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EFFECT OF GRAVITY ON A FREE-FREE FLUID-STRUCTURE SYSTEM

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ABSTRACT

In this paper, we propose a symmetric variational formulation for the eigenmode computation of a free-free elastic tank partially filled with an incompressible inviscid liquid in presence of a gravity field. The originality of this model is to take into account the strong coupling between the sloshing of the liquid free surface and the hydroelastic deformations of the tank. We will show that this allows the rigid body modes of the system to be predicted correctly.

Γ	liquid free surface
Σ_i	fluid-structure interface
Σ_f	external force application surface
Ω^S	structural domain ($\Omega^S = \Sigma_i \cup \Sigma_f$)
Ω^F	fluid domain ($\Omega^F = \Gamma \cup \Sigma_i$)
n	normal vector
τ	linear variation of normal vector: $\tau d\Sigma_0 \simeq n d\Sigma - n_0 d\Sigma_0$
Du	partial derivative matrix of u
f_D	dead loads
f_F	follower forces
g	gravity field
ρ^S	density of the structure
ρ^F	density of the fluid
H	elastic coefficient tensor of the structure
z	altitude with respect to the liquid free surface
i_x, i_y, i_z	coordinate system defined with i_z colinear to g

INTRODUCTION

In the aerospace field, the study of free structures containing internal fluids is of prime importance, for instance, to predict the in-flight stability of aircrafts with auxiliary tanks or to control the trajectory of liquid propelled launch vehicles (Abramson, 1966; Moiseyev & Romyantsev, 1968). Several symmetric formulations have been proposed to compute the linear vibrations of elastic tanks partially filled with incompressible inviscid liquids (Ohayon & Valid, 1984). However those hydroelastic models usually neglect the gravity potential energy of the fluid and therefore the sloshing of the free surface. The decoupling of both phenomena is valid except if the first coupled eigenfrequencies of the fluid-structure system are too close to the sloshing eigenfrequencies. This may occur for highly flexible tanks but also for free-free systems (since the first eigenmodes are zero-frequency rigid body modes). The former case was studied in previous articles (Schotté & Ohayon, 2001) while this paper will focus on free-free systems.

In 1995, Morand & Ohayon proposed an hydroelastic model to represent the coupling with the free surface sloshing by reintroducing the gravity in the formulation (Morand & Ohayon, 1995). A new *elastogravity* stiffness operator was then established. We will extend this formulation to free-free fluid-structure systems and show how to consider its uniform translational acceleration as an apparent gravity. It is then demonstrated that, unlike previous approximate hydroelastic models with gravity (Tong, 1966), the proposed formulation which accounts for all the prestress and "follower force" terms correctly represents the rigid body modes of the fluid-structure system and their coupling with the sloshing of the free surface. Numerical simula-

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tions and validations are presented to highlight the advantages of this model.

FREE PRESTRESSED FLUID-STRUCTURE SYSTEM

Herein we are interested in linear vibrations of pulsation ω and we then suppose that all external excitations are harmonic. Figure 1 shows the system considered here.

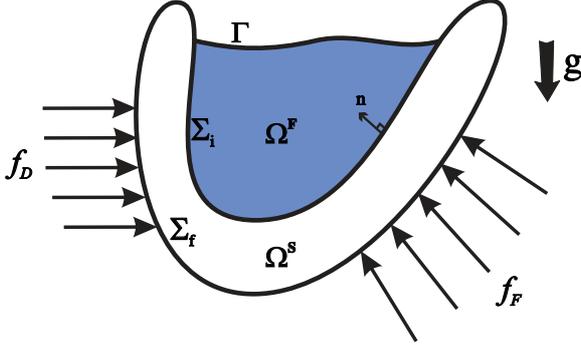


Figure 1. STUDIED MODEL

Prestressed Initial State and Apparent Gravity

The presence of gravity field g and hydrostatic pressure P on the fluid-structure interface introduces a prestress in the initial state of the system (designated by subscript $_0$). In case of a free-free fluid-structure system, this prestressed initial configuration is not static but accelerated by the external forces f_0 . The initial elastic deformation of the system will be rather described in a reference frame attached to the accelerated system. In this reference frame, the gravity is replaced with an apparent gravity that depends on the system acceleration, denoted as γ . Since the case of fluid-structure systems in rotation is very specific (Coriolis forces effect etc.) and has been treated in many other works (Greenspan, 1986), we suppose here that the rigid motion of the system due to the initial external forces is purely translational (this means that the resultant torque of f_0 with respect to the center of gravity of the system vanishes). In this case, the apparent gravity, denoted as g^{app} , is given by :

$$g^{\text{app}} = g - \gamma \quad \text{with} \quad \gamma = \frac{(m^S + m^F)g + \int_{\Sigma_{f0}} f_0 d\Sigma_0}{m^S + m^F}$$

$$g^{\text{app}} = - \frac{\int_{\Sigma_{f0}} f_0 d\Sigma_0}{m^S + m^F} \quad (1)$$

where m^S and m^F are the structure and fluid masses.

