{E}_n^S \times \tanh\left(\frac{\mathbf{E}_m}{\mathbf{E}_m^S}\right) \tanh\left(\frac{\mathbf{E}_n}{\mathbf{E}_n^S}\right), \quad (12)$$

$$\begin{aligned} \mathbf{D}_m &= \chi_{nm}^* \mathbf{E}_m^S \tanh\left(\frac{\mathbf{E}_m}{\mathbf{E}_m^S}\right) \\ &+ 2\mathbb{C}_{ijkl}^E \mathbb{Q}_{klmn} \mathbf{S}_{ij} \chi_{nm}^* \chi_{nm}^* \mathbf{E}_n^S \frac{\tanh(\mathbf{E}_n/\mathbf{E}_n^S)}{\cosh^2(\mathbf{E}_m/\mathbf{E}_m^S)} \\ &- 2\mathbb{C}_{ijkl}^E \mathbb{Q}_{klmn} \mathbb{Q}_{ijpq} \chi_{nm}^* \chi_{nm}^* \chi_{pp}^* \chi_{qq}^* \mathbf{E}_n^S \mathbf{E}_p^S \mathbf{E}_q^S \\ &\times \tanh\left(\frac{\mathbf{E}_p}{\mathbf{E}_p^S}\right) \tanh\left(\frac{\mathbf{E}_q}{\mathbf{E}_q^S}\right) \frac{\tanh(\mathbf{E}_n/\mathbf{E}_n^S)}{\cosh^2(\mathbf{E}_m/\mathbf{E}_m^S)} \end{aligned} \quad (13)$$

where \mathbf{E}^S is a saturation constant which has an electric field dimension and which characterizes the saturation of the ferroelectric cycle for high electric fields.

REMARKS

We underline here that developing constitutive equations as functions of one or more intensive variable induces more complex expressions. These laws moreover introduce new parameters (χ^* and \mathbf{E}^S) which were shown in a previous paper (Pablo and Petitjean, 2000a) to be sensitive to the ceramic own temperature. Voltage driving electrostrictive patches thus introduces additional non-linearities.

The advantages of using current as the driving input instead of voltage, discussed in a previous section, remain true.

Material Assumptions

We will suppose up to now that this study is focussed on a particular class of materials which are mechanically and electro-mechanically orthotropic. These additional assumptions can then be formulated as follows:

1. there is no coupling, through the constitutive laws, between transverse distortions and stresses on the one hand, and, plane strains and stresses on the other,
2. there is no coupling, through the constitutive laws, between transverse electric field and displacement on the one hand, and, in-plane electric field and displacement on the other,
3. electrostriction couples transverse electric displacement and field, and plane stresses and strains on the one hand, and, in-plane electric displacements and fields, and transverse stresses and strains on the other.

Let us remark that electrostrictive ceramics such as PMN-PT and piezoelectric ceramics such as PZT belong to this particular class of materials.

A Priori Assumptions for Plate Modeling

We will suppose afterwards that the structures we are interested in, are slender structures, which means that the thickness h is supposed to be of first order compared to the in-plane characteristic dimension L . One thus have: $h/L \ll 1$.

From the dynamic Equations (1), one can write:

$$\begin{aligned} \mathbf{T}_{\alpha\beta, \beta} + \mathbf{T}_{\alpha z, z} + f_{\alpha}^d &= \rho \ddot{u}_{\alpha}, \\ \mathbf{T}_{z\alpha, \alpha} + \mathbf{T}_{zz, z} + f_z^d &= \rho \ddot{u}_z. \end{aligned}$$

with $\alpha, \beta = x, y$.

From, these two equations, one can deduce that $\mathbf{T}_{\alpha\beta, \beta}$ and $\mathbf{T}_{\alpha z, z}$, and, $\mathbf{T}_{z\alpha, \alpha}$ and $\mathbf{T}_{zz, z}$ are of the same order. If we moreover underline that:

$$\begin{aligned} \mathbf{T}_{\alpha\beta, \beta} &\text{ is of order } \frac{1}{L} \|\mathbf{T}_{\alpha\beta}\|, \\ \mathbf{T}_{\alpha z, z} &\text{ is of order } \frac{1}{h} \|\mathbf{T}_{\alpha z}\|, \\ \mathbf{T}_{z\alpha, \alpha} &\text{ is of order } \frac{1}{L} \|\mathbf{T}_{z\alpha}\|, \\ \mathbf{T}_{zz, z} &\text{ is of order } \frac{1}{h} \|\mathbf{T}_{zz}\|, \end{aligned}$$

one can conclude that transverse stresses \mathbf{T}_{iz} ($i=x, y, z$) are at least of h/L order compared to in-plane stresses $\mathbf{T}_{\alpha\beta}$ ($\alpha, \beta = x, y$) and are thus negligible.

We now additionally suppose that plates are orthotropic and thus meet Kirchhoff–Love assumptions which state that every straight line in the plate that was originally perpendicular to the plate middle surface remains straight after the deformation and perpendicular to the detected middle surface (Fung, 1965). From these assumptions, one can express in-plane displacements (u_x and u_y) as linear functions of z (transverse variable), the coefficients being independent of this variable. Moreover, the transverse displacement (u_z) is constant and independent of z . One can thus write:

$$u_x(x, y, z, t) = u(x, y, t) - z w_{,x}(x, y, t) \quad (14)$$

$$u_y(x, y, z, t) = v(x, y, t) - z w_{,y}(x, y, t) \quad (15)$$

$$u_z(x, y, z, t) = w(x, y, t) \quad (16)$$

From mechanical strain–displacement relationship (Equation (4)), in-plane strains ($\mathbf{S}_P = \mathbf{S}_{\alpha\beta}$) associated with these displacements can be written as linear

functions of z :

$$\mathbf{S}_p = \mathbf{S}^0 - z\mathbf{R}^1 = \begin{pmatrix} u, x \\ v, y \\ u, y + v, x \end{pmatrix} - z \begin{pmatrix} w, xx \\ w, yy \\ 2w, xy \end{pmatrix}, \quad (17)$$

where \mathbf{S}^0 and \mathbf{R}^1 are respectively the in-plane strains deriving from membrane and the curvatures of the plate, and are independent on z . Transverse strains (\mathbf{S}_{iz} , $i=x, y, z$), derived from the displacements here defined, (Equation (17)) are, as for them, zero. These equations are the expression of the well-known Kirchhoff–Love assumptions.

Assumptions introduced in this section will allow simplifications in the theory developed hereafter.

PLATE MODELING

The purpose of the present section is to present a plate theory for electrostrictive patches, leading to the elaboration of an electrostrictive finite element.

