Modal synthesis method for poroelastic fluid-structure coupled problems including structural damping

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MODAL SYNTHESIS METHOD FOR POROELASTIC FLUID-STRUCTURE COUPLED PROBLEMS INCLUDING STRUCTURAL DAMPING

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ABSTRACT

At low frequency, modal synthesis is usually used to solve the vibrational fluid-structure interaction problem. The modal technique application for elasto-poro-acoustic complex structures requires a modal description of the poroelastic vibroacoustic behaviour. Some recent works concerning the application of modal techniques for the displacement pressure equation of the generalized Biot's model were published. Dazel proposes a modal decomposition based on the use of complex modes. This work consists in projecting the poroelastic system on a truncated basis of damped and coupled modes. In the mode calculation, the author takes into account fluid-structure interaction, viscous and thermal dissipations but structural damping is neglected.

This paper presents a modal approach allowing to reduce the size of the poroelastic matrix system and to predict more effectively the porous material vibroacoustic behaviour. Starting from Dazel's works, this modal approach is based on complex modes, modal synthesis, modal projection and modal superposition techniques. The originality of this paper is to take into account the porous material structural damping in the poroelastic mode calculation.

INTRODUCTION

The mechanical structures are subjected to a broad spectrum of dynamic excitation, for example the aerostructures can be subjected to acoustic excitations such as the jet noise, the propeller noise and the boundary layer turbulences, and also to mechanical excitations such as the propellers, engines and transmission system vibrations …. To reduce the vibrations and the acoustic radiation of these structures, various solutions can be planned. Among those, active techniques consist to measure, analyze and cancel the noise by generating a complementary noise in phase opposition. The passive techniques consist in placing between the noise source and the receiver a dissipative and damping complex structure. Finally active/passive techniques known as hybrid can also be considered.

The development of powerful numerical tools allowing to predict the vibroacoustic behaviour of these complex structures is necessary in order to increase their effectiveness. The elasto-poro-acoustic or "e-p-a" complex structures are an assembly of elastic, poroelastic and acoustic substructures. This configuration is largely used in the transportation and building industries to fight against the industrial noises. The low mass (porosity) and the capacity to dissipate the mechanical energy confer on porous materials many advantages. At low frequency, modal synthesis is usually used to solve the vibrational fluid-structure interaction problem. The modal technique application

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for e-p-a complex structures needs a modal description of the poroelastic vibroacoustic behaviour.

Some recent works concerning the application of modal techniques for the displacement pressure equation of the generalized Biot's model were published. Sgard and Atalla propose a selective modal decomposition in term of undamped and uncoupled real modes. The obtained modes take into account neither the coupling nor the dissipations. Then, the problem parameters no longer depend on frequency. Dazel proposes a modal decomposition based on the use of complex modes. This work consists in projecting the poroelastic system on a truncated basis of damped and coupled modes. In the mode calculation, the author takes into account fluid-structure interaction, viscous and thermal dissipations but structural damping dissipation is neglected.

This paper presents a modal approach allowing to reduce the size of the poroelastic matrix system and to predict more effectively the porous material vibroacoustic behaviour. Starting from Dazel's works, this modal approach is based on complex modes, modal synthesis, modal projection and modal superposition techniques. The originality of this paper is to take into account the porous material structural damping in the poroelastic mode calculation. Section 1 presents the finite element equation related to poroelastic problems and vibroacoustic response calculation by direct method. Section 2 is devoted to the description and validation of the modal approach applied to the poroelastic problems including structural damping. Section 3 presents work's conclusions and future works.

NOTATIONS

Throughout this paper, we use double and single underbars letters for tensor and vector notations. The overbar indicates the associated conjugate complex value. Dot is used to denote the first time derivation. The tilde symbol indicates that the associated quantity is complex. We use boldface capital letters for matricial notations. The symbols $\underline{\nabla} f$, $\nabla \underline{x}$ and $DIV\left(\underline{\underline{\sigma}}\right)$ denote, respectively, the scalar function gradient, the vectorial and tensorial quantity divergence. Prim is used to denote the matrix frequency dependence.

THE POROELASTIC PROBLEM

Biot [1] proposed to use the continuum mechanics formalism to model the porous material like that an equivalent homogeneous medium (the constitutive material is considered as homogeneous and isotropic). The poroelastic mediums considered in the Biot's theory are supposed to be fluid-saturated, continuums and homogeneous on the macroscopic scale. The small perturbation assumption of the elastic isotropic matrix (solid phase) and fluid ensure the linearity of the mechanical behaviour. The diffusion phenomena on the heterogeneities are neglected. The pores in the element are inter-connected between them and with outside. In agreement with Burridge and Keller's works [2], the compressible and slightly viscous fluid in tubular permanent flow is considered as no viscous on the macroscopic scale. Finally, the viscous and thermal effects are regarded as independent according to Zwikker and Kosten [3]. The Biot's theory is placed on the macroscopic scale and supposes the existence of a representative elementary volume which allows to regard the poroelastic medium on the macroscopic scale as the superposition in time and space of two coupled continuum mediums. In the generalized Biot's model proposed by Allard [4], the structural damping dissipation is modeled by a frequency independent factor, the model suggested in this paper allows to take into account the frequency dependence of the porous material structural damping. Consequently, the real and frequency independent shear modulus N (this latter is equivalent to the second Lamé's coefficient μ related to elasticity theory) becomes complex and frequency dependent, such as:

$$\tilde{N} = N\left(1 + j\eta_S\right) \tag{1}$$

where j is the imaginary number or imaginary unit defined as the root of -1 (i.e. $j = \sqrt{-1}$) and η_S is the frequency dependent structural damping. Consequently, the real and frequency independent *in vacuum* stress tensor of the porous material solid phase $\underline{\underline{\sigma}}^{S0}$ proposed by Atalla *and Al.* [5] becomes complex and frequency dependent, this latter takes the following form:

$$\underline{\underline{\tilde{\sigma}}}^{S0} = \left[\tilde{A} - \frac{\tilde{Q}^2}{\tilde{R}}\right]\nabla \underline{u}^S \, \underline{\underline{I}}_d + 2\tilde{N}\underline{\underline{\varepsilon}}^S \tag{2}$$

where $\underline{\underline{I}}_d$ is the identity tensor such as $d = 3$, Φ is the porous material porosity, $\underline{\underline{\varepsilon}}^S$ is the partial strain tensor of the porous material solid phase and $\underline{u}^S\left(x,t\right)$ is the average displacement of the porous material solid phase. The complex and frequency dependent poroelastic coefficients $\tilde{A}$, $\tilde{Q}$ and $\tilde{R}$ are according to Biot and Willis [6] of the following form:

$$\tilde{A} = \frac{\left(1-\Phi\right)^2}{\Phi}\tilde{K}_F + 2\tilde{N}\frac{\nu}{1-2\nu}, \; \tilde{Q} = \left(1-\Phi\right)\tilde{K}_F \tag{3}$$

$$\tilde{R} = \Phi\tilde{K}_F, \; \tilde{M} = \frac{\tilde{K}_F}{\Phi} \tag{4}$$

where $\tilde{A}$ is equivalent to the first Lamé's coefficient λ related to the elasticity theory, $\tilde{R}$ corresponds to the bulk modulus of the fluid occupying a Φ fraction of the porous volume, $\tilde{Q}$ corresponds to the elastic coupling coefficient, $\tilde{M}$ is the complex and frequency dependent Biot's module and ν is the Poisson's ratio, where $\tilde{K}_F$ is the complex and frequency dependent bulk modulus introduced by Champoux and Allard [7] such as:

$$\tilde{K}_F = \gamma P_0\left[\gamma - \left(\gamma-1\right)\left(1 + \frac{8\eta_{air}}{j\Lambda'^2 \, \mathrm{Pr}\, \omega\rho_F}\tilde{G}'\right)^{-1}\right]^{-1} \tag{5}$$

$$\tilde{G}' = \sqrt{1 + j\frac{\omega}{\omega_{TH}}}, \; \omega_{TH} = \frac{16\eta_{air}}{\Lambda'^2 \, \mathrm{Pr}\, \rho_F} \tag{6}$$

where $\tilde{G}'$ is the complex and frequency dependent thermal damping function, ω_{TH} the thermal characteristic frequency, Pr the Prandtl's number, Λ' the thermal characteristic length, P_0 the atmospheric pressure and γ the air specific heat ratio.