The apparent gravity, which does not depend on the presence of the gravity field g , allows the initial position of the fluid free surface¹ to be defined. Furthermore, in the reference frame attached to the system, the summation of all applied loads is null since the inertial forces associated to the apparent gravity g^{app} balance the applied loadings f_0 . To determine the initial elastic deformations u_0^e and then the prestress σ_0 in the structure, it is however necessary to "support" the unconstrained system by fixing enough degrees of freedom of the structure to make it iso-static (G eradin & Rixen, 1997). The local equations satisfied by u_0^e and σ_0 are :

$$\text{Div } \sigma_0(u_0^e) + \rho_0^S g^{\text{app}} = 0 \quad \text{in } \Omega_0^S \quad (2a)$$

$$\sigma_0 n_0 = f_0 \quad \text{on } \Sigma_{f0} \quad (2b)$$

$$\sigma_0 n_0 = -P_0^{\text{app}} n_0 \quad \text{on } \Sigma_{i0} \quad (2c)$$

where P_0^{app} is the fluid pressure in the reference frame attached to the system.

Linear hydroelastic vibrations with gravity

The apparent gravity being constant in space, we can apply the hydroelastic modelisation with constant gravity developed in previous works (Schott e & Ohayon, 1999; Schott e, 2001) to describe the linear fluid-structure vibrations in the reference frame attached to the system. We remind here that, if $\mathcal{H}^1(\Omega_0^S)$ denoted as \mathcal{C}_u , the variational formulation obtained for the deformation U^S of the structure (with respect to the initial configuration) is:

$$\exists U^S \in \mathcal{C}_u, \quad \forall \delta U \in \mathcal{C}_u,$$

$$\widehat{\mathcal{K}}(U^S, \delta U) - \omega^2 \mathcal{M}(U^S, \delta U) + \omega^2 C(\varphi, \delta U) = \tilde{f}(\delta U) \quad (3a)$$

where $\widehat{\mathcal{K}}$ is the elastogravity operator, \mathcal{M} the inertia of the structure, C the coupling at the fluid-structure interface and \tilde{f} the linear form associated with the variation of prescribed forces.

Since the fluid is incompressible and inviscid, its displacements U^F are irrotational at non-zero frequency and then can be represented by a potential φ such as $U^F = \nabla \varphi$ (on a simply connected fluid domain). With $\mathcal{C}_\varphi^* = \{\varphi \in \mathcal{H}^1(\Omega_0^F) / \int_{\Gamma_0} \varphi d\Gamma_0 = 0\}$, the variational formulation for the liquid is:

$$\exists \varphi \in \mathcal{C}_\varphi^*, \quad \forall \delta \varphi \in \mathcal{C}_\varphi^*,$$

$$\mathcal{F}(\varphi, \delta \varphi) - \frac{\omega^2}{|g^{\text{app}}|} \mathcal{S}(\varphi, \delta \varphi) + C(U^S, \delta \varphi) = 0 \quad (3b)$$

¹We have to exclude the particular case of a free fall ($f_0 = 0$) because, when $g^{\text{app}} = 0$, the position of the liquid free surface cannot be defined.

where \mathcal{F} and \mathcal{S} are the bilinear forms associated respectively with the kinetic and sloshing potential energies of the fluid (the complete expression of all those operators can be found in (Schotté & Ohayon, 1999)).

The elastogravity operator $\widehat{\mathcal{K}}$, whose symmetry is demonstrated in (Schotté, 2001), is composed of several bilinear forms: $\widehat{\mathcal{K}} = k_E + k_G + k_B + k_\Sigma^1 + k_\Sigma^2$ where

$$k_E(U^S, \delta U) = \int_{\Omega_0^s} \text{Tr} [H \varepsilon(U^S) \varepsilon(\delta U)] d\Omega_0 \quad (4a)$$

$$k_G(U^S, \delta U) = \int_{\Omega_0^s} \text{Tr} [DU^S \sigma_0^t D\delta U] d\Omega_0 \quad (4b)$$

$$k_B(U^S, \delta U) = \frac{\rho^F |g^{\text{app.}}|}{\|\Gamma_0\|} \left(\int_{\Sigma_{i0}} U^S \cdot n_0 d\Sigma_0 \right) \left(\int_{\Sigma_{i0}} \delta U \cdot n_0 d\Sigma_0 \right) \quad (4c)$$

$$k_\Sigma^1(U^S, \delta U) = -\rho^F |g^{\text{app.}}| \int_{\Sigma_{i0}} (i_z \cdot U^S) (\delta U \cdot n_0) d\Sigma_0 \quad (4d)$$

$$k_\Sigma^2(U^S, \delta U) = -\rho^F |g^{\text{app.}}| \int_{\Sigma_{i0}} z_0 \tau(U^S) \cdot \delta U d\Sigma_0 \quad (4e)$$

k_E : symmetric positive bilinear form related to the elastic stiffness of the structure,

k_G : symmetric bilinear form representing the geometric stiffness due to the prestress σ_0 in the structure in the initial state,

k_B : symmetric positive bilinear form associated with a quasi-static effect of a liquid free surface elevation,