As was previously underlined, working with electrostrictive materials implies – as for piezoceramics – coupling mechanical and electrical phenomena. The theory here developed is based on the assumption that the in-plane electric displacements and fields are negligible with respect to transverse electric displacement and field. This assumption makes it possible to obtain an explicit expression of the transverse displacement (respective field) as a function of the transverse electric field (respective displacement) and, stresses or strains. It is then possible to eliminate the unknown electrical variables in favor of the mechanical displacements and to obtain a purely mechanical problem with equivalent electric forces and modified elastic constitutive laws.

The simplification of the electromechanical problem to a purely mechanical problem is slightly different according to the driving input used. This section is thus composed of two main parts, each one being focused on the theory for one of these driving inputs.

Current Driven Actuator

In this operating configuration an out-of-plane current is imposed through the electrostrictive patch. This current then induces charges on the upper and lower ceramic surfaces which can be associated with electric displacements. We will thus, in this section, suppose that transverse electric displacements (\mathbf{D}_z) will be known for each time t on the upper and lower patch surfaces.

WEAK FORMULATION OF THE PROBLEM

When electrostrictive actuators are current driven, the electromechanical problem consists in solving, for each

time t and whatever the point $M(x, y, z)$, the mechanical motion Equation (1) meeting the constitutive laws (10) and (11), where the electric displacement \mathbf{D} satisfies the Gauss law (6).

Using the weak form of Equation (1) by projection on the mechanical test function $u^*(M)$, and considering all other equations as constraints (Equations (2)–(4), (6)–(9), (10) and (11)), one can formulate the electro-mechanical problem:

Find $u(M, t)$, $\mathbf{T}(M, t)$, $\mathbf{D}(M, t)$ and $V(M, t)$ such that:

$$\forall t \in]0, T[,$$

$$\forall u^* \in \mathcal{U}_u^0, \mathcal{U}_u^0 = \{u^*(M) \mid u^*(M) = 0 \text{ on } \Gamma_u, u^* \text{ regular}\},$$

the dynamic equation:

$$\int_{\Omega} (\mathbf{T}_{ij, j} u_i^* + f_i^d u_i^* - \rho \ddot{u}_i u_i^*) d\Omega = 0,$$

the constitutive laws:

$$\begin{cases} \mathbf{T}_{ij} = \mathbb{C}_{ijkl}^D (\mathbf{S}_{kl} - \mathbb{Q}_{klmn} \mathbf{D}_m \mathbf{D}_n), \\ E_m = \chi_{mm}^{-1} \mathbf{D}_m^s \arctan h \left(\frac{\mathbf{D}_m}{\mathbf{D}_m^s} \right) \\ - 2\mathbb{C}_{ijkl}^D \mathbb{Q}_{klmn} \mathbf{D}_n (\mathbf{S}_{ij} - \mathbb{Q}_{ijpq} \mathbf{D}_p \mathbf{D}_q) \end{cases}$$

the strain–mechanical displacement boundary conditions:

$$\mathbf{S}_{ij} = \frac{1}{2} (u_{i, j} + u_{j, i}),$$

the mechanical displacement boundary conditions:

$$u_i = u_i^d \quad \text{on } \Gamma_u \Leftrightarrow u \in \mathcal{U}_u^0,$$

the stress boundary conditions:

$$\mathbf{T}_{ij} n_j = \mathbf{F}_i^d \quad \text{on } \Gamma_F,$$

the Gauss law:

$$\mathbf{D}_{i, i} - q^d = 0 \quad \text{on } \Omega,$$

the electric potential boundary conditions:

$$V = V^d \quad \text{on } \Gamma_V,$$

the electric displacement boundary conditions:

$$\mathbf{D}_i n_i = Q^d \quad \text{on } \Gamma_Q,$$

the initial mechanical conditions:

$$u_i|_{t=0} = u_i^0 \quad \text{and} \quad \dot{u}_i|_{t=0} = \dot{u}_i^0,$$

the initial electrical conditions:

$$\mathbf{D}_i|_{t=0} = \mathbf{D}_i^0.$$

GAUSS LAW INTEGRATION

The method used to simplify the electromechanical problem consists in solving the electric problem, that is the Gauss law (Equation (6)) meeting the electric boundary conditions.

As we are using current as the driving input, we are able to impose charges to the electro-actuator. The Gauss law can then be integrated. Indeed, Gauss law can be read:

$$\text{div } \mathbf{D} = \mathbf{D}_{x,x} + \mathbf{D}_{y,y} + \mathbf{D}_{z,z} = q^d, \quad (18)$$

where q^d is body charges density imposed to the dielectric medium.

If one now elaborates constitutive equations expressing strains (\mathbf{S}) and electric displacements (\mathbf{D}) as functions of stresses (\mathbf{T}) and electric field (\mathbf{E}) in a similar way as exposed in Pablo and Petitjean (2000a), one can obtain:

$$\begin{aligned} \mathbf{D}_m &= A_{mnij} \mathbf{E}_n \mathbf{T}_{ij} + \mathbf{B}_{mn} \mathbf{E}_n, \\ \mathbf{D}_{in} &= -A_{mnij} V_{,n} \mathbf{T}_{ij} - \mathbf{B}_{mn} V_{,n}, \end{aligned} \quad (19)$$

where A and B are respectively fourth and second rank tensors expressing coefficients.

If we now take into account the material assumptions previously introduced, and the fact that transverse stresses are negligible compared to in-plane stresses, the previous equation gives:

$$\begin{aligned} \mathbf{D}_\alpha &= -A_{\alpha\beta\gamma z} V_{,\beta} \mathbf{T}_{\gamma z} - \mathbf{B}_{\alpha\beta} V_{,\beta} \quad \text{and} \\ \mathbf{D}_z &= -A_{zz\alpha\beta} V_{,z} \mathbf{T}_{\alpha,\beta} - \mathbf{B}_{zz} V_{,z} \quad (\alpha, \beta, \gamma = x, y). \end{aligned}$$

Moreover, electrostrictive patches can be regarded as plane capacitors. In such capacitors, the transverse electric field is much more greater than the in-plane field (at least of L/h order). It is thus possible to neglect the in-plane electric field with respect to the transverse one. Introducing these assumptions in the previous equation, it is then consistent to neglect the in-plane electric displacements \mathbf{D}_α ($\alpha = x, y$) compared to the transverse electric displacement \mathbf{D}_z . The Gauss law thus simplifies to:

$$\mathbf{D}_{z,z} = q^d. \quad (20)$$

Integrating this relationship according to the z variable, one can then obtain:

$$\mathbf{D}_z = q^d z + A, \quad (21)$$

where A is independent on z .

Given that no body charge density is applied in the electrostrictive actuator, and given the electric boundary conditions (electric displacements are imposed on the upper and lower metallized surfaces of the ceramic), constant A is fully determined since the patch is current driven (electric displacement \mathbf{D}_z^d is imposed through the boundary condition: $\mathbf{D}_i \cdot \mathbf{n}_i = \mathbf{Q}^d$ on Γ_Q). Equation (21) thus becomes:

$$\mathbf{D}_z = \mathbf{D}_z^d(x, y). \quad (22)$$

The Gauss law integration thus implies the transverse electric displacement to be constant through the thickness of the plate and to be equal to the imposed value.