First, one considers the displacement pressure equations of the generalized Biot's model, proposed by Atalla *and Al.* in order to highlight the associated matrix system dissymmetry.

We consider the poroelastic linear vibrations around a static balance configuration regarded as natural state (non prestressed). The assumption of the small perturbations allows to confuse the Lagrangian and Eulerian co-ordinates, let us recall that this approximation is not valid any more when the initial configuration is prestressed. Figure 1 presents the studied system geometrical configuration. The index f indicates that boundary conditions in forces (Neumann) are applied on the surface-coupling Σ^p. We suppose an unit amplitude harmonic external surface load $\underline{F}^{ext}(x,t)$ acting on the surface-coupling Σ_f^p defined by $\underline{F}^{ext}(x,t) = \underline{F}^{ext}(x)\cos(\omega t)$ and we suppose the associated solutions such as $\underline{u}^S(x,t) = \underline{u}^S(x)\cos(\omega t)$ and $p(x,t) = p(x)\cos(\omega t)$.

The system

Let $\mathbb{R}^3$ be the physical space related to the cartesian reference system and x be the geometrical point of $\mathbb{R}^3$. Let Ω^p be the $\mathbb{R}^3$ bounded domain occupied by the poroelastic medium, $\partial\Omega^p = \Sigma_f^p \cup \Sigma^p$ its boundary and $\underline{n}^p$ its outward unit normal. We define $\underline{u}^S(x,t)$ and $p(x,t)$ the porous material solid phase displacement and fluid phase interstitial pressure of a particle located in $M \in \Omega^p$ at the moment t. One defines by x_1, x_2, x_3 the M co-ordinates.

The boundary value problem

The poroelastic boundary value problem (for $\omega \neq 0$) is defined by the following equation system:

$$DIV\left(\underline{\underline{\tilde{\sigma}}}^{S0}\left(\underline{u}^S(x)\right)\right) + \omega^2\tilde{\rho}\underline{u}^S(x) + \tilde{\gamma}\,\underline{\nabla} p(x) = \underline{0} \qquad \text{in } \Omega^p \quad (7)$$

$$\Delta p(x) + \omega^2 \frac{\tilde{\rho}_{22}}{\tilde{R}} p(x) - \omega^2 \frac{\tilde{\rho}_{22}\tilde{\gamma}}{\Phi^2} \nabla \underline{u}^S(x) = 0 \qquad \text{in } \Omega^p \quad (8)$$

$$\underline{\underline{\tilde{\sigma}}}^S\left(\underline{u}^S(x)\right)\underline{n}^p \cdot \underline{n}^p = (1-\Phi)\underline{F}^{ext} \cdot \underline{n}^p \qquad \text{on } \Sigma_f^p \quad (9)$$

$$p(x)\underline{\underline{I_d}}\,\underline{n}^p \cdot \underline{n}^p = -\underline{F}^{ext} \cdot \underline{n}^p \qquad \text{on } \Sigma_f^p \quad (10)$$

The complex and frequency dependent factors $\tilde{\rho}$, $\tilde{\gamma}$ are defined by the following relations:

$$\tilde{\rho} = \tilde{\rho}_{11} - \frac{\tilde{\rho}_{12}^{\,2}}{\tilde{\rho}_{22}},\ \tilde{\gamma} = \Phi\left[\frac{\tilde{\rho}_{12}}{\tilde{\rho}_{22}} - \frac{\tilde{Q}}{\tilde{R}}\right] \quad (11)$$

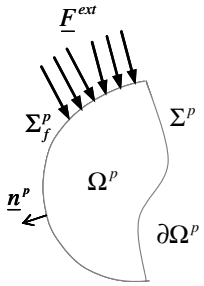

Figure 1: The studied system geometrical configuration.

where the $\tilde{\rho}_{ij}$ are the complex and frequency dependent effective densities of the generalized Biot's model such as:

$$\tilde{\rho}_{11} = \rho_{11} - j\frac{\tilde{b}}{\omega},\ \tilde{\rho}_{22} = \rho_{22} - j\frac{\tilde{b}}{\omega} \quad (12)$$

$$\tilde{\rho}_{12} = \rho_{12} + j\frac{\tilde{b}}{\omega} \quad (13)$$

where $\tilde{b}$ is the complex and frequency dependent viscous damping coefficient introduced by Johnson *and al.* [8] defined by the following relations:

$$\tilde{b} = \sigma\,\Phi^2\tilde{G},\ \tilde{G} = \sqrt{1 + j\frac{\omega}{\omega_v}} \quad (14)$$

$$\omega_v = \frac{\sigma^2\Phi^2\Lambda^2}{4\alpha^2\eta_{air}\rho_F} \quad (15)$$

where $\tilde{G}$ is the complex and frequency dependent viscous damping function, ω_v the viscous characteristic frequency, σ the air flow resistivity, Λ the viscous characteristic length and η_{air} the air dynamic viscosity.

The equations (7) and (8) are the displacement pressure form of the poroelasticity equations including the structural damping frequency dependence. This mixed form is a traditional fluid-structure coupling form. The first two terms of the equation (7) are associated to the *in vacuum* solid phase dynamic behaviour, whereas the first two terms of the equation (8) are associated to the fluid behaviour when the matrix is motionless or rigid. The equation (9) translated the normal stress continuity on the surface-coupling Σ_f^p between external and the poroelastic solid phase. The equation (10) translated the normal stress continuity on the surface-coupling Σ_f^p between external and the poroelastic fluid phase. One defines the system S constituted by the equations (7) to (10).

The variational formulation and discretization

To build the variational form of the poroelastic boundary value problem we apply the test function method. This method consists in multiplying the equations (7) and (8) respectively by a vector field of virtual displacement $\delta\underline{u}^S(x)$ and a scalar field

of virtual pressure $\delta p(x)$, integrating the obtained result on the Ω^p domain and applying the Green formula in taking into account the equations (9) to (10). Let C_{u^S} and C_p be the test function spaces of the sufficiently regular and cinematically admissible $\delta \underline{u}^S(x)$ and $\delta p(x)$ functions.