$k_\Sigma = k_\Sigma^1 + k_\Sigma^2$: symmetric bilinear form related to the "follower force" property of the liquid pressure on the fluid-structure interface.

Follower force effect

We distinguish here two kinds of external forces: the "dead loads" f^D and the follower forces f^F supposed colinear to the normal vector n . The linearized variation of external loads $\tilde{f}(\delta U)$ appearing in the right hand side member of equation (3a) is then proportional to:

$$f d\Sigma - f_0 d\Sigma_0 = \tilde{f}^D d\Sigma_0 + |\widetilde{f^F}| n_0 d\Sigma_0 + |f_0^F| \tau(U^S) d\Sigma_0$$

where $\tilde{f}^D = f^D - f_0^D$ and $|\widetilde{f^F}| = |f^F| - |f_0^F|$. The linear variation of the normal vector n , denoted as τ , is a function of U^S . In the case of prescribed follower forces, we then have to add a stiffness term k_F to the elastogravity operator, given by:

$$k_F(U^S, \delta U) = - \int_{\Sigma_{f0}} |f_0^F| \tau(U^S) \cdot \delta U d\Sigma_0 \quad (4f)$$

and to replace the right hand side member with:

$$\tilde{f}(\delta U) = \int_{\Sigma_{f0}} (\tilde{f}^D + |\widetilde{f^F}| n_0) \cdot \delta U d\Sigma_0 \quad (5)$$

The vibrations of the coupled system are free if:

$$\tilde{f}(\delta U) \equiv 0 \iff \begin{cases} f^D = f_0^D \\ |f^F| = |f_0^F| \end{cases} \quad (6)$$

Symmetry of operator k_F

First of all, let us explicitly define τ in term of U^S using the oriented surface transformation formula:

$$n d\Sigma = \det(F) F^{-t} n_0 d\Sigma_0 \quad \text{with } F = Id + DU^S \quad (7)$$

Introducing this expression in the definition of $\tau(U^S)$, we find after linearization:

$$\tau(U^S) = \text{div}(U^S) n_0 - {}^t(DU^S) n_0 \quad (8)$$

and then k_F can be written as:

$$k_F(U^S, \delta U) = \int_{\Sigma_{f0}} |f_0^F| ((DU^S)\delta U - \text{div}(U^S) \delta U) \cdot n_0 d\Sigma_0 \quad (9)$$

To study the symmetry of k_F , we will compute $k_F(U, V) - k_F(V, U)$:

$$\begin{aligned} k_F(V, U) - k_F(U, V) &= \int_{\Sigma_{f0}} |f_0^F| \left(\text{div}(U) V - (DU) V - \text{div}(V) U + (DV) U \right) \cdot n_0 d\Sigma_0 \\ &= \int_{\Sigma_{f0}} |f_0^F| \text{rot}(V \wedge U) \cdot n_0 d\Sigma_0 \\ &= \int_{\Sigma_{f0}} \text{rot}(|f_0^F| V \wedge U) \cdot n_0 d\Sigma_0 \dots \\ &\dots - \int_{\Sigma_{f0}} \left(\text{grad}(|f_0^F|) \wedge (V \wedge U) \right) \cdot n_0 d\Sigma_0 \\ &= \int_{\partial\Sigma_{f0}} |f_0^F| (V \wedge U) \cdot dl - \int_{\Sigma_{f0}} \left(\text{grad}(|f_0^F|) \wedge (V \wedge U) \right) \cdot n_0 d\Sigma_0 \end{aligned}$$

To enforce the nullity of this expression, it is sufficient that:

1. on the one hand, $|f_0^F|$ is constant on Σ_{f0} ($\text{grad}(|f_0^F|) \equiv 0$),

2. on the other hand, either Σ_{f_0} is a closed surface ($\partial\Sigma_{f_0} = \emptyset$) or the displacements of the structure are known on $\partial\Sigma_{f_0}$ (δU , cinematically admissible, is then null on $\partial\Sigma_{f_0}$). However, for free-free systems, this second option does not exist.

These both conditions are verified, for instance, in case of a pressurized tank since $|f_0^F|$ is the pressure of the gas, supposed constant in the gas volume and applied on the closed inner surface of the tank.