CONSTITUTIVE LAWS CHANGES

In this point, we introduce the solution of the electric problem obtained above, in the electromechanical constitutive equations (Equations (10) and (11)) such as to eliminate the electric variables from the problem unknown.

In order to perform this unknown elimination, we first respectively set down:

$$\begin{aligned} \mathbf{T}_p &= (\mathbf{T}_{xz} \quad \mathbf{T}_{yy} \quad \mathbf{T}_{yy})^t, \\ \mathbf{T}_T &= (\mathbf{T}_{xz} \quad \mathbf{T}_{yz} \quad \mathbf{T}_{zz})^t, \\ \mathbf{S}_p &= (\mathbf{S}_{xx} \quad \mathbf{S}_{yy} \quad 2\mathbf{S}_{xy})^t, \\ \mathbf{D}_p &= (\mathbf{D}_z \quad \mathbf{D}_y)^t \quad \text{and} \\ \mathbf{E}_p &= (\mathbf{E}_z \quad \mathbf{E}_y)^t, \end{aligned}$$

as the in-plane stresses vector, the out-of-plane stresses vector, the in-plane strains vector, the out-of-plane strains vector, and, the in-plane electric displacements and electric fields vectors.

A matrix form of constitutive laws can then be written:

$$\begin{pmatrix} \mathbf{T}_p \\ \mathbf{T}_T \\ \mathbf{E}_p \\ \mathbf{E}_z \end{pmatrix} = \begin{pmatrix} C_{PP}^D & C_{PT}^D & Q_{PP}^{\sigma D} & Q_{Pz}^{\sigma D} \\ C_{TP}^D & C_{TT}^D & Q_{TP}^{\sigma D} & Q_{Tz}^{\sigma D} \\ Q_{PP}^{ES} & Q_{PT}^{ES} & d_{PP}^D & d_{Pz}^D \\ Q_{zP}^{ES} & Q_{zT}^{ES} & d_{zP}^D & d_{zz}^D \end{pmatrix} \begin{pmatrix} \mathbf{S}_p \\ \mathbf{S}_T \\ \mathbf{D}_p \\ \mathbf{D}_z \end{pmatrix},$$

where the matrices $Q_{\alpha,\beta}^{\gamma\delta}$ and $d_{\alpha\beta}^D$ are dependent on \mathbf{D} and will not be detailed here.

Inverting out-of-plane stresses and strains, and supposing planes stresses (thin plate assumptions

implies out-of-plane stresses to be negligible), one can then write:

$$\begin{pmatrix} \mathbf{T}_P \\ \mathbf{E}_P \\ \mathbf{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{C}}_{PP}^D & \tilde{\mathbf{Q}}_{PP}^{\sigma D} & \tilde{\mathbf{Q}}_{Pz}^{\sigma D} \\ \tilde{\mathbf{Q}}_{PP}^{ES} & \tilde{d}_{PP}^D & \tilde{d}_{Pz}^D \\ \tilde{\mathbf{Q}}_{zP}^{ES} & \tilde{d}_{zP}^D & \tilde{d}_{zz}^D \end{pmatrix} \begin{pmatrix} \mathbf{S}_P \\ \mathbf{D}_P \\ \mathbf{D}_z \end{pmatrix}. \quad (23)$$

Taking moreover into account the assumptions that in-plane electric displacements are negligible compared to transverse one, Equation (23) becomes:

$$\begin{pmatrix} \mathbf{T}_P \\ \mathbf{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{C}}_{PP}^D & \tilde{\mathbf{Q}}_{Pz}^{\sigma D} \\ \tilde{\mathbf{Q}}_{zP}^{ES} & \tilde{d}_{zz}^D \end{pmatrix} \begin{pmatrix} \mathbf{S}_P \\ \mathbf{D}_z \end{pmatrix}. \quad (24)$$

We thus have:

$$\begin{aligned} \mathbf{T}_P &= \tilde{\mathbf{C}}_{PP}^D \mathbf{S}_P + \tilde{\mathbf{Q}}_{Pz}^{\sigma D} \mathbf{D}_z \\ &= \tilde{\mathbf{C}}_{PP}^D \mathbf{S}_P + \tilde{\mathbf{Q}}_{Pz}^{\sigma D} (\mathbf{D}_z^d(x, y))^2. \end{aligned} \quad (25)$$

We thus proved here that using plate assumptions, the electrical variables could be eliminated from the problem unknowns. The influence of electric phenomena is then taken into account through an imposed pre-stress. A classical mechanical variational formulation of the plate can then be used to set down the electromechanical problem where the current density is assumed to be known on the faces of the plate. This formulation uses the isolated elastic constants which does not depend on the electric state. Nevertheless, the drawback of this formulation is that the density of charges has to be known at each point of the faces of the plate.

PARTIAL CONCLUSIONS

We have proved in this paragraph that using current driven electrostrictive actuators to perform active vibration control of plate structures could lead to a simplification of the non-linear electromechanical problem if the current density could be measured at each point of the faces of the plate. Indeed, as established above, the non-linear constitutive equations reduce to linear mechanical ones with additional non-linear electrically induced prescribed stresses (known for each time t). When dealing with current driven electrostrictive actuators, plate a priori assumptions thus entitle to take into account electromechanical effects only through prescribed stresses.

Voltage Driven Actuator

We now assume the electrostrictive patches to be voltage driven. In this part we analyse the effects of thin plate assumptions on the electromechanical problem

for such a driving input. In this operating configuration a transverse voltage is imposed through the electrostrictive patch. We will thus, in this section, suppose that transverse voltage (V) is known for each time t on upper and lower patch surfaces.

WEAK FORMULATION MODIFICATION

The present problem consists in solving for each time t and whatever the point $M(x, y, z)$ the mechanical motion Equation (1) meeting the constitutive laws (12) and (13) where the electric displacement \mathbf{D} satisfies the Gauss law (6).