The Ritz-Galerkin's method allows to search an approximate solution of the S system associated variational problem in a vectorial subspace $C_{u^S,Nu^S} \times C_{p,Np}$. Let us consider a finite dimension subspace $C_{x,Nx} \subset C_x$ with $N_x \geq 1$ and a basis of it subspace $e_j = \{j = 1,...,Nx\}$. A generic element of $C_{x,Nx}$ is of the following form:

$$\hat{\underline{u}}(x) = \sum_{j=1}^{Nx} \xi_j e_j \tag{16}$$

The satisfaction of the variational properties in the vectorial subspace $C_{x,Nx}$ leads to a matrix system. In practice one selects the interpolation basis resulting from the finite element method. Then, $\hat{\underline{u}}(x)$ defines the approximate vector field value in any point of the finite element $(E_i)_{i=1...k_x}$, with k_x the element number and ξ_j defines the vectorial field nodal values of the associated finite element which will be noted $\underline{u}_N$ (degrees of freedom or d.o.f.). The retained interpolation functions in this paper are linear. The discretization of the S associated variational form gives the following matrix system:

$$\left[\tilde{\mathbf{A}}'\right]\begin{bmatrix} \underline{u}_N^S \\ \underline{p}_N \end{bmatrix} = \begin{bmatrix} \underline{F}_1^{ext} \\ \underline{F}_2^{ext} \end{bmatrix} \tag{17}$$

with

$$\left[\tilde{\mathbf{A}}'\right] = \left[\tilde{\mathbf{X}}'^p\right] - \omega^2 \left[\tilde{\mathbf{Y}}'^p\right] \tag{18}$$

$$\left[\tilde{\mathbf{X}}'^p\right] = \begin{bmatrix} \left[\tilde{\mathbf{K}}'^S\right] & -\left[\tilde{\mathbf{C}}'^S\right] \\ 0 & \left[\tilde{\mathbf{K}}'^F\right] \end{bmatrix} \tag{19}$$

$$\left[\tilde{\mathbf{Y}}'^p\right] = \begin{bmatrix} \left[\tilde{\mathbf{M}}'^S\right] & 0 \\ \left[\tilde{\mathbf{C}}'^F\right] & \left[\tilde{\mathbf{M}}'^F\right] \end{bmatrix} \tag{20}$$

where the complex and frequency dependent matrices are defined by:

$\left[\tilde{\mathbf{K}}'^S\right]$ is the $N_{u^S} \times N_{u^S}$ symmetric porous material solid phase stiffness matrix,

$\left[\tilde{\mathbf{M}}'^S\right]$ is the $N_{u^S} \times N_{u^S}$ symmetric porous material solid phase mass matrix,

$\left[\tilde{\mathbf{C}}'^S\right] = \left[\tilde{\mathbf{C}}'\right]$ is the $N_{u^S} \times N_p$ porous material volume-coupling matrix,

$\left[\tilde{\mathbf{K}}'^F\right]$ is the $N_p \times N_p$ symmetric porous material fluid phase kinetic energy matrix,

$\left[\tilde{\mathbf{M}}'^F\right]$ is the $N_p \times N_p$ symmetric porous material fluid phase compression energy matrix,

$\left[\tilde{\mathbf{C}}'^F\right] = \left[\tilde{\mathbf{C}}'\right]^T$ is the $N_p \times N_{u^S}$ porous material volume-coupling matrix,

$\underline{F}_1^{ext}$ is the N_{u^S} external surface load part acting on the solid phase porous material,

$\underline{F}_2^{ext}$ is the N_p external surface load part acting on the fluid phase porous material,

and $\left[\tilde{\mathbf{A}}'\right]$ is the $\left(N_{u^S} + N_p\right) \times \left(N_{u^S} + N_p\right)$ dissymmetric complex and frequency dependent poroelastic matrix.

In order to make the matrix $\left[\tilde{\mathbf{A}}'\right]$ in a symmetric form, first of all let us carry out the change of variable proposed by Kehr-Candille [9]: $p(x) = j\omega\, \varphi(x)$ with $\omega \neq 0$. It should be stressed that $\underline{u}^F(x) \neq \underline{\nabla}\varphi(x)$ because the fluid is rotational. After this change of variable, the poroelastic matrix $\left[\tilde{\mathbf{A}}'\right]$ takes the following form:

$$\left[\tilde{\mathbf{A}}'\right] = \left[\tilde{\mathbf{K}}'^p\right] + j\omega\left[\tilde{\mathbf{C}}'^p\right] - \omega^2\left[\tilde{\mathbf{M}}'^p\right] \tag{21}$$

where

$$\left[\tilde{\mathbf{K}}'^p\right] = \begin{bmatrix} \left[\tilde{\mathbf{K}}'^S\right] & 0 \\ 0 & \left[\tilde{\mathbf{K}}'^F\right] \end{bmatrix} \tag{22}$$

$$\left[\tilde{\mathbf{C}}'^p\right] = \begin{bmatrix} 0 & -\left[\tilde{\mathbf{C}}'\right] \\ \left[\tilde{\mathbf{C}}'\right]^T & 0 \end{bmatrix} \tag{23}$$

$$\left[\tilde{\mathbf{M}}'^p\right] = \begin{bmatrix} \left[\tilde{\mathbf{M}}'^S\right] & 0 \\ 0 & \left[\tilde{\mathbf{M}}'^F\right] \end{bmatrix} \tag{24}$$

Finally the poroelastic matrix $\left[\tilde{\mathbf{A}}'\right]$ can be put in a symmetric form by multiplying the equations relating to the poroelastic solid phase by -1. The final obtained matrix system becomes of the following form:

$$\left[\tilde{\mathbf{A}}'\right]\begin{bmatrix} \underline{u}_N^S \\ \underline{\varphi}_N \end{bmatrix} = \begin{bmatrix} -\underline{F}_1^{ext} \\ \underline{F}_2'^{ext} \end{bmatrix} \tag{25}$$

with

$$\left[\tilde{\mathbf{A}}'\right] = \begin{bmatrix} -\left[\tilde{\mathbf{K}}'^S\right] - (j\omega)^2\left[\tilde{\mathbf{M}}'^S\right] & (j\omega)\left[\tilde{\mathbf{C}}'\right] \\ (j\omega)\left[\tilde{\mathbf{C}}'\right]^T & \left[\tilde{\mathbf{K}}'^F\right] + (j\omega)^2\left[\tilde{\mathbf{M}}'^F\right] \end{bmatrix} \tag{26}$$

where $\left[\tilde{\mathbf{A}}'\right]$ is now symmetric.

The vibroacoustic response calculation by direct method

To calculate the direct vibroacoustic response of a poroelastic medium one searches for each frequency the solutions of the complex and linear system defined by the equation (25). Let us quote for example the direct method known as Gauss's elimination witch allows to solve the linear systems. The purpose of this last be to transform the system $\mathbf{A}\underline{x} = \underline{\hat{b}}$ into an equivalent system (i.e. having the same solution) having the following form $\mathbf{U}\underline{u} = \underline{\hat{b}}$, where $\mathbf{U}$ is a higher triangular matrix and $\underline{\hat{b}}$ is a suitably modified second member. Then, this last system can be solved by a retrograde substitution method. We point out the method allowing to calculate the poroelastic nodal displacement and nodal pressure by direct method. We express the matrix $\left[\tilde{\mathbf{A}}'\right]$ in the following form:

$$\left[\tilde{\mathbf{A}}'\right] = \begin{bmatrix} \tilde{\mathbf{A}}'_{u^S u^S} & \tilde{\mathbf{A}}'_{u^S \varphi} \\ \tilde{\mathbf{A}}'_{\varphi u^S} & \tilde{\mathbf{A}}'_{\varphi\varphi} \end{bmatrix} \tag{27}$$

and we consider the following problem, find $\underline{u}_N^S$ and $\underline{\varphi}_N$ such as:

$$\begin{bmatrix} \tilde{\mathbf{A}}'_{u^S u^S} & \tilde{\mathbf{A}}'_{u^S \varphi} \\ \tilde{\mathbf{A}}'_{\varphi u^S} & \tilde{\mathbf{A}}'_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \underline{u}_N^S \\ \underline{\varphi}_N \end{bmatrix} = \begin{bmatrix} -\underline{F}_1^{ext} \\ \underline{F}_2'^{ext} \end{bmatrix} \tag{28}$$

the obtained solutions are of the following form:

$$\underline{\varphi}_N = \tilde{\mathbf{B}}'^{-1} \left(\tilde{\mathbf{A}}'_{u^S u^S}{}^{-1} \underline{F}_1^{ext} + \tilde{\mathbf{A}}'_{\varphi u^S}{}^{-1} \underline{F}_2'^{ext} \right) \tag{29}$$

$$\underline{u}_N^S = -\tilde{\mathbf{A}}'_{u^S u^S}{}^{-1} \left(\underline{F}_1^{ext} + \tilde{\mathbf{A}}'_{u^S \varphi} \underline{\varphi}_N \right) \tag{30}$$

$$\underline{p}_N = j\omega\, \underline{\varphi}_N \tag{31}$$

where

$$\tilde{\mathbf{B}}' = \tilde{\mathbf{A}}'_{\varphi u^S}{}^{-1} \tilde{\mathbf{A}}'_{\varphi\varphi} - \tilde{\mathbf{A}}'_{u^S u^S}{}^{-1} \tilde{\mathbf{A}}'_{u^S \varphi} \tag{32}$$