In the general case, k_F is non-symmetric and Hibbitt has proposed to use a symmetric formulation instead: $k_F^S = 1/2(k_F + {}^t k_F)$ to preserve the conservative property of the system (Hibbitt, 1979). This approximation is also used by Mohan (Mohan, 1997). We then suppose later that k_F is symmetric.

RIGID BODY MODES OF A FREE FLUID-STRUCTURE SYSTEM

The rigid body modes of the free fluid-structure system are solutions of the following equations :

$$\begin{cases} \forall (\delta U, \delta \varphi) \in (\mathcal{C}_u, \mathcal{C}_\varphi^*), \\ \widehat{\mathcal{K}}(U^S, \delta U) = 0 \\ \mathcal{F}(\varphi, \delta \varphi) = -C(U^S, \delta \varphi) \quad \text{with } \int_{\Gamma_0} \varphi d\Gamma_0 = 0 \end{cases} \quad (10)$$

We are then interested in determining the kernel of the stiffness operator $\mathcal{K} = k_E + k_G + k_B + k_\Sigma^1 + k_\Sigma^2 + k_F$, where these different operators are defined by relations (4). Instead of doing a direct computation, we will first try to verify whether the 6 classical rigid body modes U_R (the 3 translations and the 3 rotations) belong or not to this space. In this aim, we compute the deformation energy generated by \mathcal{K} for these 6 rigid displacements U_R , defined by $E_{\mathcal{K}}(U_R) = \frac{1}{2}\mathcal{K}(U_R, U_R)$.

Rigid Translational Motions

For a translational motion, $U_R = t$ with $t \in \{i_x, i_y, i_z\}$ and

$$\begin{aligned} E_{k_E}(t) &= 0 \\ E_{k_G}(t) &= 0 \\ E_{k_F}(t) &= 0 \\ E_{k_B}(t) &= \frac{\rho^F |g^{\text{app}}|}{2 \|\Gamma_0\|} \left(t \cdot \int_{\Sigma_0} n_0 d\Sigma_0 \right)^2 \\ E_{k_\Sigma^1}(t) &= -\frac{\rho^F |g^{\text{app}}|}{2} (i_z \cdot t) \left(t \cdot \int_{\Sigma_0} n_0 d\Sigma_0 \right) \\ E_{k_\Sigma^2}(t) &= 0 \end{aligned}$$

By using the relation :

$$\int_{\Sigma_0} n_0 d\Sigma_0 - \|\Gamma_0\| i_z = \int_{\Sigma_0 \cup \Gamma_0} n_0 d\Sigma_0 = 0$$

we show that $E_{\mathcal{K}}(t) = E_{k_E}(t) + E_{k_F}(t) + E_{k_G}(t) + E_{k_B}(t) + E_{k_\Sigma^1}(t) + E_{k_\Sigma^2}(t) = 0$. The translational motions are then rigid body modes for a free fluid-structure system in presence of gravity.

Rigid Rotational Motions

A rotational motion is defined by a rotational vector θ and a center of rotation O : $U_R = \theta \wedge OM_0$ with $\theta \in \{i_x, i_y, i_z\}$, al denoted as $U_R = \Theta OM_0$, where Θ is the antisymmetric matrix associated to this rotation.

Computation of $E_{k_E}(U_R)$

$$E_{k_E}(U_R) = \frac{1}{2} \int_{\Omega_0^S} \text{Tr}[C \varepsilon(\Theta OM_0) \varepsilon(\Theta OM_0)] d\Omega_0$$

By using the following relation:

$$\varepsilon_X(\Theta OM_0) = \frac{1}{2} \left(D(\Theta OM_0) + {}^t D(\Theta OM_0) \right) = \frac{1}{2} (\Theta + {}^t \Theta) = 0$$

we simplify the previous expression and obtain:

$$E_{k_E}(U_R) = 0 \quad (11)$$

Computation of $E_{k_G}(U_R)$ Thanks to the Green formula, we integrate the expression of $E_{k_G}(U_R)$ and obtain:

$$\begin{aligned} E_{k_G}(U_R) &= \frac{1}{2} \int_{\Omega_0^S} \text{Tr}[DU_R \sigma_0 {}^t DU_R] d\Omega_0 \\ &= \frac{1}{2} \int_{\Omega_0^S} \left(\text{div}({}^t (DU_R \sigma_0) U_R) - \text{Div}(DU_R \sigma_0) \cdot U_R \right) d\Omega_0 \\ &= \frac{1}{2} \int_{\partial\Sigma_0} DU_R \sigma_0 n_0 \cdot U_R d\Sigma_0 - \frac{1}{2} \int_{\Omega_0^S} \text{Div}(DU_R \sigma_0) \cdot U_R d\Omega_0 \end{aligned}$$