Using the weak form of Equation (1) by projection on the test function $u^*(M)$, and considering all other equations as constraints (Equations (2)–(4), (6)–(9), (12) and (13)), one can formulate the electromechanical problem as follows:

Find $u(M, t)$, $\mathbf{T}(M, t)$, $\mathbf{D}(M, t)$ and $V(M, t)$ such that:

$$\begin{aligned} \forall t \notin]0, T[, \\ \forall u^* \in \mathcal{U}_a^0, \mathcal{U}_u^0 = x\{u^*(M) | u^*(M) = 0 \text{ on } \Gamma_u, u^* \text{ regular}\}, \end{aligned}$$

the dynamic equation:

$$\int_{\Omega} (\mathbf{T}_{ij,j} u_i^* + f_i^d u_i^* - \rho \ddot{u}_i u_i^*) d\Omega = 0,$$

the electromechanical constitutive equations:

$$\left\{ \begin{aligned} \mathbf{T}_{ij} &= \mathbf{C}_{ijkl}^E \mathbf{S}_{kl} - \mathbf{C}_{ijkl} \mathbf{Q}_{klmn} \chi_{mm}^* \chi_{nn}^* \mathbf{E}_m^S \mathbf{E}_n^S \\ &\quad \tanh\left(\frac{\mathbf{E}_m}{\mathbf{E}_m^S}\right) \tanh\left(\frac{\mathbf{E}_n}{\mathbf{E}_n^S}\right), \\ \mathbf{D}_m &= \chi_{mmm}^* \mathbf{E}_m^S \tanh\left(\frac{\mathbf{E}_m}{\mathbf{E}_m^S}\right) \\ &\quad + 2\mathbf{C}_{ijkl}^E \mathbf{Q}_{klmn} \mathbf{S}_{ij} \chi_{mm}^* \chi_{nn}^* \mathbf{E}_n^S \frac{\tanh(\mathbf{E}_n/\mathbf{E}_n^S)}{\cosh^2(\mathbf{E}_m/\mathbf{E}_m^S)} \\ &\quad - 2\mathbf{C}_{ijkl}^E \mathbf{Q}_{klmn} \mathbf{Q}_{ijpq} \chi_{mm}^* \chi_{nn}^* \chi_{pp}^* \chi_{qq}^* \mathbf{E}_n^S \mathbf{E}_p^S \mathbf{E}_q^S \\ &\quad \times \tanh\left(\frac{\mathbf{E}_p}{\mathbf{E}_p^S}\right) \tanh\left(\frac{\mathbf{E}_q}{\mathbf{E}_q^S}\right) \frac{\tanh(\mathbf{E}_n/\mathbf{E}_n^S)}{\cosh^2(\mathbf{E}_m/\mathbf{E}_m^S)}. \end{aligned} \right.$$

the strain–mechanical displacement relationship:

$$\mathbf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

the mechanical displacement boundary conditions:

$$u_i = u_i^d \quad \text{on } \Gamma_u \Leftrightarrow u \in \mathcal{U}_u^0,$$

the stress boundary conditions:

$$\mathbf{T}_{ij} \cdot \mathbf{n}_j = \mathbf{F}_i^d \quad \text{on } \Gamma_F,$$

the Gauss law:

$$\mathbf{D}_{i,i} - q^d = 0 \quad \text{on } \Omega,$$

the electric potential boundary conditions:

$$V = V^d \quad \text{on } \Gamma_V,$$

the electric displacement boundary conditions:

$$\mathbf{D}_i n_i = Q^d \quad \text{on } \Gamma_Q,$$

the initial mechanical conditions:

$$u_i|_{t=0} = u_i^0 \quad \text{and} \quad \dot{u}_i|_{t=0} = \dot{u}_i^0.$$

the initial electrical conditions:

$$V_i|_{t=0} = V_i^0.$$

This weak formulation for voltage driven electrostrictive actuator is moreover similar to:

Find $u(M, t)$, $\mathbf{T}(M, t)$, $\mathbf{D}(M, t)$ and $V(M, t)$ such that:

$$\forall t \in]0, T[,$$

$$\forall u^* \in \mathcal{U}_u^0, \mathcal{U}_u^0 = \{u^*(M) \mid u^*(M) = 0 \quad \text{on } \Gamma_u, u^* \text{ regular}\}.$$

the dynamic equation:

$$\int_{\Omega} (\mathbf{T}_{ij,j} u_i^* + \mathbf{f}_i^d u_i^* - \rho \ddot{u}_i u_i^*) d\Omega = 0 \quad (26)$$

the electromechanical constitutive equations:

$$\left\{ \begin{array}{l} \mathbf{T}_{ij} = \mathbb{C}_{ijkl}^D \mathbf{S}_{kl} - \mathbb{C}_{ijkl}^D \mathbb{Q}_{klmn} \mathbf{D}_m(\mathbf{S}, \mathbf{E}) \mathbf{D}_n(\mathbf{S}, \mathbf{E}), \\ \mathbf{D}_m = \chi_{mm}^* \mathbf{E}_m^s \tanh\left(\frac{\mathbf{E}_m}{\mathbf{E}_m^s}\right) \\ \quad + 2\mathbb{C}_{ijkl}^E \mathbb{Q}_{klmn} \mathbf{S}_{ij} \chi_{mm}^* \chi_{nn}^* \mathbf{E}_n^s \frac{\tanh(\mathbf{E}_n/\mathbf{E}_n^s)}{\cosh^2(\mathbf{E}_m/\mathbf{E}_m^s)} \\ \quad - 2\mathbb{C}_{ijkl}^E \mathbb{Q}_{klmn} \mathbb{Q}_{ijpq} \chi_{mm}^* \chi_{nn}^* \chi_{pp}^* \chi_{qq}^* \mathbf{E}_n^s \mathbf{E}_p^s \mathbf{E}_q^s \\ \quad \times \tanh\left(\frac{\mathbf{E}_p}{\mathbf{E}_p^s}\right) \tanh\left(\frac{\mathbf{E}_q}{\mathbf{E}_q^s}\right) \frac{\tanh(\mathbf{E}_n/\mathbf{E}_n^s)}{\cosh^2(\mathbf{E}_m/\mathbf{E}_m^s)}, \end{array} \right. \quad (27)$$

the strain–mechanical displacement relationship:

$$\mathbf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (28)$$

the mechanical displacement boundary conditions:

$$u_i = u_i^d \quad \text{on } \Gamma_u \Leftrightarrow u \in \mathcal{U}_u^{[0, T]}, \quad (29)$$

the stress boundary conditions:

$$\mathbf{T}_{i,j} n_j = \mathbf{F}_i^d \quad \text{on } \Gamma_F, \quad (30)$$

the Gauss law:

$$\mathbf{D}_{i,i} - q^d = 0 \quad \text{on } \Omega, \quad (31)$$

the electric potential boundary conditions:

$$V = V^d \quad \text{on } \Gamma_V, \quad (32)$$

the electric displacement boundary conditions:

$$\mathbf{D}_i n_i = Q^d \quad \text{on } \Gamma_Q, \quad (33)$$

the initial mechanical conditions:

$$u_i|_{t=0} = u_i^0 \quad \text{and} \quad \dot{u}_i|_{t=0} = \dot{u}_i^0 \quad (34)$$

the initial electrical conditions:

$$V_i|_{t=0} = V_i^0. \quad (35)$$

INCREMENTAL CONSTITUTIVE LAWS

We underlined in a previous section, that when electrostrictive actuators are voltage driven, additional non-linearities are introduced in the constitutive laws. However, the strongly non-linear electromechanical problem can be linearized around an operating point for each time t . This point is then characterized by its known mechanical displacements u^0 , stresses \mathbf{T}^0 , electric potential $V_{+/-}^0$ (or electric fields \mathbf{E}^0) imposed on the actuator upper and lower surfaces, and electric displacements \mathbf{D}^0 .