The "original" change of variable proposed by Kehr-Candille for the generalized Biot's model was validated for poroelastic and e-p-a complex structure problems. The referring works concerning validation step are the Debergue's works [10]. In this works, a validation of a computer code development based on the displacement pressure mixed form for the e-p-a systems is realized, numerical and experimental results are compared. The use of poroelasticity equation mixed forms allows to reduce the poroelastic matrix system size since, within the framework of the finite element method, only four degrees of freedom are necessary, three for the solid phase and one for the fluid phase. However although one passes from six degrees of freedom for the initial displacement form to four degrees of freedom for the mixed forms, the obtained poroelastic matrix systems remain of important size. To reduce efficiently the computing time and CPU memory necessary to the poroelastic system direct resolution, the development of powerful methods proves to be necessary.

In the following section, we present a modal approach allowing to solve the poroelastic problems effectively.

MODAL APPROACH

Some recent works concerning the application of modal techniques for the displacement pressure equation of the generalized Biot's model was published. Sgard and Atalla [11] propose a selective modal decomposition in term of undamped and uncoupled real modes. The obtained modes take into account neither the coupling nor the dissipations. Then, the problem parameters no longer depend on frequency. Dazel [12] proposes a modal decomposition based on the use of complex modes [13]. This work consists in projecting the poroelastic system on a truncated basis of damped and coupled modes. Although these works are very complete, the structural damping dissipation is neglected. Indeed the Dazel's developments suppose $\eta_S = 0$.

This section presents a modal approach allowing to reduce the poroelastic matrix system size and to predict more effectively the porous material vibroacoustic behaviour. These developments [14] are inspired by Dazel's works and were carried out starting from the poroelasticity equation "original" form of the generalized Biot's model proposed by Kehr-Candille and presented in the preceding section. This modal approach is based on complex modes, modal synthesis, modal projection and modal superposition techniques. The originality of this work is to take into account the porous material structural damping in the poroelastic mode calculation.

The expanded eigenvalue problem establishing

The free problem associated to the matrix system (25) is defined by:

$$\left[\tilde{\mathbf{A}}'\right] \begin{bmatrix} \underline{u}_N^S \\ \underline{\varphi}_N \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{0} \end{bmatrix} \tag{33}$$

It is noticed that each complex and frequency dependent poroelastic matrix is the product of a complex and frequency dependent scalar function by a real matrix. The low frequency observation of the multiplicative coefficients behaviour $N(1+j\eta_S)$, $\tilde{\gamma}$, $\tilde{\rho}$, $\frac{1}{\tilde{\rho}_{22}}$ and $\frac{1}{\tilde{M}}$ has allowed to highlight the function regularity, these last could be approached by an n order Taylor developments with real coefficient such as:

$$\tilde{f}(j\omega) = \sum_{i=0}^{n} f_i (j\omega)^i + \mathrm{O}\left((j\omega)^{n+1}\right) \tag{34}$$

For each matrices $\left[\tilde{\mathbf{K}}'^S\right]$, $\left[\tilde{\mathbf{M}}'^S\right]$, $\left[\tilde{\mathbf{K}}'^F\right]$, $\left[\tilde{\mathbf{M}}'^F\right]$ and $\left[\tilde{\mathbf{C}}'\right]$ we define the following developments:

$$\left[\tilde{\mathbf{K}}'^{S}\right]=\sum_{i=0}^{n}k_i^{S}\,(j\omega)^{i}\left[\mathbf{K}^{S}\right]+\mathrm{O}\left((j\omega)^{n+1}\right) \tag{35}$$

$$\left[\tilde{\mathbf{M}}'^{S}\right]=\sum_{i=0}^{n}m_i^{S}\,(j\omega)^{i}\left[\mathbf{M}^{S}\right]+\mathrm{O}\left((j\omega)^{n+1}\right) \tag{36}$$

$$\left[\tilde{\mathbf{K}}'^{F}\right]=\sum_{i=0}^{n}k_i^{F}\,(j\omega)^{i}\left[\mathbf{K}^{F}\right]+\mathrm{O}\left((j\omega)^{n+1}\right) \tag{37}$$

$$\left[\tilde{\mathbf{M}}'^{F}\right]=\sum_{i=0}^{n}m_i^{F}\,(j\omega)^{i}\left[\mathbf{M}^{F}\right]+\mathrm{O}\left((j\omega)^{n+1}\right) \tag{38}$$

$$\left[\tilde{\mathbf{C}}'\right]=\sum_{i=0}^{n}c_i\,(j\omega)^{i}\left[\mathbf{C}\right]+\mathrm{O}\left((j\omega)^{n+1}\right) \tag{39}$$

where $k_i^{S}, m_i^{S}, k_i^{F}, m_i^{F}, c_i$ are the Taylor development real coefficients respectively of the following complex and frequency dependent coefficients $N(1+j\eta_S)$, $\tilde{\rho}$, $\frac{1}{\tilde{M}}$, $\frac{1}{\tilde{\rho}_{22}}$ and $\tilde{\gamma}$. The real and independent frequency untilde matrices are defined by:

$$\left[\tilde{\mathbf{K}}'^{S}\right]=N(1+j\eta_S)\left[\mathbf{K}^{S}\right] \tag{40}$$

$$\left[\tilde{\mathbf{M}}'^{S}\right]=\tilde{\rho}\left[\mathbf{M}^{S}\right] \tag{41}$$

$$\left[\tilde{\mathbf{K}}'^{F}\right]=\frac{1}{\tilde{M}}\left[\mathbf{K}^{F}\right] \tag{42}$$

$$\left[\tilde{\mathbf{M}}'^{F}\right]=\frac{1}{\tilde{\rho}_{22}}\left[\mathbf{M}^{F}\right] \tag{43}$$

$$\left[\tilde{\mathbf{C}}'\right]=\tilde{\gamma}\left[\mathbf{C}\right] \tag{44}$$

Figures 2 to 6 present the frequency evolutions of the real and imaginary parts of the $N(1+j\eta_S)$, $\tilde{\gamma}$, $\tilde{\rho}$, $\frac{1}{\tilde{\rho}_{22}}$ and $\frac{1}{\tilde{M}}$ coefficients for nine porous material types (BK-1, BFM, FM-2, FM-3, FM-4, RGW-1, SC-1, UGW-3 and XFM) in the study frequency band (i.e. 0 to 1000 hertz). These last allow to justify the regularity assumption of the matrix multiplicative coefficient associated functions.