We then use the local equations (2) and the fact that $DU_R = \Theta$ to simplify this expression:

$$E_{k_G}(U_R) = -\frac{1}{2} \int_{\Sigma_{i0}} P_0^{\text{app.}} (\Theta n_0) \cdot U_R d\Sigma_0 \dots$$

$$\dots + \frac{1}{2} \int_{\Sigma_{f0}} (\Theta f_0) \cdot U_R d\Sigma_0 + \frac{1}{2} \int_{\Omega_0^S} \rho_0^S (\Theta g^{\text{app.}}) \cdot U_R d\Omega_0$$

The center of gravity G_0^S of the structure in its initial configuration is finally introduced to simplify this expression:

$$E_{k_G}(U_R) = \frac{1}{2} \int_{\Sigma_{i0}} P_0^{\text{app.}} n_0 \cdot (\theta \wedge (\theta \wedge OM_0)) d\Sigma_0 \dots$$

$$\dots - \frac{1}{2} \int_{\Sigma_{f0}} f_0 \cdot (\theta \wedge (\theta \wedge OM_0)) d\Sigma_0 - \frac{m^S}{2} g^{\text{app.}} \cdot (\theta \wedge (\theta \wedge OG_0^S))$$

(12)

To give a physical interpretation of this relation, we introduce the *finite* rotation $U_{\mathcal{R}}$ whose relation with U_R is given by (see figure 2):

$$U_{\mathcal{R}} = U^{(1)} + U^{(2)} \quad \text{avec} \quad \begin{cases} U^{(1)} = \theta \wedge OM_0 = U_R \\ U^{(2)} = \theta \wedge (\theta \wedge OM_0) \end{cases} \quad (13)$$

The energy $E_{k_G}(U_R)$ is opposite to the work of the prestress forces (hydrostatic pressure, external forces and weight), considered as "dead loads", during the finite rotation $U_{\mathcal{R}}$ (since the nullity of the work corresponding to $U^{(1)}$ results from the structure equilibrium in the initial state) (Morand & Ohayon, 1975).

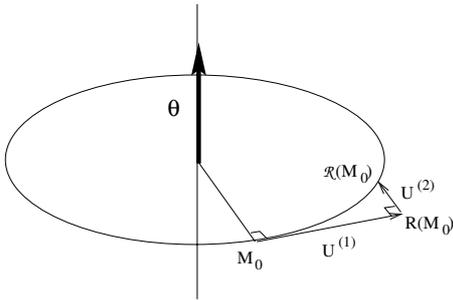


Figure 2. INFINITESIMAL AND FINITE ROTATIONS

Computation of $E_{k_{\Sigma}^2}(U_R)$ We use here the following expression of k_{Σ}^2 :

$$E_{k_{\Sigma}^2}(U_R) = \frac{1}{2} \int_{\Sigma_{i0}} P_0^{\text{app.}} \tau \cdot U_R d\Sigma_0$$

Since for a rotation of vector θ , the value of τ is:

$$\tau d\Sigma_0 = \theta \wedge n_0 d\Sigma_0 + (\theta \cdot n_0) \theta d\Sigma_0$$

the previous expression gives:

$$E_{k_{\Sigma}^2}(U_R) = \frac{1}{2} \int_{\Sigma_{i0}} P_0^{\text{app.}} (\theta \wedge n_0) \cdot (\theta \wedge OM_0) d\Sigma_0 \dots$$

$$\dots + \frac{1}{2} \int_{\Sigma_{i0}} P_0^{\text{app.}} (\theta \cdot n_0) \underbrace{(\theta \cdot (\theta \wedge OM_0))}_0 d\Sigma_0$$

Finally, we find:

$$E_{k_{\Sigma}^2}(U_R) = -\frac{1}{2} \int_{\Sigma_{i0}} P_0^{\text{app.}} n_0 \cdot (\theta \wedge (\theta \wedge OM_0)) d\Sigma_0 \quad (14)$$

Let us remark that the deformation energy associated with k_{Σ}^2 exactly compensates for the hydrostatic pressure component of the energy associated with k_G . This result seems in good agreement with physics since the aim of the operator k_{Σ}^2 is to take the following force effect into account.

Computation of $E_{k_F}(U_R)$ A similar demonstration gives:

$$E_{k_F}(U_R) = \frac{1}{2} \int_{\Sigma_{f0}} |f_0^F| n_0 \cdot (\theta \wedge (\theta \wedge OM_0)) d\Sigma_0 \quad (15)$$

Computation of $E_{k_B}(U_R)$

$$E_{k_B}(U_R) = \frac{\rho^F |g|}{2 \|\Gamma_0\|} \underbrace{\left(\int_{\Sigma_{i0}} (\theta \wedge OM_0) \cdot n_0 d\Sigma_0 \right)^2}_{(A)}$$