From this assumption, any variable v can be written as the sum of its known value at the operating point v^0 and an increment around this point \bar{v} : $v = v^0 + \bar{v}$.

Introducing this writing in the dynamic equation (Equation (26)), one then obtains, for each time t :

$$\int_{\Omega} \{[\mathbf{T}_{ij,j}^0 + \mathbf{f}_i^d = \rho \ddot{u}_i^0] u_i^* + [\bar{\mathbf{T}}_{ij,j} - \rho \bar{\ddot{u}}_i] u_i^*\} d\Omega = 0. \quad (36)$$

In order to solve this new dynamic equation, one has to establish the value of each term. This is, in particular, possible through the constitutive laws. Indeed, the values of \mathbf{T}_{ij}^0 and \mathbf{D}_i^0 are obtained by substituting all parameters and variables of the constitutive equations (Equation (27)) (i.e., χ^* , \mathbf{E}^s , \mathbf{S} and \mathbf{E}) by their values at

operating point (known values). One thus obtains:

$$\begin{aligned}
\mathbf{T}_{ij}^0 &= \mathbb{C}_{ijkl}^D \mathbf{S}_{kl}^0 \\
&\quad - \mathbb{C}_{ijkl}^D \mathbb{Q}_{klmn}^0 \mathbf{D}_m^0 (S^0, \mathbf{E}^0) D_n^0 (S^0, \mathbf{E}^0), \\
\mathbf{D}_m^0 &= \chi_{nm}^{*0} \mathbf{E}_m^{s0} \tanh\left(\frac{\mathbf{E}_m^0}{\mathbf{E}_m^{s0}}\right) \\
&\quad + 2\mathbb{C}_{ijkl}^{E0} \mathbb{Q}_{klmn}^0 S_{ij}^{*0} \chi_{nm}^{*0} \chi_{nn}^{*0} \mathbf{E}_n^{s0} \frac{\tanh(\mathbf{E}_n^0 / \mathbf{E}_n^{s0})}{\cosh^2(\mathbf{E}_m^0 / \mathbf{E}_m^{s0})} \\
&\quad - 2\mathbb{C}_{ijkl}^{E0} \mathbb{Q}_{klmn}^0 \mathbb{Q}_{ijpq}^0 \chi_{mm}^{*0} \chi_{nn}^{*0} \chi_{pp}^{*0} \chi_{qq}^{*0} \mathbf{E}_n^{s0} \mathbf{E}_p^{s0} \mathbf{E}_q^{s0} \\
&\quad \times \tanh\left(\frac{\mathbf{E}_p^0}{\mathbf{E}_p^{s0}}\right) \tanh\left(\frac{\mathbf{E}_q^0}{\mathbf{E}_q^{s0}}\right) \frac{\tanh(\mathbf{E}_n^0 / \mathbf{E}_n^{s0})}{\cosh^2(\mathbf{E}_m^0 / \mathbf{E}_m^{s0})}. \quad (37)
\end{aligned}$$

With regard to the incremental part of the stresses and the electric displacements ($\bar{\mathbf{T}}$ and $\bar{\mathbf{D}}$), one can express each of these terms as bilinear functions of strains and electric fields increments. This is carried out through a differentiation of the constitutive laws (Equation (27)) with respect to strains \mathbf{S} and electric field \mathbf{E} . One can then obtain:

$$\bar{\mathbf{T}}_{ij} = \bar{\mathbb{C}}_{ijkl}^D \bar{\mathbf{S}}_{kl} + \mathbb{d}_{ijk}^{\sigma D} \bar{\mathbf{D}}_k(\bar{\mathbf{S}}, \bar{\mathbf{E}}), \quad (39)$$

$$\bar{\mathbf{D}}_i = \mathbb{d}_{ikl}^{DS} \bar{\mathbf{S}}_{kl} + \mathbb{h}_{ij}^E \bar{\mathbf{E}}_j. \quad (40)$$

We underline here, that the coefficients $\bar{\mathbb{C}}_{ijkl}^D$, $\mathbb{d}_{ijk}^{\sigma D}$, \mathbb{d}_{ikl}^{DS} and \mathbb{h}_{ij}^E are known values calculated for parameters established at the operating point ($\chi^* = \chi^{*0}$, $\mathbf{E}^s = \mathbf{E}^{s0}$, $\mathbf{S} = \mathbf{S}^0$ and $\mathbf{E} = \mathbf{E}^0$). Coefficients thus depend of this operating point, and will thus have to be modified for each time t . We moreover underline that we don't have given here the expression of these coefficients as functions of the parameters recalled above, as they are quite complex and do not provide any essential data for the following demonstration.

Let us finally remark that one can compare these "incremental" constitutive laws (Equations (39) and (40)) to the linear piezoelectric constitutive laws. One can then apply the results of Osmont (1996), Rahmoune et al. (1998) to solve the linearized electromechanical problem. In a way to adapt this theory to electrostrictors, we will first apply the in-plane stresses assumption to the incremental constitutive laws. We will then be able to integrate in a similar way as in a previous section the Gauss law.

PLANE STRESSES ASSUMPTIONS

The first step needed to integrate the Gauss law consists in applying the in-plane stresses assumption to the incremental constitutive laws.

Let's first respectively define:

$$\begin{aligned}
\bar{\mathbf{T}}_P &= (\bar{\mathbf{T}}_{xx} \quad \bar{\mathbf{T}}_{yy} \quad \bar{\mathbf{T}}_{yy})^t, \\
\bar{\mathbf{T}}_T &= (\bar{\mathbf{T}}_{xz} \quad \bar{\mathbf{T}}_{yz} \quad \bar{\mathbf{T}}_{zz})^t, \\
\bar{\mathbf{S}}_P &= (\bar{\mathbf{S}}_{xx} \quad \bar{\mathbf{S}}_{yy} \quad 2\bar{\mathbf{S}}_{xy})^t, \\
\bar{\mathbf{S}}_T &= (2\bar{\mathbf{S}}_{xz} \quad 2\bar{\mathbf{S}}_{yz} \quad \bar{\mathbf{S}}_{zz})^t, \\
\bar{\mathbf{D}}_P &= (\bar{\mathbf{D}}_x \quad \bar{\mathbf{D}}_y)^t, \\
\bar{\mathbf{E}}_P &= (\bar{\mathbf{E}}_x \quad \bar{\mathbf{E}}_y)^t, \\
\bar{\mathbf{D}}_z &\quad \text{and} \quad \bar{\mathbf{E}}_z,
\end{aligned}$$

as the incremental in-plane stresses vector, the incremental out-of-plane stresses vector, the incremental in-plane strains vector, the incremental out-of-plane strains vector, the incremental in-plane electric displacements and incremental electric fields vectors, and, the incremental transverse electric displacements and incremental transverse electric fields.