Then, by taking into account the equations (35) to (39), the free problem defined by equation (33) can, be written itself in the following form:

$$\left(\sum_{i=0}^{n}(j\omega)^{i}\left[\mathbf{A}_i\right]+\left[R_n'\right]\right)\begin{bmatrix}\underline{u}_N^{S}\\ \underline{\varphi}_N\end{bmatrix}=\begin{bmatrix}\underline{0}\\ \underline{0}\end{bmatrix} \tag{45}$$

where n is the $j\omega$ power development order and $\left[R_n'\right]$ is the complex and ω dependent matrix corresponding to the remainder including the Taylor development neglected terms.

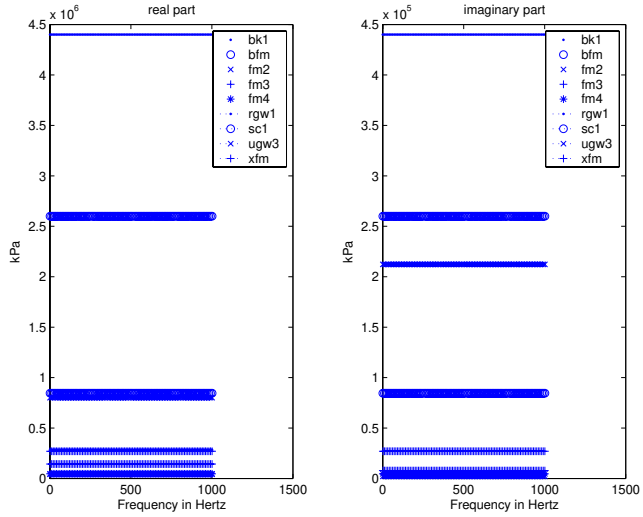

Figure 2: Frequency evolutions of the $N(1+j\eta_S)$ coefficient associated with the $\left[\tilde{\mathbf{K}}'^{S}\right]$ matrix.

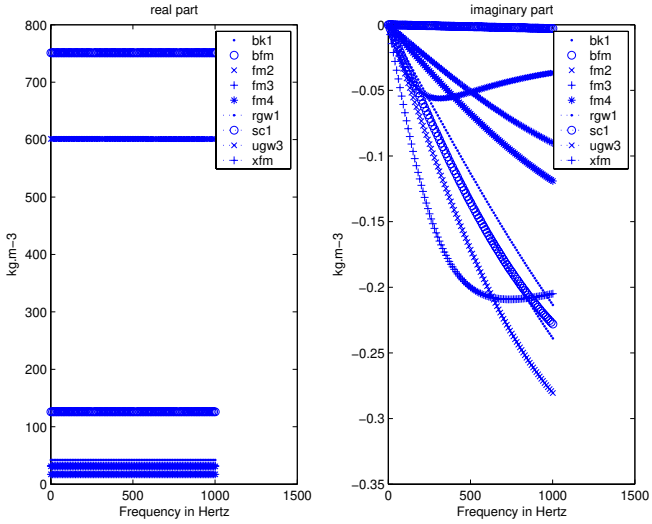

Figure 3: Frequency evolutions of the $\tilde{\rho}$ coefficient associated with the $\left[\tilde{\mathbf{M}}'^{S}\right]$ matrix.

The real and symmetric $(N_u+N_\varphi)\times(N_u+N_\varphi)$ dimension $\left[\mathbf{A}_i\right]$ matrices are defined by:

$$\left[\mathbf{A}_0\right]=\begin{bmatrix}-k_0^{S}\left[\mathbf{K}^{S}\right] & [0]\\ [0] & k_0^{F}\left[\mathbf{K}^{F}\right]\end{bmatrix} \tag{46}$$

$$\left[\mathbf{A}_1\right]=\begin{bmatrix}-k_1^{S}\left[\mathbf{K}^{S}\right] & c_0\left[\mathbf{C}\right]\\ c_0\left[\mathbf{C}\right]^{T} & k_1^{F}\left[\mathbf{K}^{F}\right]\end{bmatrix} \tag{47}$$

$$\left[\mathbf{A}_2\right]=\begin{bmatrix} -m_0^S\left[\mathbf{M}^S\right] & c_1\left[\mathbf{C}\right] \\ c_1\left[\mathbf{C}\right]^T & k_2^F\left[\mathbf{K}^F\right]+m_0^F\left[\mathbf{M}^F\right] \end{bmatrix} \tag{48}$$

$$\left[\mathbf{A}_3\right]=\begin{bmatrix} -k_3^S\left[\mathbf{K}^S\right]-m_1^S\left[\mathbf{M}^S\right] & c_2\left[\mathbf{C}\right] \\ c_2\left[\mathbf{C}\right]^T & k_3^F\left[\mathbf{K}^F\right]+m_1^F\left[\mathbf{M}^F\right] \end{bmatrix} \tag{49}$$

$\forall i \text{ even} \geq 2$

$$\left[\mathbf{A}_i\right]=\begin{bmatrix} -m_{i-2}^S\left[\mathbf{M}^S\right] & c_{i-1}\left[\mathbf{C}\right] \\ c_{i-1}\left[\mathbf{C}\right]^T & k_i^F\left[\mathbf{K}^F\right]+m_{i-2}^F\left[\mathbf{M}^F\right] \end{bmatrix} \tag{50}$$

$\forall i \text{ odd} \geq 2$

$$\left[\mathbf{A}_i\right]=\begin{bmatrix} -k_i^S\left[\mathbf{K}^S\right]-m_{i-2}^S\left[\mathbf{M}^S\right] & c_{i-1}\left[\mathbf{C}\right] \\ c_{i-1}\left[\mathbf{C}\right]^T & k_i^F\left[\mathbf{K}^F\right]+m_{i-2}^F\left[\mathbf{M}^F\right] \end{bmatrix} \tag{51}$$

The construction of the expanded eigenvalue problem is based on the use of Duncan transformation. We define the $n-1$ order expanded Duncan transformation the following application:

$$\underline{U}^p=\begin{Bmatrix} (j\omega)^{n-1}\underline{u}^p \\ \vdots \\ (j\omega)\underline{u}^p \\ \underline{u}^p \end{Bmatrix}, \text{ with } \underline{u}^p=\begin{bmatrix} \underline{u}_N^S \\ \underline{\varphi}_N \end{bmatrix} \tag{52}$$

We define the expanded eigenvalue problem (in neglecting the remainder) by the following matrix system:

$$\left[\mathbf{B}_1\right]\underline{U}^p+s\left[\mathbf{B}_2\right]\underline{U}^p=\begin{bmatrix} \underline{0} \\ \underline{0} \end{bmatrix} \tag{53}$$

with $s\in\mathbb{C}$ and $\left[\mathbf{B}_1\right],\left[\mathbf{B}_2\right]$ are the real symmetric $n\left(N_u+N_\varphi\right)\times n\left(N_u+N_\varphi\right)$ dimension matrices defined by:

$$\left[\mathbf{B}_1\right]=\begin{bmatrix} [0] & \cdots & \cdots & -\left[\mathbf{A}_n\right] & [0] \\ \vdots & & \iddots & -\left[\mathbf{A}_{n-1}\right] & [0] \\ \vdots & \iddots & \iddots & \vdots & \vdots \\ -\left[\mathbf{A}_n\right] & -\left[\mathbf{A}_{n-1}\right] & \cdots & -\left[\mathbf{A}_2\right] & [0] \\ [0] & [0] & \cdots & [0] & \left[\mathbf{A}_0\right] \end{bmatrix} \tag{54}$$

$$\left[\mathbf{B}_2\right]=\begin{bmatrix} [0] & \cdots & \cdots & [0] & \left[\mathbf{A}_n\right] \\ \vdots & & \iddots & \left[\mathbf{A}_d\right] & \left[\mathbf{A}_{n-1}\right] \\ \vdots & \iddots & \iddots & \iddots & \vdots \\ [0] & \left[\mathbf{A}_n\right] & \iddots & \iddots & \left[\mathbf{A}_2\right] \\ \left[\mathbf{A}_n\right] & \left[\mathbf{A}_{n-1}\right] & \cdots & \left[\mathbf{A}_2\right] & \left[\mathbf{A}_1\right] \end{bmatrix} \tag{55}$$