Some vector analysis formulae are used to simplify the term denoted as (A). We obtain:

$$(A) = \int_{\Omega_0^F} \theta \cdot \underbrace{\text{rot}(OM_0)}_0 d\Omega_0 + (i_z \wedge \theta) \cdot \left(\int_{\Gamma_0} OM_0 d\Gamma_0 \right)$$

We introduce C_0 , the center of the liquid free surface (in its initial configuration), defined by $\|\Gamma_0\| OC_0 = \int_{\Gamma_0} OM_0 d\Gamma_0$, and finally find for $E_{k_B}(U_R)$:

$$E_{k_B}(U_R) = \frac{\rho^F |g^{\text{app}}| \|\Gamma_0\|}{2} \left((i_z \wedge \theta) \cdot OC_0 \right)^2 \quad (16)$$

Computation of $E_{k_\Sigma^1}(U_R)$

$$\begin{aligned} E_{k_\Sigma^1}(U_R) &= -\frac{\rho^F |g^{\text{app}}|}{2} \int_{\Sigma_{i_0}} \left(OM_0 \cdot (i_z \wedge \theta) \right) \left((n_0 \wedge \theta) \cdot OM_0 \right) d\Sigma_0 \\ &= -\frac{\rho^F |g^{\text{app}}|}{2} \int_{\Omega_0^F} \underbrace{\text{div} \left(\left(OM_0 \cdot (\theta \wedge i_z) \right) (\theta \wedge OM_0) \right)}_{(A)} d\Omega_0 \dots \\ &\dots - \frac{\rho^F |g^{\text{app}}|}{2} \int_{\Gamma_0} \underbrace{\left(OM_0 \cdot (i_z \wedge \theta) \right)^2}_{(B)} d\Gamma_0 \end{aligned}$$

The first term (A) can be simplified to give:

$$(A) = (\theta \wedge i_z) \cdot \left(\theta \wedge \int_{\Omega_0^F} OM_0 d\Omega_0 \right)$$

If the center of gravity G_0^F of the fluid domain in its initial configuration is introduced, this relation gives:

$$(A) = -\|\Omega_0^F\| i_z \cdot \left(\theta \wedge (\theta \wedge OG_0^F) \right)$$

The second term (B) is simplified by introducing the point C_0 previously defined. We obtain:

$$(B) = \|\Gamma_0\| \left(OC_0 \cdot (i_z \wedge \theta) \right)^2 + {}^t(i_z \wedge \theta) \mathfrak{S}_\Gamma^C(i_z \wedge \theta)$$

where \mathfrak{S}_Γ^C is the inertia tensor of the free surface Γ_0 with respect to its center C_0 , defined by:

$$\mathfrak{S}_\Gamma^C = \int_{\Gamma_0} C_0 M_0 {}^t(C_0 M_0) d\Gamma_0 \quad (17)$$

Finally, the energy $E_{k_\Sigma^1}(U_R)$ is written:

$$\begin{aligned} E_{k_\Sigma^1}(U_R) &= -\frac{m^F}{2} g^{\text{app}} \cdot \left(\theta \wedge (\theta \wedge OG_0^F) \right) \dots \\ &\dots - \frac{\rho^F |g^{\text{app}}|}{2} \left(\|\Gamma_0\| \left(OC_0 \cdot (i_z \wedge \theta) \right)^2 + {}^t(i_z \wedge \theta) \mathfrak{S}_\Gamma^C(i_z \wedge \theta) \right) \end{aligned} \quad (18)$$

Computation of $E_{\mathcal{X}}(U_R)$ We assemble the various energy computed previously to give the following expression of $E_{\mathcal{X}}(U_R)$:

$$\begin{aligned} E_{\mathcal{X}}(U_R) &= -\frac{\rho^F |g^{\text{app}}|}{2} {}^t(i_z \wedge \theta) \mathfrak{S}_\Gamma^C(i_z \wedge \theta) \dots \\ &\dots - \frac{m}{2} g^{\text{app}} \cdot \left(\theta \wedge (\theta \wedge OG_0) \right) - \frac{1}{2} \int_{\Sigma_{f_0}} f_{D0} \cdot (\theta \wedge (\theta \wedge OM_0)) d\Sigma_0 \end{aligned} \quad (19)$$

where G_0 is the center of gravity of the fluid-structure system in its initial configuration and m its mass.