Differentiating the constitutive laws (12) and (13), and applying the previous definition of incremental vectors, one can write:

$$\begin{pmatrix} \bar{\mathbf{T}}_P \\ \bar{\mathbf{T}}_T \\ \bar{\mathbf{D}}_P \\ \bar{\mathbf{D}}_z \end{pmatrix} = \begin{pmatrix} \bar{\mathbb{C}}_{PP}^E & \bar{\mathbb{C}}_{PT}^E & \mathbb{d}_{PP}^{\sigma E} & \mathbb{d}_{Pz}^{\sigma E} \\ \bar{\mathbb{C}}_{TP}^E & \bar{\mathbb{C}}_{TT}^E & \mathbb{d}_{TP}^{\sigma E} & \mathbb{d}_{Tz}^{\sigma E} \\ \mathbb{d}_{PP}^{DS} & \mathbb{d}_{PT}^{DS} & \mathbb{h}_{PP}^E & \mathbb{h}_{Pz}^E \\ \mathbb{d}_{zP}^{DS} & \mathbb{d}_{zT}^{DS} & \mathbb{h}_{zP}^E & \mathbb{h}_{zz}^E \end{pmatrix} \begin{pmatrix} \bar{\mathbf{S}}_P \\ \bar{\mathbf{S}}_T \\ \bar{\mathbf{E}}_P \\ \bar{\mathbf{E}}_z \end{pmatrix}.$$

where sub-matrices are dependent on \mathbf{E} .

If we now invert out-of-plane stresses and strains and apply the plane stresses assumption to this matrix form equation, one can rewrite the incremental constitutive equation as follows:

$$\begin{pmatrix} \bar{\mathbf{T}}_P \\ \bar{\mathbf{D}}_P \\ \bar{\mathbf{D}}_z \end{pmatrix} = \begin{pmatrix} \bar{\mathbb{C}}_{PP}^E & \tilde{\mathbb{d}}_{PP}^{\sigma E} & \tilde{\mathbb{d}}_{Pz}^{\sigma E} \\ \tilde{\mathbb{d}}_{PP}^{DS} & \tilde{\mathbb{h}}_{PP}^E & \tilde{\mathbb{h}}_{Pz}^E \\ \tilde{\mathbb{d}}_{zP}^{DS} & \tilde{\mathbb{h}}_{zP}^E & \tilde{\mathbb{h}}_{zz}^E \end{pmatrix} \begin{pmatrix} \bar{\mathbf{S}}_P \\ \bar{\mathbf{E}}_P \\ \bar{\mathbf{E}}_z \end{pmatrix} \quad (41)$$

We now focus our study on mechanically and electrically orthotropic materials (previously underlined). We moreover recall that electro-actuators such as electrostrictive and piezoelectric patches can be regarded as plane capacitors where transverse electric field is much more greater than in-plane fields, which thus can be neglected.

From Equation (41), one can then, in particular, read:

$$\bar{\mathbf{D}}_z = \tilde{\mathbb{d}}_{zP}^{DS} \bar{\mathbf{S}}_P + \tilde{\mathbb{h}}_{zz}^E \bar{\mathbf{E}}_z. \quad (42)$$

Finally, proceeding in a similar way for constitutive laws (10) and (11), one obtains, with the plane stresses

assumption:

$$\bar{\mathbf{T}}_P = \tilde{\mathbf{C}}_{PP}^D \bar{\mathbf{S}}_P + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \bar{\mathbf{D}}_z. \quad (43)$$

Given the incremental electric displacement relationship (42), we are now able to integrate the Gauss law and thus find a solution for the electrical problem, which will then be introduced in the stress constitutive law (43).

GAUSS LAW INTEGRATION

As previously proved, in-plane electric displacements (\mathbf{D}_x and \mathbf{D}_y) are negligible compared to transverse one (\mathbf{D}_z). Integrating the simplified Gauss law (Equation (20)) with respect to the z variable, one can then read:

$$\begin{aligned} \mathbf{D}_{z,z} &= \mathbf{D}_{z,z}^0 + \bar{\mathbf{D}}_{z,z} = q^d \\ &\Downarrow \\ \bar{\mathbf{D}}_z &= \int_{-h/2}^z q^d dz - \mathbf{D}_z^0 + A_0, \end{aligned} \quad (44)$$

where A_0 is a constant independent of z which is to be fixed.

We derived in the previous paragraph, the incremental electromechanical constitutive laws. From Equation (42) and given the electric field–voltage relationship (Equation (9)) one can write:

$$\bar{\mathbf{D}}_z = \tilde{\mathfrak{d}}_{zP}^{DS} \bar{\mathbf{S}}_P + \tilde{\mathfrak{h}}_{zz}^E \bar{\mathbf{E}} = \tilde{\mathfrak{d}}_{zP}^{DS} \bar{\mathbf{S}}_P - \tilde{\mathfrak{h}}_{zz}^E \bar{V}_{,z}.$$

Substituting this equation in Equation (44) gives:

$$\bar{V}_{,z} = \frac{1}{\tilde{\mathfrak{h}}_{zz}^E} \left[\mathbf{D}_z^0 - A_0 + \tilde{\mathfrak{d}}_{zP}^{DS} \bar{\mathbf{S}}_P - \int_{-h/2}^z q^d dz \right]. \quad (45)$$

If we now substitute Kirchhoff–Love plane strains (Equation (17)) in this expression, and integrate with respect to the z variable, we can write:

$$\begin{aligned} \bar{V} &= \int_{-h/2}^z \left\{ \frac{1}{\tilde{\mathfrak{h}}_{zz}^E} \left[\tilde{\mathfrak{d}}_{zP}^{DS} \bar{\mathbf{S}}^0 - z \tilde{\mathfrak{d}}_{zP}^{DS} \bar{\mathbf{R}}^1 \right] \right\} dz \\ &+ \int_{-h/2}^z \left\{ \frac{1}{\tilde{\mathfrak{h}}_{zz}^E} \left[\mathbf{D}_z^0 - A_0 - \int_{-h/2}^z q^d dz \right] \right\} dz \\ &+ \bar{V} \left(-\frac{h}{2} \right). \end{aligned} \quad (46)$$