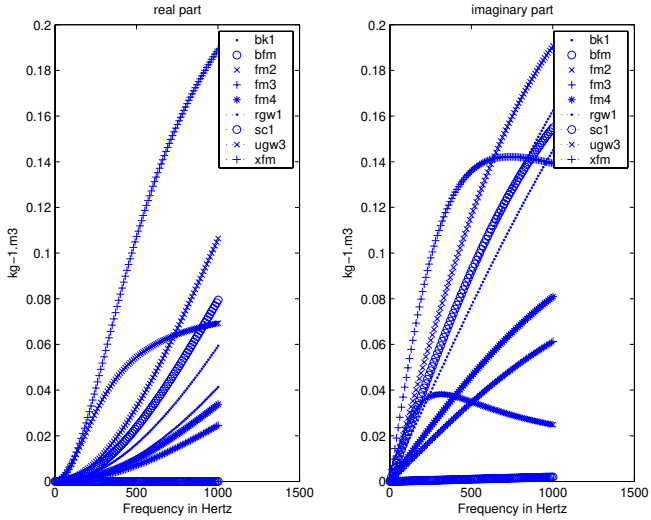

Figure 4: Frequency evolutions of the $\frac{1}{\tilde{M}}$ coefficient associated with the $\left[\tilde{\mathbf{K}}'^F\right]$ matrix.

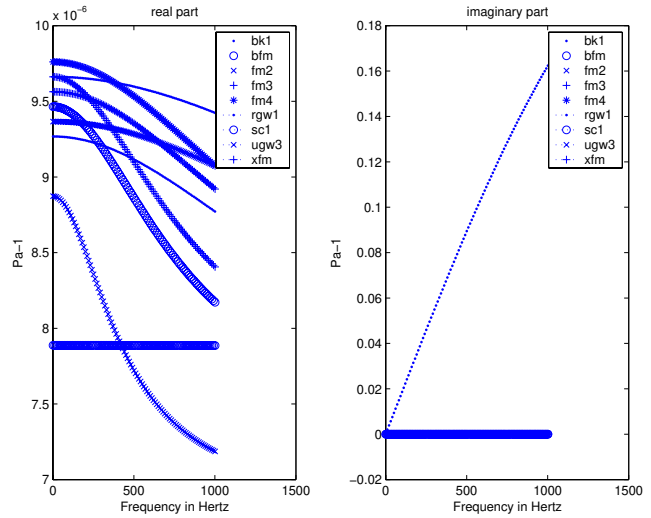

Figure 5: Frequency evolutions of the $\frac{1}{\tilde{\rho}_{22}}$ coefficient associated with the $\left[\tilde{\mathbf{M}}'^F\right]$ matrix.

We define the poroelastic complex modes as the solutions $\left\{s,\underline{\Psi}^p\right\}$ of the following $n\left(N_u+N_\varphi\right)$ dimension expanded eigenvalue problem:

$$\left[\mathbf{B}_1\right]\underline{\Psi}^p+s\left[\mathbf{B}_2\right]\underline{\Psi}^p=\begin{bmatrix} \underline{0} \\ \underline{0} \end{bmatrix} \tag{56}$$

If $\left\{s_i,\underline{\Psi}_i^p\right\}$ and $\left\{s_j,\underline{\Psi}_j^p\right\}$ are two eigenvalue problem solutions, the application of the principle of orthogonality gives the following equations:

$$\underline{\Psi}_j^{pT}\left[\mathbf{B}_1\right]\underline{\Psi}_i^p = -s_i\delta_{ij} \tag{57}$$

$$\underline{\Psi}_j^{pT}\left[\mathbf{B}_2\right]\underline{\Psi}_i^p = \delta_{ij} \tag{58}$$

Where δ_{ij} is the Kronecker delta function (i.e. $\delta_{ij}=0$ for $i\neq j$ or $\delta_{ij}=1$ for $i=j$).

If $\left\{s,\underline{\Psi}^p\right\}$ is a complex mode, $\left\{s,\underline{\Psi}^p\right\}$ and the conjugate $\left\{\overline{s},\overline{\underline{\Psi}}^p\right\}$ are two eigenvalue problem solutions, then the application of the conjugation principle gives the following equations:

$$\left[\mathbf{B}_1\right]\underline{\Psi}^p + s\left[\mathbf{B}_2\right]\underline{\Psi}^p = \begin{bmatrix}\underline{0}\\ \underline{0}\end{bmatrix} \tag{59}$$

$$\left[\mathbf{B}_1\right]\overline{\underline{\Psi}}^p + \overline{s}\left[\mathbf{B}_2\right]\overline{\underline{\Psi}}^p = \begin{bmatrix}\underline{0}\\ \underline{0}\end{bmatrix} \tag{60}$$

Taking into account the fact that the $\left[\mathbf{A}_i\right]$ matrix is invertible and that the $\left[\mathbf{B}_1\right],\left[\mathbf{B}_2\right]$ matrices are symmetric, the eigenvalue calculation can be carried out using traditional eigenvalue search algorithms.

The poroelastic modes

The Dazel's works made it possible to define a selection criterion allowing to retain only the representative poroelastic modes. Among these modes, one distinguishes two categories. The first corresponds to the modes with eigenvalues having a no null imaginary part and the second corresponds to the modes with eigenvalues having a null imaginary part. The first category is called complex modes and the second category is called real modes. First, We consider the complex modes with complex eigenvalues having strictly negative real and positive imaginary parts. We define the complex eigenvalues by $s=-\alpha+j\omega$ with $\alpha>0$ and $\omega>0$. Then, we consider the real modes with strictly negative real eigenvalues. We define the real eigenvalue by $s=-\alpha$ with $\alpha>0$.

The retained complex modes for the modal superposition will be the modes such as:

$$\left|\omega\right|<\omega_c,\ \xi<\xi_c \text{ wih } \xi=\frac{\alpha}{\omega} \text{ and } \xi_c=\frac{\alpha_c}{\omega_c} \tag{61}$$

The real modes retained for the modal superposition will be the modes such as:

$$\left|\alpha\right|<\alpha_c \tag{62}$$

with ξ_c, ω_c and α_c the limit values imposed by the criterion. ω_c corresponds to the excitation spectrum maximum frequency and ξ_c depends on the material properties, boundary conditions and excitation type.

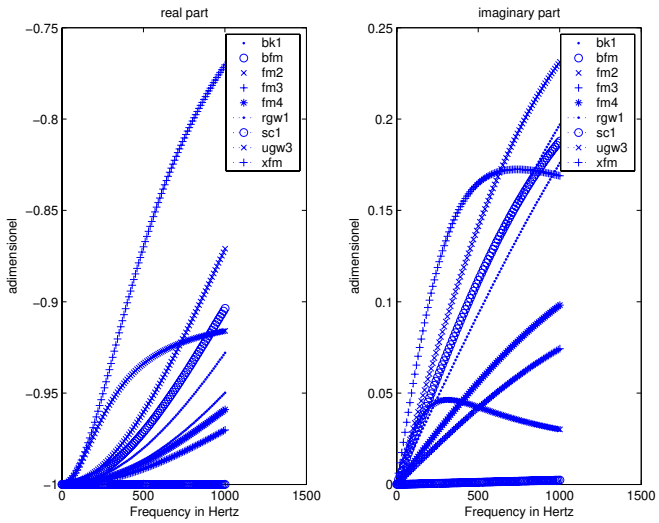

Figure 6: Frequency evolutions of the $\tilde{\gamma}$ coefficient associated with the $\left[\tilde{\mathbf{C}}'\right]$ matrix.