Using the definition of g^{app} , equation (1), we can give a final expression of this energy:

$$\begin{aligned} E_{\mathcal{X}}(U_R) &= -\frac{\rho^F |g^{\text{app}}|}{2} {}^t(i_z \wedge \theta) \mathfrak{S}_\Gamma^C(i_z \wedge \theta) \dots \\ &\dots + \frac{1}{2} \left(\int_{\Sigma_{f_0}} |f_0^F| n_0 d\Sigma_0 \right) \cdot (\theta \wedge (\theta \wedge OG_0)) \dots \\ &\dots - \frac{1}{2} \int_{\Sigma_{f_0}} f_{D0} \cdot (\theta \wedge (\theta \wedge GM_0)) d\Sigma_0 \end{aligned} \quad (20)$$

The term depending on follower forces f^F is null if f_0^F naturally satisfies the condition of symmetry for k_F ($|f_0^F|$ constant and Σ_{f_0} closed surface). Otherwise, we can eliminate this term by placing the center of rotation O on the system center of gravity G . On the other hand, we remark that the term depending on the dead loads f^D is not cancelled.

We can now give the following conclusions :

1. If all external loads f_0 are not follower forces, the rotational movements are no longer rigid body modes for the system. We remark that the position of the dead loads in relation to the system center of gravity determines their stabilizing or destabilizing effect on the system. This result was given by Kreis & Klein in (Kreis & Klein, 1992).
2. If the prestresses in the initial state are only due to follower forces, additional stiffness terms k_Σ and k_F nullify the deformation energy generated by the geometric stiffness k_G .

Then, the rotational movement with respect to the gravity axis i_z is still a zero-frequency rigid body mode for the system.

3. However, the two rotations with respect to horizontal axes i_x and i_y are no longer energyless movement because a fluid-structure system prestressed by the gravity is not invariant during such rotations : when the structure rotates, the liquid free surface remains horizontal by action of the gravity forces which generate this energy.

Table 1 summarizes the contribution of each term of the stiffness operator to the deformation energy of the system.

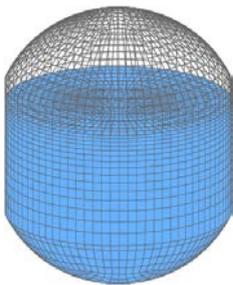
	$E_{k_E} + E_{k_G^g} + E_{k_G^{f_0}} + E_{k_F} + E_{k_B} + E_{k_\Sigma^1} + E_{k_\Sigma^2} = E_{\mathcal{X}}$
$T_{x \text{ or } y}$	$0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$
T_z	$0 + 0 + 0 + 0 + \textcircled{1} + (-\textcircled{1}) + 0 = 0$
R_z	$0 + \textcircled{2} + \textcircled{3} + (-\textcircled{3}) + 0 + 0 + (-\textcircled{2}) = 0$
$R_{x \text{ or } y}$	$0 + \textcircled{2} + \textcircled{3} + (-\textcircled{3}) + \textcircled{1} + \textcircled{4} - \textcircled{1} + (-\textcircled{2}) = \textcircled{4}$

Table 1. SUMMARY OF THE DIFFERENT CONTRIBUTIONS TO THE DEFORMATION ENERGY ASSOCIATED TO THE RIGID BODY MOTIONS

NUMERICAL EXAMPLES

Test case

To illustrate this result, the following test case has been studied: The first free eigenfrequencies, obtained with the hydroelas-



$R = 2.7 \text{ m}$
 $L = 6.7 \text{ m}$
 $h = 4.5 \text{ m}$
 $\rho^f = 1138.7 \text{ kg/m}^3$
 $e = 5. \text{ mm}$
 $\rho^s = 4450. \text{ kg/m}^3$
 $E = 8.14 \cdot 10^{10} \text{ S.I.}$
 $\nu = 0.33$

Figure 3. MESH OF TEST CASE TANK

tic modelisation with gravity presented here, are shown in table 2. The study of these results leads to the following remarks:

1. The x and y -translational modes have zero frequency, as predicted by the theory.

2. The translational and rotational modes in relation to the z axis have quasi-zero frequencies. This error appears because, numerically, the differences between respectively E_{k_B} and $E_{k_\Sigma^1}$, and $E_{k_G^g}$ and $E_{k_\Sigma^2}$ are not exactly zero.
3. As expected, the rotational motions with respect to x and y axes have non-zero frequencies. These eigenmodes are the result of a coupling between the rotation of the structure and the first fluid sloshing modes (the first sloshing eigenfrequencies are summarized in table 3). Figure 4 illustrates the modal participation of the first 48 sloshing modes to the ninth mode (0.59 Hz): it highlights the coupling between the rotational movement of the structure and principally the first and tenth sloshing modes of the fluid.
4. The other computed eigenfrequencies are equal to the sloshing frequencies because, in this frequency range, the elastic modes of the structure are not yet excited.

mode		freq. (Hz)
1 and 2	trans. x and y	0.
3 and 4	trans. z and rot. z	~ 0 .
5	rot. x + sloshing 1	0.395
6	rot. y + sloshing 2	0.395
7 and 8	sloshing 3 or 4	0.53
9	rot. x + sloshing 1 et 10	0.59
10	rot. y + sloshing 2 et 11	0.59
11	sloshing 5	0.595
12 and 13	sloshing 6 or 7	0.62
...		...