Let us remark that from this expression, electric potential and field are respectively quadratic and linear functions of z through the thickness of the plate. When

electrostrictive patches are voltage driven, an electrical potential is applied on upper and lower ceramic surfaces. In a similar way, a potential increment \bar{V} could be applied on these surfaces with $\bar{V} = \bar{V}_-$ for $z = -h/2$ and $\bar{V} = \bar{V}^+$ for $z = h/2$. These voltage increment boundary conditions substituted in Equation (46) then give us the value of the constant A_0 :

$$\begin{aligned} A_0 &= \frac{1}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} \left[\left\langle \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \right\rangle \bar{\mathbf{S}}^0 - \left\langle z \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \right\rangle \bar{\mathbf{R}}^1 \right] \\ &+ \mathbf{D}_z^0 - \frac{1}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} \left[\left\langle \bar{q}^d 1 / \tilde{\mathfrak{h}}_{zz}^E \right\rangle + \frac{1}{h} (\bar{V}^+ - \bar{V}_-) \right], \end{aligned} \quad (47)$$

where we set down: $1/h \int_{-h/2}^{h/2} a dz = \langle a \rangle$ the mean value of any variable a and $\int_{-h/2}^z q^d dz = \bar{q}^d$ to simplify expressions.

Replacing this constant value in Equations (44) and (45), and taking into account the electric field–electric potential relationship (Equation (9)), one respectively obtains the electric displacement and the field increments as linear functions of membrane in-plane strains and curvatures increments:

$$\bar{\mathbf{D}}_z = \Phi_0^D \bar{\mathbf{S}}^0 - \Phi_1^D \bar{\mathbf{R}}^1 + \bar{\mathbf{D}}_z^{\text{elec}}, \quad (48)$$

$$\bar{\mathbf{E}}_z = \Phi_0^E \bar{\mathbf{S}}^0 - \Phi_1^E \bar{\mathbf{R}}^1 + \bar{\mathbf{E}}_z^{\text{elec}}. \quad (49)$$

In these expressions, we set down:

$$\Phi_0^D = \frac{\left\langle \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \right\rangle}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} \quad \text{and} \quad \Phi_1^D = \frac{\left\langle z \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \right\rangle}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle},$$

$$\bar{\mathbf{D}}_z^{\text{elec}} = \bar{q}^d - \frac{\left\langle \bar{q}^d / \tilde{\mathfrak{h}}_{zz}^E \right\rangle}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} - \frac{1}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} \frac{\bar{V}^+ - \bar{V}_-}{h},$$

$$\Phi_0^E = \frac{1}{\tilde{\mathfrak{h}}_{zz}^E} \left[\frac{\left\langle \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \right\rangle}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} - \tilde{\mathfrak{d}}_{zP}^{DS} \right],$$

$$\Phi_1^E = \frac{1}{\tilde{\mathfrak{h}}_{zz}^E} \left[\frac{\left\langle z \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \right\rangle}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} - z \tilde{\mathfrak{d}}_{zP}^{DS} \right], \quad \text{and,}$$

$$\bar{\mathbf{E}}_z^{\text{elec}} = \frac{1}{\tilde{\mathfrak{h}}_{zz}^E} \left[\bar{q}^d - \frac{\left\langle \bar{q}^d / \tilde{\mathfrak{h}}_{zz}^E \right\rangle}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} - \frac{1}{\langle 1/\tilde{\mathfrak{h}}_{zz}^E \rangle} \frac{\bar{V}^+ - \bar{V}_-}{h} \right].$$

SIMPLIFICATION OF INCREMENTAL CONSTITUTIVE LAWS

If we now substitute the bilinear electric displacements obtained above (Equation (48)) in the incremental

stresses constitutive law (43), one can then write the constitutive laws as bilinear functions of membrane in-plane strains and curvatures increments:

$$\begin{cases} \bar{\mathbf{T}}_P = \Phi_0^\sigma \bar{\mathbf{S}}^0 - \Phi_1^\sigma \bar{\mathbf{R}}^1 + \bar{\mathbf{T}}_P^{\text{elec}} \\ \bar{\mathbf{D}}_z = \Phi_0^D \bar{\mathbf{S}}^0 - \Phi_1^D \bar{\mathbf{R}}^1 + \bar{\mathbf{D}}_z^{\text{elec}} \end{cases} \quad (50)$$

where we set:

$$\begin{aligned} \Phi_0^\sigma &= \tilde{\mathbf{C}}_{PP}^D + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \Phi_0^D = \tilde{\mathbf{C}}_{PP}^D + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \frac{\langle \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \rangle}{\langle 1 / \tilde{\mathfrak{h}}_{zz}^E \rangle}, \\ \Phi_1^\sigma &= z \tilde{\mathbf{C}}_{PP}^D + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \Phi_1^D = z \tilde{\mathbf{C}}_{PP}^D + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \frac{\langle z \tilde{\mathfrak{d}}_{zP}^{DS} / \tilde{\mathfrak{h}}_{zz}^E \rangle}{\langle 1 / \tilde{\mathfrak{h}}_{zz}^E \rangle}, \\ \bar{\mathbf{T}}_P^{\text{elec}} &= \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \bar{\mathbf{D}}_z^{\text{elec}} = \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \left[\bar{\mathbf{q}}^d - \frac{\langle \bar{\mathbf{q}}^d / \tilde{\mathfrak{h}}_{zz}^E \rangle}{\langle 1 / \tilde{\mathfrak{h}}_{zz}^E \rangle}, - \frac{\bar{V}^+ - \bar{V}^-}{\langle 1 / \tilde{\mathfrak{h}}_{zz}^E \rangle h} \right]. \end{aligned}$$

In the particular case of homogeneous materials and null electrical volumic charges, this law simplifies in:

$$\bar{\mathbf{T}}_P = \left(\tilde{\mathbf{C}}_{PP}^D + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \tilde{\mathfrak{d}}_{zP}^{DS} \right) \bar{\mathbf{S}}^0 - z \tilde{\mathbf{C}}_{PP}^D \bar{\mathbf{R}}^1 - \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \tilde{\mathfrak{h}}_{zz}^E \frac{\bar{V}^+ - \bar{V}^-}{h}. \quad (51)$$

$$\bar{\mathbf{D}}_z = \tilde{\mathfrak{d}}_{zP}^{DS} \bar{\mathbf{S}}^0 - \tilde{\mathfrak{h}}_{zz}^E \frac{\bar{V}^+ - \bar{V}^-}{h}. \quad (52)$$

Let us remark that the coefficient of the membrane in-plane strains increments of Equation (51) ($\tilde{\mathbf{C}}_{PP}^D + \tilde{\mathfrak{d}}_{Pz}^{\sigma D} \tilde{\mathfrak{d}}_{zP}^{DS}$) is in fact equivalent to the short-circuited elastic constant ($\tilde{\mathbf{C}}_{PP}^E$). This can thus underline that the membrane stresses are governed by the short-circuited elastic constants and that bending stresses are governed by the isolated elastic constants.