The vibroacoustic response calculation by modal synthesis method

To calculate the vibroacoustic response of the poroelastic structure subjected to the harmonic external surface load $\underline{F}^{ext}(x,t)$ by modal projection and modal superposition we suppose the existence of an eigenvector basis. The latter is constituted by the eigenvectors $\underline{\Psi}_i^p$ associated to the r first retained eigenvalues. First of all we rewrite the expanded eigenvalue problem (53) to which we add to the second member relating to the $\underline{F}^{ext}(x,t)$ associated terms, the equation takes the following form:

$$\left[\mathbf{B}_1\right]\underline{U}^p + j\omega\left[\mathbf{B}_2\right]\underline{U}^p = \begin{bmatrix}\underline{0}\\ \vdots\\ \underline{0}\\ -\underline{F}_1^{ext}\\ \underline{F}_2'^{ext}\end{bmatrix} \tag{63}$$

For more readable reading one note:

$$\underline{G}^{ext} = \left\{\underline{0}\ \cdots\ \underline{0}\ -\underline{F}_1^{ext}\ \ \underline{F}_2'^{ext}\right\}^T \tag{64}$$

Now we project the vector $\underline{U}^p$ on the r first modes truncated basis $\underline{\Psi}_i^p$ (Ritz-Galerkin's method). Then the projection is defined by the following equation:

$$\underline{U}^p = \sum_{i=1}^{r}\underline{\Psi}_i^p q_i = \left[\underline{\Psi}^p\right]\underline{q} \tag{65}$$

where $\left[\underline{\Psi}^p\right]=\left[\underline{\Psi}_1^p \cdots \underline{\Psi}_r^p\right]$ is the truncated modal matrix constituted by the r eigenvectors $\underline{\Psi}_i^p$ and $\underline{q}=\{q_1 \cdots q_r\}^T$ is the modal co-ordinate vector (or generalized co-ordinate vector).

While replacing $\underline{U}^p$ by his development defined by the preceding equation in the equation (63) and by projecting the result on the truncated basis $\underline{\Psi}_i^p$, we obtain the following equation:

$$\underline{\Psi}_i^{pT}\left[\mathbf{B}_1\right]\underline{\Psi}_i^p q_i + j\omega\,\underline{\Psi}_i^{pT}\left[\mathbf{B}_2\right]\underline{\Psi}_i^p q_i = \underline{\Psi}_i^{pT}\underline{G}^{ext} \tag{66}$$

The uncoupled equations allowing to calculate each q_i component are of the following form:

$$q_i = \left(\underline{\Psi}_i^{pT}\left[\mathbf{B}_1\right]\underline{\Psi}_i^p + j\omega\,\underline{\Psi}_i^{pT}\left[\mathbf{B}_2\right]\underline{\Psi}_i^p\right)^{-1}\underline{\Psi}_i^{pT}\underline{G}^{ext} \tag{67}$$

Finally, we can recompose the vibroacoustic response:

$$\underline{U}^p = \sum_{i=1}^{r}\left(\Pi_i^p\underline{\Psi}_i^{pT}\underline{G}^{ext}\underline{\Psi}_i^p + \bar{\Pi}_i^p\underline{\bar{\Psi}}_i^{pT}\underline{G}^{ext}\underline{\bar{\Psi}}_i^p\right) \tag{68}$$

where

$$\Pi_i^p = \left(\underline{\Psi}_i^{pT}\left[\mathbf{B}_1\right]\underline{\Psi}_i^p + j\omega\,\underline{\Psi}_i^{pT}\left[\mathbf{B}_2\right]\underline{\Psi}_i^p\right)^{-1} \tag{69}$$

$$\bar{\Pi}_i^p = \left(\underline{\bar{\Psi}}_i^{pT}\left[\mathbf{B}_1\right]\underline{\Psi}_i^p + j\omega\,\underline{\bar{\Psi}}_i^{pT}\left[\mathbf{B}_2\right]\underline{\Psi}_i^p\right)^{-1} \tag{70}$$

Numerical implementation and validation

The coupling modeling between the poroelastic material solid and fluid phases is not "standard" since contrary to the "traditional" fluid-structure interaction problems (example: elasto-acoustic …) for which interaction acts on a surface-coupling, poroelastic interaction acts on a volume-coupling. This characteristic comes owing to the fact that the physical quantities observed on a macroscopic scale are averaged quantities on a representative elementary volume. Moreover, porous material constitutive matrices are complex and frequency dependent.

The stiffness, mass and elasto-acoustic, elasto-poroelastic, acousto-poroelastic, poroelastic-poroelastic surface-coupling matrices can be calculated by the NASTRAN® software. However, the calculation of the poroelastic volume-coupling matrix is not possible.

To calculate this matrix, a FORTRAN program has been developed. The matrix is read and inserted in the matrix system by DMAP developments (NASTRAN® internal programming language).

To validate the modal approach, a MATLAB® program has been developed. This last allows to calculate the poroelastic vibroacoustic behaviour by direct integration and modal superposition. The porous material structural damping modeling η_S allows to calculate the vibroacoustic behaviour by considering this last null, constant or frequency dependent.

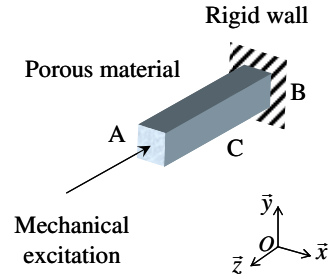

Figure 7: Geometrical configuration of the studied system.

Properties	Material : RGW-1
Porosity, Φ	0.94
Flow resistivity, σ ($kNm^{-4}s^{-1}$)	40
Tortuosity, α	1.06
Viscous characteristic length, Λ (m)	56×10^{-6}
Thermal characteristic length, Λ' (m)	110×10^{-6}
Matrix density, ρ_S (kgm^{-3})	130
In vacuum shear modulus, N (kPa)	2.2×10^{3}
Poisson's factor, ν	0
Structural damping, η_S	0.1

Table 1: RGW-1 porous material properties.

First of all we present the configuration used to validate the developed program. Then we present the comparative study results, calculated from direct integration, between case with structural damping and case without structural damping. Finally, we present the comparative study results, calculated for $\eta_S = 0.1$ and for $\eta_S = af + bf^3$, between direct integration and modal superposition methods, where a and b are two scalar constants such as $a = 5.10^{-5}$ and $b = 5.10^{-10}$, f is the frequency.

The considered system consists of a porous material, supposed one-dimensional, free on the A boundary, stuck on the B rigid boundary and guided on the C boundaries. Figure 7 presents the studied system configuration. The properties of porous material are gathered in table 1.

Only the corresponding force value is imposed on the porous material solid phase degrees of freedom.

We suppose the fictitious excitation associated with $\underline{G}^{ext}$ acting only on the porous material solid phase such as:

$$\underline{G}^{ext} = \left\{\underline{0} \quad \cdots \quad \underline{0} \quad -\underline{F}^{ext} \quad \underline{0}\right\}^T \tag{71}$$

The HEXA poroelastic element number allowing to check the convergence criterion in the study frequency band (i.e. 0 to 1000 hertz) is 6. The reference poroelastic element used is an eight-node parallelepiped and the interpolations functions are linear.