Table 2. FIRST FREE HYDROELASTIC EIGENFREQUENCIES WITH GRAVITY

Comparison with a benchmark

We have tried to validate this aspect of our modeling by comparison with results available in the literature. A benchmark was proposed by Kreis & Klein in 1991 to study the coupling between rotational motion of the tank and sloshing of the fluid (Kreis & Klein, 1991). The proposed model is illustrated by figure 5. The tank, whose walls (b) except the bottom (A) are rigid, is suspended from a pivot such as the only rigid body mode of the structure is the rotational motion with respect to the horizontal axis.

mode	freq. (Hz)
1 and 2	0.41
3 and 4	0.53
5	0.595
6 and 7	0.62
8 and 9	0.70
10 and 11	0.705
12 and 13	0.77
...	...

Table 3. FIRST SLOSHING EIGENFREQUENCIES

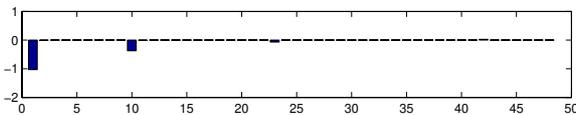
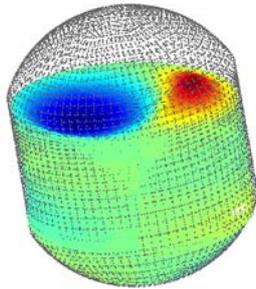
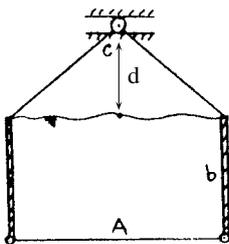


Figure 4. SLOSHING MODES PARTICIPATION TO 9th FREE HYDROELASTIC MODE



$A = 1. \text{ m}$
 $b = 0.5 \text{ m}$
 $d = 1. \text{ m}$
 $\rho^f = 1000. \text{ kg/m}^3$
 $e = 5. \text{ cm}$
 $\rho^s = 3000. \text{ kg/m}^3$
 $E = 10^{10} \text{ S.I.}$
 $\nu = 0.3$

Figure 5. KREIS & KLEIN BENCHMARK MODEL

Kreis & Klein's method is particularly adapted to represent the rigid mode behaviour of the system by including in the model explicit rigid degrees of freedom. Moreover, the fluid is considered through approximations as a subsystem coupled via a pendulum motion to the structure. The results we obtain with our modeling on a three-dimensional finite element model are

in good agreement with their results: Kreis & Klein predict an eigenmode coupling the rotation of the structure and the first sloshing mode of the fluid for a frequency of 0.99 Hz (see figure 6) when we find the same eigenmode with a difference of 8% on the frequency (see figure 7). According to the author's knowledge, no experimental measurements have been compared to the numerical simulation results published in this benchmark and it could be the aim of future works.

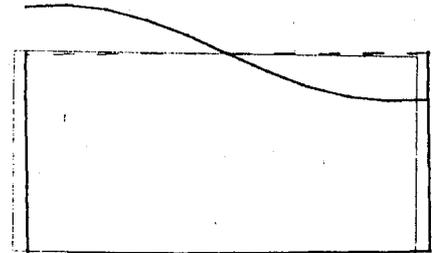


Figure 6. EIGENMODE FROM REF. (Kreis & Klein, 1991) (0.99 Hz)

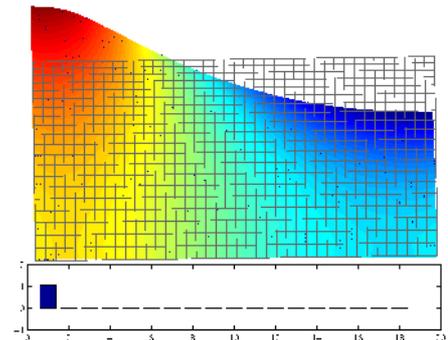


Figure 7. SAME EIGENMODE COMPUTED WITH OUR MODELING (0.91 Hz)

CONCLUSION

We have shown that the hydroelastic modeling with gravity presented in previous papers (Schotté & Ohayon, 1999; Schotté & Ohayon, 2001) can be adapted to study the particular case of free-free fluid-structure systems by replacing the gravity by an apparent gravity depending on the system acceleration. The study of the rigid body modes has highlighted the prime importance of distinguishing the follower forces from the dead loads in the external forces applied to the system: depending on whether dead loads are applied or not to the system, the rotational motion

with respect to the gravity axis can or cannot be a zero-frequency rigid body mode for the system. We have also demonstrated that, due to the presence of gravity, the rotational motions with respect to horizontal axes are not zero-frequency rigid body modes but are coupled with the sloshing movements of the fluid free surface. Some validations of this aspect have been engaged but future works will consolidate this result.

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