PARTIAL CONCLUSIONS

In this part we have shown that when electrostrictive ceramics are voltage driven, the non-linear electromechanical problem could be linearized around an operating point (Equation (36)) for each time t , each unknown being written as the sum of an established term at operating point and an incremental term. We moreover proved that an incremental constitutive law could be deduced from classical ones (Equations (12) and (13)) and that these ones were linear functions of strains and electric fields increments. Finally, these linear equations were reduced to linear functions of plane strains only. The electromechanical problem was thus reduced to a simple mechanical problem with modified stiffness coefficients and additional prescribed stresses.

FINITE ELEMENT METHOD

It was highlighted in the last section that whatever the electrostrictive actuators driving input chosen, a plate theory for orthotropic materials with a priori Kirchhoff–Love assumptions reduces the electromechanical non-linear problem to a simple non-linear mechanical problem through a modification of stiffness coefficients or additional applied prescribed stresses.

When electrostrictive patches are current driven, the plane stresses can be written as linear functions of plane strains (Equation (25)). This law is then similar to a classical mechanical constitutive law using isolated elastic constants, with prescribed stresses dependent on the imposed electric displacement. Classical plate finite element can thus be used to simulate electrostrictive thin plate behavior. But in this case, the surface charges are to be known at each point of the faces of the plate.

When electrostrictive patches are voltage driven, the non-linear problem is linearized around an operating point. Electromechanical constitutive laws are then split up into an established part (calculated by introducing displacement and voltage known values in constitutive equations) and an incremental part (linear function of strains and electric field increments). Kirchhoff–Love plate assumptions then allow to express these incremental constitutive laws as linear functions of strains only through modification of stiffness coefficients and additional prescribed stresses (Equation (50)). One can thus conclude that in a similar way as for current driven actuator, classical plate finite element can be used here to simulate electrostrictive thin plate behavior. Furthermore, the particular constitutive law (51) can be taken into account through the use of a stratified plate finite element with three layers.

Finally, whatever the electrostrictive actuators driving input chosen, a plate theory with a priori Kirchhoff–Love assumptions leads to use classical mechanical plate finite elements. No special electromechanical electrostrictive finite element thus needs to be developed.

This observation will be the subject of the second part of this paper which will be devoted to numerical and experimental results, and presented at ICASST'01 (Pablo et al., 2001a).

APPLICATION TO ACTIVE VIBRATION CONTROL

Applications of plates active vibration control mainly use patches which are metallized on the upper and lower faces. These surfaces are thus electric equipotentials which implies that the distribution of electric charges on these faces cannot be known. In fact, only the mean value $[\mathbf{D}_z]$ of these charges over the surfaces can be prescribed through the measurement of the driving

current. If one supposes the patch to be homogeneous, then the mean value of the stresses $[T]$ over the patch is related to the mean value of the strains $[S]$ over this patch, using the isolated elastic constants through the relationship:

$$[T_P] = \tilde{\mathbf{C}}_{PP}^D [S_P] + \tilde{\mathbf{d}}_{Pz}^D [D_z]. \quad (53)$$

Furthermore, deviations of stresses $T - [T]$ and strains $S - [S]$ from their mean values are related together using the short-circuited elastic constants through the relationship:

$$(T_P - [T_P]) = \tilde{\mathbf{C}}_{PP}^E (S^0 - [S^0]) - z \tilde{\mathbf{C}}_{PP}^D (R^1 - [R^1]). \quad (54)$$

Given that one can write $T = (T - [T]) + [T]$, these last two relationships can be combined to obtain:

$$T_P = \tilde{\mathbf{C}}_{PP}^D S^0 - \tilde{z}_{PP}^D R^1 + \tilde{\mathbf{d}}_{Pz}^D [D_z] + (\tilde{\mathbf{C}}_{PP}^D - \tilde{\mathbf{C}}_{PP}^E)(S^0 - [S^0]). \quad (55)$$

It then seems that, in practice, it will not be possible to avoid the influence of electric field on constitutive law in the patch and therefore, in the control law.

However, if one wants a patch to be efficient on a vibration it has to be placed on an antinode of this vibration. However, at such a place, the deviation is minimum. If L is the characteristic in-plane dimension of the patch and λ the wave length, the deviation of stresses or strains are of order L/λ with respect to their mean value. So, if the difference between the isolated constants $\tilde{\mathbf{C}}_{PP}^D$ and the short-circuited ones $\tilde{\mathbf{C}}_{PP}^E$ is small enough, the expression $(\tilde{\mathbf{C}}_{PP}^D - \tilde{\mathbf{C}}_{PP}^E)(S^0 - [S^0])$ may be neglected. One then obtains the current driven model in which the surface charges are supposed to be constant over the patch.

This approximation, if consistent, allows to simplify the control process, making it possible to use the current as driving input.

Further details on the current driven actuator will be reported in the second part of this paper.

SUMMARY AND CONCLUSIONS

This paper aims at exposing the elaboration of thin electrostrictive plate finite element in order to simulate the dynamic behavior of thin plate structures coupled with electrostrictive patches as actuators.

It was first shown that current driving electrostrictors would be valuable since this input avoids strain saturations observed for high electric field magnitudes when the ceramic is voltage driven. Moreover, on the one hand, the constitutive laws obtained for current driven ceramics are only dependent on two parameters which are not ceramic temperature sensitive (isolated

parameters). On the other hand, voltage driving these ceramics introduces additional parameters in constitutive laws that were shown to be dependent on ceramic temperature (Pablo and Petitjean, 2000b).

In a second part, governing equations and assumptions needed for developing the thin electrostrictive plate finite element were formulated. From these equations, plate theories, based on Osmond (1996) were respectively established for current driven and voltage driven actuators. In one hand, when ceramics were current driven, it was proved that the non-linear electromechanical problem could be reduced to a simple mechanical problem with additional prescribed stresses dependent on applied electric displacements. On the other hand, when actuators were voltage driven, we demonstrated that the non-linear electromechanical problem could be linearized around an operating point, unknowns being split up into a well known part and an incremental one. This linearization led to incremental constitutive laws which are linear functions of strains and electric field increments. Application of thin plate assumptions then reduced these electromechanical equations to simple mechanical laws with specific stiffness coefficients depending on the electric field and electrically induced prescribed stresses.

From these results, the elaboration of new plate electrostrictive finite elements were no longer necessary since classical ones could take into account non-linear electrostrictive effects through prescribed stresses and modifications of stiffness coefficients. It is moreover important to highlight here that similar results for orthotropic plates could be obtained using Mindlin-Reissner plate assumptions instead of the Kirchhoff-Love ones postulated in this paper.

Finally, we propose to use these plate models to simplify the design of current driven control laws for the reduction of vibration of plates with electrostrictive patches. Further details will be given in the second part of this paper.

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