In this study, the approximation orders of the matrices multiplicative coefficients associated functions are close to 3.

In first step, we compare the nodal displacement (eleventh d.o.f. on the surface A) and nodal pressure (thirty-seventh d.o.f.), calculated by direct method, between with and without structural damping for $\eta_S = 0.1$ and $\eta_S = af + bf^3$. These results are presented Figures 8 and 9.

Figures 8 and 9 illustrate the structural damping influence on nodal displacement and the nodal interstitial pressure respectively for $\eta_S = 0.1$ and $\eta_S = af + bf^3$. The comparison between the direct solutions shows logically that the peaks are pronounced in the structural damping neglected case. We do not note a frequency shift of the response peaks.

In second step we compare for $\eta_S = 0.1$ and $\eta_S = af + bf^3$ the nodal displacement (eleventh d.o.f. on the surface A) and nodal pressure (thirty-seventh d.o.f.) obtained by direct integration and modal superposition. These results are presented Figure 10 and 11.

Figures 10 and 11 illustrate the reduction obtained using the modal synthesis method. We realize the modal superposition on a truncated modal basis constituted by one real mode and the first complex modes. We selected 5 pairs of complex modes for $\eta_S = 0.1$ and 4 pairs of complex modes for $\eta_S = af + bf^3$. The number of degree of freedom or d.o.f. is for $\eta_S = 0.1$ of 11 (10 complex modes and 1 real mode) and for $\eta_S = af + bf^3$ of 9 (8 complex modes and 1 real mode). The size of the initial system is of 52 d.o.f..

CONCLUSIONS AND FUTURE WORKS

This paper has presented a modal analysis approach based on modal synthesis and complex modes to reduce the size of the poroelastic matrix system and to predict more effectively the vibroacoustic behaviour of porous material in low frequency domain. This latter has the originality to include the structural damping in the poroelastic mode calculation and consists in projecting the poroelastic system on a truncated basis of damped and coupled modes to reduce the poroelastic problem size.

First to improve the method, a validation step will be realized for three-dimensional structures.

Moreover, the influence of the system configuration (constitutive material, boundary conditions and excitation) on the method as on the poroelastic modes selection criterion will be studied. If the selection criterion remains valid then the automation of the modes selection process could be realized.

Finally, one will be able to couple this method with the traditional modal synthesis methods in order to solve effectively elasto-poro-acoustic problems by modal techniques.

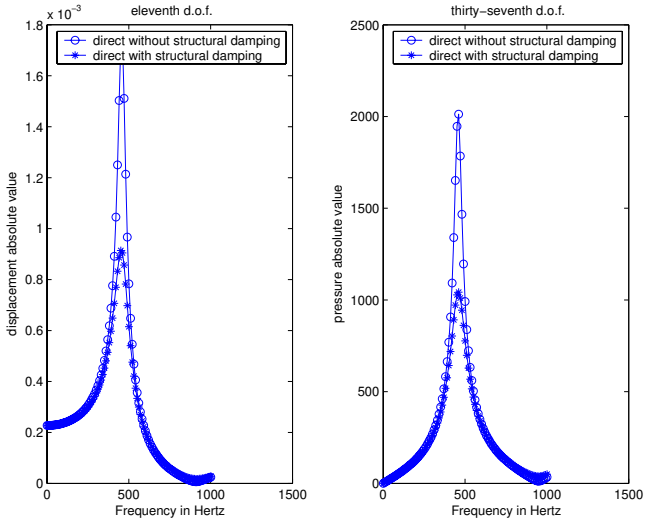

Figure 8: Nodal displacement and nodal pressure calculation by direct method with (star curve) and without (circle curve) structural damping for $\eta_S = 0.1$.

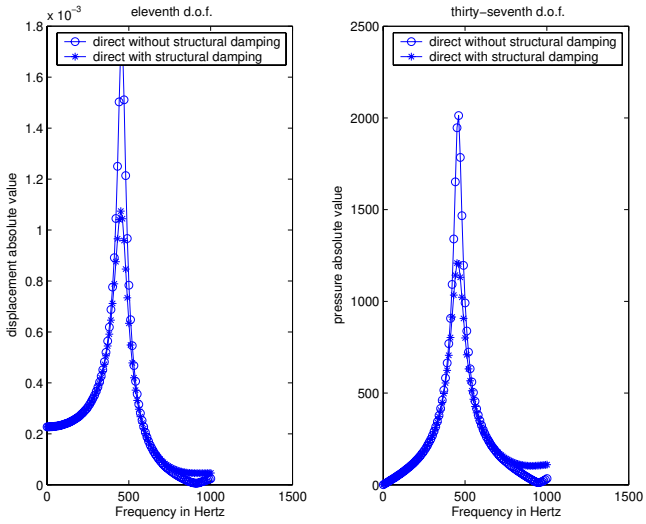

Figure 9: Nodal displacement and nodal pressure calculation by direct method with (star curve) and without (circle curve) structural damping for $\eta_S = af + bf^3$.

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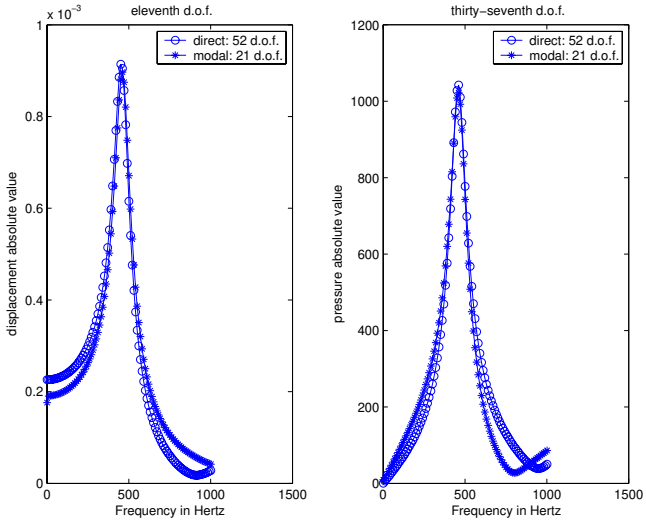

Figure 10: Nodal displacement and nodal pressure calculation by modal superposition (star curve) and direct integration (circle curve) for $\eta_S = 0.1$.

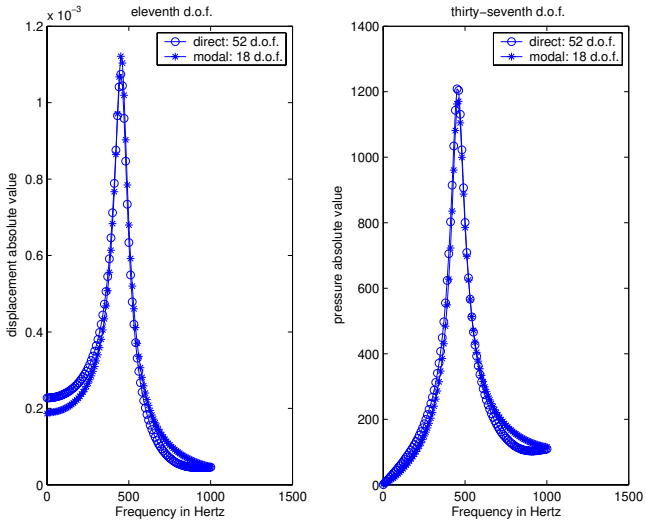

Figure 11: Nodal displacement and nodal pressure calculation by modal superposition (star curve) and direct integration (circle curve) for $\eta_S = af + bf^3$.